

# CAS integration tests. Progress report

## Giac 1.9.0-11 via sagemath 9.6 vs. Giac 1.7.0 via sagemath 9.3

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19 Test file number 39	773
20 Test file number 40	788
21 Test file number 41	795
22 Test file number 42	797
23 Test file number 46	803
24 Test file number 50	818
25 Test file number 51	821
26 Test file number 52	823
27 Test file number 53	842
28 Test file number 54	843
29 Test file number 55	844
30 Test file number 58	845
31 Test file number 59	846
32 Test file number 60	850
33 Test file number 61	876
34 Test file number 62	877
35 Test file number 63	878
36 Test file number 64	878
37 Test file number 65	879
38 Test file number 66	900
39 Test file number 68	911
40 Test file number 69	915
41 Test file number 70	919
42 Test file number 72	941
43 Test file number 73	958
44 Test file number 74	1059
45 Test file number 75	1119

46 Test file number 76	1127
47 Test file number 78	1221
48 Test file number 79	1222
49 Test file number 82	1244
50 Test file number 83	1248
51 Test file number 85	1248
52 Test file number 89	1249
53 Test file number 91	1254
54 Test file number 92	1254
55 Test file number 93	1265
56 Test file number 94	1265
57 Test file number 95	1293
58 Test file number 96	1294
59 Test file number 97	1297
60 Test file number 98	1302
61 Test file number 99	1316
62 Test file number 101	1319
63 Test file number 102	1326
64 Test file number 103	1329
65 Test file number 104	1383
66 Test file number 105	1419
67 Test file number 106	1423
68 Test file number 108	1479
69 Test file number 110	1480
70 Test file number 113	1481
71 Test file number 115	1491
72 Test file number 118	1492

73 Test file number 120	1494
74 Test file number 121	1496
75 Test file number 122	1553
76 Test file number 126	1637
77 Test file number 129	1697
78 Test file number 134	1700
79 Test file number 135	1702
80 Test file number 136	1713
81 Test file number 137	1714
82 Test file number 139	1728
83 Test file number 141	1737
84 Test file number 156	1753
85 Test file number 163	1755
86 Test file number 164	1757
87 Test file number 170	1781
88 Test file number 173	1781
89 Test file number 176	1802
90 Test file number 179	1802
91 Test file number 180	1803
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100Test file number 210

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# 1 Table summary of progress report

Table 1: Table summary of progress report

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1	1	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: integrate (cos(x*2)*exp(ln(-sin
2	10	3	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (pi/x/2)
3	10	231	1 (pass)	0 (not solved)
4	10	398	1 (pass)	0 (not solved)
5	10	592	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -1/2*((( - i)*pi+sqrt(2)*atan((1
6	11	11	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT: sage2:=int(sage0, x)::O
7	11	19	1 (pass)	0 (not solved)
8	11	20	1 (pass)	0 (not solved)
9	11	30	1 (pass)	0 (not solved)
10	13	785	1 (pass)	0 (not solved)
11	13	786	1 (pass)	0 (not solved)
12	13	787	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
13	13	791	1 (pass)	0 (not solved)
14	13	792	1 (pass)	0 (not solved)
15	13	793	1 (pass)	0 (not solved)
16	13	794	1 (pass)	0 (not solved)
17	13	795	1 (pass)	0 (not solved)
18	13	797	1 (pass)	0 (not solved)
19	13	798	1 (pass)	0 (not solved)
20	13	799	1 (pass)	0 (not solved)
21	13	800	1 (pass)	0 (not solved)
22	13	801	1 (pass)	0 (not solved)
23	13	802	1 (pass)	0 (not solved)
24	13	803	1 (pass)	0 (not solved)
25	13	833	1 (pass)	0 (not solved)
26	13	834	1 (pass)	0 (not solved)
27	13	835	1 (pass)	0 (not solved)
28	13	839	1 (pass)	0 (not solved)
29	13	840	1 (pass)	0 (not solved)
30	13	841	1 (pass)	0 (not solved)
31	13	842	1 (pass)	0 (not solved)
32	13	843	1 (pass)	0 (not solved)
33	13	845	1 (pass)	0 (not solved)
34	13	846	1 (pass)	0 (not solved)
35	13	847	1 (pass)	0 (not solved)
36	13	848	1 (pass)	0 (not solved)
37	13	849	1 (pass)	0 (not solved)
38	13	850	1 (pass)	0 (not solved)
39	13	851	1 (pass)	0 (not solved)
40	13	1145	1 (pass)	-1 (time out)
41	13	1146	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
42	13	1147	1 (pass)	-1 (time out)
43	13	1148	1 (pass)	-1 (time out)
44	13	1169	1 (pass)	-1 (time out)
45	14	556	1 (pass)	-1 (time out)
46	14	565	1 (pass)	-1 (time out)
47	14	574	1 (pass)	-1 (time out)
48	14	606	1 (pass)	-1 (time out)
49	14	615	1 (pass)	-1 (time out)
50	14	624	1 (pass)	-1 (time out)
51	14	655	1 (pass)	-1 (time out)
52	14	664	1 (pass)	-1 (time out)
53	14	722	1 (pass)	-1 (time out)
54	14	775	1 (pass)	-1 (time out)
55	14	801	1 (pass)	-1 (time out)
56	14	813	1 (pass)	-1 (time out)
57	17	20	1 (pass)	-1 (time out)
58	17	21	1 (pass)	-1 (time out)
59	17	22	1 (pass)	-1 (time out)
60	17	23	1 (pass)	-1 (time out)
61	17	25	1 (pass)	-1 (time out)
62	17	27	1 (pass)	-1 (time out)
63	17	28	1 (pass)	-1 (time out)
64	17	29	1 (pass)	-1 (time out)
65	17	30	1 (pass)	-1 (time out)
66	17	32	1 (pass)	-1 (time out)
67	17	53	1 (pass)	-1 (time out)
68	17	59	1 (pass)	-1 (time out)
69	17	60	1 (pass)	-1 (time out)
70	25	41	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
71	25	50	1 (pass)	0 (not solved)
72	25	51	1 (pass)	0 (not solved)
73	25	59	1 (pass)	0 (not solved)
74	25	60	1 (pass)	0 (not solved)
75	25	61	1 (pass)	0 (not solved)
76	25	95	1 (pass)	0 (not solved)
77	25	96	1 (pass)	0 (not solved)
78	25	100	1 (pass)	0 (not solved)
79	25	106	1 (pass)	0 (not solved)
80	25	107	1 (pass)	0 (not solved)
81	25	108	1 (pass)	0 (not solved)
82	25	113	1 (pass)	0 (not solved)
83	25	114	1 (pass)	0 (not solved)
84	25	115	1 (pass)	0 (not solved)
85	25	117	1 (pass)	0 (not solved)
86	25	122	1 (pass)	0 (not solved)
87	25	123	1 (pass)	0 (not solved)
88	25	129	1 (pass)	0 (not solved)
89	25	130	1 (pass)	0 (not solved)
90	25	1922	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
91	25	1923	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
92	25	1925	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
93	25	1926	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
94	25	1928	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
95	25	1929	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
96	25	1930	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
97	25	1931	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
98	25	1932	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
99	25	1935	1 (pass)	0 (not solved)
100	25	1936	1 (pass)	0 (not solved)
101	25	1937	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
102	25	1938	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
103	25	1939	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
104	25	1940	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
105	25	1941	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
106	25	1942	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
107	25	1943	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
108	25	1949	1 (pass)	0 (not solved)
109	25	1951	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0
110	25	1952	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
111	25	1953	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0
112	25	1954	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
113	25	1955	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
114	25	1956	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
115	25	2021	1 (pass)	0 (not solved)
116	25	2022	1 (pass)	0 (not solved)
117	25	2023	1 (pass)	0 (not solved)
118	25	2040	1 (pass)	0 (not solved)
119	25	2806	1 (pass)	0 (not solved)
120	25	2807	1 (pass)	0 (not solved)
121	25	2817	1 (pass)	0 (not solved)
122	25	2818	1 (pass)	0 (not solved)
123	25	2829	1 (pass)	0 (not solved)
124	25	2830	1 (pass)	0 (not solved)
125	25	2835	1 (pass)	0 (not solved)
126	25	2836	1 (pass)	0 (not solved)
127	25	2839	1 (pass)	0 (not solved)
128	25	2988	1 (pass)	0 (not solved)
129	25	2990	1 (pass)	0 (not solved)
130	25	3061	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
131	25	3062	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
132	25	3064	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
133	27	530	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0
134	27	538	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0
135	27	964	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
136	27	968	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
137	27	969	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
138	27	970	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
139	27	971	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
140	27	972	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
141	27	973	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
142	27	974	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
143	27	975	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
144	27	977	1 (pass)	0 (not solved)
145	27	978	1 (pass)	0 (not solved)
146	27	979	1 (pass)	0 (not solved)
147	27	980	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
148	27	981	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
149	27	982	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
150	27	983	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
151	27	984	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
152	27	985	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
153	28	15	1 (pass)	-1 (time out)
154	30	98	1 (pass)	0 (not solved)
155	30	110	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
156	30	111	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
157	30	124	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
158	30	125	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
159	30	252	1 (pass)	0 (not solved)
160	30	253	1 (pass)	0 (not solved)
161	30	254	1 (pass)	0 (not solved)
162	30	257	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
163	30	258	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
164	30	259	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
165	30	260	1 (pass)	0 (not solved)
166	30	261	1 (pass)	0 (not solved)
167	30	264	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
168	30	265	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
169	30	266	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
170	30	267	1 (pass)	0 (not solved)
171	30	268	1 (pass)	0 (not solved)
172	30	284	1 (pass)	0 (not solved)
173	30	285	1 (pass)	0 (not solved)
174	30	286	1 (pass)	0 (not solved)
175	30	310	1 (pass)	0 (not solved)
176	30	311	1 (pass)	0 (not solved)
177	31	139	1 (pass)	0 (not solved)
178	31	140	1 (pass)	0 (not solved)
179	31	141	1 (pass)	0 (not solved)
180	31	143	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
181	31	144	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
182	31	148	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT: sage2:=int(sage0, x)::O
183	31	150	1 (pass)	0 (not solved)
184	31	151	1 (pass)	0 (not solved)
185	31	152	1 (pass)	0 (not solved)
186	31	153	1 (pass)	0 (not solved)
187	31	154	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
188	31	156	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
189	31	157	1 (pass)	0 (not solved)
190	31	158	1 (pass)	0 (not solved)
191	32	17	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
192	32	18	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
193	32	19	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
194	32	20	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
195	32	67	1 (pass)	0 (not solved)
196	33	190	1 (pass)	0 (not solved)
197	33	191	1 (pass)	0 (not solved)
198	33	192	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
199	33	193	1 (pass)	0 (not solved)
200	33	194	1 (pass)	0 (not solved)
201	33	195	1 (pass)	0 (not solved)
202	33	196	1 (pass)	0 (not solved)
203	33	197	1 (pass)	0 (not solved)
204	33	198	1 (pass)	0 (not solved)
205	33	199	1 (pass)	0 (not solved)
206	33	200	1 (pass)	0 (not solved)
207	33	201	1 (pass)	0 (not solved)
208	33	202	1 (pass)	0 (not solved)
209	33	203	1 (pass)	0 (not solved)
210	33	204	1 (pass)	0 (not solved)
211	33	205	1 (pass)	0 (not solved)
212	33	206	1 (pass)	0 (not solved)
213	33	207	1 (pass)	0 (not solved)
214	33	212	1 (pass)	0 (not solved)
215	33	213	1 (pass)	0 (not solved)
216	33	214	1 (pass)	0 (not solved)
217	33	217	1 (pass)	0 (not solved)
218	33	218	1 (pass)	0 (not solved)
219	33	618	1 (pass)	0 (not solved)
220	33	619	1 (pass)	-1 (time out)
221	33	803	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(16*d^7*\exp(1)^6*\exp(2)^2-$
222	33	804	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
223	33	805	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-2*d)^5*(-1/2*(-2*d*\exp(1)-2*s)$
224	33	806	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(12*d)^4*(-1/2*(-2*d*\exp(1)-2*s)$
225	33	807	1 (pass)	0 (not solved)
226	33	808	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(960*d)^2*(-1/2*(-2*d*\exp(1)-2*s)$
227	33	809	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3840*d)*(-1/2*(-2*d*\exp(1)-2*s)$
228	33	810	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(53760*(-1/2*(-2*d*\exp(1)-2*s)^2)$
229	33	811	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(860160*(-1/2*(-2*d*\exp(1)-2*s)$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
230	33	812	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-5160960*(-1/2*(-2*d*\exp(1))-2$
231	33	813	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-20643840*(-1/2*(-2*d*\exp(1)-$
232	33	814	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(454164480*(-1/2*(-2*d*\exp(1)-$
233	33	815	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3633315840*(-1/2*(-2*d*\exp(1)$
234	33	831	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*\exp(2)*\operatorname{atan}((-1/2*(-2*d*\exp$
235	33	832	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
236	33	833	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(7*(-1/2*(-2*d*\exp(1)-2*\sqrt{d}$
237	33	834	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(36*(-1/2*(-2*d*\exp(1)-2*\sqrt{d}$
238	33	835	1 (pass)	0 (not solved)
239	33	843	1 (pass)	0 (not solved)
240	33	856	1 (pass)	0 (not solved)
241	33	859	1 (pass)	0 (not solved)
242	33	862	1 (pass)	0 (not solved)
243	33	863	1 (pass)	0 (not solved)
244	33	864	1 (pass)	0 (not solved)
245	33	865	1 (pass)	0 (not solved)
246	33	866	1 (pass)	0 (not solved)
247	33	867	1 (pass)	0 (not solved)
248	33	868	1 (pass)	0 (not solved)
249	33	869	1 (pass)	0 (not solved)
250	33	870	1 (pass)	0 (not solved)
251	33	871	1 (pass)	0 (not solved)
252	33	872	1 (pass)	0 (not solved)
253	33	873	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
254	33	874	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
255	33	875	1 (pass)	-1 (time out)
256	33	876	1 (pass)	-1 (time out)
257	33	877	1 (pass)	-1 (time out)
258	33	878	1 (pass)	-1 (time out)
259	33	879	1 (pass)	0 (not solved)
260	33	880	1 (pass)	0 (not solved)
261	33	881	1 (pass)	0 (not solved)
262	33	882	1 (pass)	0 (not solved)
263	33	883	1 (pass)	0 (not solved)
264	33	884	1 (pass)	0 (not solved)
265	33	885	1 (pass)	0 (not solved)
266	33	886	1 (pass)	0 (not solved)
267	33	887	1 (pass)	0 (not solved)
268	33	888	1 (pass)	0 (not solved)
269	33	889	1 (pass)	0 (not solved)
270	33	890	1 (pass)	0 (not solved)
271	33	891	1 (pass)	0 (not solved)
272	33	892	1 (pass)	0 (not solved)
273	33	895	1 (pass)	0 (not solved)
274	33	897	1 (pass)	0 (not solved)
275	33	899	1 (pass)	0 (not solved)
276	33	902	1 (pass)	0 (not solved)
277	33	904	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
278	33	905	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
279	33	906	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
280	33	907	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
281	33	908	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
282	33	909	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
283	33	910	1 (pass)	-1 (time out)
284	33	914	1 (pass)	0 (not solved)
285	33	915	1 (pass)	0 (not solved)
286	33	917	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
287	33	919	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
288	33	921	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
289	33	922	1 (pass)	0 (not solved)
290	33	923	1 (pass)	0 (not solved)
291	33	924	1 (pass)	0 (not solved)
292	33	938	1 (pass)	0 (not solved)
293	33	939	1 (pass)	0 (not solved)
294	33	940	1 (pass)	0 (not solved)
295	33	942	1 (pass)	0 (not solved)
296	33	944	1 (pass)	0 (not solved)
297	33	998	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: (5*exp(2)^4*d^4*exp(1)-30*exp(
298	33	999	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: (2*exp(2)^3*d^3*exp(1)-8*exp(2

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
299	33	1000	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3*\exp(2)^2*d^2*\exp(1)-7*\exp(2)$
300	33	1001	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(\exp(2)*d*\exp(1)-d*\exp(1)^3)/c$
301	33	1002	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)*1/2/c/\exp(2)*\ln(x^2*\exp$
302	33	1003	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2/c*1/2/d/\sqrt{-\exp(1)^2+\exp(2)}$
303	33	1004	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^2/(c*\exp(2)*d^2*\exp(1)-$
304	33	1005	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)/(c*d^3*\exp(1)^2-c*d^3*e$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
305	33	1006	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3*\exp(2)*\exp(1)+4*\exp(1)^3)/$
306	33	1007	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/4*x^4*c^6*\exp(2)^6*\exp(1)^7$
307	33	1008	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/3*x^3*c^4*\exp(2)^4*\exp(1)^6$
308	33	1009	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/2*x^2*c^2*\exp(2)^2*\exp(1)^5$
309	33	1010	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $x*\exp(1)^4/c^2/\exp(2)^2+((\exp($
310	33	1011	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^3*1/2/c^2/\exp(2)^2*\ln(x$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
311	33	1012	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(x*\exp(2)-2*x*\exp(1))^2-d*\exp($
312	33	1013	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-x*1/2/c^2/d/(-x^2*\exp(2)-2*x*$
313	33	1014	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(x*\exp(2)+d*\exp(1))/(-2*c^2*ex$
314	33	1015	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^4/(c^2*\exp(2)^2*d^4*\exp$
315	33	1016	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(\exp(1)*x+d)^{-1}/\exp(1)*\exp(1)$
316	33	1017	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/4*x^4*c^9*\exp(2)^9*\exp(1)^9$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
317	33	1018	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/3*x^3*c^6*exp(2)^6*exp(1)^8$
318	33	1019	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(1/2*x^2*c^3*exp(2)^3*exp(1)^7$
319	33	1020	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $x*exp(1)^6/c^3/exp(2)^3+(3*ex$
320	33	1021	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $exp(1)^5*1/2/c^3/exp(2)^3*\ln(x$
321	33	1022	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3*x^3*exp(2)^3-8*x^3*exp(2)*e$
322	33	1023	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3*x^3*exp(2)^2+9*x^2*exp(2)*d$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
323	33	1024	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3*x^3*exp(2)^2+2*x^3*exp(2))*$
324	33	1025	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3*x^3*exp(2)^2-9*x^2*exp(2))*$
325	33	1026	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3*x^3*exp(2)^3-9*x^2*exp(2))^$
326	33	1027	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $exp(1)^6/(c^3*exp(2)^3*d^6*exp$
327	33	1028	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(exp(1)*x+d)^{-1}/exp(1)*exp(1)^$
328	33	1033	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $4*1/4/exp(1)*sqrt(c*d^2+2*c*d*$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
329	33	1035	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(-(-c*\sqrt{c*d^2+2*c*d*x}*ex$
330	33	1036	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-3*c*(\sqrt{c*d^2+2*c*d*x}*e$
331	33	1037	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^{2*(2*(-(\exp(1)*x+d)^{-1/}$
332	33	1038	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((105*c*(\sqrt{c*d^2+2*c*d*x}*$
333	33	1043	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((16*c*\exp(1)^4*1/96/\exp(1)^$
334	33	1044	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
335	33	1045	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $4*c*1/4/\exp(1)*\sqrt{c*d^2+2*c*d}$
336	33	1046	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(-c*\sqrt{c*\exp(2)})/2/\exp(2)*$
337	33	1047	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^{2*(2*(-\exp(1)*x+d)^{-1/2})}$
338	33	1048	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(-(15*c)^{2*(\sqrt{c*d^2+2*c*d})}$
339	33	1049	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-105*c)^{2*(\sqrt{c*d^2+2*c*d})}$
340	33	1054	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(((192*c^2*\exp(1))^{10*1/1920})$
341	33	1055	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
342	33	1056	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((16*c^2*\exp(1))^4*1/96/\exp(1)$
343	33	1057	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(2*c^2*\exp(1)*1/8/\exp(1)*x+2$
344	33	1058	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
345	33	1059	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(-c^2*\sqrt{c*\exp(2)})/2/\exp(2)$
346	33	1060	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(-(-15*c^3*(\sqrt{c*d^2+2*c*d}$
347	33	1061	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-105*c^3*(\sqrt{c*d^2+2*c*d}$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
348	33	1067	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2/d/\sqrt{c*\exp(1)^2-c*\exp(2)}$ *
349	33	1068	1 (pass)	0 (not solved)
350	33	1069	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-3*\sqrt{c*\exp(2)*x^2+c*d}$
351	33	1070	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-15*\sqrt{c*\exp(2)*x^2+c*d}$
352	33	1075	1 (pass)	0 (not solved)
353	33	1076	1 (pass)	0 (not solved)
354	33	1077	1 (pass)	0 (not solved)
355	33	1078	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
356	33	1085	1 (pass)	0 (not solved)
357	33	1086	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
358	33	1087	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
359	33	1088	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
360	33	1094	1 (pass)	0 (not solved)
361	33	1095	1 (pass)	0 (not solved)
362	33	1096	1 (pass)	0 (not solved)
363	33	1596	1 (pass)	0 (not solved)
364	33	1597	1 (pass)	0 (not solved)
365	33	1598	1 (pass)	0 (not solved)
366	33	1599	1 (pass)	0 (not solved)
367	33	1600	1 (pass)	0 (not solved)
368	33	1601	1 (pass)	0 (not solved)
369	33	1603	1 (pass)	0 (not solved)
370	33	1604	1 (pass)	0 (not solved)
371	33	1605	1 (pass)	0 (not solved)
372	33	1606	1 (pass)	0 (not solved)
373	33	1607	1 (pass)	0 (not solved)
374	33	1608	1 (pass)	0 (not solved)
375	33	1609	1 (pass)	0 (not solved)
376	33	1610	1 (pass)	0 (not solved)
377	33	1611	1 (pass)	0 (not solved)
378	33	1613	1 (pass)	0 (not solved)
379	33	1618	1 (pass)	0 (not solved)
380	33	1619	1 (pass)	0 (not solved)
381	33	1620	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
382	33	1871	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^2/(d*\exp(1)^{4*a}-d*\exp(1)$
383	33	1872	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(a*\exp(2)-d^{2*c})/(2*a^{2*d}^{2*ex}$
384	33	1873	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-c^{2*d}^{4+c*d}^{2*\exp(1)^{2*a+c*d}$
385	33	1883	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^2/(d^{2*\exp(1)^{5*a}^{2-2*d}$
386	33	1884	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(\exp(1)*x+d)^{-1}/\exp(1)*\exp(1)$
387	33	1895	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1)^3/(d^{3*\exp(1)^{7*a}^{3-3*d}$
388	33	1896	1 (pass)	0 (not solved)
389	33	1897	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
390	33	1898	1 (pass)	0 (not solved)
391	33	1899	1 (pass)	-1 (time out)
392	33	1912	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
393	33	1913	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
394	33	1916	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: exp(1) *(2*(-(exp(1)*x+d))^-1/ex
395	33	1923	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
396	33	1924	1 (pass)	-1 (time out)
397	33	1925	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
398	33	1927	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(1) * (2 * (-\exp(1) * x + d)^{-1} / \exp(x))$
399	33	1936	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
400	33	1937	1 (pass)	-1 (time out)
401	33	1938	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
402	33	1939	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
403	33	1940	1 (pass)	-1 (time out)
404	33	1951	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
405	33	1961	1 (pass)	0 (not solved)
406	33	1972	1 (pass)	0 (not solved)
407	33	2000	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
408	33	2001	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $((4*a^7*c^2*d^2*\exp(2))^{7-2*a^7})$
409	33	2002	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $((-4*a^6*c^2*d^2*\exp(2))^{6+2*a^6})$
410	33	2003	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $((4*a^5*c^2*d^2*\exp(2))^{5-2*a^5})$
411	33	2005	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((4*a^4*c*d*\exp(2))^{4+2*a^4*s})$
412	33	2006	1 (pass)	-1 (time out)
413	33	2007	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((4*a^6*c*d*\exp(2))^{6+2*a^6*s})$
414	33	2008	1 (pass)	-1 (time out)
415	33	2009	1 (pass)	-1 (time out)
416	33	2010	1 (pass)	-1 (time out)
417	33	2011	1 (pass)	-1 (time out)
418	33	2012	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
419	33	2013	1 (pass)	-1 (time out)
420	33	2014	1 (pass)	-1 (time out)
421	33	2015	1 (pass)	-1 (time out)
422	33	2016	1 (pass)	-1 (time out)
423	33	2017	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((12*a^7*c*d*exp(1)*exp(2))^7$
424	33	2018	1 (pass)	-1 (time out)
425	33	2019	1 (pass)	-1 (time out)
426	33	2020	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
427	33	2021	1 (pass)	-1 (time out)
428	33	2022	1 (pass)	-1 (time out)
429	33	2024	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(12*a^6*c*d*exp(1)^2*exp(2))^6+$
430	33	2025	1 (pass)	-1 (time out)
431	33	2026	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-12*a^8*c*d*exp(1))^2*exp(2)$
432	33	2027	1 (pass)	0 (not solved)
433	33	2028	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
434	33	2029	1 (pass)	0 (not solved)
435	33	2030	1 (pass)	0 (not solved)
436	33	2031	1 (pass)	0 (not solved)
437	33	2032	1 (pass)	0 (not solved)
438	33	2033	1 (pass)	0 (not solved)
439	33	2034	1 (pass)	0 (not solved)
440	33	2035	1 (pass)	0 (not solved)
441	33	2036	1 (pass)	0 (not solved)
442	33	2037	1 (pass)	0 (not solved)
443	33	2038	1 (pass)	0 (not solved)
444	33	2039	1 (pass)	0 (not solved)
445	33	2040	1 (pass)	0 (not solved)
446	33	2041	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x);O
447	33	2042	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
448	33	2043	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
449	33	2044	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
450	33	2045	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
451	33	2046	1 (pass)	-1 (time out)
452	33	2047	1 (pass)	0 (not solved)
453	33	2048	1 (pass)	0 (not solved)
454	33	2049	1 (pass)	0 (not solved)
455	33	2050	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
456	33	2051	1 (pass)	-1 (time out)
457	33	2052	1 (pass)	-1 (time out)
458	33	2053	1 (pass)	-1 (time out)
459	33	2054	1 (pass)	-1 (time out)
460	33	2055	1 (pass)	-1 (time out)
461	33	2056	1 (pass)	-1 (time out)
462	33	2057	1 (pass)	0 (not solved)
463	33	2058	1 (pass)	0 (not solved)
464	33	2059	1 (pass)	0 (not solved)
465	33	2060	1 (pass)	0 (not solved)
466	33	2061	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
467	33	2062	1 (pass)	0 (not solved)
468	33	2063	1 (pass)	0 (not solved)
469	33	2064	1 (pass)	0 (not solved)
470	33	2065	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
471	33	2066	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
472	33	2067	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
473	33	2068	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
474	33	2069	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
475	33	2070	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
476	33	2071	1 (pass)	-1 (time out)
477	33	2072	1 (pass)	-1 (time out)
478	33	2073	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
479	33	2074	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
480	33	2075	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
481	33	2076	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
482	33	2077	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
483	33	2078	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
484	33	2079	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
485	33	2080	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
486	33	2081	1 (pass)	0 (not solved)
487	33	2082	1 (pass)	0 (not solved)
488	33	2354	1 (pass)	-1 (time out)
489	33	2399	1 (pass)	-1 (time out)
490	33	2407	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
491	33	2409	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*exp(2)*2/2/sqrt(-c*d^2*exp(2
492	33	2421	1 (pass)	0 (not solved)
493	33	2422	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
494	33	2425	1 (pass)	0 (not solved)
495	33	2426	1 (pass)	0 (not solved)
496	34	714	1 (pass)	0 (not solved)
497	34	715	1 (pass)	0 (not solved)
498	34	716	1 (pass)	0 (not solved)
499	34	717	1 (pass)	0 (not solved)
500	34	718	1 (pass)	0 (not solved)
501	34	719	1 (pass)	0 (not solved)
502	34	720	1 (pass)	0 (not solved)
503	34	721	1 (pass)	0 (not solved)
504	34	722	1 (pass)	0 (not solved)
505	34	723	1 (pass)	0 (not solved)
506	34	724	1 (pass)	0 (not solved)
507	34	725	1 (pass)	0 (not solved)
508	34	726	1 (pass)	0 (not solved)
509	34	727	1 (pass)	0 (not solved)
510	34	728	1 (pass)	0 (not solved)
511	34	1458	1 (pass)	0 (not solved)
512	34	1462	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
513	34	1767	1 (pass)	0 (not solved)
514	34	1768	1 (pass)	0 (not solved)
515	34	1769	1 (pass)	0 (not solved)
516	34	1770	1 (pass)	0 (not solved)
517	34	1771	1 (pass)	0 (not solved)
518	34	1772	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
519	34	1774	1 (pass)	0 (not solved)
520	34	1775	1 (pass)	0 (not solved)
521	34	1776	1 (pass)	0 (not solved)
522	34	1777	1 (pass)	0 (not solved)
523	34	1778	1 (pass)	0 (not solved)
524	34	1779	1 (pass)	0 (not solved)
525	34	1780	1 (pass)	0 (not solved)
526	34	1781	1 (pass)	0 (not solved)
527	34	1783	1 (pass)	0 (not solved)
528	34	2029	1 (pass)	0 (not solved)
529	34	2033	1 (pass)	0 (not solved)
530	34	2034	1 (pass)	0 (not solved)
531	34	2035	1 (pass)	0 (not solved)
532	34	2036	1 (pass)	0 (not solved)
533	34	2037	1 (pass)	0 (not solved)
534	34	2039	1 (pass)	0 (not solved)
535	34	2041	1 (pass)	0 (not solved)
536	34	2175	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
537	34	2176	1 (pass)	-1 (time out)
538	34	2177	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-c*sqrt(-c*exp(2))*g/2/c/ex

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
539	34	2178	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((100663296*exp(2))^2*(sqrt(-
540	34	2179	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0
541	34	2180	1 (pass)	-1 (time out)
542	34	2181	1 (pass)	-1 (time out)
543	34	2182	1 (pass)	-1 (time out)
544	34	2186	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
545	34	2187	1 (pass)	-1 (time out)
546	34	2188	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(-4*exp(1)^3*c^2*g*1/16/exp(
547	34	2189	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -4*exp(1)*c*g*1/4/exp(1)^3*sqr
548	34	2191	1 (pass)	-1 (time out)
549	34	2192	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
550	34	2193	1 (pass)	-1 (time out)
551	34	2194	1 (pass)	-1 (time out)
552	34	2198	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
553	34	2199	1 (pass)	-1 (time out)
554	34	2200	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((96*exp(1)^5*c^5*g*1/768/e
555	34	2201	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((16*exp(1)^4*c^4*g*1/96/exp
556	34	2202	1 (pass)	-1 (time out)
557	34	2203	1 (pass)	-1 (time out)
558	34	2204	1 (pass)	-1 (time out)
559	34	2205	1 (pass)	-1 (time out)
560	34	2206	1 (pass)	-1 (time out)
561	34	2207	1 (pass)	-1 (time out)
562	34	2208	1 (pass)	-1 (time out)
563	34	2212	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
564	34	2213	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
565	34	2214	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-1024*exp(2))*(sqrt(-c*exp(
566	34	2215	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*((-73728*exp(2))^2*(sqrt(-c*e
567	34	2216	1 (pass)	0 (not solved)
568	34	2221	1 (pass)	0 (not solved)
569	34	2229	1 (pass)	0 (not solved)
570	34	2231	1 (pass)	0 (not solved)
571	34	2232	1 (pass)	0 (not solved)
572	34	2233	1 (pass)	-1 (time out)
573	34	2234	1 (pass)	0 (not solved)
574	34	2235	1 (pass)	0 (not solved)
575	34	2236	1 (pass)	-2 (exception) Exception raised: AttributeError >> type
576	34	2237	1 (pass)	0 (not solved)
577	34	2238	1 (pass)	0 (not solved)
578	34	2239	1 (pass)	0 (not solved)
579	34	2240	1 (pass)	-1 (time out)
580	34	2241	1 (pass)	0 (not solved)
581	34	2242	1 (pass)	0 (not solved)
582	34	2243	1 (pass)	0 (not solved)
583	34	2244	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
584	34	2245	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
585	34	2246	1 (pass)	-1 (time out)
586	34	2247	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Warning
587	34	2248	1 (pass)	-1 (time out)
588	34	2249	1 (pass)	-1 (time out)
589	34	2250	1 (pass)	0 (not solved)
590	34	2251	1 (pass)	0 (not solved)
591	34	2252	1 (pass)	0 (not solved)
592	34	2253	1 (pass)	0 (not solved)
593	34	2254	1 (pass)	0 (not solved)
594	34	2255	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
595	34	2256	1 (pass)	-1 (time out)
596	34	2257	1 (pass)	-1 (time out)
597	34	2258	1 (pass)	-1 (time out)
598	34	2259	1 (pass)	-1 (time out)
599	34	2260	1 (pass)	-1 (time out)
600	34	2261	1 (pass)	-1 (time out)
601	34	2262	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
602	34	2263	1 (pass)	-1 (time out)
603	34	2264	1 (pass)	-1 (time out)
604	34	2265	1 (pass)	0 (not solved)
605	34	2266	1 (pass)	0 (not solved)
606	34	2267	1 (pass)	0 (not solved)
607	34	2268	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
608	34	2269	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
609	34	2270	1 (pass)	-1 (time out)
610	34	2271	1 (pass)	-1 (time out)
611	34	2272	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
612	34	2273	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
613	34	2274	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
614	34	2275	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
615	34	2276	1 (pass)	-1 (time out)
616	34	2277	1 (pass)	-1 (time out)
617	34	2278	1 (pass)	-1 (time out)
618	34	2279	1 (pass)	-1 (time out)
619	34	2280	1 (pass)	-1 (time out)
620	34	2281	1 (pass)	-1 (time out)
621	34	2282	1 (pass)	-1 (time out)
622	34	2283	1 (pass)	-1 (time out)
623	34	2284	1 (pass)	-1 (time out)
624	34	2494	1 (pass)	-1 (time out)
625	35	99	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/8*(\exp(2)^3+2*(-2*d*\exp(1)-2)$
626	35	100	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/24*((-1/2*(-2*d*\exp(1)-2*\sqrt{2})*\exp(2)^3+2*(-2*d*\exp(1)-2))$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
627	35	101	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/192*((-1/2*(-2*d*\exp(1)-2*sq$
628	35	102	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(4*d^5*\exp(2)^3-4*d^5*\exp($
629	35	103	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(12*d^9*\exp(1)^4*\exp(2)^2-$
630	35	104	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(-12*d^8*\exp(1)^4*\exp(2)^2$
631	35	105	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(12*d^7*\exp(1)^4*\exp(2)^2-$
632	35	106	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(-12*d^6*\exp(1)^4*\exp(2)^2$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
633	35	107	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(12*d^5*\exp(1)^4*\exp(2)^2-$
634	35	108	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-3/8*d^4*\text{sign}(d)*\text{asin}(x*\exp(2))$
635	35	109	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(12*d^3*\exp(1)^4*\exp(2)^2-$
636	35	110	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/8*(d^2*\exp(2)^3+2*d^2*(-2*d*$
637	35	111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/24*((-1/2*(-2*d*\exp(1))-2*\text{sq}$
638	35	112	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/192*((-1/2*(-2*d*\exp(1))-2*\text{sq}$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
639	35	113	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/960 * ((-1/2 * (-2 * d * \exp(1) - 2 * s$
640	35	114	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/1920 * ((-1/2 * (-2 * d * \exp(1) - 2 * s$
641	35	115	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/13440 * ((-1/2 * (-2 * d * \exp(1) - 2 * s$
642	35	116	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/215040 * ((-1/2 * (-2 * d * \exp(1) - 2 * s$
643	35	119	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-3/2 * d^3 * \text{sign}(d) * \text{asin}(x * \exp(2))$
644	35	120	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $3/2 * d^2 * \text{sign}(d) * \text{asin}(x * \exp(2)) /$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
645	35	121	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-d*\text{sign}(d)*\text{asin}(x*\exp(2)/d/\exp$
646	35	122	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\text{sign}(d)*\text{asin}(x*\exp(2)/d/\exp(1)$
647	35	123	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*\exp(2)*\text{atan}((-1/2*(-2*d*\exp$
648	35	124	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\exp(2)*\ln(1/2*\text{abs}(-2*d*\exp(1)$
649	35	125	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-x*\exp(2)^3/d^3/(-2*d*\exp(1)-2$
650	35	126	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/8*(\exp(2)^3+2*(-2*d*\exp(1)-2$
651	35	157	1 (pass)	0 (not solved)
652	35	158	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
653	35	159	1 (pass)	0 (not solved)
654	35	160	1 (pass)	0 (not solved)
655	35	161	1 (pass)	0 (not solved)
656	35	162	1 (pass)	0 (not solved)
657	35	167	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
658	35	168	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
659	35	169	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
660	35	170	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
661	35	172	1 (pass)	0 (not solved)
662	35	173	1 (pass)	0 (not solved)
663	35	174	1 (pass)	0 (not solved)
664	35	175	1 (pass)	0 (not solved)
665	35	176	1 (pass)	0 (not solved)
666	35	177	1 (pass)	0 (not solved)
667	35	178	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
668	35	179	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3*d^{2*(-1/2*(-2*d*\exp(1)-2*sq$
669	35	180	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-d*(-1/2*(-2*d*\exp(1)-2*sqrt($
670	35	181	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-(-1/2*(-2*d*\exp(1)-2*sqrt(d^$
671	35	182	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(3*(-1/2*(-2*d*\exp(1)-2*sqrt(d$
672	35	183	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(2*\exp(1)*\exp(2)^5+5*(-1/2*(-2$
673	35	184	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(7*(-1/2*(-2*d*\exp(1)-2*sqrt(d$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
674	35	185	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-9*(-1/2*(-2*d*\exp(1)-2*\sqrt{d}))$
675	35	188	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-162*d^3*(-1/2*(-2*d*\exp(1)-2$
676	35	189	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(84*d^2*(-1/2*(-2*d*\exp(1)-2*s$
677	35	190	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-30*d*(-1/2*(-2*d*\exp(1)-2*s$
678	35	191	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-96*(-1/2*(-2*d*\exp(1)-2*\sqrt{d}))$
679	35	192	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(6*\exp(1)*\exp(2)^7+12*(-1/2*(-$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
680	35	193	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(12*(-1/2*(-2*d*\exp(1)-2*\sqrt{d}))$
681	35	194	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(6*\exp(1)*\exp(2)^8+12*(-1/2*(-$
682	35	198	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-162*d^7*(-1/2*(-2*d*\exp(1)-2$
683	35	199	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(84*d^6*(-1/2*(-2*d*\exp(1)-2*s$
684	35	200	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-30*d^5*(-1/2*(-2*d*\exp(1)-2*$
685	35	201	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-288*d^4*(-1/2*(-2*d*\exp(1)-2$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
686	35	202	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(6*d^3*(-1/2*(-2*d*\exp(1)-2*sq$
687	35	203	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(12*d^2*(-1/2*(-2*d*\exp(1)-2*s$
688	35	204	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-54*d*(-1/2*(-2*d*\exp(1)-2*sq$
689	35	205	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/4*(-2*d*\exp(1)-2*sqrt(d^2-x^$
690	35	206	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/16*(-2*d*(-1/2*(-2*d*\exp(1)-$
691	35	207	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/512*(256*d^4*(-1/2*(-2*d*\exp$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
692	35	208	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/65536*(-81920*d^9*(-1/2*(-2*$
693	35	209	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/480*((-1/2*(-2*d*exp(1)-2*sq$
694	35	437	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
695	35	438	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
696	35	439	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
697	35	440	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
698	35	442	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-2*\exp(1)*a*\exp(2)+2*\exp(1$
699	35	443	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((2*\exp(1)^2*a*\exp(2)-2*\exp($
700	35	444	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-2*\exp(1)^3*a*\exp(2)+2*\exp$
701	35	445	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((2*\exp(1)^4*a*\exp(2)-2*\exp($
702	35	446	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
703	35	447	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
704	35	448	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
705	35	449	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
706	35	451	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
707	35	452	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
708	35	453	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((2*\exp(1)^2*a^2*\exp(2)^2-4*$
709	35	454	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-2*\exp(1)^3*a^2*\exp(2)^2+4$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
710	35	455	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((2*\exp(1)^4*a^2*\exp(2)^2-4*$
711	35	456	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-2*\exp(1)^5*a^2*\exp(2)^2+4$
712	35	457	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
713	35	458	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
714	35	459	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
715	35	460	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
716	35	462	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
717	35	463	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
718	35	464	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
719	35	465	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
720	35	466	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*((-2*\exp(1)^3*a^3*\exp(2)^3+6$
721	35	467	1 (pass)	-1 (time out)
722	35	468	1 (pass)	-1 (time out)
723	35	469	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
724	35	470	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
725	35	471	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
726	35	553	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(d^2 * g^2 - 2 * d * \exp(1) * g * f + \exp(1))$
727	35	554	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(-g^2 * d^2 * \exp(1) + g * d * \exp(1))^2$
728	35	555	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(2 * \exp(2)^2 * d * g * f - 3 * \exp(2)^2 * \dots)$
729	35	556	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(\exp(2)^3 * d * g * f - 2 * \exp(2)^3 * \dots)$

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
730	35	565	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(d^2 \exp(1)^2 * g^2 - 2 * d * \exp(1)^3)$
731	35	566	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-\exp(1) * x + d)^{-1} / \exp(1) * g^2 * d$
732	35	567	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-3 * \exp(2)^2 * d^2 * \exp(1) * g^2 + 12)$
733	35	568	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-2 * \exp(2)^3 * d^2 * \exp(1) * g^2 + 10)$
734	35	577	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(d^2 * \exp(1)^4 * g^2 - 2 * d * \exp(1)^4)$
735	35	578	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-((\exp(1) * x + d)^{-1} / \exp(1) * g^2 * d)$
736	35	618	1 (pass)	-1 (time out)
737	35	657	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
738	35	658	1 (pass)	0 (not solved)
739	35	659	1 (pass)	0 (not solved)
740	35	660	1 (pass)	0 (not solved)
741	35	661	1 (pass)	0 (not solved)
742	35	665	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
743	35	666	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
744	35	667	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
745	35	668	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
746	35	669	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
747	35	672	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
748	35	673	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
749	35	674	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
750	35	675	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
751	35	676	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
752	35	679	1 (pass)	0 (not solved)
753	35	680	1 (pass)	0 (not solved)
754	35	681	1 (pass)	0 (not solved)
755	35	682	1 (pass)	0 (not solved)
756	35	683	1 (pass)	0 (not solved)
757	35	689	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
758	35	690	1 (pass)	0 (not solved)
759	35	691	1 (pass)	0 (not solved)
760	35	692	1 (pass)	-1 (time out)
761	35	693	1 (pass)	0 (not solved)
762	35	700	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
763	35	701	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
764	35	702	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
765	35	703	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
766	35	704	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
767	35	783	1 (pass)	0 (not solved)
768	35	784	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
769	35	785	1 (pass)	0 (not solved)
770	35	786	1 (pass)	0 (not solved)
771	35	787	1 (pass)	0 (not solved)
772	35	847	1 (pass)	-1 (time out)
773	35	848	1 (pass)	-1 (time out)
774	35	849	1 (pass)	-1 (time out)
775	35	861	1 (pass)	-1 (time out)
776	36	6	1 (pass)	-1 (time out)
777	38	4	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(4*A*d*exp(2)^3-4*A*d*exp($
778	38	5	1 (pass)	0 (not solved)
779	38	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-8*A*(-1/2*(-2*d*exp(1)-2*sqrt$
780	38	7	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(8*A*(-1/2*(-2*d*exp(1)-2*sqrt$
781	38	8	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
782	38	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-960*A*(-1/2*(-2*d*exp(1)-2*s$
783	38	14	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(-4*A*exp(2)^2-4*C*d^2*exp$
784	38	16	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(-2*A*(-1/2*(-2*d*exp(1)-2*sqr$
785	38	17	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(64*C*d^2*(-1/2*(-2*d*exp(1)-2$
786	38	195	1 (pass)	-1 (time out)
787	38	206	1 (pass)	-1 (time out)
788	38	207	1 (pass)	-1 (time out)
789	39	268	1 (pass)	0 (not solved)
790	39	271	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
791	39	272	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
792	39	275	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
793	39	278	1 (pass)	0 (not solved)
794	39	279	1 (pass)	0 (not solved)
795	39	280	1 (pass)	0 (not solved)
796	39	283	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
797	39	284	1 (pass)	0 (not solved)
798	39	285	1 (pass)	0 (not solved)
799	39	640	1 (pass)	0 (not solved)
800	39	641	1 (pass)	0 (not solved)
801	39	642	1 (pass)	0 (not solved)
802	39	643	1 (pass)	0 (not solved)
803	39	644	1 (pass)	0 (not solved)
804	39	653	1 (pass)	0 (not solved)
805	39	654	1 (pass)	0 (not solved)
806	39	655	1 (pass)	0 (not solved)
807	39	656	1 (pass)	0 (not solved)
808	39	657	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
809	39	658	1 (pass)	0 (not solved)
810	39	994	1 (pass)	0 (not solved)
811	39	1000	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
812	39	1002	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
813	40	193	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\left(d^2 \exp(2)^3\right)^{1/4} \cdot \text{abs}(d)$ *
814	40	194	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(-2 \cdot (d^2 \exp(2)^3)^{1/4}) \cdot \text{abs}($
815	40	198	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT: sage2:=int(sage0, x)::O
816	40	218	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT: sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
817	40	219	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(4*b^5*c*exp(1)*exp(2)^5+2*b^5$
818	40	222	1 (pass)	-1 (time out)
819	40	223	1 (pass)	-1 (time out)
820	40	224	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
821	40	274	1 (pass)	-1 (time out)
822	40	275	1 (pass)	-1 (time out)
823	41	81	1 (pass)	0 (not solved)
824	41	83	1 (pass)	0 (not solved)
825	41	85	1 (pass)	0 (not solved)
826	42	40	1 (pass)	-1 (time out)
827	42	41	1 (pass)	-1 (time out)
828	42	56	1 (pass)	-1 (time out)
829	42	57	1 (pass)	-1 (time out)
830	42	58	1 (pass)	-1 (time out)
831	42	59	1 (pass)	-1 (time out)
832	46	98	1 (pass)	0 (not solved)
833	46	99	1 (pass)	0 (not solved)
834	46	100	1 (pass)	0 (not solved)
835	46	101	1 (pass)	0 (not solved)
836	46	102	1 (pass)	0 (not solved)
837	46	103	1 (pass)	0 (not solved)
838	46	104	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
839	46	105	1 (pass)	0 (not solved)
840	46	106	1 (pass)	0 (not solved)
841	46	107	1 (pass)	0 (not solved)
842	46	108	1 (pass)	0 (not solved)
843	46	109	1 (pass)	0 (not solved)
844	46	110	1 (pass)	0 (not solved)
845	46	111	1 (pass)	0 (not solved)
846	46	112	1 (pass)	0 (not solved)
847	46	113	1 (pass)	0 (not solved)
848	46	114	1 (pass)	0 (not solved)
849	46	115	1 (pass)	0 (not solved)
850	46	116	1 (pass)	0 (not solved)
851	46	117	1 (pass)	0 (not solved)
852	46	453	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
853	46	454	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
854	46	596	1 (pass)	-1 (time out)
855	50	49	1 (pass)	0 (not solved)
856	50	55	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
857	50	60	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0]
858	51	43	1 (pass)	0 (not solved)
859	51	44	1 (pass)	-1 (time out)
860	51	413	1 (pass)	-1 (time out)
861	52	276	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
862	52	277	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
863	52	278	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
864	52	296	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
865	52	297	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
866	52	298	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
867	52	300	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
868	52	301	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
869	52	442	1 (pass)	-1 (time out)
870	52	457	1 (pass)	0 (not solved)
871	52	458	1 (pass)	-1 (time out)
872	52	459	1 (pass)	-1 (time out)
873	52	476	1 (pass)	0 (not solved)
874	52	477	1 (pass)	-1 (time out)
875	52	478	1 (pass)	-1 (time out)
876	52	498	1 (pass)	0 (not solved)
877	52	504	1 (pass)	0 (not solved)
878	52	526	1 (pass)	0 (not solved)
879	52	527	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
880	52	532	1 (pass)	0 (not solved)
881	52	533	1 (pass)	0 (not solved)
882	52	536	1 (pass)	0 (not solved)
883	52	537	1 (pass)	0 (not solved)
884	52	717	1 (pass)	-1 (time out)
885	52	811	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
886	52	812	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
887	52	891	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
888	52	899	1 (pass)	0 (not solved)
889	53	82	1 (pass)	-1 (time out)
890	54	32	1 (pass)	-1 (time out)
891	54	39	1 (pass)	-1 (time out)
892	55	624	1 (pass)	0 (not solved)
893	58	186	1 (pass)	-1 (time out)
894	58	187	1 (pass)	-1 (time out)
895	59	33	1 (pass)	-1 (time out)
896	59	66	1 (pass)	-1 (time out)
897	59	106	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
898	59	177	1 (pass)	-1 (time out)
899	59	230	1 (pass)	-1 (time out)
900	59	238	1 (pass)	-1 (time out)
901	59	239	1 (pass)	-1 (time out)
902	60	31	1 (pass)	-1 (time out)
903	60	32	1 (pass)	-1 (time out)
904	60	33	1 (pass)	-1 (time out)
905	60	34	1 (pass)	-1 (time out)
906	60	37	1 (pass)	-1 (time out)
907	60	38	1 (pass)	-1 (time out)
908	60	39	1 (pass)	-1 (time out)
909	60	40	1 (pass)	-1 (time out)
910	60	41	1 (pass)	-1 (time out)
911	60	44	1 (pass)	-1 (time out)
912	60	45	1 (pass)	-1 (time out)
913	60	46	1 (pass)	-1 (time out)
914	60	90	1 (pass)	-1 (time out)
915	60	91	1 (pass)	-1 (time out)
916	60	97	1 (pass)	-1 (time out)
917	60	98	1 (pass)	-1 (time out)
918	60	99	1 (pass)	-1 (time out)
919	60	135	1 (pass)	-1 (time out)
920	60	136	1 (pass)	-1 (time out)
921	60	137	1 (pass)	-1 (time out)
922	60	138	1 (pass)	-1 (time out)
923	60	141	1 (pass)	-1 (time out)
924	60	142	1 (pass)	-1 (time out)
925	60	143	1 (pass)	-1 (time out)
926	60	144	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
927	60	145	1 (pass)	-1 (time out)
928	60	148	1 (pass)	-1 (time out)
929	60	149	1 (pass)	-1 (time out)
930	60	150	1 (pass)	-1 (time out)
931	60	177	1 (pass)	-1 (time out)
932	60	192	1 (pass)	-1 (time out)
933	60	199	1 (pass)	-1 (time out)
934	60	224	1 (pass)	0 (not solved)
935	60	225	1 (pass)	0 (not solved)
936	60	234	1 (pass)	0 (not solved)
937	60	235	1 (pass)	0 (not solved)
938	60	236	1 (pass)	0 (not solved)
939	61	28	1 (pass)	-1 (time out)
940	61	43	1 (pass)	0 (not solved)
941	62	455	1 (pass)	-1 (time out)
942	63	634	1 (pass)	-1 (time out)
943	64	271	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT: Evaluat
944	65	340	1 (pass)	-1 (time out)
945	65	355	1 (pass)	-1 (time out)
946	65	371	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
947	65	372	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
948	65	373	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
949	65	374	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
950	65	375	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
951	65	376	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
952	65	377	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
953	65	385	1 (pass)	0 (not solved)
954	65	386	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
955	65	387	1 (pass)	0 (not solved)
956	65	388	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
957	65	389	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
958	65	390	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
959	65	397	1 (pass)	0 (not solved)
960	65	398	1 (pass)	0 (not solved)
961	65	399	1 (pass)	0 (not solved)
962	65	400	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
963	65	401	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
964	65	402	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
965	65	403	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
966	65	410	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
967	65	411	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
968	65	413	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
969	65	414	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
970	65	415	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
971	65	416	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
972	65	424	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
973	65	425	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
974	65	428	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
975	65	429	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
976	65	430	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
977	65	437	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
978	65	438	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
979	65	441	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
980	65	442	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
981	65	443	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
982	66	22	1 (pass)	-1 (time out)
983	66	106	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
984	66	128	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
985	66	129	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
986	66	130	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
987	66	131	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
988	66	132	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
989	66	133	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
990	66	162	1 (pass)	-1 (time out)
991	66	187	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
992	66	193	1 (pass)	-1 (time out)
993	66	257	1 (pass)	-1 (time out)
994	66	258	1 (pass)	-1 (time out)
995	66	259	1 (pass)	-1 (time out)
996	66	263	1 (pass)	-1 (time out)
997	66	264	1 (pass)	-1 (time out)
998	66	265	1 (pass)	-1 (time out)
999	66	268	1 (pass)	-1 (time out)
1000	68	33	1 (pass)	-1 (time out)
1001	68	34	1 (pass)	-1 (time out)
1002	68	35	1 (pass)	-1 (time out)
1003	68	37	1 (pass)	-1 (time out)
1004	68	38	1 (pass)	-1 (time out)
1005	68	39	1 (pass)	-1 (time out)
1006	69	122	1 (pass)	0 (not solved)
1007	69	217	1 (pass)	-1 (time out)
1008	70	108	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1009	70	123	1 (pass)	-1 (time out)
1010	70	124	1 (pass)	-1 (time out)
1011	70	125	1 (pass)	-1 (time out)
1012	70	126	1 (pass)	-1 (time out)
1013	70	132	1 (pass)	-1 (time out)
1014	70	134	1 (pass)	-1 (time out)
1015	70	135	1 (pass)	-1 (time out)
1016	70	136	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1017	70	137	1 (pass)	-1 (time out)
1018	70	138	1 (pass)	-1 (time out)
1019	70	146	1 (pass)	-1 (time out)
1020	70	147	1 (pass)	-1 (time out)
1021	70	148	1 (pass)	-1 (time out)
1022	70	149	1 (pass)	-1 (time out)
1023	70	150	1 (pass)	-1 (time out)
1024	70	151	1 (pass)	-1 (time out)
1025	70	152	1 (pass)	-1 (time out)
1026	70	153	1 (pass)	-1 (time out)
1027	70	154	1 (pass)	-1 (time out)
1028	70	155	1 (pass)	-1 (time out)
1029	70	167	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1030	70	168	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1031	70	178	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1032	70	181	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1033	70	192	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1034	70	343	1 (pass)	-1 (time out)
1035	70	344	1 (pass)	-1 (time out)
1036	70	473	1 (pass)	0 (not solved)
1037	70	476	1 (pass)	0 (not solved)
1038	70	477	1 (pass)	0 (not solved)
1039	70	478	1 (pass)	0 (not solved)
1040	70	506	1 (pass)	0 (not solved)
1041	70	509	1 (pass)	0 (not solved)
1042	70	510	1 (pass)	0 (not solved)
1043	70	511	1 (pass)	0 (not solved)
1044	70	630	1 (pass)	-1 (time out)
1045	70	631	1 (pass)	-1 (time out)
1046	72	2	1 (pass)	-1 (time out)
1047	72	27	1 (pass)	-1 (time out)
1048	72	36	1 (pass)	-1 (time out)
1049	72	92	1 (pass)	0 (not solved)
1050	72	93	1 (pass)	-1 (time out)
1051	72	94	1 (pass)	-1 (time out)
1052	72	95	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1053	72	96	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1054	72	97	1 (pass)	-1 (time out)
1055	72	98	1 (pass)	-1 (time out)
1056	72	99	1 (pass)	-1 (time out)
1057	72	100	1 (pass)	-1 (time out)
1058	72	101	1 (pass)	-1 (time out)
1059	72	102	1 (pass)	-1 (time out)
1060	72	103	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1061	72	104	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1062	72	105	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1063	72	107	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1064	72	108	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1065	72	109	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1066	72	110	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1067	72	111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1068	72	112	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1069	72	113	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1070	72	114	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1071	72	140	1 (pass)	-1 (time out)
1072	72	161	1 (pass)	-1 (time out)
1073	73	49	1 (pass)	-1 (time out)
1074	73	50	1 (pass)	-1 (time out)
1075	73	51	1 (pass)	-1 (time out)
1076	73	56	1 (pass)	-1 (time out)
1077	73	57	1 (pass)	-1 (time out)
1078	73	58	1 (pass)	-1 (time out)
1079	73	59	1 (pass)	-1 (time out)
1080	73	60	1 (pass)	-1 (time out)
1081	73	65	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1082	73	75	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1083	73	86	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1084	73	89	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1085	73	90	1 (pass)	0 (not solved)
1086	73	153	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1087	73	154	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1088	73	155	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1089	73	156	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1090	73	161	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1091	73	162	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1092	73	163	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1093	73	164	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1094	73	165	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1095	73	170	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1096	73	171	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1097	73	172	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1098	73	173	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1099	73	174	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1100	73	290	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1101	73	291	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1102	73	292	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1103	73	293	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1104	73	294	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1105	73	295	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1106	73	296	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1107	73	297	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1108	73	298	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1109	73	299	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1110	73	300	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1111	73	301	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1112	73	302	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1113	73	303	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1114	73	304	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1115	73	305	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x);O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1116	73	306	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1117	73	307	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1118	73	308	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1119	73	309	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1120	73	310	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1121	73	311	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1122	73	312	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1123	73	313	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1124	73	315	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1125	73	317	1 (pass)	-1 (time out)
1126	73	318	1 (pass)	-1 (time out)
1127	73	319	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1128	73	320	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1129	73	321	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1130	73	322	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1131	73	323	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1132	73	324	1 (pass)	-1 (time out)
1133	73	325	1 (pass)	-1 (time out)
1134	73	326	1 (pass)	-1 (time out)
1135	73	327	1 (pass)	-1 (time out)
1136	73	328	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1137	73	329	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1138	73	330	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1139	73	331	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1140	73	332	1 (pass)	-1 (time out)
1141	73	333	1 (pass)	-1 (time out)
1142	73	334	1 (pass)	-1 (time out)
1143	73	335	1 (pass)	-1 (time out)
1144	73	336	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1145	73	337	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1146	73	338	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1147	73	339	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1148	73	340	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1149	73	341	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1150	73	342	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1151	73	343	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1152	73	344	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1153	73	345	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1154	73	346	1 (pass)	-1 (time out)
1155	73	347	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1156	73	348	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1157	73	349	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1158	73	350	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1159	73	351	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1160	73	352	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1161	73	353	1 (pass)	-1 (time out)
1162	73	354	1 (pass)	-1 (time out)
1163	73	355	1 (pass)	-1 (time out)
1164	73	356	1 (pass)	-1 (time out)
1165	73	357	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1166	73	358	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1167	73	359	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1168	73	360	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1169	73	361	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1170	73	362	1 (pass)	-1 (time out)
1171	73	363	1 (pass)	-1 (time out)
1172	73	364	1 (pass)	-1 (time out)
1173	73	365	1 (pass)	-1 (time out)
1174	73	366	1 (pass)	-1 (time out)
1175	73	367	1 (pass)	-1 (time out)
1176	73	368	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1177	73	369	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1178	73	370	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1179	73	371	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1180	73	372	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1181	73	373	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1182	73	374	1 (pass)	-1 (time out)
1183	73	375	1 (pass)	-1 (time out)
1184	73	376	1 (pass)	-1 (time out)
1185	73	377	1 (pass)	-1 (time out)
1186	73	378	1 (pass)	-1 (time out)
1187	73	379	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1188	73	380	1 (pass)	-1 (time out)
1189	73	381	1 (pass)	-1 (time out)
1190	73	382	1 (pass)	-1 (time out)
1191	73	383	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1192	73	384	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1193	73	385	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1194	73	387	1 (pass)	0 (not solved)
1195	73	388	1 (pass)	0 (not solved)
1196	73	389	1 (pass)	-1 (time out)
1197	73	390	1 (pass)	-1 (time out)
1198	73	391	1 (pass)	-1 (time out)
1199	73	392	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1200	73	393	1 (pass)	0 (not solved)
1201	73	395	1 (pass)	0 (not solved)
1202	73	396	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1203	73	397	1 (pass)	-1 (time out)
1204	73	398	1 (pass)	-1 (time out)
1205	73	399	1 (pass)	-1 (time out)
1206	73	400	1 (pass)	-1 (time out)
1207	73	401	1 (pass)	0 (not solved)
1208	73	402	1 (pass)	0 (not solved)
1209	73	522	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1210	73	523	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1211	73	524	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1212	73	525	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1213	73	526	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1214	73	527	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1215	73	528	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1216	73	529	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1217	73	530	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1218	73	531	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1219	73	532	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
1220	73	533	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1221	73	534	1 (pass)	-1 (time out)
1222	73	535	1 (pass)	-1 (time out)
1223	73	536	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1224	73	537	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1225	73	538	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1226	73	539	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1227	73	540	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1228	73	541	1 (pass)	-1 (time out)
1229	73	542	1 (pass)	-1 (time out)
1230	73	543	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1231	73	544	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1232	73	545	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1233	73	546	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1234	73	547	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT: sage2:=int(sage0, x)::O
1235	73	548	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
1236	73	549	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
1237	73	550	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1238	73	551	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1239	73	552	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1240	73	553	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1241	73	554	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1242	73	556	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1243	73	557	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1244	73	558	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1245	73	559	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1246	73	560	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1247	73	561	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1248	73	562	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1249	73	563	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1250	73	568	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1251	73	569	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1252	73	570	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1253	74	1	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1254	74	2	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1255	74	3	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1256	74	4	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1257	74	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1258	74	6	1 (pass)	-1 (time out)
1259	74	7	1 (pass)	-1 (time out)
1260	74	8	1 (pass)	-1 (time out)
1261	74	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1262	74	10	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1263	74	11	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1264	74	12	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1265	74	13	1 (pass)	-1 (time out)
1266	74	14	1 (pass)	-1 (time out)
1267	74	15	1 (pass)	-1 (time out)
1268	74	16	1 (pass)	-1 (time out)
1269	74	17	1 (pass)	-1 (time out)
1270	74	18	1 (pass)	-1 (time out)
1271	74	19	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1272	74	20	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1273	74	21	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1274	74	22	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1275	74	23	1 (pass)	-1 (time out)
1276	74	24	1 (pass)	-1 (time out)
1277	74	25	1 (pass)	-1 (time out)
1278	74	26	1 (pass)	-1 (time out)
1279	74	27	1 (pass)	-1 (time out)
1280	74	28	1 (pass)	-1 (time out)
1281	74	29	1 (pass)	-1 (time out)
1282	74	30	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1283	74	31	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1284	74	32	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1285	74	33	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1286	74	34	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1287	74	35	1 (pass)	-1 (time out)
1288	74	36	1 (pass)	-1 (time out)
1289	74	37	1 (pass)	-1 (time out)
1290	74	38	1 (pass)	-1 (time out)
1291	74	39	1 (pass)	-1 (time out)
1292	74	40	1 (pass)	-1 (time out)
1293	74	41	1 (pass)	-1 (time out)
1294	74	42	1 (pass)	-1 (time out)
1295	74	43	1 (pass)	-1 (time out)
1296	74	44	1 (pass)	-1 (time out)
1297	74	45	1 (pass)	-1 (time out)
1298	74	46	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1299	74	47	1 (pass)	0 (not solved)
1300	74	48	1 (pass)	0 (not solved)
1301	74	49	1 (pass)	0 (not solved)
1302	74	50	1 (pass)	-1 (time out)
1303	74	51	1 (pass)	-1 (time out)
1304	74	52	1 (pass)	-1 (time out)
1305	74	53	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1306	74	54	1 (pass)	0 (not solved)
1307	74	55	1 (pass)	0 (not solved)
1308	74	56	1 (pass)	0 (not solved)
1309	74	57	1 (pass)	-1 (time out)
1310	74	58	1 (pass)	-1 (time out)
1311	74	59	1 (pass)	-1 (time out)
1312	74	60	1 (pass)	-1 (time out)
1313	74	61	1 (pass)	-1 (time out)
1314	74	62	1 (pass)	0 (not solved)
1315	74	63	1 (pass)	0 (not solved)
1316	74	174	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1317	74	175	1 (pass)	-1 (time out)
1318	74	176	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1319	74	177	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1320	74	325	1 (pass)	-1 (time out)
1321	74	326	1 (pass)	-1 (time out)
1322	74	327	1 (pass)	-1 (time out)
1323	74	328	1 (pass)	-1 (time out)
1324	74	332	1 (pass)	-1 (time out)
1325	74	333	1 (pass)	-1 (time out)
1326	74	334	1 (pass)	-1 (time out)
1327	74	335	1 (pass)	-1 (time out)
1328	74	346	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x);O
1329	74	445	1 (pass)	-1 (time out)
1330	74	446	1 (pass)	-1 (time out)
1331	74	447	1 (pass)	-1 (time out)
1332	74	448	1 (pass)	-1 (time out)
1333	74	449	1 (pass)	-1 (time out)
1334	74	450	1 (pass)	-1 (time out)
1335	74	451	1 (pass)	-1 (time out)
1336	74	452	1 (pass)	-1 (time out)
1337	74	455	1 (pass)	-1 (time out)
1338	74	456	1 (pass)	-1 (time out)
1339	74	457	1 (pass)	-1 (time out)
1340	74	458	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1341	74	459	1 (pass)	-1 (time out)
1342	74	460	1 (pass)	-1 (time out)
1343	74	461	1 (pass)	-1 (time out)
1344	74	462	1 (pass)	-1 (time out)
1345	74	463	1 (pass)	-1 (time out)
1346	74	485	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1347	74	565	1 (pass)	-1 (time out)
1348	74	566	1 (pass)	-1 (time out)
1349	74	567	1 (pass)	-1 (time out)
1350	74	697	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1351	74	698	1 (pass)	-1 (time out)
1352	74	699	1 (pass)	-1 (time out)
1353	74	936	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1354	74	1040	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1355	74	1041	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1356	74	1235	1 (pass)	-1 (time out)
1357	74	1236	1 (pass)	-1 (time out)
1358	75	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1359	75	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1360	75	7	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1361	75	8	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1362	75	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1363	75	10	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1364	75	11	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1365	75	13	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1366	75	14	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1367	75	19	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1368	75	20	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1369	75	21	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1370	75	22	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1371	75	23	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1372	75	24	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1373	76	14	1 (pass)	-1 (time out)
1374	76	81	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1375	76	82	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1376	76	83	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1377	76	84	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1378	76	85	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1379	76	86	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1380	76	87	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1381	76	88	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1382	76	89	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1383	76	90	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1384	76	91	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1385	76	92	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1386	76	93	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1387	76	94	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1388	76	95	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1389	76	96	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1390	76	97	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1391	76	98	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1392	76	99	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1393	76	100	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1394	76	101	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1395	76	102	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1396	76	103	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1397	76	104	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1398	76	105	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1399	76	106	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1400	76	107	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1401	76	108	1 (pass)	-1 (time out)
1402	76	109	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1403	76	110	1 (pass)	-1 (time out)
1404	76	111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1405	76	112	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1406	76	113	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1407	76	114	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1408	76	115	1 (pass)	-1 (time out)
1409	76	116	1 (pass)	-1 (time out)
1410	76	117	1 (pass)	-1 (time out)
1411	76	118	1 (pass)	-1 (time out)
1412	76	119	1 (pass)	-1 (time out)
1413	76	120	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1414	76	121	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1415	76	122	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1416	76	123	1 (pass)	-1 (time out)
1417	76	124	1 (pass)	-1 (time out)
1418	76	125	1 (pass)	-1 (time out)
1419	76	126	1 (pass)	-1 (time out)
1420	76	127	1 (pass)	-1 (time out)
1421	76	128	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1422	76	129	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1423	76	130	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1424	76	131	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1425	76	132	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1426	76	133	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1427	76	134	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1428	76	135	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1429	76	136	1 (pass)	-1 (time out)
1430	76	137	1 (pass)	-1 (time out)
1431	76	138	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1432	76	139	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1433	76	140	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1434	76	141	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1435	76	142	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1436	76	143	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1437	76	144	1 (pass)	-1 (time out)
1438	76	145	1 (pass)	-1 (time out)
1439	76	146	1 (pass)	-1 (time out)
1440	76	147	1 (pass)	-1 (time out)
1441	76	148	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1442	76	149	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1443	76	150	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1444	76	151	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1445	76	152	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1446	76	153	1 (pass)	-1 (time out)
1447	76	155	1 (pass)	-1 (time out)
1448	76	156	1 (pass)	-1 (time out)
1449	76	157	1 (pass)	-1 (time out)
1450	76	158	1 (pass)	-1 (time out)
1451	76	159	1 (pass)	-1 (time out)
1452	76	160	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1453	76	161	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1454	76	162	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1455	76	163	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1456	76	164	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1457	76	165	1 (pass)	-1 (time out)
1458	76	166	1 (pass)	-1 (time out)
1459	76	167	1 (pass)	-1 (time out)
1460	76	168	1 (pass)	-1 (time out)
1461	76	169	1 (pass)	-1 (time out)
1462	76	170	1 (pass)	-1 (time out)
1463	76	171	1 (pass)	-1 (time out)
1464	76	172	1 (pass)	-1 (time out)
1465	76	173	1 (pass)	-1 (time out)
1466	76	174	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1467	76	175	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1468	76	176	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/
1469	76	177	1 (pass)	0 (not solved)
1470	76	178	1 (pass)	0 (not solved)
1471	76	179	1 (pass)	0 (not solved)
1472	76	180	1 (pass)	-1 (time out)
1473	76	181	1 (pass)	-1 (time out)
1474	76	182	1 (pass)	-1 (time out)
1475	76	183	1 (pass)	-1 (time out)
1476	76	184	1 (pass)	0 (not solved)
1477	76	185	1 (pass)	0 (not solved)
1478	76	186	1 (pass)	0 (not solved)
1479	76	187	1 (pass)	-1 (time out)
1480	76	188	1 (pass)	-1 (time out)
1481	76	189	1 (pass)	-1 (time out)
1482	76	190	1 (pass)	-1 (time out)
1483	76	191	1 (pass)	-1 (time out)
1484	76	192	1 (pass)	0 (not solved)
1485	76	193	1 (pass)	0 (not solved)
1486	76	194	1 (pass)	0 (not solved)
1487	76	222	1 (pass)	-1 (time out)
1488	76	223	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1489	76	286	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1490	76	287	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1491	76	288	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1492	76	289	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1493	76	290	1 (pass)	-1 (time out)
1494	76	291	1 (pass)	-1 (time out)
1495	76	292	1 (pass)	-1 (time out)
1496	76	293	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1497	76	294	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1498	76	295	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1499	76	296	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1500	76	297	1 (pass)	-1 (time out)
1501	76	298	1 (pass)	-1 (time out)
1502	76	299	1 (pass)	-1 (time out)
1503	76	300	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1504	76	301	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1505	76	302	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1506	76	303	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1507	76	304	1 (pass)	-1 (time out)
1508	76	305	1 (pass)	-1 (time out)
1509	76	306	1 (pass)	-1 (time out)
1510	76	307	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1511	76	308	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1512	76	309	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
1513	76	310	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1514	76	311	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1515	76	312	1 (pass)	-1 (time out)
1516	76	313	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1517	76	314	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1518	76	315	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1519	76	317	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1520	76	318	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1521	76	319	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1522	76	320	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1523	76	321	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1524	76	322	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1525	76	323	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1526	76	324	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1527	76	325	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1528	76	326	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1529	78	7	1 (pass)	0 (not solved)
1530	78	16	1 (pass)	0 (not solved)
1531	79	122	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1532	79	123	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1533	79	132	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1534	79	133	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1535	79	144	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1536	79	145	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1537	79	153	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1538	79	324	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1539	79	325	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1540	79	334	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1541	79	335	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1542	79	345	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1543	79	346	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1544	79	354	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1545	79	457	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1546	79	458	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1547	79	459	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1548	79	461	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1549	79	462	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1550	79	463	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1551	79	464	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1552	79	465	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1553	79	466	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1554	79	467	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1555	79	468	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1556	79	469	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1557	79	470	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1558	79	472	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1559	79	473	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1560	79	474	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1561	79	475	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1562	79	476	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1563	79	477	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1564	79	482	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1565	79	483	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1566	79	484	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1567	79	485	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1568	79	486	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1569	79	487	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1570	82	55	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
1571	82	141	1 (pass)	-1 (time out)
1572	82	151	1 (pass)	-1 (time out)
1573	82	162	1 (pass)	-1 (time out)
1574	82	271	1 (pass)	0 (not solved)
1575	82	272	1 (pass)	0 (not solved)
1576	82	275	1 (pass)	0 (not solved)
1577	82	276	1 (pass)	0 (not solved)
1578	83	22	1 (pass)	-1 (time out)
1579	85	43	1 (pass)	0 (not solved)
1580	89	109	1 (pass)	-1 (time out)
1581	89	110	1 (pass)	-1 (time out)
1582	89	111	1 (pass)	-1 (time out)
1583	89	117	1 (pass)	-1 (time out)
1584	89	118	1 (pass)	-1 (time out)
1585	89	119	1 (pass)	-1 (time out)
1586	89	120	1 (pass)	-1 (time out)
1587	89	223	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1588	91	1	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1589	92	79	1 (pass)	-1 (time out)
1590	92	80	1 (pass)	-1 (time out)
1591	92	81	1 (pass)	-1 (time out)
1592	92	86	1 (pass)	-1 (time out)
1593	92	87	1 (pass)	-1 (time out)
1594	92	88	1 (pass)	-1 (time out)
1595	92	89	1 (pass)	-1 (time out)
1596	92	90	1 (pass)	-1 (time out)
1597	92	94	1 (pass)	-1 (time out)
1598	92	95	1 (pass)	-1 (time out)
1599	92	96	1 (pass)	-1 (time out)
1600	92	97	1 (pass)	-1 (time out)
1601	92	98	1 (pass)	-1 (time out)
1602	92	99	1 (pass)	-1 (time out)
1603	92	114	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1604	92	122	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1605	93	91	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1606	94	79	1 (pass)	-1 (time out)
1607	94	80	1 (pass)	-1 (time out)
1608	94	81	1 (pass)	-1 (time out)
1609	94	82	1 (pass)	-1 (time out)
1610	94	86	1 (pass)	-1 (time out)
1611	94	87	1 (pass)	-1 (time out)
1612	94	88	1 (pass)	-1 (time out)
1613	94	89	1 (pass)	-1 (time out)
1614	94	90	1 (pass)	-1 (time out)
1615	94	91	1 (pass)	-1 (time out)
1616	94	95	1 (pass)	-1 (time out)
1617	94	96	1 (pass)	-1 (time out)
1618	94	97	1 (pass)	-1 (time out)
1619	94	98	1 (pass)	-1 (time out)
1620	94	99	1 (pass)	-1 (time out)
1621	94	100	1 (pass)	-1 (time out)
1622	94	101	1 (pass)	-1 (time out)
1623	94	117	1 (pass)	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1624	94	118	1 (pass)	-2 (exception)  Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
1625	94	124	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1626	94	377	1 (pass)	-1 (time out)
1627	94	378	1 (pass)	-1 (time out)
1628	94	379	1 (pass)	-1 (time out)
1629	94	380	1 (pass)	-1 (time out)
1630	94	381	1 (pass)	-1 (time out)
1631	94	385	1 (pass)	-1 (time out)
1632	94	386	1 (pass)	-1 (time out)
1633	94	387	1 (pass)	-1 (time out)
1634	94	388	1 (pass)	-1 (time out)
1635	94	389	1 (pass)	-1 (time out)
1636	94	390	1 (pass)	-1 (time out)
1637	94	394	1 (pass)	-1 (time out)
1638	94	395	1 (pass)	-1 (time out)
1639	94	396	1 (pass)	-1 (time out)
1640	94	397	1 (pass)	-1 (time out)
1641	94	398	1 (pass)	-1 (time out)
1642	94	399	1 (pass)	-1 (time out)
1643	94	400	1 (pass)	-1 (time out)
1644	94	416	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1645	94	417	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1646	94	424	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
1647	95	51	1 (pass)	0 (not solved)
1648	95	52	1 (pass)	0 (not solved)
1649	96	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1650	96	10	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1651	96	11	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1652	96	12	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1653	97	6	1 (pass)	-1 (time out)
1654	97	8	1 (pass)	-1 (time out)
1655	97	13	1 (pass)	-1 (time out)
1656	97	14	1 (pass)	-1 (time out)
1657	97	16	1 (pass)	-1 (time out)
1658	97	18	1 (pass)	-1 (time out)
1659	97	19	1 (pass)	-1 (time out)
1660	97	20	1 (pass)	-1 (time out)
1661	98	2	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1662	98	4	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1663	98	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1664	98	8	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1665	98	17	1 (pass)	0 (not solved)
1666	98	18	1 (pass)	0 (not solved)
1667	98	19	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1668	98	20	1 (pass)	0 (not solved)
1669	98	22	1 (pass)	0 (not solved)
1670	98	30	1 (pass)	0 (not solved)
1671	98	31	1 (pass)	0 (not solved)
1672	98	32	1 (pass)	0 (not solved)
1673	98	33	1 (pass)	-1 (time out)
1674	98	34	1 (pass)	-1 (time out)
1675	98	35	1 (pass)	-1 (time out)
1676	98	36	1 (pass)	-1 (time out)
1677	98	37	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1678	98	38	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1679	99	28	1 (pass)	-1 (time out)
1680	99	33	1 (pass)	-1 (time out)
1681	101	467	1 (pass)	0 (not solved)
1682	101	515	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1683	101	522	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1684	101	528	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1685	101	529	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1686	101	530	1 (pass)	-1 (time out)
1687	101	536	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1688	101	540	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1689	101	541	1 (pass)	-1 (time out)
1690	101	542	1 (pass)	-1 (time out)
1691	101	543	1 (pass)	-1 (time out)
1692	101	673	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
1693	101	674	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1694	101	675	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1695	101	676	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1696	102	11	1 (pass)	-1 (time out)
1697	102	22	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1698	102	23	1 (pass)	-1 (time out)
1699	102	32	1 (pass)	-1 (time out)
1700	102	34	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1701	102	43	1 (pass)	-1 (time out)
1702	103	214	1 (pass)	0 (not solved)
1703	103	215	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1704	103	216	1 (pass)	0 (not solved)
1705	103	217	1 (pass)	0 (not solved)
1706	103	222	1 (pass)	0 (not solved)
1707	103	223	1 (pass)	0 (not solved)
1708	103	224	1 (pass)	0 (not solved)
1709	103	230	1 (pass)	0 (not solved)
1710	103	231	1 (pass)	0 (not solved)
1711	103	232	1 (pass)	0 (not solved)
1712	103	233	1 (pass)	0 (not solved)
1713	103	234	1 (pass)	0 (not solved)
1714	103	235	1 (pass)	0 (not solved)
1715	103	236	1 (pass)	0 (not solved)
1716	103	237	1 (pass)	0 (not solved)
1717	103	238	1 (pass)	0 (not solved)
1718	103	239	1 (pass)	0 (not solved)
1719	103	240	1 (pass)	0 (not solved)
1720	103	241	1 (pass)	0 (not solved)
1721	103	242	1 (pass)	0 (not solved)
1722	103	243	1 (pass)	0 (not solved)
1723	103	244	1 (pass)	0 (not solved)
1724	103	245	1 (pass)	0 (not solved)
1725	103	246	1 (pass)	0 (not solved)
1726	103	247	1 (pass)	0 (not solved)
1727	103	387	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1728	103	388	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1729	103	398	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1730	103	399	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1731	103	409	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1732	103	410	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1733	103	655	1 (pass)	0 (not solved)
1734	103	656	1 (pass)	0 (not solved)
1735	103	657	1 (pass)	0 (not solved)
1736	103	658	1 (pass)	0 (not solved)
1737	103	659	1 (pass)	0 (not solved)
1738	103	660	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1739	103	661	1 (pass)	0 (not solved)
1740	103	662	1 (pass)	0 (not solved)
1741	103	665	1 (pass)	-1 (time out)
1742	103	667	1 (pass)	-1 (time out)
1743	103	668	1 (pass)	-1 (time out)
1744	103	669	1 (pass)	-1 (time out)
1745	103	670	1 (pass)	-1 (time out)
1746	103	671	1 (pass)	0 (not solved)
1747	103	672	1 (pass)	0 (not solved)
1748	103	673	1 (pass)	0 (not solved)
1749	103	674	1 (pass)	0 (not solved)
1750	103	958	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1751	103	1046	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1752	103	1054	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1753	104	112	1 (pass)	0 (not solved)
1754	104	113	1 (pass)	0 (not solved)
1755	104	114	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1756	104	115	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1757	104	116	1 (pass)	0 (not solved)
1758	104	117	1 (pass)	0 (not solved)
1759	104	118	1 (pass)	0 (not solved)
1760	104	119	1 (pass)	0 (not solved)
1761	104	120	1 (pass)	0 (not solved)
1762	104	121	1 (pass)	0 (not solved)
1763	104	122	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1764	104	123	1 (pass)	0 (not solved)
1765	104	124	1 (pass)	0 (not solved)
1766	104	125	1 (pass)	0 (not solved)
1767	104	126	1 (pass)	0 (not solved)
1768	104	127	1 (pass)	0 (not solved)
1769	104	128	1 (pass)	0 (not solved)
1770	104	129	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1771	104	130	1 (pass)	0 (not solved)
1772	104	131	1 (pass)	0 (not solved)
1773	104	132	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1774	104	133	1 (pass)	0 (not solved)
1775	104	134	1 (pass)	0 (not solved)
1776	104	135	1 (pass)	0 (not solved)
1777	104	136	1 (pass)	0 (not solved)
1778	104	137	1 (pass)	0 (not solved)
1779	104	138	1 (pass)	0 (not solved)
1780	104	139	1 (pass)	0 (not solved)
1781	104	140	1 (pass)	0 (not solved)
1782	104	141	1 (pass)	0 (not solved)
1783	104	142	1 (pass)	0 (not solved)
1784	104	143	1 (pass)	0 (not solved)
1785	104	144	1 (pass)	0 (not solved)
1786	104	145	1 (pass)	0 (not solved)
1787	104	146	1 (pass)	0 (not solved)
1788	104	147	1 (pass)	0 (not solved)
1789	104	148	1 (pass)	0 (not solved)
1790	104	149	1 (pass)	0 (not solved)
1791	104	150	1 (pass)	0 (not solved)
1792	104	151	1 (pass)	0 (not solved)
1793	104	152	1 (pass)	0 (not solved)
1794	104	153	1 (pass)	0 (not solved)
1795	104	373	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1796	104	374	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
1797	104	475	1 (pass)	0 (not solved)
1798	105	50	1 (pass)	-1 (time out)
1799	105	57	1 (pass)	-1 (time out)
1800	105	58	1 (pass)	-1 (time out)
1801	105	64	1 (pass)	-1 (time out)
1802	105	65	1 (pass)	-1 (time out)
1803	106	1	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1804	106	2	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1805	106	3	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1806	106	4	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1807	106	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1808	106	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1809	106	7	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1810	106	8	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1811	106	9	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1812	106	10	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1813	106	11	1 (pass)	0 (not solved)
1814	106	12	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1815	106	13	1 (pass)	-1 (time out)
1816	106	14	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1817	106	15	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1818	106	30	1 (pass)	-1 (time out)
1819	106	33	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1820	106	34	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1821	106	35	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1822	106	36	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1823	106	37	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1824	106	39	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1825	106	43	1 (pass)	-1 (time out)
1826	106	46	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1827	106	47	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1828	106	48	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1829	106	49	1 (pass)	-1 (time out)
1830	106	58	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1831	106	59	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1832	106	60	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1833	106	71	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1834	106	72	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1835	106	73	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1836	106	83	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1837	106	84	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1838	106	85	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1839	106	92	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1840	106	93	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1841	106	94	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1842	106	105	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1843	106	106	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1844	106	116	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1845	106	117	1 (pass)	-1 (time out)
1846	106	118	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1847	106	121	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1848	106	128	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1849	106	129	1 (pass)	0 (not solved)
1850	106	130	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1851	106	133	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1852	106	140	1 (pass)	-1 (time out)
1853	106	141	1 (pass)	0 (not solved)
1854	106	142	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1855	106	145	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1856	106	188	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1857	106	189	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1858	106	190	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1859	106	194	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1860	106	198	1 (pass)	-1 (time out)
1861	106	201	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1862	106	202	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1863	106	203	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1864	106	204	1 (pass)	-1 (time out)
1865	106	211	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1866	106	212	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1867	106	213	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1868	106	214	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1869	106	215	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1870	106	216	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1871	106	224	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1872	106	225	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1873	106	226	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1874	106	227	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1875	106	228	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1876	106	229	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1877	106	237	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1878	106	238	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1879	106	239	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1880	106	240	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1881	106	241	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1882	106	242	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1883	106	253	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1884	106	271	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1885	106	272	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1886	106	283	1 (pass)	0 (not solved)
1887	106	296	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1888	106	323	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1889	106	374	1 (pass)	-1 (time out)
1890	106	375	1 (pass)	-1 (time out)
1891	106	381	1 (pass)	-1 (time out)
1892	106	382	1 (pass)	-1 (time out)
1893	106	384	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1894	106	429	1 (pass)	-1 (time out)
1895	106	430	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1896	106	435	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1897	106	436	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1898	106	441	1 (pass)	-1 (time out)
1899	106	448	1 (pass)	-1 (time out)
1900	106	449	1 (pass)	-1 (time out)
1901	108	45	1 (pass)	0 (not solved)
1902	110	23	1 (pass)	0 (not solved)
1903	113	1	1 (pass)	0 (not solved)
1904	113	9	1 (pass)	0 (not solved)
1905	113	10	1 (pass)	0 (not solved)
1906	113	11	1 (pass)	0 (not solved)
1907	113	12	1 (pass)	0 (not solved)
1908	113	13	1 (pass)	0 (not solved)
1909	113	15	1 (pass)	0 (not solved)
1910	113	18	1 (pass)	0 (not solved)
1911	113	45	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1912	113	48	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1913	113	49	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1914	113	51	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1915	113	52	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1916	113	53	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1917	113	54	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1918	113	56	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1919	113	57	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1920	115	228	1 (pass)	0 (not solved)
1921	118	123	1 (pass)	0 (not solved)
1922	118	142	1 (pass)	0 (not solved)
1923	118	244	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1924	120	207	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1925	120	300	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1926	120	330	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
1927	121	1	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1928	121	2	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1929	121	3	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1930	121	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1931	121	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1932	121	11	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1933	121	12	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1934	121	14	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1935	121	15	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1936	121	16	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1937	121	17	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1938	121	21	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1939	121	22	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1940	121	23	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1941	121	31	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1942	121	32	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1943	121	42	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1944	121	43	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1945	121	44	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1946	121	45	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1947	121	46	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1948	121	47	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1949	121	48	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1950	121	49	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1951	121	50	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1952	121	51	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1953	121	52	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1954	121	53	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1955	121	54	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1956	121	55	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1957	121	56	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1958	121	57	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1959	121	58	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1960	121	59	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1961	121	60	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1962	121	61	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1963	121	62	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1964	121	63	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1965	121	64	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1966	121	90	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1967	121	91	1 (pass)	-1 (time out)
1968	121	92	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1969	121	93	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1970	121	98	1 (pass)	-1 (time out)
1971	121	99	1 (pass)	-1 (time out)
1972	121	100	1 (pass)	-1 (time out)
1973	121	106	1 (pass)	-1 (time out)
1974	121	107	1 (pass)	-1 (time out)
1975	121	108	1 (pass)	-1 (time out)
1976	121	109	1 (pass)	-1 (time out)
1977	121	113	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1978	121	114	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1979	121	115	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
1980	121	116	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1981	121	119	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1982	121	120	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1983	121	121	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1984	121	122	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1985	121	123	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1986	121	124	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1987	121	125	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1988	121	126	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1989	121	127	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1990	121	128	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1991	121	129	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
1992	121	130	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1993	121	143	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
1994	121	147	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1995	121	148	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1996	121	149	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1997	121	150	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
1998	121	151	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
1999	121	152	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2000	121	153	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2001	121	154	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2002	121	155	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2003	121	156	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2004	121	157	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2005	121	158	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2006	121	159	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2007	121	160	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2008	121	161	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2009	121	162	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2010	121	163	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2011	121	164	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2012	121	165	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2013	122	1	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2014	122	2	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2015	122	3	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2016	122	4	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2017	122	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2018	122	10	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2019	122	11	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2020	122	12	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2021	122	13	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2022	122	14	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2023	122	15	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2024	122	16	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2025	122	21	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2026	122	22	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2027	122	23	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2028	122	24	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2029	122	25	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2030	122	26	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2031	122	27	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2032	122	28	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2033	122	29	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2034	122	34	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2035	122	35	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2036	122	36	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2037	122	37	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2038	122	42	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2039	122	43	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2040	122	44	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2041	122	45	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2042	122	52	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2043	122	53	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2044	122	54	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2045	122	55	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2046	122	68	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2047	122	69	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2048	122	70	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2049	122	75	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2050	122	76	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2051	122	77	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2052	122	78	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2053	122	83	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2054	122	84	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2055	122	85	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2056	122	89	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2057	122	90	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2058	122	91	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2059	122	92	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2060	122	96	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2061	122	97	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2062	122	98	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2063	122	99	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2064	122	103	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2065	122	104	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2066	122	105	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2067	122	106	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2068	122	107	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2069	122	108	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2070	122	109	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2071	122	111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2072	122	112	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2073	122	113	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2074	122	114	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2075	122	115	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2076	122	116	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2077	122	119	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2078	122	120	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2079	122	121	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2080	122	122	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2081	122	123	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2082	122	124	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2083	122	125	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2084	122	126	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2085	122	130	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2086	122	131	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2087	122	132	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2088	122	133	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2089	122	134	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2090	122	135	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2091	122	136	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2092	122	137	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2093	122	138	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2094	122	139	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2095	122	140	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2096	122	141	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2097	122	142	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2098	122	143	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2099	122	144	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2100	122	145	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2101	122	146	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2102	122	147	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2103	122	148	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2104	122	149	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2105	122	150	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2106	122	168	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2107	122	169	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2108	122	171	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2109	122	172	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2110	122	179	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2111	122	184	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2112	122	185	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2113	122	186	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2114	122	187	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2115	122	188	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2116	122	189	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2117	122	193	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2118	122	194	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2119	122	195	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2120	122	196	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2121	122	197	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2122	122	198	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2123	122	202	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2124	122	203	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2125	122	204	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2126	122	205	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2127	122	206	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2128	122	207	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2129	122	210	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2130	122	211	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2131	122	212	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2132	122	213	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2133	122	217	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2134	122	218	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2135	122	219	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2136	122	220	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2137	122	225	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2138	122	226	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2139	122	227	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2140	122	228	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2141	122	238	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2142	122	244	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2143	122	245	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2144	122	246	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2145	122	247	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2146	122	248	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2147	122	252	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2148	122	253	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2149	122	254	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2150	122	255	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2151	122	258	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2152	122	259	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2153	122	260	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2154	122	261	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2155	126	5	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2156	126	6	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2157	126	7	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2158	126	18	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2159	126	19	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2160	126	20	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2161	126	31	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2162	126	32	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2163	126	33	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2164	126	44	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2165	126	45	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2166	126	46	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2167	126	57	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2168	126	58	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2169	126	59	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2170	126	67	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2171	126	68	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2172	126	69	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2173	126	70	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2174	126	71	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2175	126	72	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2176	126	80	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2177	126	81	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2178	126	82	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2179	126	93	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2180	126	94	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2181	126	95	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2182	126	96	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2183	126	98	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2184	126	106	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2185	126	107	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2186	126	108	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2187	126	110	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2188	126	111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2189	126	119	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2190	126	120	1 (pass)	0 (not solved)
2191	126	121	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2192	126	123	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2193	126	124	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2194	126	152	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2195	126	153	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2196	126	154	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2197	126	155	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2198	126	165	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2199	126	166	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2200	126	167	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2201	126	168	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2202	126	169	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2203	126	180	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2204	126	181	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2205	126	193	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2206	126	278	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2207	126	290	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2208	126	291	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2209	126	311	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2210	126	312	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2211	126	313	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2212	126	314	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2213	126	315	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2214	126	316	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2215	126	324	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2216	126	325	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2217	126	326	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2218	126	327	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2219	126	328	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2220	126	329	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2221	126	337	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2222	126	338	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2223	126	339	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2224	126	340	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2225	126	341	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2226	126	342	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2227	126	350	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2228	126	351	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2229	126	352	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2230	126	353	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2231	126	354	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2232	126	355	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2233	126	363	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2234	126	364	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2235	126	365	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2236	126	366	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2237	126	367	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2238	126	368	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2239	126	376	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2240	126	377	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2241	126	378	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2242	126	379	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2243	126	380	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2244	126	381	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2245	126	389	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2246	126	390	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2247	126	391	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2248	126	392	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2249	126	402	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2250	126	403	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2251	126	404	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2252	126	405	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x);O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2253	126	406	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
2254	126	407	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
2255	126	415	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2256	126	416	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2257	126	417	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2258	126	428	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2259	126	429	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2260	126	430	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2261	126	452	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2262	126	453	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2263	126	454	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2264	126	455	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2265	126	456	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2266	126	459	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2267	129	16	1 (pass)	0 (not solved)
2268	129	19	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2269	129	20	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2270	134	13	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O
2271	134	14	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0 , x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2272	134	15	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x);O
2273	135	72	1 (pass)	-1 (time out)
2274	135	77	1 (pass)	-1 (time out)
2275	135	78	1 (pass)	-1 (time out)
2276	135	79	1 (pass)	-1 (time out)
2277	135	93	1 (pass)	-1 (time out)
2278	135	165	1 (pass)	0 (not solved)
2279	135	166	1 (pass)	0 (not solved)
2280	135	167	1 (pass)	0 (not solved)
2281	135	181	1 (pass)	0 (not solved)
2282	135	195	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2283	135	196	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2284	135	198	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2285	135	200	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2286	135	214	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2287	135	216	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2288	135	218	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2289	135	233	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2290	135	234	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2291	135	235	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2292	135	250	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2293	135	251	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2294	135	252	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2295	136	229	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2296	136	236	1 (pass)	-1 (time out)
2297	137	19	1 (pass)	-1 (time out)
2298	137	20	1 (pass)	-1 (time out)
2299	137	21	1 (pass)	-1 (time out)
2300	137	28	1 (pass)	-1 (time out)
2301	137	29	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2302	137	30	1 (pass)	-1 (time out)
2303	137	76	1 (pass)	-1 (time out)
2304	137	77	1 (pass)	-1 (time out)
2305	137	78	1 (pass)	-1 (time out)
2306	137	93	1 (pass)	-1 (time out)
2307	137	94	1 (pass)	-1 (time out)
2308	137	95	1 (pass)	-1 (time out)
2309	137	96	1 (pass)	-1 (time out)
2310	137	142	1 (pass)	-1 (time out)
2311	137	143	1 (pass)	-1 (time out)
2312	137	144	1 (pass)	-1 (time out)
2313	137	150	1 (pass)	-1 (time out)
2314	137	151	1 (pass)	-1 (time out)
2315	137	152	1 (pass)	-1 (time out)
2316	137	153	1 (pass)	-1 (time out)
2317	137	160	1 (pass)	-1 (time out)
2318	137	161	1 (pass)	-1 (time out)
2319	137	162	1 (pass)	-1 (time out)
2320	137	275	1 (pass)	-1 (time out)
2321	137	375	1 (pass)	-1 (time out)
2322	139	13	1 (pass)	-1 (time out)
2323	139	14	1 (pass)	-1 (time out)
2324	139	15	1 (pass)	-1 (time out)
2325	139	19	1 (pass)	-1 (time out)
2326	139	20	1 (pass)	-1 (time out)
2327	139	21	1 (pass)	-1 (time out)
2328	139	70	1 (pass)	-1 (time out)
2329	139	71	1 (pass)	-1 (time out)
2330	139	72	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2331	139	96	1 (pass)	-1 (time out)
2332	139	97	1 (pass)	-1 (time out)
2333	139	98	1 (pass)	-1 (time out)
2334	139	101	1 (pass)	-1 (time out)
2335	139	123	1 (pass)	-1 (time out)
2336	139	124	1 (pass)	-1 (time out)
2337	139	125	1 (pass)	-1 (time out)
2338	141	139	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2339	141	140	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2340	141	141	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/
2341	141	142	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (2*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2342	141	165	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2343	141	166	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2344	141	167	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2345	141	168	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2346	141	169	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2347	141	170	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2348	141	171	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2349	141	172	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2350	141	173	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2351	141	174	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2352	141	175	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2353	141	176	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/
2354	141	182	1 (pass)	0 (not solved)
2355	141	183	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2356	141	337	1 (pass)	0 (not solved)
2357	141	605	1 (pass)	-1 (time out)
2358	141	606	1 (pass)	-1 (time out)
2359	141	637	1 (pass)	0 (not solved)
2360	141	641	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2361	141	643	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2362	141	717	1 (pass)	0 (not solved)
2363	141	751	1 (pass)	0 (not solved)
2364	141	900	1 (pass)	-1 (time out)
2365	156	69	1 (pass)	-1 (time out)
2366	156	76	1 (pass)	-1 (time out)
2367	156	81	1 (pass)	-1 (time out)
2368	156	82	1 (pass)	-1 (time out)
2369	156	87	1 (pass)	-1 (time out)
2370	163	287	1 (pass)	-1 (time out)
2371	163	288	1 (pass)	-1 (time out)
2372	164	66	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2373	164	76	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2374	164	77	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2375	164	80	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2376	164	81	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2377	164	100	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2378	164	101	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2379	164	102	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2380	164	105	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2381	164	106	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2382	164	108	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2383	164	109	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2384	164	111	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2385	164	117	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2386	164	118	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2387	164	119	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2388	164	275	1 (pass)	-1 (time out)
2389	164	352	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2390	164	362	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2391	164	363	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2392	164	366	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2393	164	367	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2394	164	372	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2395	164	375	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2396	164	376	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2397	164	377	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2398	164	380	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2399	164	381	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2400	164	383	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2401	164	384	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2402	164	385	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2403	164	386	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2404	164	392	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2405	164	393	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2406	164	394	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2407	164	479	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage3:=type(sage2)::OU
2408	170	75	1 (pass)	-1 (time out)
2409	173	229	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2410	173	230	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2411	173	231	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2412	173	232	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2413	173	233	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2414	173	235	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2415	173	236	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2416	173	237	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2417	173	238	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2418	173	239	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2419	173	240	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2420	173	241	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2421	173	242	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2422	173	243	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2423	173	244	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2424	173	245	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2425	173	246	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2426	173	247	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2427	173	248	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2428	173	249	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2429	173	250	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2430	173	251	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2431	173	252	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2432	173	253	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2433	173	254	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2434	176	34	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2435	179	82	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2436	180	139	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2437	180	141	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2438	180	145	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2439	180	147	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2440	185	3	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2441	185	4	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
2442	185	5	1 (pass)	-1 (time out)
2443	185	6	1 (pass)	-1 (time out)
2444	185	7	1 (pass)	-1 (time out)
2445	185	86	1 (pass)	0 (not solved)
2446	185	117	1 (pass)	0 (not solved)
2447	185	827	1 (pass)	-1 (time out)
2448	185	833	1 (pass)	-1 (time out)
2449	185	1000	1 (pass)	0 (not solved)
2450	185	1012	1 (pass)	0 (not solved)
2451	185	1037	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, x)::O
2452	194	248	1 (pass)	0 (not solved)
2453	194	258	1 (pass)	0 (not solved)
2454	196	696	1 (pass)	-1 (time out)
2455	196	724	1 (pass)	-1 (time out)
2456	197	12	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(x*ln((1+(sqrt(exp(1)*x^2+
2457	197	274	1 (pass)	0 (not solved)
2458	198	1	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2459	198	2	1 (pass)	0 (not solved)
2460	198	3	1 (pass)	0 (not solved)
2461	198	4	1 (pass)	0 (not solved)
2462	198	5	1 (pass)	0 (not solved)
2463	198	6	1 (pass)	0 (not solved)
2464	198	8	1 (pass)	0 (not solved)
2465	198	9	1 (pass)	0 (not solved)
2466	198	10	1 (pass)	0 (not solved)
2467	198	11	1 (pass)	0 (not solved)
2468	198	12	1 (pass)	0 (not solved)
2469	198	14	1 (pass)	0 (not solved)
2470	198	16	1 (pass)	0 (not solved)
2471	198	20	1 (pass)	0 (not solved)
2472	198	22	1 (pass)	0 (not solved)
2473	198	35	1 (pass)	0 (not solved)
2474	198	36	1 (pass)	0 (not solved)
2475	198	37	1 (pass)	0 (not solved)
2476	198	38	1 (pass)	0 (not solved)
2477	198	44	1 (pass)	0 (not solved)
2478	198	45	1 (pass)	0 (not solved)
2479	198	46	1 (pass)	0 (not solved)
2480	198	47	1 (pass)	0 (not solved)
2481	198	50	1 (pass)	0 (not solved)
2482	198	51	1 (pass)	0 (not solved)
2483	198	52	1 (pass)	0 (not solved)
2484	198	53	1 (pass)	0 (not solved)
2485	198	55	1 (pass)	0 (not solved)
2486	198	57	1 (pass)	0 (not solved)
2487	198	58	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2488	198	59	1 (pass)	0 (not solved)
2489	198	60	1 (pass)	0 (not solved)
2490	198	62	1 (pass)	0 (not solved)
2491	198	63	1 (pass)	0 (not solved)
2492	198	64	1 (pass)	0 (not solved)
2493	198	65	1 (pass)	0 (not solved)
2494	198	67	1 (pass)	0 (not solved)
2495	198	68	1 (pass)	0 (not solved)
2496	198	83	1 (pass)	0 (not solved)
2497	198	84	1 (pass)	0 (not solved)
2498	198	85	1 (pass)	0 (not solved)
2499	198	87	1 (pass)	0 (not solved)
2500	198	88	1 (pass)	0 (not solved)
2501	198	89	1 (pass)	0 (not solved)
2502	198	90	1 (pass)	0 (not solved)
2503	198	91	1 (pass)	0 (not solved)
2504	198	92	1 (pass)	0 (not solved)
2505	198	94	1 (pass)	0 (not solved)
2506	198	96	1 (pass)	0 (not solved)
2507	198	97	1 (pass)	0 (not solved)
2508	198	99	1 (pass)	0 (not solved)
2509	198	102	1 (pass)	0 (not solved)
2510	198	103	1 (pass)	0 (not solved)
2511	198	104	1 (pass)	0 (not solved)
2512	198	105	1 (pass)	0 (not solved)
2513	198	107	1 (pass)	0 (not solved)
2514	198	108	1 (pass)	0 (not solved)
2515	198	128	1 (pass)	0 (not solved)
2516	198	129	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2517	198	130	1 (pass)	0 (not solved)
2518	198	131	1 (pass)	0 (not solved)
2519	198	132	1 (pass)	0 (not solved)
2520	198	133	1 (pass)	0 (not solved)
2521	198	134	1 (pass)	0 (not solved)
2522	198	135	1 (pass)	0 (not solved)
2523	198	137	1 (pass)	0 (not solved)
2524	198	138	1 (pass)	0 (not solved)
2525	198	139	1 (pass)	0 (not solved)
2526	198	140	1 (pass)	0 (not solved)
2527	198	141	1 (pass)	0 (not solved)
2528	198	142	1 (pass)	0 (not solved)
2529	198	143	1 (pass)	0 (not solved)
2530	198	144	1 (pass)	0 (not solved)
2531	198	145	1 (pass)	0 (not solved)
2532	198	147	1 (pass)	0 (not solved)
2533	198	148	1 (pass)	0 (not solved)
2534	198	149	1 (pass)	0 (not solved)
2535	198	150	1 (pass)	0 (not solved)
2536	198	151	1 (pass)	0 (not solved)
2537	198	152	1 (pass)	0 (not solved)
2538	198	153	1 (pass)	0 (not solved)
2539	198	154	1 (pass)	0 (not solved)
2540	198	155	1 (pass)	0 (not solved)
2541	198	156	1 (pass)	0 (not solved)
2542	198	157	1 (pass)	0 (not solved)
2543	198	159	1 (pass)	0 (not solved)
2544	198	160	1 (pass)	0 (not solved)
2545	198	161	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2546	198	162	1 (pass)	0 (not solved)
2547	198	163	1 (pass)	0 (not solved)
2548	198	164	1 (pass)	0 (not solved)
2549	198	165	1 (pass)	0 (not solved)
2550	198	167	1 (pass)	0 (not solved)
2551	198	168	1 (pass)	0 (not solved)
2552	198	169	1 (pass)	0 (not solved)
2553	198	170	1 (pass)	0 (not solved)
2554	198	171	1 (pass)	0 (not solved)
2555	198	172	1 (pass)	0 (not solved)
2556	198	173	1 (pass)	0 (not solved)
2557	198	174	1 (pass)	0 (not solved)
2558	198	176	1 (pass)	0 (not solved)
2559	198	177	1 (pass)	0 (not solved)
2560	198	178	1 (pass)	0 (not solved)
2561	198	179	1 (pass)	0 (not solved)
2562	198	180	1 (pass)	0 (not solved)
2563	198	181	1 (pass)	0 (not solved)
2564	198	182	1 (pass)	0 (not solved)
2565	198	183	1 (pass)	0 (not solved)
2566	198	189	1 (pass)	0 (not solved)
2567	198	193	1 (pass)	0 (not solved)
2568	198	268	1 (pass)	0 (not solved)
2569	198	272	1 (pass)	0 (not solved)
2570	198	273	1 (pass)	0 (not solved)
2571	198	274	1 (pass)	0 (not solved)
2572	198	292	1 (pass)	0 (not solved)
2573	198	293	1 (pass)	0 (not solved)
2574	198	294	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2575	198	295	1 (pass)	0 (not solved)
2576	198	296	1 (pass)	0 (not solved)
2577	198	297	1 (pass)	0 (not solved)
2578	198	299	1 (pass)	0 (not solved)
2579	198	300	1 (pass)	0 (not solved)
2580	203	49	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2581	203	50	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2582	203	51	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2583	203	52	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2584	203	54	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2585	203	55	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2586	203	56	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2587	203	57	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2588	203	58	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2589	206	63	1 (pass)	-1 (time out)
2590	206	66	1 (pass)	-1 (time out)
2591	209	521	1 (pass)	0 (not solved)
2592	209	1760	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2593	209	2024	1 (pass)	0 (not solved)
2594	209	2308	1 (pass)	-1 (time out)
2595	209	2358	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2596	209	2359	1 (pass)	0 (not solved)
2597	209	2384	1 (pass)	0 (not solved)
2598	209	2385	1 (pass)	0 (not solved)
2599	209	2406	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2600	209	2407	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2601	209	2408	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2602	209	2525	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2603	209	2526	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2604	209	2651	1 (pass)	0 (not solved)
2605	209	2652	1 (pass)	0 (not solved)
2606	209	2672	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2607	209	2673	1 (pass)	0 (not solved)
2608	209	2706	1 (pass)	-1 (time out)
2609	209	2707	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2610	209	2724	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2611	209	2725	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2612	209	2893	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2613	209	2897	1 (pass)	-1 (time out)
2614	209	2936	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2615	209	2937	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2616	209	2962	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2617	209	2963	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, x)::O
2618	209	3016	1 (pass)	-1 (time out)
2619	209	3033	1 (pass)	-1 (time out)
2620	209	3078	1 (pass)	-1 (time out)
2621	209	3138	1 (pass)	-1 (time out)
2622	210	16	1 (pass)	0 (not solved)
2623	210	76	1 (pass)	-1 (time out)
2624	210	88	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: (71*exp(25/4)+ln(2)+12)*1/2/sq
2625	210	159	1 (pass)	0 (not solved)
2626	210	182	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2627	210	206	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-15 \cdot \exp(2)/62 \cdot 2 \cdot 1/10/\sqrt{-\exp}$
2628	210	253	1 (pass)	-1 (time out)
2629	210	351	1 (pass)	0 (not solved)
2630	210	379	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2631	210	539	1 (pass)	0 (not solved)
2632	210	549	1 (pass)	0 (not solved)
2633	210	567	1 (pass)	0 (not solved)
2634	210	614	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: sageVARx* $\exp(6)/\exp(6)+15/\ln(1)$
2635	210	636	1 (pass)	-2 (exception) Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT: sage2OUTPUT: Unable
2636	210	802	1 (pass)	-1 (time out)
2637	210	809	1 (pass)	0 (not solved)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2638	210	817	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-25*((-4*\exp(6)+4*\exp(3)^2)/(e$
2639	210	850	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(2401*\exp(8)-3)/\exp(4)/2401*1/$
2640	210	886	1 (pass)	-1 (time out)
2641	210	962	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2*(4*sageVARx^2+4*sageVARx*\exp$
2642	210	1011	1 (pass)	-1 (time out)
2643	210	1030	1 (pass)	-1 (time out)
2644	210	1093	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2645	210	1123	1 (pass)	-1 (time out)
2646	210	1164	1 (pass)	-1 (time out)
2647	210	1174	1 (pass)	-1 (time out)
2648	210	1236	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $8*((-576*\sqrt{(576*\exp(4))^5+88$

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
2649	210	1241	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 5*sageVARx*exp(30)/exp(30)+((3
2650	210	1269	1 (pass)	-1 (time out)
2651	210	1280	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -5000*exp(5)/5000/sqrt(exp(10)
2652	210	1290	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 6/sqrt(-exp(3)^2+exp(6))*atan(
2653	210	1366	1 (pass)	-1 (time out)
2654	210	1380	1 (pass)	0 (not solved)
2655	210	1492	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 4*sageVARx*exp(6)/exp(6)+(-160
2656	210	1522	1 (pass)	-1 (time out)
2657	210	1598	1 (pass)	-1 (time out)
2658	210	1622	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -3*(sqrt(exp(5)^2-exp(10))*(4*

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
2659	210	1628	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2660	210	1666	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 5*(sageVARx^3+4*sageVARx^2+((-
2661	210	1680	1 (pass)	-1 (time out)
2662	210	1697	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2663	210	1702	1 (pass)	-1 (time out)
2664	210	1725	1 (pass)	0 (not solved)
2665	210	1830	1 (pass)	-1 (time out)
2666	210	1909	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: (-2*exp(8)+2*exp(4)^2)/(exp(8)
2667	210	1911	1 (pass)	0 (not solved)
2668	210	1920	1 (pass)	-1 (time out)
2669	210	1932	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 75*((exp(2)^2-exp(4))/225*ln(2

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2670	210	1952	1 (pass)	-1 (time out)
2671	210	2020	1 (pass)	0 (not solved)
2672	210	2076	1 (pass)	0 (not solved)
2673	210	2118	1 (pass)	-1 (time out)
2674	210	2176	1 (pass)	0 (not solved)
2675	210	2209	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-10/\sqrt{\exp(2)^2 - \exp(4)} \cdot \ln(s)$
2676	210	2215	1 (pass)	-1 (time out)
2677	210	2217	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-1/2/\sqrt{-\exp(25)^2 + \exp(50)}$ *
2678	210	2232	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $8*((-48*\exp(4)^2 + 48*\exp(8)))/(5)$
2679	210	2289	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-3*(3*\ln(\exp(2)+4)-13)*1/6/\sqrt{\quad}$
2680	210	2423	1 (pass)	-1 (time out)
2681	210	2440	1 (pass)	-1 (time out)
2682	210	2443	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
2683	210	2504	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-sageVARx -(\exp(20)*\exp(8)*\exp($
2684	210	2509	1 (pass)	0 (not solved)
2685	210	2578	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2/3*(2*sageVARx^2-10*sageVARx*$
2686	210	2602	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2/\sqrt{\exp(10)-\exp(5)^2}*(-i)$
2687	210	2727	1 (pass)	-1 (time out)
2688	210	2742	1 (pass)	-1 (time out)
2689	210	2762	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/2*(8*sageVARx/4+(72*\exp(2))*e$
2690	210	2781	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*((-100*\exp(6)*\ln(2))^2+20*ex$
2691	210	2785	1 (pass)	-1 (time out)

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2692	210	2800	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $4*(\exp(2)^{-3})*1/44/\sqrt{\exp(2)^{\wedge}}$
2693	210	2904	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*((-157464*\exp(1)^3+157464*e$
2694	210	2928	1 (pass)	0 (not solved)
2695	210	2992	1 (pass)	-1 (time out)
2696	210	3042	1 (pass)	0 (not solved)
2697	210	3164	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2698	210	3227	1 (pass)	-1 (time out)
2699	210	3251	1 (pass)	-1 (time out)
2700	210	3265	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\ln(5*\exp(3))*(-1/\text{sageVARx}+(-1$
2701	210	3356	1 (pass)	-1 (time out)
2702	210	3377	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\ln(3)*\ln(2)*8*2*1/32/\sqrt{-ex$

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2703	210	3389	1 (pass)	-1 (time out)
2704	210	3415	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $9*(\text{sageVARx}^3 + \text{sageVARx}^2 * \exp(2))$
2705	210	3426	1 (pass)	0 (not solved)
2706	210	3434	1 (pass)	0 (not solved)
2707	210	3467	1 (pass)	-1 (time out)
2708	210	3518	1 (pass)	0 (not solved)
2709	210	3593	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/8*((-16*\exp(8) + 16*\exp(4)^2)/$
2710	210	3658	1 (pass)	-1 (time out)
2711	210	3709	1 (pass)	0 (not solved)
2712	210	3720	1 (pass)	-1 (time out)
2713	210	3734	1 (pass)	-1 (time out)
2714	210	3744	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-24*1/2/\sqrt{-\exp(1)^2 + \exp(2)}$
2715	210	3827	1 (pass)	0 (not solved)
2716	210	3866	1 (pass)	-1 (time out)
2717	210	3906	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2718	210	3982	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*((-exp(4)*exp(5)*exp(exp(4)))$
2719	210	3986	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $4*((-39366000*exp(4))^5+3936600$
2720	210	4005	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2721	210	4044	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-3*((-exp(4)+exp(2)^2)/(ln(9*e$
2722	210	4070	1 (pass)	0 (not solved)
2723	210	4108	1 (pass)	-1 (time out)
2724	210	4175	1 (pass)	0 (not solved)
2725	210	4339	1 (pass)	-1 (time out)
2726	210	4383	1 (pass)	-1 (time out)
2727	210	4399	1 (pass)	0 (not solved)
2728	210	4455	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\ln(2)^{-1/2*(8*sageVARx*ln(2)/$

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2729	210	4483	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $10*((-50000*\exp(1)*\exp(4))^6+80$
2730	210	4504	1 (pass)	0 (not solved)
2731	210	4543	1 (pass)	-1 (time out)
2732	210	4555	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(64*\exp(2)-64*\exp(1)^2)/(\exp(2)$
2733	210	4701	1 (pass)	-1 (time out)
2734	210	4704	1 (pass)	0 (not solved)
2735	210	4775	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2736	210	4793	1 (pass)	0 (not solved)
2737	210	4795	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-9/4*((2*\exp(4)-2*\exp(2))^2)/10$
2738	210	4846	1 (pass)	0 (not solved)
2739	210	4884	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*(3*\exp(5)+2*\exp(5/2))*1/44/$

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#	test file #	integral #	Giac 1.9.0-11 via sage-math 9.6	Giac 1.7.0 via sagemath 9.3
2740	210	4972	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-880*1/220/\sqrt{-\exp(8)^2+\exp($
2741	210	4998	1 (pass)	-1 (time out)
2742	210	5042	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-84*\exp(2)/5*2*1/8/\sqrt{-\exp(1$
2743	210	5062	1 (pass)	-1 (time out)
2744	210	5130	1 (pass)	0 (not solved)
2745	210	5234	1 (pass)	-1 (time out)
2746	210	5246	1 (pass)	-1 (time out)
2747	210	5450	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $4*\ln(6)^{-1/5*((-\exp(5)+\exp(3))*$
2748	210	5504	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-5*(25*sageVARx*\exp(10)*1/25/e$
2749	210	5513	1 (pass)	-1 (time out)
2750	210	5534	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-1/2/\sqrt{(\exp(16)^2-\exp(32))*1$

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2751	210	5560	1 (pass)	0 (not solved)
2752	210	5568	1 (pass)	-1 (time out)
2753	210	5601	1 (pass)	0 (not solved)
2754	210	5621	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2/5*(17*sageVARx+20*1/2/\sqrt{-$
2755	210	5667	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-9*((900*\exp(6)*\exp(3)-24300*e$
2756	210	5707	1 (pass)	0 (not solved)
2757	210	5790	1 (pass)	0 (not solved)
2758	210	5800	1 (pass)	-1 (time out)
2759	210	5839	1 (pass)	0 (not solved)
2760	210	5861	1 (pass)	0 (not solved)
2761	210	5894	1 (pass)	-1 (time out)
2762	210	5980	1 (pass)	-1 (time out)
2763	210	6107	1 (pass)	-1 (time out)
2764	210	6111	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*(3*sageVARx^2-1/sageVARx+9*$
2765	210	6193	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-4/sageVARx+(2*\exp(3/2)*\ln(3)+$

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2766	210	6197	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2767	210	6209	1 (pass)	0 (not solved)
2768	210	6216	1 (pass)	-1 (time out)
2769	210	6258	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: sageVARx*exp(2)/exp(2)+exp(7)*
2770	210	6277	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -9*exp(16/exp(2))/18/sqrt(-exp
2771	210	6278	1 (pass)	0 (not solved)
2772	210	6321	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: sageVARx-1/sqrt(-exp(1)^2+exp(
2773	210	6397	1 (pass)	-1 (time out)
2774	210	6398	1 (pass)	-1 (time out)
2775	210	6461	1 (pass)	0 (not solved)
2776	210	6521	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: 2*(2*i*sqrt(-4*exp(1)^12-16*ex

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2777	210	6534	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/3*(\exp(2)*\exp(2)*1/2/\exp(4))$ *
2778	210	6594	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(\exp(3)+25)*\ln(3)*3*2*1/2/\sqrt{\quad}$
2779	210	6625	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $16*(16*\exp(3)+47)*2*1/2560/\sqrt{\quad}$
2780	210	6700	1 (pass)	0 (not solved)
2781	210	6722	1 (pass)	0 (not solved)
2782	210	6725	1 (pass)	0 (not solved)
2783	210	6847	1 (pass)	-1 (time out)
2784	210	6850	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-288*\exp(15)/48/\sqrt{-\exp(15)}$ <sup>^</sup>
2785	210	6858	1 (pass)	-1 (time out)
2786	210	6885	1 (pass)	-1 (time out)
2787	210	6911	1 (pass)	-1 (time out)
2788	210	6953	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2789	210	7061	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $128*(2*sageVARx/16+(-6*\exp(32))$
2790	210	7195	1 (pass)	-1 (time out)
2791	210	7256	1 (pass)	-1 (time out)
2792	210	7282	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2/3*(288*sageVARx/144+9*1/2/sa$
2793	210	7297	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $8*(7*sageVARx/196+(60*\exp(2))-2$
2794	210	7312	1 (pass)	-1 (time out)
2795	210	7353	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $75*\exp(4)/450/\sqrt{\exp(1)^2-\exp$
2796	210	7479	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-25*((\exp(4)-\exp(2)^2)/(16*\exp$

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2797	210	7551	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(\exp(5) + 4 \ln((7 \ln(2) - 1) \ln(2)))$
2798	210	7560	1 (pass)	0 (not solved)
2799	210	7561	1 (pass)	0 (not solved)
2800	210	7805	1 (pass)	-1 (time out)
2801	210	7835	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2802	210	7917	1 (pass)	0 (not solved)
2803	210	7950	1 (pass)	0 (not solved)
2804	210	7956	1 (pass)	0 (not solved)
2805	210	7960	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-1/2/\sqrt{-\exp(10)+\exp(5)^2} * 1$
2806	210	7969	1 (pass)	-1 (time out)
2807	210	7981	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV

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Table 1 – continued from previous page

#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2808	210	8012	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: sageVARx*exp(5)+(-exp(6))*exp(3)
2809	210	8026	1 (pass)	-1 (time out)
2810	210	8040	1 (pass)	-1 (time out)
2811	210	8091	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: -5*exp(3)/5/sqrt(-exp(3)^2+exp
2812	210	8173	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0, sageV
2813	210	8184	1 (pass)	0 (not solved)
2814	210	8211	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: (-8*exp(6)*ln(2))*exp(2)*exp(1)
2815	210	8257	1 (pass)	-1 (time out)
2816	210	8286	1 (pass)	-1 (time out)
2817	210	8346	1 (pass)	-1 (time out)
2818	210	8361	1 (pass)	-1 (time out)
2819	210	8377	1 (pass)	-1 (time out)
2820	210	8396	1 (pass)	0 (not solved)
2821	210	8409	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2822	210	8422	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2823	210	8454	1 (pass)	0 (not solved)
2824	210	8502	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-2*\ln(\ln(5)^{2/4})^{-1}*(-12*\exp($
2825	210	8692	1 (pass)	0 (not solved)
2826	210	8749	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $(10*\exp(2)-10*\exp(1)^2)/\exp(2)$
2827	210	8766	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: sageVARx*exp(4)+4*sageVARx-exp
2828	210	8826	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2829	210	8869	1 (pass)	0 (not solved)
2830	210	8892	1 (pass)	-1 (time out)
2831	210	9094	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2832	210	9153	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-4*(\text{sageVARx}*\exp(32)/\exp(32)+2)$
2833	210	9157	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-15/4*((100*\exp(2))*\exp(1)-500*$
2834	210	9163	1 (pass)	0 (not solved)
2835	210	9181	1 (pass)	0 (not solved)
2836	210	9189	1 (pass)	0 (not solved)
2837	210	9199	1 (pass)	0 (not solved)
2838	210	9327	1 (pass)	-1 (time out)
2839	210	9331	1 (pass)	-1 (time out)
2840	210	9335	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $1/4*(7*\text{sageVARx}+(-8*\exp(1)+2*e)$
2841	210	9374	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $\exp(8)*\ln(5)*8/16/\sqrt{-\exp(8)}$
2842	210	9385	1 (pass)	0 (not solved)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2843	210	9448	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-(30*\exp(6)*\ln(2)+15*\exp(6)*\ln$
2844	210	9489	1 (pass)	-1 (time out)
2845	210	9497	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $144*\exp(34)*2*1/6/\sqrt{-16*\exp$
2846	210	9580	1 (pass)	-1 (time out)
2847	210	9605	1 (pass)	0 (not solved)
2848	210	9626	1 (pass)	-2 (exception) Exception raised: TypeError >> An error occurred running a Giac command: INPUT: sage2:=int(sage0, sageV
2849	210	9704	1 (pass)	0 (not solved)
2850	210	9755	1 (pass)	-1 (time out)
2851	210	9776	1 (pass)	0 (not solved)
2852	210	9830	1 (pass)	0 (not solved)
2853	210	9875	1 (pass)	-1 (time out)
2854	210	9889	1 (pass)	0 (not solved)
2855	210	9890	1 (pass)	-1 (time out)
2856	210	9903	1 (pass)	0 (not solved)
2857	210	9911	1 (pass)	0 (not solved)
2858	210	9986	1 (pass)	0 (not solved)
2859	210	10039	1 (pass)	0 (not solved)
2860	210	10074	1 (pass)	-1 (time out)

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#	test file #	integral #	Giac 1.9.0-11 via sagemath 9.6	Giac 1.7.0 via sagemath 9.3
2861	210	10124	1 (pass)	-1 (time out)
2862	210	10142	1 (pass)	0 (not solved)
2863	210	10188	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-\ln(5)^{-1} \ln(2 \ln(5)) * 2 * (5 * \text{sage}$
2864	210	10236	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $-5 * (\text{sageVARx} - 8 * 1/4 / \sqrt{-\exp(1$
2865	210	10249	1 (pass)	0 (not solved)
2866	210	10315	1 (pass)	-2 (exception) Exception raised: NotImplementedError >> Unable to parse Giac output: $2 * (\text{sageVARx} + \exp(20) * \exp(40) / \text{ex}$

## 2 Test file number 1

Test folder name:

test\_cases/0\_Independent\_test\_suites/1\_Apostol\_Problems

### 2.1 Problem number 9

$$\int \cos(2x) \sqrt{4 - \sin(2x)} dx$$

Optimal antiderivative

$$-\frac{(4 - \sin(2x))^{\frac{3}{2}}}{3}$$

command

```
integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{3}(-\sin(2x) + 4)^{\frac{3}{2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 3 Test file number 10

Test folder name:

test\_cases/0\_Independent\_test\_suites/10\_Timofeev\_Problems

### 3.1 Problem number 3

$$\int \sec(2ax) dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(2ax))}{2a}$$

command

```
integrate(sec(2*a*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\left|\frac{1}{\sin(2ax)} + \sin(2ax) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(2ax)} + \sin(2ax) - 2\right|\right)}{8a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 3.2 Problem number 231

$$\int \frac{1}{(9 + 3x - 5x^2 + x^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(3-x)(1+x)}{8(x^3-5x^2+3x+9)^{3/2}} + \frac{5(3-x)^2(1+x)}{64(x^3-5x^2+3x+9)^{3/2}} - \frac{15(3-x)^3(1+x)}{256(x^3-5x^2+3x+9)^{3/2}} + \frac{15(3-x)^3(1+x)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{1+x}}{2}\right)}{512(x^3-5x^2+3x+9)^{3/2}}$$

command

`integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15 \log(\sqrt{x+1} + 2)}{1024 \operatorname{sgn}(x-3)} + \frac{15 \log(|\sqrt{x+1} - 2|)}{1024 \operatorname{sgn}(x-3)} + \frac{1}{32 \sqrt{x+1} \operatorname{sgn}(x-3)} + \frac{7(x+1)^{3/2} - 36 \sqrt{x+1}}{256(x-3)^2 \operatorname{sgn}(x-3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 3.3 Problem number 398

$$\int \frac{1}{\sqrt[3]{\tan(5x)}} dx$$

Optimal antiderivative

$$\frac{3 \ln\left(1 + \tan^{2/3}(5x)\right)}{20} - \frac{\ln(1 + \tan^2(5x))}{20} - \frac{\arctan\left(\frac{(1 - 2(\tan^{2/3}(5x)))\sqrt{3}}{3}\right)}{10} \sqrt{3}$$

command

`integrate(1/tan(5*x)^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{10} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2 \tan(5x)^{2/3} - 1\right)\right) - \frac{1}{20} \log\left(\tan(5x)^{4/3} - \tan(5x)^{2/3} + 1\right) + \frac{1}{10} \log\left(\tan(5x)^{2/3} + 1\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\tan(5x)^{1/3}} dx$$

### 3.4 Problem number 592

$$\int \frac{\cosh(x)(-\cosh(2x) + \tanh(x))}{\sqrt{\sinh(2x)} (\sinh^2(x) + \sinh(2x))} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right)}{6} + \frac{\arctan\left(\operatorname{sech}(x) \sqrt{\cosh(x) \sinh(x)}\right) \sqrt{2}}{3} - \frac{\operatorname{arctanh}\left(\operatorname{sech}(x) \sqrt{\cosh(x) \sinh(x)}\right) \sqrt{2}}{3} + \frac{\cosh(x)}{\sqrt{\sinh(2x)}}$$

command

`integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x, algorithm="gi`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \arctan\left(\sqrt{e^{4x}-1} - e^{2x}\right) + \frac{1}{6} \sqrt{2} \log\left(-\sqrt{e^{4x}-1} + e^{2x}\right) + \frac{\sqrt{2}}{\sqrt{e^{4x}-1} - e^{2x} + 1} + \frac{1}{6} \arctan\left(\frac{1}{4} \sqrt{2} \left(3 \sqrt{e^{4x}-1} - 3e^{2x} - 1\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 4 Test file number 11

Test folder name:

`test_cases/0_Independent_test_suites/11_Welz_Problems`

### 4.1 Problem number 11

$$\int \left( \frac{1}{\sqrt{2} (1+x)^2 \sqrt{-i+x^2}} + \frac{1}{\sqrt{2} (1+x)^2 \sqrt{i+x^2}} \right) dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{i+x}{\sqrt{1-i} \sqrt{x^2-i}}\right) \sqrt{2}}{2(1-i)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{i-x}{\sqrt{1+i} \sqrt{x^2+i}}\right) \sqrt{2}}{2(1+i)^{\frac{3}{2}}} + \frac{\left(-\frac{1}{4} - \frac{i}{4}\right) \sqrt{x^2-i} \sqrt{2}}{1+x} + \frac{\left(-\frac{1}{4} + \frac{i}{4}\right) \sqrt{x^2+i} \sqrt{2}}{1+x}$$

command

`integrate(1/2/(1+x)^2*2^(1/2)/(-1+x^2)^(1/2)+1/2/(1+x)^2*2^(1/2)/(1+x^2)^(1/2),x, algorithm="`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{-(i-1) \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( \frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) + (2i-2)x + 2i + 2}{\left( \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( \frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) - 2x \right)^2 - 4 \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( \frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) + 8x - 4} \right) + \sqrt{2} \left( \frac{(i+1) \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( -\frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) - (2i+2)x - 2i + 2}{\left( \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( -\frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) - 2x \right)^2 - 4 \sqrt{2x^2 + 2\sqrt{x^4 + 1}} \left( -\frac{i}{x^2 + \sqrt{x^4 + 1}} + 1 \right) + 8x - 4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 4.2 Problem number 19

$$\int \frac{(x + \sqrt{a + x^2})^b}{\sqrt{a + x^2}} dx$$

Optimal antiderivative

$$\frac{(x + \sqrt{x^2 + a})^b}{b}$$

command

`integrate((x+(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(x + \sqrt{x^2 + a})^b}{b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^b}{\sqrt{x^2 + a}} dx$$

### 4.3 Problem number 20

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^b}{\sqrt{a + x^2}} dx$$

Optimal antiderivative

$$-\frac{\left(x - \sqrt{x^2 + a}\right)^b}{b}$$

command

```
integrate((x-(x^2+a)^(1/2))^b/(x^2+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(x - \sqrt{x^2 + a}\right)^b}{b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^b}{\sqrt{x^2 + a}} dx$$

### 4.4 Problem number 30

$$\int \frac{\left(x + \sqrt{b + x^2}\right)^a}{\sqrt{b + x^2}} dx$$

Optimal antiderivative

$$\frac{\left(x + \sqrt{x^2 + b}\right)^a}{a}$$

command

```
integrate((x+(x^2+b)^(1/2))^a/(x^2+b)^(1/2),x, algorithm="giac")
```



Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(x + \sqrt{x^2 + b})^a}{a}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(x + \sqrt{x^2 + b})^a}{\sqrt{x^2 + b}} dx$$

## 5 Test file number 13

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/13\_1.1.1.2-a+b\_x-  
^m-c+d\_x-^n

### 5.1 Problem number 785

$$\int \frac{a + bx}{x^2 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{(bx + a)^2}{2ax\sqrt{cx^2}}$$

command

`integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2bx + a}{2\sqrt{c}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.2 Problem number 786

$$\int \frac{a + bx}{x^3 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{a}{3x^2 \sqrt{cx^2}} - \frac{b}{2x \sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3bx + 2a}{6\sqrt{c}x^3 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx + a}{\sqrt{cx^2} x^3} dx$$

## 5.3 Problem number 787

$$\int \frac{a + bx}{x^4 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{a}{4x^3 \sqrt{cx^2}} - \frac{b}{3x^2 \sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx + 3a}{12\sqrt{c}x^4 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.4 Problem number 791

$$\int \frac{a + bx}{(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(bx + a)^2}{2acx\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2bx + a}{2c^{\frac{3}{2}}x^2\text{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.5 Problem number 792

$$\int \frac{a + bx}{x(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a}{3cx^2\sqrt{cx^2}} - \frac{b}{2cx\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3bx + 2a}{6c^{\frac{3}{2}}x^3\text{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx + a}{(cx^2)^{\frac{3}{2}}x} dx$$

### 5.6 Problem number 793

$$\int \frac{a + bx}{x^2 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a}{4cx^3\sqrt{cx^2}} - \frac{b}{3cx^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx + 3a}{12c^{\frac{3}{2}}x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.7 Problem number 794

$$\int \frac{a + bx}{x^3 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a}{5cx^4\sqrt{cx^2}} - \frac{b}{4cx^3\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{5bx + 4a}{20c^{\frac{3}{2}}x^5\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx + a}{(cx^2)^{\frac{3}{2}} x^3} dx$$

### 5.8 Problem number 795

$$\int \frac{a + bx}{x^4 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a}{6cx^5\sqrt{cx^2}} - \frac{b}{5cx^4\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6bx + 5a}{30c^{\frac{3}{2}}x^6\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.9 Problem number 797

$$\int \frac{x^2(a + bx)}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(bx + a)^2}{2ac^2x\sqrt{cx^2}}$$

command

```
integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2bx + a}{2c^{\frac{5}{2}}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.10 Problem number 798

$$\int \frac{x(a+bx)}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{3c^2x^2\sqrt{cx^2}} - \frac{b}{2c^2x\sqrt{cx^2}}$$

command

```
integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3bx+2a}{6c^{\frac{5}{2}}x^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx+a)x}{(cx^2)^{\frac{5}{2}}} dx$$

### 5.11 Problem number 799

$$\int \frac{a+bx}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{4c^2x^3\sqrt{cx^2}} - \frac{b}{3c^2x^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx+3a}{12c^{\frac{5}{2}}x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.12 Problem number 800

$$\int \frac{a + bx}{x (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{5c^2x^4\sqrt{cx^2}} - \frac{b}{4c^2x^3\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{5bx + 4a}{20c^{\frac{5}{2}}x^5\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx + a}{(cx^2)^{\frac{5}{2}}x} dx$$

### 5.13 Problem number 801

$$\int \frac{a + bx}{x^2 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{6c^2x^5\sqrt{cx^2}} - \frac{b}{5c^2x^4\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6bx + 5a}{30c^{\frac{5}{2}}x^6\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.14 Problem number 802

$$\int \frac{a + bx}{x^3 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{7c^2x^6\sqrt{cx^2}} - \frac{b}{6c^2x^5\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{7bx + 6a}{42c^{\frac{5}{2}}x^7\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx + a}{(cx^2)^{\frac{5}{2}}x^3} dx$$

### 5.15 Problem number 803

$$\int \frac{a + bx}{x^4 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a}{8c^2x^7\sqrt{cx^2}} - \frac{b}{7c^2x^6\sqrt{cx^2}}$$

command

```
integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8bx + 7a}{56c^{\frac{5}{2}}x^8\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



### 5.16 Problem number 833

$$\int \frac{(a + bx)^2}{x^2 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{2ab}{\sqrt{cx^2}} - \frac{a^2}{2x\sqrt{cx^2}} + \frac{b^2 x \ln(x)}{\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \log(|x|)}{\sqrt{c} \operatorname{sgn}(x)} - \frac{4ab\sqrt{c}x + a^2\sqrt{c}}{2cx^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.17 Problem number 834

$$\int \frac{(a + bx)^2}{x^3 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{(bx + a)^3}{3ax^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3b^2\sqrt{c}x^2 + 3ab\sqrt{c}x + a^2\sqrt{c}}{3cx^3 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)^2}{\sqrt{cx^2} x^3} dx$$

### 5.18 Problem number 835

$$\int \frac{(a + bx)^2}{x^4 \sqrt{cx^2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{4x^3 \sqrt{cx^2}} - \frac{2ab}{3x^2 \sqrt{cx^2}} - \frac{b^2}{2x \sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6b^2\sqrt{c}x^2 + 8ab\sqrt{c}x + 3a^2\sqrt{c}}{12cx^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.19 Problem number 839

$$\int \frac{(a + bx)^2}{(cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2ab}{c\sqrt{cx^2}} - \frac{a^2}{2cx\sqrt{cx^2}} + \frac{b^2x \ln(x)}{c\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2b^2 \log(|x|)}{\sqrt{c} \operatorname{sgn}(x)} - \frac{4ab\sqrt{c}x + a^2\sqrt{c}}{cx^2 \operatorname{sgn}(x)}}{2c}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.20 Problem number 840

$$\int \frac{(a + bx)^2}{x (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(bx + a)^3}{3acx^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3b^2\sqrt{c}x^2 + 3ab\sqrt{c}x + a^2\sqrt{c}}{3c^2x^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)^2}{(cx^2)^{\frac{3}{2}}x} dx$$

## 5.21 Problem number 841

$$\int \frac{(a + bx)^2}{x^2 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{4cx^3\sqrt{cx^2}} - \frac{2ab}{3cx^2\sqrt{cx^2}} - \frac{b^2}{2cx\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6b^2\sqrt{c}x^2 + 8ab\sqrt{c}x + 3a^2\sqrt{c}}{12c^2x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.22 Problem number 842

$$\int \frac{(a + bx)^2}{x^3 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{5cx^4\sqrt{cx^2}} - \frac{ab}{2cx^3\sqrt{cx^2}} - \frac{b^2}{3cx^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{10b^2\sqrt{c}x^2 + 15ab\sqrt{c}x + 6a^2\sqrt{c}}{30c^2x^5\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)^2}{(cx^2)^{\frac{3}{2}} x^3} dx$$

### 5.23 Problem number 843

$$\int \frac{(a + bx)^2}{x^4 (cx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{6cx^5\sqrt{cx^2}} - \frac{2ab}{5cx^4\sqrt{cx^2}} - \frac{b^2}{4cx^3\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15b^2\sqrt{c}x^2 + 24ab\sqrt{c}x + 10a^2\sqrt{c}}{60c^2x^6\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.24 Problem number 845

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2ab}{c^2\sqrt{cx^2}} - \frac{a^2}{2c^2x\sqrt{cx^2}} + \frac{b^2x \ln(x)}{c^2\sqrt{cx^2}}$$

command

```
integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \log(|x|)}{c^{\frac{5}{2}} \operatorname{sgn}(x)} - \frac{4ab\sqrt{c}x + a^2\sqrt{c}}{2c^3x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.25 Problem number 846

$$\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(bx+a)^3}{3ac^2x^2\sqrt{cx^2}}$$

command

```
integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3b^2\sqrt{c}x^2 + 3ab\sqrt{c}x + a^2\sqrt{c}}{3c^3x^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx+a)^2x}{(cx^2)^{\frac{5}{2}}} dx$$

### 5.26 Problem number 847

$$\int \frac{(a + bx)^2}{(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}} - \frac{2ab}{3c^2x^2\sqrt{cx^2}} - \frac{b^2}{2c^2x\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6b^2\sqrt{c}x^2 + 8ab\sqrt{c}x + 3a^2\sqrt{c}}{12c^3x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.27 Problem number 848

$$\int \frac{(a + bx)^2}{x(cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}} - \frac{ab}{2c^2x^3\sqrt{cx^2}} - \frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{10b^2\sqrt{c}x^2 + 15ab\sqrt{c}x + 6a^2\sqrt{c}}{30c^3x^5\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)^2}{(cx^2)^{\frac{5}{2}}x} dx$$

### 5.28 Problem number 849

$$\int \frac{(a + bx)^2}{x^2 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}} - \frac{2ab}{5c^2x^4\sqrt{cx^2}} - \frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15b^2\sqrt{c}x^2 + 24ab\sqrt{c}x + 10a^2\sqrt{c}}{60c^3x^6\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 5.29 Problem number 850

$$\int \frac{(a + bx)^2}{x^3 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}} - \frac{ab}{3c^2x^5\sqrt{cx^2}} - \frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{21b^2\sqrt{c}x^2 + 35ab\sqrt{c}x + 15a^2\sqrt{c}}{105c^3x^7\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)^2}{(cx^2)^{\frac{5}{2}} x^3} dx$$

## 5.30 Problem number 851

$$\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}} - \frac{2ab}{7c^2x^6\sqrt{cx^2}} - \frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

command

```
integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{28b^2\sqrt{c}x^2 + 48ab\sqrt{c}x + 21a^2\sqrt{c}}{168c^3x^8\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 5.31 Problem number 1145

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5a^2cx(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{3}{2}}}{24} + \frac{x(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{5}{2}}}{6} \\ & + \frac{5a^6c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{c}(-bx+a)}\right)}{8b} + \frac{5a^4c^2x\sqrt{bx+a}\sqrt{-bcx+ac}}{16} \end{aligned}$$

command

```
integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$240 \left( \frac{2ac \log\left(\left| \frac{-\sqrt{bx+a}\sqrt{-c} + \sqrt{-(bx+a)c+2ac}}{\sqrt{-c}} \right| \right) - \sqrt{-(bx+a)c+2ac}\sqrt{bx+a}}{\sqrt{-c}} \right) a^5 c^2 - 120 \left( \frac{2a^2c \log\left(\left| \frac{-\sqrt{bx+a}\sqrt{-c} + \sqrt{-(bx+a)c+2ac}}{\sqrt{-c}} \right| \right) - \sqrt{-(bx+a)c+2ac}\sqrt{bx+a}}{\sqrt{-c}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 5.32 Problem number 1146

$$\int (a + bx)^{3/2} (ac - bcx)^{3/2} dx$$

Optimal antiderivative

$$\frac{x(bx + a)^{\frac{3}{2}} (-bcx + ac)^{\frac{3}{2}}}{4} + \frac{3a^4 c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{c} \sqrt{bx + a}}{\sqrt{c} (-bx + a)}\right)}{4b} + \frac{3a^2 cx \sqrt{bx + a} \sqrt{-bcx + ac}}{8}$$

command

`integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$24 \left( \frac{2ac \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c + 2ac}}{\sqrt{-c}} \right| \right) - \sqrt{-(bx+a)c + 2ac} \sqrt{bx+a}}{\sqrt{-c}} \right) a^3 c - 12 \left( \frac{2a^2 c \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c + 2ac}}{\sqrt{-c}} \right| \right) - \sqrt{-(bx+a)c + 2ac} \sqrt{bx+a}}{\sqrt{-c}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 5.33 Problem number 1147

$$\int \sqrt{a + bx} \sqrt{ac - bcx} dx$$

Optimal antiderivative

$$\frac{a^2 \arctan\left(\frac{\sqrt{c} \sqrt{bx + a}}{\sqrt{c} (-bx + a)}\right) \sqrt{c}}{b} + \frac{x \sqrt{bx + a} \sqrt{-bcx + ac}}{2}$$

command

`integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^2 c \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c + 2ac}}{\sqrt{-c}} \right| \right) + \sqrt{-(bx+a)c + 2ac} \sqrt{bx+a} (bx - 2a) - 2 \left( \frac{2ac \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c + 2ac}}{\sqrt{-c}} \right| \right) - \sqrt{-(bx+a)c + 2ac} \sqrt{bx+a}}{\sqrt{-c}} \right)}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 5.34 Problem number 1148

$$\int \frac{1}{\sqrt{a+bx} \sqrt{ac-bcx}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left( \frac{\sqrt{c} \sqrt{bx+a}}{\sqrt{c}(-bx+a)} \right)}{b\sqrt{c}}$$

command

```
integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log \left( \left| -\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c+2ac} \right| \right)}{b\sqrt{-c}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 5.35 Problem number 1169

$$\int \frac{1}{\sqrt{a+bx} \sqrt{-ad+bdx}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{bx+a}}{\sqrt{bdx-ad}} \right)}{b\sqrt{d}}$$

command

```
integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log \left( \left| -\sqrt{bx+a} \sqrt{d} + \sqrt{(bx+a)d-2ad} \right| \right)}{b\sqrt{d}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6 Test file number 14

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.1\_Linear/14\_1.1.1.3-a+b\_x-  
^m-c+d\_x-^n-e+f\_x-^p

### 6.1 Problem number 556

$$\int \frac{\sqrt{a+bx} \sqrt{c+dx}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad+bc)^2(ad+bc)(7a^2d^2+2abcd+7b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{9}{2}}c^{\frac{9}{2}}} \\ & - \frac{\sqrt{bx+a}\sqrt{dx+c}}{5x^5} - \frac{(ad+bc)\sqrt{bx+a}\sqrt{dx+c}}{40acx^4} \\ & + \frac{(7a^2d^2-2abcd+7b^2c^2)\sqrt{bx+a}\sqrt{dx+c}}{240a^2c^2x^3} \\ & - \frac{(ad+bc)(35a^2d^2-46abcd+35b^2c^2)\sqrt{bx+a}\sqrt{dx+c}}{960a^3c^3x^2} \\ & + \frac{(105a^4d^4-40a^3bcd^3-34a^2b^2c^2d^2-40ab^3c^3d+105b^4c^4)\sqrt{bx+a}\sqrt{dx+c}}{1920a^4c^4x} \end{aligned}$$

command

```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 6.2 Problem number 565

$$\int \frac{\sqrt{a+bx} (c+dx)^{3/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad + bc)^3 (3a^2d^2 + 6abcd + 7b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{bx+a}}{\sqrt{a} \sqrt{dx+c}}\right)}{128a^{\frac{9}{2}}c^{\frac{7}{2}}} - \frac{(dx+c)^{\frac{3}{2}} \sqrt{bx+a}}{5x^5} \\ & - \frac{(3ad + bc) \sqrt{bx+a} \sqrt{dx+c}}{40ax^4} + \frac{\left(\frac{7b^2c}{a} - 12bd - \frac{3ad^2}{c}\right) \sqrt{bx+a} \sqrt{dx+c}}{240x^3a} \\ & - \frac{(-15a^3d^3 + 9a^2bcd^2 - 61ab^2c^2d + 35b^3c^3) \sqrt{bx+a} \sqrt{dx+c}}{960a^3c^2x^2} \\ & + \frac{(-45a^4d^4 + 30a^3bcd^3 + 36a^2b^2c^2d^2 - 190ab^3c^3d + 105b^4c^4) \sqrt{bx+a} \sqrt{dx+c}}{1920a^4c^3x} \end{aligned}$$

command

```
integrate((d*x+c)^(3/2)*(b*x+a)^(1/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 6.3 Problem number 574

$$\int \frac{\sqrt{a+bx} (c+dx)^{5/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{7}{2}}}{5acx^5} - \frac{(-ad+bc)^4 (3ad+7bc) \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{bx+a}}{\sqrt{a} \sqrt{dx+c}}\right)}{128a^{\frac{9}{2}}c^{\frac{5}{2}}} \\ & - \frac{(-ad+bc)^2 (3ad+7bc) (dx+c)^{\frac{3}{2}} \sqrt{bx+a}}{192a^3c^2x^2} + \frac{(-ad+bc) (3ad+7bc) (dx+c)^{\frac{5}{2}} \sqrt{bx+a}}{240a^2c^2x^3} \\ & + \frac{(3ad+7bc) (dx+c)^{\frac{7}{2}} \sqrt{bx+a}}{40ac^2x^4} + \frac{(-ad+bc)^3 (3ad+7bc) \sqrt{bx+a} \sqrt{dx+c}}{128a^4c^2x} \end{aligned}$$

command

```
integrate((d*x+c)^(5/2)*(b*x+a)^(1/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.4 Problem number 606

$$\int \frac{(a+bx)^{3/2} \sqrt{c+dx}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad+bc)^3 (7a^2d^2 + 6abcd + 3b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{7}{2}}c^{\frac{9}{2}}} - \frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{5x^5} \\ & - \frac{(ad+3bc)\sqrt{bx+a}\sqrt{dx+c}}{40cx^4} - \frac{\left(\frac{3b^2c}{a} + 12bd - \frac{7ad^2}{c}\right)\sqrt{bx+a}\sqrt{dx+c}}{240cx^3} \\ & + \frac{(-35a^3d^3 + 61a^2bcd^2 - 9ab^2c^2d + 15b^3c^3)\sqrt{bx+a}\sqrt{dx+c}}{960a^2c^3x^2} \\ & - \frac{(-105a^4d^4 + 190a^3bcd^3 - 36a^2b^2c^2d^2 - 30ab^3c^3d + 45b^4c^4)\sqrt{bx+a}\sqrt{dx+c}}{1920a^3c^4x} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.5 Problem number 615

$$\int \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{8a^2c^2x^4} - \frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{2}}}{5acx^5} \\ & + \frac{3(-ad+bc)^4(ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{7}{2}}c^{\frac{7}{2}}} \\ & + \frac{(-ad+bc)^2(ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{64a^2c^3x^2} + \frac{(-ad+bc)(ad+bc)(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{16a^3c^3x^3} \\ & - \frac{3(-ad+bc)^3(ad+bc)\sqrt{bx+a}\sqrt{dx+c}}{128a^3c^3x} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.6 Problem number 624

$$\int \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5cx^5} + \frac{3(-ad+bc)^5 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{7}{2}}c^{\frac{5}{2}}} \\ & + \frac{(-ad+bc)^3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{64a^2c^2x^2} - \frac{(-ad+bc)^2(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{80a^2c^2x^3} \\ & - \frac{3(-ad+bc)(dx+c)^{\frac{7}{2}}\sqrt{bx+a}}{40c^2x^4} - \frac{3(-ad+bc)^4\sqrt{bx+a}\sqrt{dx+c}}{128a^3c^2x} \end{aligned}$$

command

```
integrate((b*x+a)^(3/2)*(d*x+c)^(5/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.7 Problem number 655

$$\int \frac{(a+bx)^{5/2}\sqrt{c+dx}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ad+bc)(7ad+3bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{48a^3c^3x^3} + \frac{(7ad+3bc)(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{40a^2c^2x^4} \\ & - \frac{(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}}{5acx^5} - \frac{(-ad+bc)^4(7ad+3bc) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{5}{2}}c^{\frac{9}{2}}} \\ & + \frac{(-ad+bc)^2(7ad+3bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{64a^2c^4x^2} + \frac{(-ad+bc)^3(7ad+3bc)\sqrt{bx+a}\sqrt{dx+c}}{128a^2c^4x} \end{aligned}$$

command

```
integrate((b*x+a)^(5/2)*(d*x+c)^(1/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.8 Problem number 664

$$\int \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{x^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{8c^2x^4} - \frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{2}}}{5cx^5} \\ & - \frac{3(-ad+bc)^5 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{128a^{\frac{5}{2}}c^{\frac{7}{2}}} - \frac{(-ad+bc)^3(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{64ac^3x^2} \\ & - \frac{(-ad+bc)^2(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{16c^3x^3} + \frac{3(-ad+bc)^4\sqrt{bx+a}\sqrt{dx+c}}{128a^2c^3x} \end{aligned}$$

command

```
integrate((b*x+a)^(5/2)*(d*x+c)^(3/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.9 Problem number 722

$$\int \frac{(c+dx)^{5/2}}{x^6\sqrt{a+bx}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc)^3 (3a^2d^2 + 14abcd + 63b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right) - \frac{c(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{5ax^5} + \frac{c(-13ad+9bc)\sqrt{bx+a}\sqrt{dx+c}}{40a^2x^4} - \frac{(93a^2d^2 - 148abcd + 63b^2c^2)\sqrt{bx+a}\sqrt{dx+c}}{240a^3x^3} + \frac{(-15a^3d^3 + 481a^2bcd^2 - 749ab^2c^2d + 315b^3c^3)\sqrt{bx+a}\sqrt{dx+c}}{960a^4cx^2} - \frac{(-45a^4d^4 - 90a^3bcd^3 + 1564a^2b^2c^2d^2 - 2310ab^3c^3d + 945b^4c^4)\sqrt{bx+a}\sqrt{dx+c}}{1920a^5c^2x}}$$

command

```
integrate((d*x+c)^(5/2)/x^6/(b*x+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 6.10 Problem number 775

$$\int \frac{(c+dx)^{5/2}}{x^5(a+bx)^{3/2}} dx$$

Optimal antiderivative

$$\frac{5(-ad + bc)^2 (-a^2d^2 - 14abcd + 63b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right) - \frac{c(dx+c)^{\frac{3}{2}}}{4ax^4\sqrt{bx+a}} + \frac{b(-15a^3d^3 + 839a^2bcd^2 - 1785ab^2c^2d + 945b^3c^3)\sqrt{dx+c}}{192a^5c\sqrt{bx+a}} + \frac{c(-11ad+9bc)\sqrt{dx+c}}{24a^2x^3\sqrt{bx+a}} - \frac{(-59ad+63bc)(-ad+bc)\sqrt{dx+c}}{96a^3x^2\sqrt{bx+a}} + \frac{(-ad+bc)(15a^2d^2 - 322abcd + 315b^2c^2)\sqrt{dx+c}}{192a^4cx\sqrt{bx+a}}}$$

command

```
integrate((d*x+c)^(5/2)/x^5/(b*x+a)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 6.11 Problem number 801

$$\int \frac{(c+dx)^{5/2}}{x^4(a+bx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c(dx+c)^{\frac{3}{2}}}{3ax^3(bx+a)^{\frac{3}{2}}} + \frac{5(-ad+bc)(a^2d^2-14abcd+21b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{8a^{\frac{11}{2}}\sqrt{c}} \\ & -\frac{7b(-7ad+15bc)(-ad+bc)\sqrt{dx+c}}{24a^4(bx+a)^{\frac{3}{2}}} + \frac{3c(-ad+bc)\sqrt{dx+c}}{4a^2x^2(bx+a)^{\frac{3}{2}}} \\ & -\frac{(-11ad+21bc)(-ad+bc)\sqrt{dx+c}}{8a^3x(bx+a)^{\frac{3}{2}}} - \frac{b(113a^2d^2-420abcd+315b^2c^2)\sqrt{dx+c}}{24a^5\sqrt{bx+a}} \end{aligned}$$

command

`integrate((d*x+c)^(5/2)/x^4/(b*x+a)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 6.12 Problem number 813

$$\int \frac{1}{x^3(a+bx)^{5/2}(c+dx)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b(-21a^2d^2-6abcd+35b^2c^2)}{12a^3c^2(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}} - \frac{1}{2acx^2(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}} \\ & + \frac{\frac{7ad}{4} + \frac{7bc}{4}}{a^2c^2x(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}} - \frac{5(7a^2d^2+10abcd+7b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{bx+a}}{\sqrt{a}\sqrt{dx+c}}\right)}{4a^{\frac{9}{2}}c^{\frac{9}{2}}} \\ & + \frac{b(7a^3d^3-3a^2bcd^2-55ab^2c^2d+35b^3c^3)}{4a^4c^2(-ad+bc)^2(dx+c)^{\frac{3}{2}}\sqrt{bx+a}} \\ & + \frac{d(-35a^4d^4+48a^3bcd^3+18a^2b^2c^2d^2-200ab^3c^3d+105b^4c^4)\sqrt{bx+a}}{12a^4c^3(-ad+bc)^3(dx+c)^{\frac{3}{2}}} \\ & + \frac{d(ad+bc)(105a^4d^4-340a^3bcd^3+406a^2b^2c^2d^2-340ab^3c^3d+105b^4c^4)\sqrt{bx+a}}{12a^4c^4(-ad+bc)^4\sqrt{dx+c}} \end{aligned}$$

command

```
integrate(1/x^3/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7 Test file number 17

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.1_Linear/17_1.1.1.6_P-x-
a+b_x-^m-c+d_x-^n-e+f_x-^p
```

### 7.1 Problem number 20

$$\int \sqrt{a+bx} \sqrt{ac-bcx} (e+fx)^3 (A+Bx+Cx^2) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A(6a^2b^2e f^2 + 8b^4e^3) + a^2(a^2f^2(Bf + 3Ce) + 2b^2e^2(3Bf + Ce))) x \sqrt{bx+a} \sqrt{-bcx+ac}}{16b^4} \\ & - \frac{(8a^2C f^2 - b^2(3C e^2 - 7f(2Af + Be))) (fx+e)^2 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{70b^4f} \\ & + \frac{(-7Bf + 3Ce) (fx+e)^3 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{42b^2f} \\ & - \frac{C(fx+e)^4 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{7b^2f} \\ & - \frac{(64a^4C f^4 + 16a^2b^2f^2(15C e^2 + 7f(Af + 3Be)) - 8b^4e^2(3C e^2 - 7f(12Af + Be)) + 3b^2f(a^2f^2(35Bf + 41Ce))}{840b^6f} \\ & + \frac{a^2(A(6a^2b^2e f^2 + 8b^4e^3) + a^2(a^2f^2(Bf + 3Ce) + 2b^2e^2(3Bf + Ce))) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right) \sqrt{c} \sqrt{bx+a}}{16b^5 \sqrt{-b^2cx^2 + a^2c}} \end{aligned}$$

command

```
integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.2 Problem number 21

$$\int \sqrt{a+bx} \sqrt{ac-bcx} (e+fx)^2 (A+Bx+Cx^2) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2A(a^2b^2f^2 + 4b^4e^2) + a^2(a^2Cf^2 + 2b^2e(2Bf + Ce))) x \sqrt{bx+a} \sqrt{-bcx+ac}}{16b^4} \\ & + \frac{(-2Bf + Ce)(fx + e)^2 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{10b^2f} \\ & - \frac{C(fx + e)^3 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{6b^2f} \\ & - \frac{(16a^2f^2(Bf + 2Ce) - 8b^2e(Ce^2 - 2f(5Af + Be)) + 3f(5a^2Cf^2 - b^2(2Ce^2 - 2f(5Af + 2Be)))) x (-b^2x^2 + a^2)}{120b^4f} \\ & + \frac{a^2(2A(a^2b^2f^2 + 4b^4e^2) + a^2(a^2Cf^2 + 2b^2e(2Bf + Ce))) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right) \sqrt{c} \sqrt{bx+a} \sqrt{-bcx+ac}}{16b^5\sqrt{-b^2cx^2 + a^2c}} \end{aligned}$$

command

```
integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.3 Problem number 22

$$\int \sqrt{a+bx} \sqrt{ac-bcx} (e+fx) (A+Bx+Cx^2) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4Ab^2e + a^2(Bf + Ce)) x \sqrt{bx+a} \sqrt{-bcx+ac}}{8b^2} \\ & - \frac{C(fx + e)^2 (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{5b^2f} \\ & - \frac{(8a^2Cf^2 - 4b^2(3Ce^2 - 5f(Af + Be)) - 3b^2f(-5Bf + 3Ce)x) (-b^2x^2 + a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{60b^4f} \\ & + \frac{a^2(4Ab^2e + a^2(Bf + Ce)) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right) \sqrt{c} \sqrt{bx+a} \sqrt{-bcx+ac}}{8b^3\sqrt{-b^2cx^2 + a^2c}} \end{aligned}$$

command

```
integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.4 Problem number 23

$$\int \sqrt{a+bx} \sqrt{ac-bcx} (A+Bx+Cx^2) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(4A + \frac{a^2C}{b^2}\right) x \sqrt{bx+a} \sqrt{-bcx+ac}}{8} - \frac{B(-b^2x^2+a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{3b^2} \\ & - \frac{Cx(-b^2x^2+a^2) \sqrt{bx+a} \sqrt{-bcx+ac}}{4b^2} \\ & + \frac{a^2(4Ab^2+a^2C) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2+a^2c}}\right) \sqrt{c} \sqrt{bx+a} \sqrt{-bcx+ac}}{8b^3 \sqrt{-b^2cx^2+a^2c}} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$24 \left( \frac{2ac \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c+2ac}}{\sqrt{-c}} \right|\right)}{\sqrt{-c}} - \sqrt{-(bx+a)c+2ac} \sqrt{bx+a} \right) Aab^2 - 12 \left( \frac{2a^2c \log\left(\left| \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c+2ac}}{\sqrt{-c}} \right|\right)}{\sqrt{-c}} - \sqrt{-(bx+a)c+2ac} \sqrt{bx+a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 7.5 Problem number 25

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx} \sqrt{ac - bcx} (e + fx)^2} dx$$

Optimal antiderivative

$$\frac{f \left( A + \frac{e(-Bf + Ce)}{f^2} \right) (-b^2x^2 + a^2)}{(-a^2f^2 + b^2e^2) (fx + e) \sqrt{bx + a} \sqrt{-bcx + ac}} + \frac{C \arctan \left( \frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}} \right) \sqrt{-b^2cx^2 + a^2c}}{b f^2 \sqrt{c} \sqrt{bx + a} \sqrt{-bcx + ac}}$$

$$+ \frac{(a^2f^2(-Bf + 2Ce) - b^2(-Ae f^2 + C e^3)) \arctan \left( \frac{(b^2ex + a^2f)\sqrt{c}}{\sqrt{-a^2f^2 + b^2e^2} \sqrt{-b^2cx^2 + a^2c}} \right) \sqrt{-b^2cx^2 + a^2c}}{f^2 (-a^2f^2 + b^2e^2)^{\frac{3}{2}} \sqrt{c} \sqrt{bx + a} \sqrt{-bcx + ac}}$$

command

`integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( Ba^2b\sqrt{-c} f^3 - 2Ca^2b\sqrt{-c} f^2e - Ab^3\sqrt{-c} f^2e + Cb^3\sqrt{-c} e^3 \right) \arctan \left( \frac{\left( \sqrt{bx + a} \sqrt{-c} - \sqrt{-(bx + a)c + 2ac} \right)^2 f - 2bce}{2 \sqrt{a^2f^2 - b^2e^2} c} \right)}{(a^2f^4 - b^2f^2e^2) \sqrt{a^2f^2 - b^2e^2} c} +$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 7.6 Problem number 27

$$\int \frac{(e + fx)^3 (A + Bx + Cx^2)}{\sqrt{a + bx} \sqrt{ac - bcx}} dx$$

Optimal antiderivative

$$\frac{(16a^2C f^2 - b^2(3C e^2 - 5f(4Af + 3Be))) (fx + e)^2 (-b^2x^2 + a^2)}{60b^4 f \sqrt{bx + a} \sqrt{-bcx + ac}}$$

$$+ \frac{(-5Bf + Ce) (fx + e)^3 (-b^2x^2 + a^2)}{20b^2 f \sqrt{bx + a} \sqrt{-bcx + ac}} - \frac{C (fx + e)^4 (-b^2x^2 + a^2)}{5b^2 f \sqrt{bx + a} \sqrt{-bcx + ac}}$$

$$\frac{(64a^4C f^4 + 16a^2b^2 f^2(13C e^2 + 5f(Af + 3Be)) - 4b^4e^2(3C e^2 - 5f(16Af + 3Be)) + b^2f(a^2f^2(45Bf + 71Ce))}{120b^6 f \sqrt{bx + a} \sqrt{-bcx + ac}}$$

$$+ \frac{(4A(3a^2b^2e f^2 + 2b^4e^3) + a^2(3a^2f^2(Bf + 3Ce) + 4b^2e^2(3Bf + Ce))) \arctan \left( \frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}} \right) \sqrt{-b^2cx^2 + a^2c}}{8b^5 \sqrt{c} \sqrt{bx + a} \sqrt{-bcx + ac}}$$

command

```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( 2 \left( 3 \left( \frac{4(bx+a)Cf^3}{c} - \frac{16Cac^4f^3 - 5Bbc^4f^3 - 15Cbc^4f^2e}{c^5} \right) (bx+a) + \frac{88Ca^2c^4f^3 - 45Babc^4f^3 + 20Ab^2c^4f^3 - 135Cabc^4f^2e + 60Bb^2c^4}{c^5} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.7 Problem number 28

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx} \sqrt{ac-bcx}} dx$$

Optimal antiderivative

$$\frac{(-4Bf+Ce)(fx+e)^2(-b^2x^2+a^2)}{12b^2f\sqrt{bx+a}\sqrt{-bcx+ac}} - \frac{C(fx+e)^3(-b^2x^2+a^2)}{4b^2f\sqrt{bx+a}\sqrt{-bcx+ac}}$$

$$- \frac{(16a^2f^2(Bf+2Ce) - 4b^2e(Ce^2 - 4f(3Af+Be)) + f(9a^2Cf^2 - b^2(2Ce^2 - 4f(3Af+2Be))))x(-b^2x^2+a^2)}{24b^4f\sqrt{bx+a}\sqrt{-bcx+ac}}$$

$$+ \frac{(4A(a^2b^2f^2 + 2b^4e^2) + a^2(3a^2Cf^2 + 4b^2e(2Bf+Ce))) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2+a^2c}}\right) \sqrt{-b^2cx^2+a^2c}}{8b^5\sqrt{c}\sqrt{bx+a}\sqrt{-bcx+ac}}$$

command

```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( 2 \left( \frac{3(bx+a)Cf^2}{c} - \frac{9Cac^3f^2 - 4Bbc^3f^2 - 8Cbc^3fe}{c^4} \right) (bx+a) + \frac{27Ca^2c^3f^2 - 16Babc^3f^2 + 12Ab^2c^3f^2 - 32Cabc^3fe + 24Bb^2c^3fe + 12Cb^2c^3fe}{c^4} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.8 Problem number 29

$$\int \frac{(e + fx)(A + Bx + Cx^2)}{\sqrt{a + bx} \sqrt{ac - bcx}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{C(fx + e)^2(-b^2x^2 + a^2)}{3b^2f\sqrt{bx + a}\sqrt{-bcx + ac}} \\ & -\frac{(4a^2Cf^2 - 2b^2(Ce^2 - 3f(Af + Be)) - b^2f(-3Bf + Ce)x)(-b^2x^2 + a^2)}{6b^4f\sqrt{bx + a}\sqrt{-bcx + ac}} \\ & + \frac{(2Ab^2e + a^2(Bf + Ce)) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right) \sqrt{-b^2cx^2 + a^2c}}{2b^3\sqrt{c}\sqrt{bx + a}\sqrt{-bcx + ac}} \end{aligned}$$

command

```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\left(\frac{2(bx+a)Cf}{c} - \frac{4Cac^2f - 3Bbc^2f - 3Cbc^2e}{c^3}\right)(bx + a) + \frac{3(2Ca^2c^2f - Babc^2f + 2Ab^2c^2f - Cabc^2e + 2Bb^2c^2e)}{c^3}\right) \sqrt{-(bx + a)c + 2a}}{6b^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.9 Problem number 30

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx} \sqrt{ac - bcx}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-b^2x^2 + a^2)}{b^2\sqrt{bx + a}\sqrt{-bcx + ac}} - \frac{Cx(-b^2x^2 + a^2)}{2b^2\sqrt{bx + a}\sqrt{-bcx + ac}} \\ & + \frac{(2Ab^2 + a^2C) \arctan\left(\frac{bx\sqrt{c}}{\sqrt{-b^2cx^2 + a^2c}}\right) \sqrt{-b^2cx^2 + a^2c}}{2b^3\sqrt{c}\sqrt{bx + a}\sqrt{-bcx + ac}} \end{aligned}$$

command

`integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-(bx+a)c+2ac} \sqrt{bx+a} \left( \frac{(bx+a)C}{c} - \frac{Cac-2Bbc}{c^2} \right) + \frac{2(Ca^2+2Ab^2) \log\left( \frac{-\sqrt{bx+a} \sqrt{-c} + \sqrt{-(bx+a)c+2ac}}{\sqrt{-c}} \right)}{\sqrt{-c}}}{2b^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.10 Problem number 32

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx} \sqrt{ac-bcx} (e+fx)^2} dx$$

Optimal antiderivative

$$\frac{f \left( A + \frac{e(-Bf+Ce)}{f^2} \right) (-b^2x^2 + a^2)}{(-a^2f^2 + b^2e^2) (fx + e) \sqrt{bx+a} \sqrt{-bcx+ac}} + \frac{C \arctan \left( \frac{bx\sqrt{c}}{\sqrt{-b^2cx^2+a^2c}} \right) \sqrt{-b^2cx^2+a^2c}}{b f^2 \sqrt{c} \sqrt{bx+a} \sqrt{-bcx+ac}}$$

$$+ \frac{(a^2f^2(-Bf+2Ce) - b^2(-Aef^2 + Ce^3)) \arctan \left( \frac{(b^2ex+a^2f)\sqrt{c}}{\sqrt{-a^2f^2+b^2e^2} \sqrt{-b^2cx^2+a^2c}} \right) \sqrt{-b^2cx^2+a^2c}}{f^2 (-a^2f^2 + b^2e^2)^{\frac{3}{2}} \sqrt{c} \sqrt{bx+a} \sqrt{-bcx+ac}}$$

command

`integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( Ba^2b\sqrt{-c} f^3 - 2Ca^2b\sqrt{-c} f^2e - Ab^3\sqrt{-c} f^2e + Cb^3\sqrt{-c} e^3 \right) \arctan \left( \frac{\left( \sqrt{bx+a} \sqrt{-c} - \sqrt{-(bx+a)c+2ac} \right)^2_{f-2bce}}{2 \sqrt{a^2f^2 - b^2e^2} c} \right)}{(a^2f^4 - b^2f^2e^2) \sqrt{a^2f^2 - b^2e^2} c} +$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 7.11 Problem number 53

$$\int \frac{\sqrt{c+dx} (A+Bx+Cx^2)}{(a+bx)^4 \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{(-cf+de)(b^2(A d^2 e^2 - 2cde(-Af+Be) + c^2(5A f^2 - 6Bef + 8C e^2)) + ab(d^2 e(-4Af+Be) - c^2 f(-Bf+4C e^2)))}{3b(-ad+bc)(-af+be)(bx+a)^3} + \frac{(A b^2 - a(bB - aC)) (dx+c)^{\frac{3}{2}} \sqrt{fx+e}}{3b(-ad+bc)(-af+be)(bx+a)^3} + \frac{(4a^3 Cdf - b^3(-5Acf - 3Ade + 6Bce) + a b^2(-8Adf + Bcf + 3Bde + 12cCe) - a^2 b(-2Bdf + 7cCf + 9Cde)) \sqrt{fx+e}}{12b^2(-ad+bc)(-af+be)^2(bx+a)^2} + \frac{(8a^4 C d^2 f^2 - 2a^3 bdf(-2Bdf + 7cCf + 13Cde) - b^4(3A d^2 e^2 - 2cde(-2Af + 3Be) - 3c^2(5A f^2 - 6Bef + 8C e^2))) \sqrt{fx+e}}{4b(-ad+bc)^2(-af+be)^2(bx+a)}$$

command

```
integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 7.12 Problem number 59

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3 \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal antiderivative

$$\frac{(b^2(3A d^2 e^2 - 2cde(-Af+2Be) + c^2(3A f^2 - 4Bef + 8C e^2)) + ab(d^2 e(-8Af+Be) - c^2 f(-Bf+8Ce) - 2c^2 f^2)) \sqrt{dx+c} \sqrt{fx+e}}{4b(-ad+bc)^2(-af+be)^2(bx+a)} + \frac{(A b^2 - a(bB - aC)) \sqrt{dx+c} \sqrt{fx+e}}{2b(-ad+bc)(-af+be)(bx+a)^2} + \frac{(2a^3 Cdf + a b^2(-6Adf + Bcf + Bde + 8cCe) - b^3(4Bce - 3A(cf+de)) + a^2 b(2Bdf - 5C(cf+de))) \sqrt{dx+c}}{4b(-ad+bc)^2(-af+be)^2(bx+a)}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 7.13 Problem number 60

$$\int \frac{A + Bx + Cx^2}{(a + bx)^4 \sqrt{c + dx} \sqrt{e + fx}} dx$$

Optimal antiderivative

$$\begin{aligned} & (b^3(5A d^3 e^3 - 3c d^2 e^2(-Af + 2Be) + c^2 de(3A f^2 - 4Bef + 8C e^2)) + c^3 f(5A f^2 - 6Bef + 8C e^2)) + a b^2(d^3 e^2(- \\ & \frac{(A b^2 - a(bB - aC)) \sqrt{dx + c} \sqrt{fx + e}}{3b(-ad + bc)(-af + be)(bx + a)^3} \\ & + \frac{(2a^3 Cdf + a b^2(-10A df + Bcf + Bde + 12cCe) - b^3(6Bce - 5A(cf + de)) + a^2 b(4Bdf - 7C(cf + de))) \sqrt{dx + c}}{12b(-ad + bc)^2(-af + be)^2(bx + a)^2} \\ & + \frac{(4a^4 C d^2 f^2 + 8a^3 bdf(Bdf - 2C(cf + de)) - b^4(15A d^2 e^2 - 2cde(-7Af + 9Be) + 3c^2(5A f^2 - 6Bef + 8C e^2)) - \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 8 Test file number 25

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/25\_1.1.3.2-c\_x-  
^m-a+b\_x^n-p

### 8.1 Problem number 41

$$\int \frac{1}{x^2 \sqrt{bx^2}} dx$$

Optimal antiderivative

$$-\frac{1}{2x\sqrt{bx^2}}$$

command

`integrate(1/x^2/(b*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2\sqrt{b}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.2 Problem number 50

$$\int \frac{1}{(bx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2bx\sqrt{bx^2}}$$

command

`integrate(1/(b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2b^{\frac{3}{2}}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.3 Problem number 51

$$\int \frac{1}{x^2 (bx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4bx^3\sqrt{bx^2}}$$

command

```
integrate(1/x^2/(b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4b^{\frac{3}{2}}x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.4 Problem number 59

$$\int \frac{x^2}{(bx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2b^2x\sqrt{bx^2}}$$

command

```
integrate(x^2/(b*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2b^{\frac{5}{2}}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.5 Problem number 60

$$\int \frac{1}{(bx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4b^2x^3\sqrt{bx^2}}$$

command

```
integrate(1/(b*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4b^{\frac{5}{2}}x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.6 Problem number 61

$$\int \frac{1}{x^2(bx^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{6b^2x^5\sqrt{bx^2}}$$

command

```
integrate(1/x^2/(b*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6b^{\frac{5}{2}}x^6\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.7 Problem number 95

$$\int \frac{1}{(bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2}{7bx^2\sqrt{bx^3}}$$

command

```
integrate(1/(b*x^3)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{7\sqrt{bx}bx^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.8 Problem number 96

$$\int \frac{1}{(bx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2bx\sqrt{bx^2}}$$

command

```
integrate(1/(b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2b^{\frac{3}{2}}x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 8.9 Problem number 100

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^4}{11b\sqrt{\frac{b}{x^3}}}$$

command

```
integrate(1/(b/x^3)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{bx}x^5}{11b^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{\frac{3}{2}}} dx$$

## 8.10 Problem number 106

$$\int \sqrt[3]{\frac{b}{x^2}} dx$$

Optimal antiderivative

$$3\left(\frac{b}{x^2}\right)^{\frac{1}{3}} x$$

command

```
integrate((b/x^2)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3x\left(\frac{b}{x^2}\right)^{\frac{1}{3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^2}\right)^{\frac{1}{3}} dx$$

## 8.11 Problem number 107

$$\int \sqrt[3]{\frac{b}{x^3}} dx$$

Optimal antiderivative

$$\left(\frac{b}{x^3}\right)^{\frac{1}{3}} x \ln(x)$$

command

```
integrate((b/x^3)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$+\infty$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^3}\right)^{\frac{1}{3}} dx$$

## 8.12 Problem number 108

$$\int \sqrt[3]{\frac{b}{x^4}} dx$$

Optimal antiderivative

$$-3\left(\frac{b}{x^4}\right)^{\frac{1}{3}} x$$

command

```
integrate((b/x^4)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-3x\left(\frac{b}{x^4}\right)^{\frac{1}{3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^4}\right)^{\frac{1}{3}} dx$$



### 8.13 Problem number 113

$$\int \left(\frac{b}{x^2}\right)^{2/3} dx$$

Optimal antiderivative

$$-3\left(\frac{b}{x^2}\right)^{\frac{2}{3}} x$$

command

```
integrate((b/x^2)^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-3x\left(\frac{b}{x^2}\right)^{\frac{2}{3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^2}\right)^{\frac{2}{3}} dx$$

### 8.14 Problem number 114

$$\int \left(\frac{b}{x^3}\right)^{2/3} dx$$

Optimal antiderivative

$$-\left(\frac{b}{x^3}\right)^{\frac{2}{3}} x$$

command

```
integrate((b/x^3)^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-x\left(\frac{b}{x^3}\right)^{\frac{2}{3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^3}\right)^{\frac{2}{3}} dx$$

### 8.15 Problem number 115

$$\int \left(\frac{b}{x^4}\right)^{2/3} dx$$

Optimal antiderivative

$$-\frac{3\left(\frac{b}{x^4}\right)^{\frac{2}{3}} x}{5}$$

command

```
integrate((b/x^4)^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3}{5} x \left(\frac{b}{x^4}\right)^{\frac{2}{3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(\frac{b}{x^4}\right)^{\frac{2}{3}} dx$$

### 8.16 Problem number 117

$$\int \frac{1}{\sqrt[3]{bx^4}} dx$$

Optimal antiderivative

$$-\frac{3x}{(bx^4)^{\frac{1}{3}}}$$

command

```
integrate(1/(b*x^4)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3x}{(bx^4)^{\frac{1}{3}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(bx^4)^{\frac{1}{3}}} dx$$

## 8.17 Problem number 122

$$\int \frac{1}{\sqrt[3]{\frac{b}{x^2}}} dx$$

Optimal antiderivative

$$\frac{3x}{5 \left(\frac{b}{x^2}\right)^{\frac{1}{3}}}$$

command

```
integrate(1/(b/x^2)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3x}{5 \left(\frac{b}{x^2}\right)^{\frac{1}{3}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{b}{x^2}\right)^{\frac{1}{3}}} dx$$

## 8.18 Problem number 123

$$\int \frac{1}{\sqrt[3]{\frac{b}{x^3}}} dx$$

Optimal antiderivative

$$\frac{x}{2 \left(\frac{b}{x^3}\right)^{\frac{1}{3}}}$$

command

```
integrate(1/(b/x^3)^(1/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{2 \left(\frac{b}{x^3}\right)^{\frac{1}{3}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{\frac{1}{3}}} dx$$

## 8.19 Problem number 129

$$\int \frac{1}{\left(\frac{b}{x^2}\right)^{2/3}} dx$$

Optimal antiderivative

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}$$

command

```
integrate(1/(b/x^2)^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3x}{7\left(\frac{b}{x^2}\right)^{2/3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{b}{x^2}\right)^{2/3}} dx$$

## 8.20 Problem number 130

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{2/3}} dx$$

Optimal antiderivative

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}$$

command

```
integrate(1/(b/x^3)^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{3\left(\frac{b}{x^3}\right)^{2/3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{b}{x^3}\right)^{2/3}} dx$$

### 8.21 Problem number 1922

$$\int \frac{x^4}{\sqrt{a + \frac{b}{x^2}}} dx$$

Optimal antiderivative

$$\frac{8b^2x\sqrt{a + \frac{b}{x^2}}}{15a^3} - \frac{4bx^3\sqrt{a + \frac{b}{x^2}}}{15a^2} + \frac{x^5\sqrt{a + \frac{b}{x^2}}}{5a}$$

command

```
integrate(x^4/(a+b/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8b^{\frac{5}{2}}\operatorname{sgn}(x)}{15a^3} + \frac{\sqrt{ax^2 + b}b^2}{a^3\operatorname{sgn}(x)} + \frac{3(ax^2 + b)^{\frac{5}{2}} - 10(ax^2 + b)^{\frac{3}{2}}b}{15a^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 8.22 Problem number 1923

$$\int \frac{x^2}{\sqrt{a + \frac{b}{x^2}}} dx$$

Optimal antiderivative

$$-\frac{2bx\sqrt{a + \frac{b}{x^2}}}{3a^2} + \frac{x^3\sqrt{a + \frac{b}{x^2}}}{3a}$$

command

```
integrate(x^2/(a+b/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2b^{\frac{3}{2}}\operatorname{sgn}(x)}{3a^2} + \frac{(ax^2 + b)^{\frac{3}{2}}}{3a^2\operatorname{sgn}(x)} - \frac{\sqrt{ax^2 + b}b}{a^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 8.23 Problem number 1925

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} x^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{\sqrt{b}}$$

command

```
integrate(1/(a+b/x^2)^(1/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{ax^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 8.24 Problem number 1926

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} x^4} dx$$

Optimal antiderivative

$$\frac{a \operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{2b^{\frac{3}{2}}} - \frac{\sqrt{a + \frac{b}{x^2}}}{2bx}$$

command

```
integrate(1/(a+b/x^2)^(1/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} b} + \frac{\sqrt{ax^2+b} a}{bx^2}$$


---


$$2 \operatorname{asgn}(x)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.25 Problem number 1928

$$\int \frac{1}{\sqrt{2 + \frac{b}{x^2}} x^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccsch}\left(\frac{x\sqrt{2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

command

`integrate(1/x^2/(2+b/x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{2x^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.26 Problem number 1929

$$\int \frac{1}{\sqrt{2 - \frac{b}{x^2}} x^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccsc}\left(\frac{x\sqrt{2}}{\sqrt{b}}\right)}{\sqrt{b}}$$

command

```
integrate(1/x^2/(2-b/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{2x^2-b}}{\sqrt{b}}\right)}{\sqrt{b}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.27 Problem number 1930

$$\int \frac{x^3}{\left(a + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{15b^2 \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{15b^2}{8a^3\sqrt{a + \frac{b}{x^2}}} - \frac{5bx^2}{8a^2\sqrt{a + \frac{b}{x^2}}} + \frac{x^4}{4a\sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(x^3/(a+b/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(x^2\left(\frac{2x^2}{a\operatorname{sgn}(x)} - \frac{5b}{a^2\operatorname{sgn}(x)}\right) - \frac{15b^2}{a^3\operatorname{sgn}(x)}\right)x}{8\sqrt{ax^2+b}} + \frac{15b^2\log(|b|\operatorname{sgn}(x))}{16a^{7/2}} - \frac{15b^2\log\left(\left|-\sqrt{a}x + \sqrt{ax^2+b}\right|\right)}{8a^{7/2}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 8.28 Problem number 1931

$$\int \frac{x}{\left(a + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{3b \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{3b}{2a^2 \sqrt{a + \frac{b}{x^2}}} + \frac{x^2}{2a \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(x/(a+b/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x\left(\frac{x^2}{a \operatorname{sgn}(x)} + \frac{3b}{a^2 \operatorname{sgn}(x)}\right)}{2 \sqrt{ax^2 + b}} - \frac{3b \log(|b|) \operatorname{sgn}(x)}{4a^{5/2}} + \frac{3b \log\left(\left|-\sqrt{a}x + \sqrt{ax^2 + b}\right|\right)}{2a^{5/2} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.29 Problem number 1932

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(1/(a+b/x^2)^(3/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|b|)\operatorname{sgn}(x)}{2a^{\frac{3}{2}}} - \frac{x}{\sqrt{ax^2+b}\operatorname{asgn}(x)} - \frac{\log\left(\left|-\sqrt{a}x + \sqrt{ax^2+b}\right|\right)}{a^{\frac{3}{2}}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 8.30 Problem number 1935

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^7} dx$$

Optimal antiderivative

$$-\frac{\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}}{3b^3} + \frac{a^2}{b^3\sqrt{a + \frac{b}{x^2}}} + \frac{2a\sqrt{a + \frac{b}{x^2}}}{b^3}$$

command

`integrate(1/(a+b/x^2)^(3/2)/x^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2x}{\sqrt{ax^2+b}b^3\operatorname{sgn}(x)} - \frac{2\left(3\left(\sqrt{a}x - \sqrt{ax^2+b}\right)^4 a^{\frac{3}{2}} - 12\left(\sqrt{a}x - \sqrt{ax^2+b}\right)^2 a^{\frac{3}{2}}b + 5a^{\frac{3}{2}}b^2\right)}{3\left(\left(\sqrt{a}x - \sqrt{ax^2+b}\right)^2 - b\right)^3 b^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^7} dx$$

### 8.31 Problem number 1936

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^9} dx$$

Optimal antiderivative

$$\frac{a\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}}{b^4} - \frac{\left(a + \frac{b}{x^2}\right)^{\frac{5}{2}}}{5b^4} - \frac{a^3}{b^4\sqrt{a + \frac{b}{x^2}}} - \frac{3a^2\sqrt{a + \frac{b}{x^2}}}{b^4}$$

command

`integrate(1/(a+b/x^2)^(3/2)/x^9,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{a^3 x}{\sqrt{ax^2 + b} b^4 \operatorname{sgn}(x)} + \frac{2 \left( 5 \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^8 a^{\frac{5}{2}} - 30 \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^6 a^{\frac{5}{2}} b + 80 \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^4 a^{\frac{5}{2}} b^2 - 50 \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^2 a^{\frac{5}{2}} b^3 + 5 a^{\frac{5}{2}} b^4 \right) \operatorname{sgn}(x)}{5 \left( \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^2 - b \right)^5 b^3 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^9} dx$$

### 8.32 Problem number 1937

$$\int \frac{x^4}{\left(a + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{8b^2 x}{5a^3 \sqrt{a + \frac{b}{x^2}}} - \frac{2bx^3}{5a^2 \sqrt{a + \frac{b}{x^2}}} + \frac{x^5}{5a \sqrt{a + \frac{b}{x^2}}} + \frac{16b^2 x \sqrt{a + \frac{b}{x^2}}}{5a^4}$$

command

`integrate(x^4/(a+b/x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{16 b^{\frac{5}{2}} \operatorname{sgn}(x)}{5 a^4} + \frac{b^3}{\sqrt{ax^2 + b} a^4 \operatorname{sgn}(x)} + \frac{(ax^2 + b)^{\frac{5}{2}} a^{16} - 5 (ax^2 + b)^{\frac{3}{2}} a^{16} b + 15 \sqrt{ax^2 + b} a^{16} b^2}{5 a^{20} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 8.33 Problem number 1938

$$\int \frac{x^2}{\left(a + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4bx}{3a^2\sqrt{a + \frac{b}{x^2}}} + \frac{x^3}{3a\sqrt{a + \frac{b}{x^2}}} - \frac{8bx\sqrt{a + \frac{b}{x^2}}}{3a^3}$$

command

```
integrate(x^2/(a+b/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8b^{\frac{3}{2}}\operatorname{sgn}(x)}{3a^3} - \frac{b^2}{\sqrt{ax^2 + b}a^3\operatorname{sgn}(x)} + \frac{(ax^2 + b)^{\frac{3}{2}}a^6 - 6\sqrt{ax^2 + b}a^6b}{3a^9\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 8.34 Problem number 1939

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{x}{a\sqrt{a + \frac{b}{x^2}}} + \frac{2x\sqrt{a + \frac{b}{x^2}}}{a^2}$$

command

```
integrate(1/(a+b/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{ax^2 + b}}{a\operatorname{sgn}(x)} + \frac{b}{\sqrt{ax^2 + b}a\operatorname{sgn}(x)} - \frac{2\sqrt{b}\operatorname{sgn}(x)}{a^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 8.35 Problem number 1940

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^2} dx$$

Optimal antiderivative

$$-\frac{1}{ax \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(1/(a+b/x^2)^(3/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\operatorname{sgn}(x)}{a\sqrt{b}} - \frac{1}{\sqrt{ax^2 + b} \operatorname{asgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 8.36 Problem number 1941

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^4} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{x \sqrt{a + \frac{b}{x^2}}}\right)}{b^{3/2}} + \frac{1}{bx \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(1/(a+b/x^2)^(3/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(\sqrt{b} \operatorname{arctan}\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\right) \operatorname{sgn}(x)}{\sqrt{-b} b^{3/2}} + \frac{\operatorname{arctan}\left(\frac{\sqrt{ax^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b} \operatorname{bsgn}(x)} + \frac{1}{\sqrt{ax^2 + b} \operatorname{bsgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.37 Problem number 1942

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^6} dx$$

Optimal antiderivative

$$\frac{3a \operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{2b^{\frac{5}{2}}} + \frac{1}{bx^3\sqrt{a + \frac{b}{x^2}}} - \frac{3\sqrt{a + \frac{b}{x^2}}}{2b^2x}$$

command

```
integrate(1/(a+b/x^2)^(3/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3a \operatorname{arctan}\left(\frac{\sqrt{ax^2 + b}}{\sqrt{-b}}\right)}{2\sqrt{-b} b^2 \operatorname{sgn}(x)} - \frac{3(ax^2 + b)a - 2ab}{2\left((ax^2 + b)^{\frac{3}{2}} - \sqrt{ax^2 + b} b\right) b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.38 Problem number 1943

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{3/2} x^8} dx$$

Optimal antiderivative

$$-\frac{15a^2 \operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{8b^{\frac{7}{2}}} + \frac{1}{bx^5\sqrt{a + \frac{b}{x^2}}} - \frac{5\sqrt{a + \frac{b}{x^2}}}{4b^2x^3} + \frac{15a\sqrt{a + \frac{b}{x^2}}}{8b^3x}$$

command

```
integrate(1/(a+b/x^2)^(3/2)/x^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 a^2 \arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{8 \sqrt{-b} b^3 \operatorname{sgn}(x)} + \frac{a^2}{\sqrt{ax^2+b} b^3 \operatorname{sgn}(x)} + \frac{7 (ax^2+b)^{\frac{3}{2}} a^2 - 9 \sqrt{ax^2+b} a^2 b}{8 a^2 b^3 x^4 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 8.39 Problem number 1949

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2} x^7} dx$$

Optimal antiderivative

$$\frac{a^2}{3b^3 \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}} - \frac{2a}{b^3 \sqrt{a + \frac{b}{x^2}}} - \frac{\sqrt{a + \frac{b}{x^2}}}{b^3}$$

command

`integrate(1/(a+b/x^2)^(5/2)/x^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{x \left( \frac{5a^2x^2}{b^3 \operatorname{sgn}(x)} + \frac{6a}{b^2 \operatorname{sgn}(x)} \right)}{3 (ax^2 + b)^{\frac{3}{2}}} + \frac{2 \sqrt{a}}{\left( \left( \sqrt{a} x - \sqrt{ax^2 + b} \right)^2 - b \right) b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{\frac{5}{2}} x^7} dx$$

## 8.40 Problem number 1951

$$\int \frac{x^2}{\left(a + \frac{b}{x^2}\right)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2bx}{3a^2 \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}} + \frac{x^3}{3a \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}} + \frac{8bx}{3a^3 \sqrt{a + \frac{b}{x^2}}} - \frac{16bx \sqrt{a + \frac{b}{x^2}}}{3a^4}$$

command

```
integrate(x^2/(a+b/x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 b^{\frac{3}{2}} \operatorname{sgn}(x)}{3 a^4} - \frac{9 (a x^2 + b) b^2 - b^3}{3 (a x^2 + b)^{\frac{3}{2}} a^4 \operatorname{sgn}(x)} + \frac{(a x^2 + b)^{\frac{3}{2}} a^8 - 9 \sqrt{a x^2 + b} a^8 b}{3 a^{12} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 8.41 Problem number 1952

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{x}{3a \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}}} - \frac{4x}{3a^2 \sqrt{a + \frac{b}{x^2}}} + \frac{8x \sqrt{a + \frac{b}{x^2}}}{3a^3}$$

command

```
integrate(1/(a+b/x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8 \sqrt{b} \operatorname{sgn}(x)}{3 a^3} + \frac{\sqrt{a x^2 + b}}{a^3 \operatorname{sgn}(x)} + \frac{6 (a x^2 + b) b - b^2}{3 (a x^2 + b)^{\frac{3}{2}} a^3 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



### 8.42 Problem number 1953

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2} x^2} dx$$

Optimal antiderivative

$$-\frac{2b}{3a^2 \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^3} - \frac{1}{a \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x}$$

command

```
integrate(1/(a+b/x^2)^(5/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \operatorname{sgn}(x)}{3 a^2 \sqrt{b}} - \frac{3 a x^2 + 2 b}{3 (a x^2 + b)^{\frac{3}{2}} a^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 8.43 Problem number 1954

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2} x^4} dx$$

Optimal antiderivative

$$-\frac{1}{3a \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^3}$$

command

```
integrate(1/(a+b/x^2)^(5/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\operatorname{sgn}(x)}{3 a b^{\frac{3}{2}}} - \frac{1}{3 (a x^2 + b)^{\frac{3}{2}} a \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 8.44 Problem number 1955

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2} x^6} dx$$

Optimal antiderivative

$$\frac{1}{3b \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^3} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{b^{\frac{5}{2}}} + \frac{1}{b^2 x \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(1/(a+b/x^2)^(5/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(3\sqrt{b} \operatorname{arctan}\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4\sqrt{-b}\right) \operatorname{sgn}(x)}{3\sqrt{-b} b^{\frac{5}{2}}} + \frac{\operatorname{arctan}\left(\frac{\sqrt{ax^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2 \operatorname{sgn}(x)} + \frac{3ax^2 + 4b}{3(ax^2 + b)^{\frac{3}{2}} b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.45 Problem number 1956

$$\int \frac{1}{\left(a + \frac{b}{x^2}\right)^{5/2} x^8} dx$$

Optimal antiderivative

$$\frac{1}{3b \left(a + \frac{b}{x^2}\right)^{\frac{3}{2}} x^5} + \frac{5a \operatorname{arctanh}\left(\frac{\sqrt{b}}{x\sqrt{a + \frac{b}{x^2}}}\right)}{2b^{\frac{7}{2}}} + \frac{5}{3b^2 x^3 \sqrt{a + \frac{b}{x^2}}} - \frac{5\sqrt{a + \frac{b}{x^2}}}{2b^3 x}$$

command

```
integrate(1/(a+b/x^2)^(5/2)/x^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{5a \arctan\left(\frac{\sqrt{ax^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b}b^3\operatorname{sgn}(x)} - \frac{6(ax^2+b)a+ab}{3(ax^2+b)^{\frac{3}{2}}b^3\operatorname{sgn}(x)} - \frac{\sqrt{ax^2+b}}{2b^3x^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 8.46 Problem number 2021

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7} dx$$

Optimal antiderivative

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9b^2} + \frac{2a\sqrt{a+\frac{b}{x^3}}}{3b^2}$$

command

```
integrate(1/x^7/(a+b/x^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\left(a+\frac{b}{x^3}\right)^{\frac{3}{2}}}{9b^2} + \frac{2\sqrt{a+\frac{b}{x^3}}a}{3b^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a+\frac{b}{x^3}}x^7} dx$$

## 8.47 Problem number 2022

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{10}} dx$$

Optimal antiderivative

$$\frac{4a\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}}}{9b^3} - \frac{2\left(a + \frac{b}{x^3}\right)^{\frac{5}{2}}}{15b^3} - \frac{2a^2\sqrt{a + \frac{b}{x^3}}}{3b^3}$$

command

`integrate(1/x^10/(a+b/x^3)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\sqrt{a + \frac{b}{x^3}} a^2}{3b^3} - \frac{2\left(3\left(a + \frac{b}{x^3}\right)^{\frac{5}{2}} - 10\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}} a\right)}{45b^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{10}} dx$$

## 8.48 Problem number 2023

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{13}} dx$$

Optimal antiderivative

$$-\frac{2a^2\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}}}{3b^4} + \frac{2a\left(a + \frac{b}{x^3}\right)^{\frac{5}{2}}}{5b^4} - \frac{2\left(a + \frac{b}{x^3}\right)^{\frac{7}{2}}}{21b^4} + \frac{2a^3\sqrt{a + \frac{b}{x^3}}}{3b^4}$$

command

`integrate(1/x^13/(a+b/x^3)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a + \frac{b}{x^3}} a^3}{3b^4} - \frac{2\left(5\left(a + \frac{b}{x^3}\right)^{\frac{7}{2}} - 21\left(a + \frac{b}{x^3}\right)^{\frac{5}{2}}a + 35\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}}a^2\right)}{105b^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a + \frac{b}{x^3}} x^{13}} dx$$

### 8.49 Problem number 2040

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{3/2} x^4} dx$$

Optimal antiderivative

$$\frac{2}{3b\sqrt{a + \frac{b}{x^3}}}$$

command

`integrate(1/(a+b/x^3)^(3/2)/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3\sqrt{a + \frac{b}{x^3}} b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(a + \frac{b}{x^3}\right)^{\frac{3}{2}} x^4} dx$$

### 8.50 Problem number 2806

$$\int \frac{1}{(c(a + bx)^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2bc(bx + a)\sqrt{c(bx + a)^2}}$$

command

```
integrate(1/(c*(b*x+a)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2(bx+a)^2bc^{\frac{3}{2}}\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.51 Problem number 2807

$$\int \frac{1}{(c(a+bx)^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4bc^2(bx+a)^3\sqrt{c(bx+a)^2}}$$

command

```
integrate(1/(c*(b*x+a)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4(bx+a)^4bc^{\frac{5}{2}}\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.52 Problem number 2817

$$\int \frac{1}{(c(a+bx)^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2}{7bc(bx+a)^2\sqrt{c(bx+a)^3}}$$

command

```
integrate(1/(c*(b*x+a)^3)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2c^2}{7(bcx+ac)^{\frac{7}{2}}\text{bsgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.53 Problem number 2818

$$\int \frac{1}{(c(a+bx)^3)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2}{13bc^2(bx+a)^5\sqrt{c(bx+a)^3}}$$

command

```
integrate(1/(c*(b*x+a)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2c^4}{13(bcx+ac)^{\frac{13}{2}}\text{bsgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 8.54 Problem number 2829

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(bx+a)^3}{4bc\sqrt{\frac{c}{(bx+a)^2}}}$$

command

```
integrate(1/(c/(b*x+a)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 (bx^2 + 2 ax)a^2 + (bx^2 + 2 ax)^2 b}{4 c^{\frac{3}{2}} \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{c}{(bx+a)^2}\right)^{\frac{3}{2}}} dx$$

### 8.55 Problem number 2830

$$\int \frac{1}{\left(\frac{c}{(a+bx)^2}\right)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(bx + a)^5}{6b c^2 \sqrt{\frac{c}{(bx + a)^2}}}$$

command

```
integrate(1/(c/(b*x+a)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^5 \sqrt{c} x^6 + 6 ab^4 \sqrt{c} x^5 + 15 a^2 b^3 \sqrt{c} x^4 + 20 a^3 b^2 \sqrt{c} x^3 + 15 a^4 b \sqrt{c} x^2 + 6 a^5 \sqrt{c} x}{6 c^3 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{c}{(bx+a)^2}\right)^{\frac{5}{2}}} dx$$



## 8.56 Problem number 2835

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(bx+a)^4}{11bc \sqrt{\frac{c}{(bx+a)^3}}}$$

command

```
integrate(1/(c/(b*x+a)^3)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( 693 \sqrt{bcx+ac} a^5 - \frac{1155 \left( 3 \sqrt{bcx+ac} ac - (bcx+ac)^{\frac{3}{2}} \right) a^4}{c} + \frac{462 \left( 15 \sqrt{bcx+ac} a^2 c^2 - 10 (bcx+ac)^{\frac{3}{2}} ac + 3 (bcx+ac)^{\frac{5}{2}} \right) a^3}{c^2} - \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{c}{(bx+a)^3}\right)^{\frac{3}{2}}} dx$$

## 8.57 Problem number 2836

$$\int \frac{1}{\left(\frac{c}{(a+bx)^3}\right)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(bx+a)^7}{17b c^2 \sqrt{\frac{c}{(bx+a)^3}}}$$

command

```
integrate(1/(c/(b*x+a)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( 109395 \sqrt{bcx + ac} a^8 - \frac{291720 \left( 3 \sqrt{bcx + ac} ac - (bcx + ac)^{\frac{3}{2}} \right) a^7}{c} + \frac{204204 \left( 15 \sqrt{bcx + ac} a^2 c^2 - 10 (bcx + ac)^{\frac{3}{2}} ac + 3 (bcx + ac)^{\frac{5}{2}} \right)}{c^2} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left( \frac{c}{(bx+a)^3} \right)^{\frac{5}{2}}} dx$$

**8.58 Problem number 2839**

$$\int \frac{1}{\left( \frac{c}{(a+bx)^{3/2}} \right)^{2/3}} dx$$

Optimal antiderivative

$$\frac{bx + a}{2b \left( \frac{c}{(bx+a)^{\frac{3}{2}}} \right)^{\frac{2}{3}}}$$

command

```
integrate(1/(c/(b*x+a)^(3/2))^(2/3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$+\infty$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left( \frac{c}{(bx+a)^{\frac{3}{2}}} \right)^{\frac{2}{3}}} dx$$

**8.59 Problem number 2988**

$$\int \frac{x}{\sqrt{a + b \sqrt{\frac{c}{x}}}} dx$$

Optimal antiderivative

$$\frac{35b^4c^2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{32a^{\frac{9}{2}}} - \frac{7bc^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{12a^2\left(\frac{c}{x}\right)^{\frac{3}{2}}} + \frac{35b^2cx\sqrt{a+b\sqrt{\frac{c}{x}}}}{48a^3} + \frac{x^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{2a} - \frac{35b^3c^2\sqrt{a+b\sqrt{\frac{c}{x}}}}{32a^4\sqrt{\frac{c}{x}}}$$

command

`integrate(x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{105b^4c^4 \log(c^2|b|)}{\sqrt{ac}a^4} - \frac{105b^4c^4 \log\left(\left| -bc^2 - 2\sqrt{ac}\left(\sqrt{ac}\sqrt{cx} - \sqrt{ac^2x + \sqrt{cx}bc^2}\right)\right|\right)}{\sqrt{ac}a^4}}{192c^{\frac{3}{2}}\operatorname{sgn}(x)} - 2\sqrt{ac^2x + \sqrt{cx}bc^2}\left(2\sqrt{cx}\left(4\sqrt{\frac{c}{x}}\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x}{\sqrt{b\sqrt{\frac{c}{x}} + a}} dx$$

**8.60 Problem number 2990**

$$\int \frac{1}{\sqrt{a+b\sqrt{\frac{c}{x}}}x} dx$$

Optimal antiderivative

$$\frac{4 \operatorname{arctanh}\left(\frac{\sqrt{a+b\sqrt{\frac{c}{x}}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

command

`integrate(1/x/(a+b*(c/x)^(1/2))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{c} \left( \frac{\sqrt{ac} \log\left(\sqrt{|a||c|} |b||c|\right)}{ac} - \frac{\sqrt{ac} \log\left(-\sqrt{ac} bc - 2 \left(\sqrt{ac} \sqrt{cx} - \sqrt{ac^2x + \sqrt{cx} bc^2}\right) a\right)}{ac} \right)}{\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{b\sqrt{\frac{c}{x}} + ax}} dx$$

### 8.61 Problem number 3061

$$\int \frac{x^2}{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-231b^6d^3 + 1260ab^4cd^2 - 1680a^2b^2c^2d + 320a^3c^3) \operatorname{arctanh}\left(\frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a}\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}\right)}{512a^{\frac{13}{2}}} \\ & - \frac{11bd^3\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{30a^2\left(\frac{d}{x}\right)^{\frac{5}{2}}} + \frac{bd^2(-77b^2d+156ac)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{160a^4\left(\frac{d}{x}\right)^{\frac{3}{2}}} \\ & + \frac{(385b^4d^2 - 1176ab^2cd + 400a^2c^2)x\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{640a^5} \\ & - \frac{(-99b^2d+100ac)x^2\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{240a^3} + \frac{x^3\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{3a} \\ & - \frac{7bd(165b^4d^2 - 680ab^2cd + 528a^2c^2)\sqrt{a+\frac{c}{x}+b\sqrt{\frac{d}{x}}}}{1280a^6\sqrt{\frac{d}{x}}} \end{aligned}$$

command

```
integrate(x^2/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \sqrt{ad^2x + \sqrt{dx} bd^2 + cd^2} \left( 2 \sqrt{dx} \left( 4 \sqrt{dx} \left( 2 \sqrt{dx} \left( 8 \sqrt{dx} \left( \frac{11b}{a^2d^2} - \frac{10\sqrt{dx}}{ad^3} \right) - \frac{99a^3b^2d^2 - 100a^4cd}{a^6d^3} \right) + \frac{3(77a^2b^3}{a^6d^3} \right) \right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 8.62 Problem number 3062

$$\int \frac{x}{\sqrt{a + b\sqrt{\frac{d}{x}} + \frac{c}{x}}} dx$$

Optimal antiderivative

$$\frac{(35b^4d^2 - 120ab^2cd + 48a^2c^2) \operatorname{arctanh} \left( \frac{2a+b\sqrt{\frac{d}{x}}}{2\sqrt{a} \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}} \right)}{64a^{\frac{9}{2}}} - \frac{7bd^2 \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}{12a^2 \left(\frac{d}{x}\right)^{\frac{3}{2}}} - \frac{(-35b^2d + 36ac) x \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}{48a^3} + \frac{x^2 \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}{2a} + \frac{5bd(-21b^2d + 44ac) \sqrt{a + \frac{c}{x} + b\sqrt{\frac{d}{x}}}}{96a^4 \sqrt{\frac{d}{x}}}$$

command

```
integrate(x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{ad^2x + \sqrt{dx} bd^2 + cd^2} \left( 2 \sqrt{dx} \left( 4 \sqrt{dx} \left( \frac{7b}{a^2} - \frac{6 \sqrt{dx}}{ad} \right) - \frac{35 ab^2 d^2 - 36 a^2 cd}{a^4 d} \right) + \frac{5 (21 b^3 d^3 - 44 abcd^2)}{a^4 d} \right) + \frac{3 (35 b^4 d^4}{$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 8.63 Problem number 3064

$$\int \frac{1}{\sqrt{a + b \sqrt{\frac{d}{x}} + \frac{c}{x}}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{2a + b \sqrt{\frac{d}{x}}}{2\sqrt{a} \sqrt{a + \frac{c}{x} + b \sqrt{\frac{d}{x}}}} \right)}{\sqrt{a}}$$

command

`integrate(1/x/(a+c/x+b*(d/x)^(1/2))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{d} \left( \frac{\sqrt{ad} \log \left( \left| -\sqrt{ad} bd - 2 \left( \sqrt{ad} \sqrt{dx} - \sqrt{ad^2x + \sqrt{dx} bd^2 + cd^2} \right) a \right| \right)}{ad} - \frac{\sqrt{ad} \log \left( \left| -\sqrt{ad} bd + 2 \sqrt{cd^2} a \right| \right)}{ad} \right)}{\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9 Test file number 27

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.3\_General/27\_1.1.3.4-e\_x-  
 $\hat{m}-a+b_x\hat{n}-\hat{p}-c+d_x\hat{n}-\hat{q}$

### 9.1 Problem number 530

$$\int \sqrt{ex} (a + bx^3)^{3/2} (A + Bx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(6Ab - aB)(ex)^{\frac{3}{2}}(bx^3 + a)^{\frac{3}{2}}}{36be} + \frac{B(ex)^{\frac{3}{2}}(bx^3 + a)^{\frac{5}{2}}}{9be} \\ & + \frac{a^2(6Ab - aB) \operatorname{arctanh}\left(\frac{(ex)^{\frac{3}{2}}\sqrt{b}}{e^{\frac{3}{2}}\sqrt{bx^3 + a}}\right) \sqrt{e}}{24b^{\frac{3}{2}}} + \frac{a(6Ab - aB)(ex)^{\frac{3}{2}}\sqrt{bx^3 + a}}{24be} \end{aligned}$$

command

`integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{72} \left( 6\sqrt{bx^3 + a} \left(2x^3 + \frac{a}{b}\right) Bax^{\frac{3}{2}} + 6\sqrt{bx^3 + a} \left(2x^3 + \frac{a}{b}\right) Abx^{\frac{3}{2}} + \left(2\left(4x^3 + \frac{a}{b}\right)x^3 - \frac{3a^2}{b^2}\right) \sqrt{bx^3 + a} Bbx^{\frac{3}{2}} + \right. \\ & \left. (B^2a^6 + 4ABA^5b + 4A^2a^4b^2)e^{\frac{1}{2}} \log\left(\left|(Ba^3x^{\frac{3}{2}} + 2Aa^2bx^{\frac{3}{2}}\right)\sqrt{b} + \sqrt{B^2a^7 + 4ABA^6b + 4A^2a^5b^2 + (Ba^3x^{\frac{3}{2}} + 2Aa^2bx^{\frac{3}{2}})^2}\right|\right) \right) \\ & \frac{1}{24b^{\frac{3}{2}}|Ba^3 + 2Aa^2b|} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 9.2 Problem number 538

$$\int \sqrt{ex} (a + bx^3)^{5/2} (A + Bx^3) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5a(8Ab - aB)(ex)^{\frac{3}{2}}(bx^3 + a)^{\frac{3}{2}}}{288be} + \frac{(8Ab - aB)(ex)^{\frac{3}{2}}(bx^3 + a)^{\frac{5}{2}}}{72be} + \frac{B(ex)^{\frac{3}{2}}(bx^3 + a)^{\frac{7}{2}}}{12be} \\ & + \frac{5a^3(8Ab - aB) \operatorname{arctanh}\left(\frac{(ex)^{\frac{3}{2}}\sqrt{b}}{e^{\frac{3}{2}}\sqrt{bx^3 + a}}\right) \sqrt{e}}{192b^{\frac{3}{2}}} + \frac{5a^2(8Ab - aB)(ex)^{\frac{3}{2}}\sqrt{bx^3 + a}}{192be} \end{aligned}$$

command

```
integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{576} \left( 48 \sqrt{bx^3 + a} \left( 2x^3 + \frac{a}{b} \right) Ba^2 x^{\frac{3}{2}} + 96 \sqrt{bx^3 + a} \left( 2x^3 + \frac{a}{b} \right) Aabx^{\frac{3}{2}} + 16 \left( 2 \left( 4x^3 + \frac{a}{b} \right) x^3 - \frac{3a^2}{b^2} \right) \sqrt{bx^3 + a} \right. \\ \left. (25B^2a^8 + 240ABa^7b + 576A^2a^6b^2) e^{\frac{1}{2}} \log \left( \left| \left( 5Ba^4x^{\frac{3}{2}} + 24Aa^3bx^{\frac{3}{2}} \right) \sqrt{b} + \sqrt{25B^2a^9 + 240ABa^8b + 576A^2a^6b^2} \right| \right) \right. \\ \left. \right) / 192 b^{\frac{3}{2}} |5Ba^4 + 24Aa^3b|$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 9.3 Problem number 964

$$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x} dx$$

Optimal antiderivative

$$\frac{a \operatorname{arctanh} \left( \frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{\sqrt{c}} - \frac{b \sqrt{c + \frac{d}{x^2}}}{d}$$

command

```
integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{a \log \left( \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 \right)}{2 \sqrt{c} \operatorname{sgn}(x)} + \frac{2b\sqrt{c}}{\left( \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 - d \right) \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



#### 9.4 Problem number 968

$$\int \frac{\left(a + \frac{b}{x^2}\right) x^4}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal antiderivative

$$-\frac{2d(-4ad + 5bc)x\sqrt{c + \frac{d}{x^2}}}{15c^3} + \frac{(-4ad + 5bc)x^3\sqrt{c + \frac{d}{x^2}}}{15c^2} + \frac{ax^5\sqrt{c + \frac{d}{x^2}}}{5c}$$

command

```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(5bcd^{\frac{3}{2}} - 4ad^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15c^3} - \frac{(bcd - ad^2)\sqrt{cx^2 + d}}{c^3\operatorname{sgn}(x)} + \frac{3(cx^2 + d)^{\frac{5}{2}}a + 5(cx^2 + d)^{\frac{3}{2}}bc - 10(cx^2 + d)^{\frac{3}{2}}ad}{15c^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 9.5 Problem number 969

$$\int \frac{\left(a + \frac{b}{x^2}\right) x^2}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal antiderivative

$$\frac{(-2ad + 3bc)x\sqrt{c + \frac{d}{x^2}}}{3c^2} + \frac{ax^3\sqrt{c + \frac{d}{x^2}}}{3c}$$

command

```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(3bc\sqrt{d} - 2ad^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3c^2} + \frac{(cx^2 + d)^{\frac{3}{2}}a}{3c^2\operatorname{sgn}(x)} + \frac{\sqrt{cx^2 + d}(bc - ad)}{c^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9.6 Problem number 970

$$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{\sqrt{d}} + \frac{ax\sqrt{c + \frac{d}{x^2}}}{c}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(bc \operatorname{arctan}\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + a\sqrt{-d}\sqrt{d}\right)\operatorname{sgn}(x)}{c\sqrt{-d}} + \frac{b \operatorname{arctan}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{-d}}\right)}{\sqrt{-d}} + \frac{\sqrt{cx^2 + d} a}{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9.7 Problem number 971

$$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^2} dx$$

Optimal antiderivative

$$\frac{(-2ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{2d^{\frac{3}{2}}} - \frac{b\sqrt{c + \frac{d}{x^2}}}{2dx}$$

command

`integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(bc^2-2acd) \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d} + \frac{\sqrt{cx^2+d}bc}{dx^2}}{2 \operatorname{csgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9.8 Problem number 972

$$\int \frac{a + \frac{b}{x^2}}{\sqrt{c + \frac{d}{x^2}} x^4} dx$$

Optimal antiderivative

$$\frac{c(-4ad + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{\frac{5}{2}}} - \frac{b\sqrt{c + \frac{d}{x^2}}}{4dx^3} + \frac{(-4ad + 3bc)\sqrt{c + \frac{d}{x^2}}}{8d^2x}$$

command

`integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(3bc^3-4ac^2d) \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{3(cx^2+d)^{\frac{3}{2}}bc^3-4(cx^2+d)^{\frac{3}{2}}ac^2d-5\sqrt{cx^2+d}bc^3d+4\sqrt{cx^2+d}ac^2d^2}{c^2d^2x^4}}{8 \operatorname{csgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.9 Problem number 973

$$\int \frac{\left(a + \frac{b}{x^2}\right) x^3}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3d(-5ad + 4bc) \operatorname{arctanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{\frac{7}{2}}} + \frac{3d(-5ad + 4bc)}{8c^3 \sqrt{c + \frac{d}{x^2}}} + \frac{(-5ad + 4bc) x^2}{8c^2 \sqrt{c + \frac{d}{x^2}}} + \frac{a x^4}{4c \sqrt{c + \frac{d}{x^2}}}$$

command

```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(x^2 \left(\frac{2ax^2}{c \operatorname{sgn}(x)} + \frac{4bc^4 \operatorname{sgn}(x) - 5ac^3 d \operatorname{sgn}(x)}{c^5}\right) + \frac{3(4bc^3 d \operatorname{sgn}(x) - 5ac^2 d^2 \operatorname{sgn}(x))}{c^5}\right) x}{8 \sqrt{cx^2 + d}} - \frac{3(4bcd \log(|d|) - 5ad^2 \log(|d|)) \operatorname{sgn}(x)}{16c^{\frac{7}{2}}} + \frac{3(4bcd - 5ad^2) \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + d}\right|\right)}{8c^{\frac{7}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.10 Problem number 974

$$\int \frac{\left(a + \frac{b}{x^2}\right) x}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-3ad + 2bc) \operatorname{arctanh}\left(\frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{\frac{5}{2}}} + \frac{3ad - 2bc}{2c^2 \sqrt{c + \frac{d}{x^2}}} + \frac{a x^2}{2c \sqrt{c + \frac{d}{x^2}}}$$

command

```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x \left( \frac{ax^2}{c \operatorname{sgn}(x)} - \frac{2bc^2 \operatorname{sgn}(x) - 3acd \operatorname{sgn}(x)}{c^3} \right)}{2 \sqrt{cx^2 + d}} + \frac{(2bc \log(|d|) - 3ad \log(|d|)) \operatorname{sgn}(x)}{4c^{\frac{5}{2}}} - \frac{(2bc - 3ad) \log \left( \left| -\sqrt{c}x + \sqrt{cx^2 + d} \right| \right)}{2c^{\frac{5}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9.11 Problem number 975

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x} dx$$

Optimal antiderivative

$$\frac{a \operatorname{arctanh} \left( \frac{\sqrt{c + \frac{d}{x^2}}}{\sqrt{c}} \right)}{c^{\frac{3}{2}}} + \frac{-ad + bc}{cd \sqrt{c + \frac{d}{x^2}}}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log(|d|) \operatorname{sgn}(x)}{2c^{\frac{3}{2}}} + \frac{(bc \operatorname{sgn}(x) - ad \operatorname{sgn}(x))x}{\sqrt{cx^2 + d} cd} - \frac{a \log \left( \left| -\sqrt{c}x + \sqrt{cx^2 + d} \right| \right)}{c^{\frac{3}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.12 Problem number 977

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^5} dx$$

Optimal antiderivative

$$-\frac{b\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3} + \frac{c(-ad + bc)}{d^3 \sqrt{c + \frac{d}{x^2}}} + \frac{(-ad + 2bc) \sqrt{c + \frac{d}{x^2}}}{d^3}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(bc^2 - acd)x}{\sqrt{cx^2 + d} d^3 \operatorname{sgn}(x)} - \frac{2 \left( 3 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 bc^{\frac{3}{2}} - 3 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 a \sqrt{c} d - 12 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 bc^{\frac{3}{2}} d + 6 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 d^2 \operatorname{sgn}(x) \right)}{3 \left( \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 - d \right)^3 d^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^5} dx$$

### 9.13 Problem number 978

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^7} dx$$

Optimal antiderivative

$$\frac{(-ad + 3bc) \left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^4} - \frac{b\left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4} - \frac{c^2(-ad + bc)}{d^4 \sqrt{c + \frac{d}{x^2}}} - \frac{c(-2ad + 3bc) \sqrt{c + \frac{d}{x^2}}}{d^4}$$

command

`integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(bc^3 - ac^2d)x}{\sqrt{cx^2 + d} d^4 \operatorname{sgn}(x)} + \frac{2 \left( 15 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^8 bc^{\frac{5}{2}} - 15 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^8 ac^{\frac{3}{2}} d - 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^6 bc^{\frac{5}{2}} d + 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^6 ac^{\frac{3}{2}} d - 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 bc^{\frac{5}{2}} d + 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 ac^{\frac{3}{2}} d - 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 bc^{\frac{5}{2}} d + 90 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^2 ac^{\frac{3}{2}} d - 90 bc^{\frac{5}{2}} d + 90 ac^{\frac{3}{2}} d \right)}{d^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^7} dx$$

### 9.14 Problem number 979

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^9} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c(-ad + 2bc) \left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^5} + \frac{(-ad + 4bc) \left(c + \frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^5} \\ & -\frac{b \left(c + \frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5} + \frac{c^3(-ad + bc)}{d^5 \sqrt{c + \frac{d}{x^2}}} + \frac{c^2(-3ad + 4bc) \sqrt{c + \frac{d}{x^2}}}{d^5} \end{aligned}$$

command

`integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(bc^4 - ac^3d)x}{\sqrt{cx^2 + d} d^5 \operatorname{sgn}(x)} + \frac{2 \left( 35 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^{12} bc^{\frac{7}{2}} - 35 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^{12} ac^{\frac{5}{2}} d - 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^{10} bc^{\frac{7}{2}} d + 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^{10} ac^{\frac{5}{2}} d - 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^8 bc^{\frac{7}{2}} d + 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^8 ac^{\frac{5}{2}} d - 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^6 bc^{\frac{7}{2}} d + 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^6 ac^{\frac{5}{2}} d - 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 bc^{\frac{7}{2}} d + 280 \left( \sqrt{c} x - \sqrt{cx^2 + d} \right)^4 ac^{\frac{5}{2}} d - 280 bc^{\frac{7}{2}} d + 280 ac^{\frac{5}{2}} d \right)}{d^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}} x^9} dx$$

### 9.15 Problem number 980

$$\int \frac{\left(a + \frac{b}{x^2}\right) x^4}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4d(-6ad + 5bc)x}{15c^3\sqrt{c + \frac{d}{x^2}}} + \frac{(-6ad + 5bc)x^3}{15c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^5}{5c\sqrt{c + \frac{d}{x^2}}} - \frac{8d(-6ad + 5bc)x\sqrt{c + \frac{d}{x^2}}}{15c^4}$$

command

```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8(5bcd^2 - 6ad^3)\operatorname{sgn}(x)}{15c^4\sqrt{d}} - \frac{bcd^2 - ad^3}{\sqrt{cx^2 + d}c^4\operatorname{sgn}(x)} + \frac{3(cx^2 + d)^{\frac{5}{2}}ac^{16} + 5(cx^2 + d)^{\frac{3}{2}}bc^{17} - 15(cx^2 + d)^{\frac{3}{2}}ac^{16}d - 30\sqrt{cx^2 + d}bc^{17}d + 45\sqrt{cx^2 + d}ac^{16}d^2}{15c^{20}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.16 Problem number 981

$$\int \frac{\left(a + \frac{b}{x^2}\right) x^2}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(-4ad + 3bc)x}{3c^2\sqrt{c + \frac{d}{x^2}}} + \frac{ax^3}{3c\sqrt{c + \frac{d}{x^2}}} + \frac{2(-4ad + 3bc)x\sqrt{c + \frac{d}{x^2}}}{3c^3}$$

command

```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$-\frac{2(3bcd - 4ad^2)\operatorname{sgn}(x)}{3c^3\sqrt{d}} + \frac{bcd - ad^2}{\sqrt{cx^2 + d}c^3\operatorname{sgn}(x)} + \frac{(cx^2 + d)^{\frac{3}{2}}ac^6 + 3\sqrt{cx^2 + d}bc^7 - 6\sqrt{cx^2 + d}ac^6d}{3c^9\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.17 Problem number 982

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2ad - bc}{c^2x\sqrt{c + \frac{d}{x^2}}} + \frac{ax}{c\sqrt{c + \frac{d}{x^2}}}$$

command

`integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(bc - 2ad)\operatorname{sgn}(x)}{c^2\sqrt{d}} + \frac{\sqrt{cx^2 + d}a}{c^2\operatorname{sgn}(x)} - \frac{bc - ad}{\sqrt{cx^2 + d}c^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.18 Problem number 983

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2}x^2} dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{d^{\frac{3}{2}}} + \frac{-ad + bc}{cdx\sqrt{c + \frac{d}{x^2}}}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d} d \operatorname{sgn}(x)} - \frac{\left(bc\sqrt{d} \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + bc\sqrt{-d} - a\sqrt{-d} d\right) \operatorname{sgn}(x)}{c\sqrt{-d} d^{\frac{3}{2}}} + \frac{bc - ad}{\sqrt{cx^2+d} cd \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 9.19 Problem number 984

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^4} dx$$

Optimal antiderivative

$$\frac{(-2ad + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{2d^{\frac{5}{2}}} - \frac{b}{2d x^3 \sqrt{c + \frac{d}{x^2}}} + \frac{2ad - 3bc}{2d^2 x \sqrt{c + \frac{d}{x^2}}}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(3bc - 2ad) \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{2\sqrt{-d} d^2 \operatorname{sgn}(x)} - \frac{3(cx^2+d)bc - 2(cx^2+d)ad - 2bcd + 2ad^2}{2\left((cx^2+d)^{\frac{3}{2}} - \sqrt{cx^2+d} d\right) d^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 9.20 Problem number 985

$$\int \frac{a + \frac{b}{x^2}}{\left(c + \frac{d}{x^2}\right)^{3/2} x^6} dx$$

Optimal antiderivative

$$\frac{3c(-4ad + 5bc) \operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c + \frac{d}{x^2}}}\right)}{8d^{\frac{7}{2}}} - \frac{b}{4dx^5\sqrt{c + \frac{d}{x^2}}} + \frac{4ad - 5bc}{4d^2x^3\sqrt{c + \frac{d}{x^2}}} + \frac{3(-4ad + 5bc)\sqrt{c + \frac{d}{x^2}}}{8d^3x}$$

command

```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(5bc^2 - 4acd) \operatorname{arctan}\left(\frac{\sqrt{cx^2 + d}}{\sqrt{-d}}\right)}{8\sqrt{-d}d^3\operatorname{sgn}(x)} + \frac{bc^2 - acd}{\sqrt{cx^2 + d}d^3\operatorname{sgn}(x)} + \frac{7(cx^2 + d)^{\frac{3}{2}}bc^2 - 4(cx^2 + d)^{\frac{3}{2}}acd - 9\sqrt{cx^2 + d}bc^2d + 4\sqrt{cx^2 + d}acd^2}{8c^2d^3x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 10 Test file number 28

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.3_General/28_1.1.3.6-g_x-
~m-a+b_x^n-~p-c+d_x^n-~q-e+f_x^n-~r
```

## 10.1 Problem number 15

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 c^2 (aBc + 3A(ad + bc)) x^{1+n} (ex)^m}{1 + m + n} \\ & + \frac{3ac(aBc(ad + bc) + A(a^2 d^2 + 3abcd + b^2 c^2)) x^{1+2n} (ex)^m}{1 + m + 2n} \\ & + \frac{(3aBc(a^2 d^2 + 3abcd + b^2 c^2) + A(a^3 d^3 + 9a^2 bc d^2 + 9a b^2 c^2 d + b^3 c^3)) x^{1+3n} (ex)^m}{1 + m + 3n} \\ & + \frac{(a^3 B d^3 + 9a b^2 cd(Ad + Bc) + 3a^2 b d^2(Ad + 3Bc) + b^3 c^2(3Ad + Bc)) x^{1+4n} (ex)^m}{1 + m + 4n} \\ & + \frac{3bd(a^2 B d^2 + b^2 c(Ad + Bc) + abd(Ad + 3Bc)) x^{1+5n} (ex)^m}{1 + m + 5n} \\ & + \frac{b^2 d^2 (Abd + 3aBd + 3bBc) x^{1+6n} (ex)^m}{1 + m + 6n} + \frac{b^3 B d^3 x^{1+7n} (ex)^m}{1 + m + 7n} + \frac{a^3 A c^3 (ex)^{1+m}}{e(1 + m)} \end{aligned}$$

command

```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 11 Test file number 30

Test folder name:

```
test_cases/1_Algebraic_functions/1.1_Binomial_products/1.1.4_Improper/30_1.1.4.2-c_x-
^m-a_x^j+b_x^n^p
```

## 11.1 Problem number 98

$$\int \frac{1}{\sqrt{ax + bx^4}} dx$$

Optimal antiderivative

$$x \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right) \sqrt{\frac{\left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}} \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right) \text{EllipticF} \left( \sqrt{1 - \frac{\left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right)^2}{\left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}, \frac{\sqrt{6}}{4} \right) \\ \frac{3 \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 - \sqrt{3}) \right) a^{\frac{1}{3}} \sqrt{bx^4 + ax} \sqrt{\frac{b^{\frac{1}{3}} x \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x \right)}{\left( a^{\frac{1}{3}} + b^{\frac{1}{3}} x (1 + \sqrt{3}) \right)^2}}}{1}$$

command

`integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} \sqrt{bx^4 + ax} x - \frac{a \arctan \left( \frac{\sqrt{b + \frac{a}{x^3}}}{\sqrt{-b}} \right)}{3 \sqrt{-b}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{bx^4 + ax}} dx$$

## 11.2 Problem number 110

$$\int \frac{x^3}{(b\sqrt{x} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\frac{693b^5 \operatorname{arctanh} \left( \frac{\sqrt{a} \sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{64a^{\frac{13}{2}}} - \frac{4x^3}{a\sqrt{b\sqrt{x} + ax}} \\ + \frac{693b^4 \sqrt{b\sqrt{x} + ax}}{64a^6} + \frac{231b^2 x \sqrt{b\sqrt{x} + ax}}{40a^4} - \frac{99b x^{\frac{3}{2}} \sqrt{b\sqrt{x} + ax}}{20a^3} \\ + \frac{22x^2 \sqrt{b\sqrt{x} + ax}}{5a^2} - \frac{231b^3 \sqrt{x} \sqrt{b\sqrt{x} + ax}}{32a^5}$$

command

```
integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{320} \sqrt{ax + b\sqrt{x}} \left( 2 \left( 4 \left( 2 \sqrt{x} \left( \frac{8\sqrt{x}}{a^2} - \frac{19b}{a^3} \right) + \frac{71b^2}{a^4} \right) \sqrt{x} - \frac{515b^3}{a^5} \right) \sqrt{x} + \frac{2185b^4}{a^6} \right) \\ & + \frac{693b^5 \log \left( \left| -2\sqrt{a} \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{128a^{\frac{13}{2}}} \\ & + \frac{4b^6}{\left( a \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + \sqrt{a}b \right) a^6} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 11.3 Problem number 111

$$\int \frac{x^2}{(b\sqrt{x} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{35b^3 \operatorname{arctanh} \left( \frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}} \right)}{4a^{\frac{9}{2}}} - \frac{4x^2}{a\sqrt{b\sqrt{x} + ax}} \\ & + \frac{35b^2 \sqrt{b\sqrt{x} + ax}}{4a^4} + \frac{14x \sqrt{b\sqrt{x} + ax}}{3a^2} - \frac{35b\sqrt{x} \sqrt{b\sqrt{x} + ax}}{6a^3} \end{aligned}$$

command

```
integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{12} \sqrt{ax + b\sqrt{x}} \left( 2 \sqrt{x} \left( \frac{4\sqrt{x}}{a^2} - \frac{11b}{a^3} \right) + \frac{57b^2}{a^4} \right) \\ & + \frac{35b^3 \log \left( \left| -2\sqrt{a} \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right| \right)}{8a^{\frac{9}{2}}} \\ & + \frac{4b^4}{\left( a \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + \sqrt{a}b \right) a^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 11.4 Problem number 124

$$\int \frac{x^{5/2}}{(b\sqrt{x} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{315b^4 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{\frac{11}{2}}} - \frac{4x^{\frac{5}{2}}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} \\ & - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} \end{aligned}$$

command

`integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{32} \sqrt{ax + b\sqrt{x}} \left( 2 \left( 4\sqrt{x} \left( \frac{2\sqrt{x}}{a^2} - \frac{5b}{a^3} \right) + \frac{41b^2}{a^4} \right) \sqrt{x} - \frac{187b^3}{a^5} \right) \\ & - \frac{315b^4 \log\left(\left| -2\sqrt{a} \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) - b \right|\right)}{64a^{\frac{11}{2}}} \\ & - \frac{4b^5}{\left( a \left( \sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}} \right) + \sqrt{a}b \right) a^5} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 11.5 Problem number 125

$$\int \frac{x^{3/2}}{(b\sqrt{x} + ax)^{3/2}} dx$$

Optimal antiderivative

$$\frac{15b^2 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x} + ax}}\right)}{2a^{7/2}} - \frac{4x^{3/2}}{a\sqrt{b\sqrt{x} + ax}} - \frac{15b\sqrt{b\sqrt{x} + ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x} + ax}}{a^2}$$

command

```
integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \sqrt{ax + b\sqrt{x}} \left( \frac{2\sqrt{x}}{a^2} - \frac{7b}{a^3} \right) - \frac{15b^2 \log\left(\left| -2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b \right|\right)}{4a^{7/2}} - \frac{\left(a\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) + \sqrt{a}b\right)a^3}{4b^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 11.6 Problem number 252

$$\int \frac{x^4}{\sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$\frac{16a^2\sqrt{bx^3 + ax^2}}{35b^3} - \frac{32a^3\sqrt{bx^3 + ax^2}}{35b^4x} - \frac{12ax\sqrt{bx^3 + ax^2}}{35b^2} + \frac{2x^2\sqrt{bx^3 + ax^2}}{7b}$$

command

```
integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32a^{7/2}\operatorname{sgn}(x)}{35b^4} + \frac{2\left(5(bx + a)^{7/2} - 21(bx + a)^{5/2}a + 35(bx + a)^{3/2}a^2 - 35\sqrt{bx + a}a^3\right)}{35b^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\sqrt{bx^3 + ax^2}} dx$$



### 11.7 Problem number 253

$$\int \frac{x^3}{\sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$-\frac{8a\sqrt{bx^3 + ax^2}}{15b^2} + \frac{16a^2\sqrt{bx^3 + ax^2}}{15b^3x} + \frac{2x\sqrt{bx^3 + ax^2}}{5b}$$

command

`integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{16a^{\frac{5}{2}}\operatorname{sgn}(x)}{15b^3} + \frac{2\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}a^2\right)}{15b^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\sqrt{bx^3 + ax^2}} dx$$

### 11.8 Problem number 254

$$\int \frac{x^2}{\sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^3 + ax^2}}{3b} - \frac{4a\sqrt{bx^3 + ax^2}}{3b^2x}$$

command

`integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^{\frac{3}{2}}\operatorname{sgn}(x)}{3b^2} + \frac{2\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)}{3b^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^2}{\sqrt{bx^3 + ax^2}} dx$$

### 11.9 Problem number 257

$$\int \frac{1}{x\sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{bx^3 + ax^2}}{ax^2}$$

command

```
integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a} + \frac{\sqrt{bx+a} b}{ax} - \frac{b \operatorname{sgn}(x)}{b \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 11.10 Problem number 258

$$\int \frac{1}{x^2\sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$-\frac{3b^2 \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{4a^{\frac{5}{2}}} - \frac{\sqrt{bx^3 + ax^2}}{2x^3 a} + \frac{3b\sqrt{bx^3 + ax^2}}{4a^2 x^2}$$

command

```
integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3b^3 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^2} + \frac{3(bx+a)^{\frac{3}{2}} b^3 - 5\sqrt{bx+a} ab^3}{a^2 b^2 x^2} - \frac{b \operatorname{sgn}(x)}{4 b \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 11.11 Problem number 259

$$\int \frac{1}{x^3 \sqrt{ax^2 + bx^3}} dx$$

Optimal antiderivative

$$\frac{5b^3 \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{8a^{\frac{7}{2}}} - \frac{\sqrt{bx^3 + ax^2}}{3ax^4} + \frac{5b\sqrt{bx^3 + ax^2}}{12a^2x^3} - \frac{5b^2\sqrt{bx^3 + ax^2}}{8a^3x^2}$$

command

```
integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{15b^4 \operatorname{arctan}\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15(bx+a)^{\frac{5}{2}}b^4 - 40(bx+a)^{\frac{3}{2}}ab^4 + 33\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24b\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 11.12 Problem number 260

$$\int \frac{x^6}{(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2x^4}{b\sqrt{bx^3 + ax^2}} - \frac{16a\sqrt{bx^3 + ax^2}}{5b^3} + \frac{32a^2\sqrt{bx^3 + ax^2}}{5b^4x} + \frac{12x\sqrt{bx^3 + ax^2}}{5b^2}$$

command

```
integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{32a^{\frac{5}{2}}\operatorname{sgn}(x)}{5b^4} + \frac{2a^3}{\sqrt{bx+a}b^4\operatorname{sgn}(x)} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16} - 5(bx+a)^{\frac{3}{2}}ab^{16} + 15\sqrt{bx+a}a^2b^{16}\right)}{5b^{20}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^6}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

### 11.13 Problem number 261

$$\int \frac{x^5}{(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2x^3}{b\sqrt{bx^3 + ax^2}} + \frac{8\sqrt{bx^3 + ax^2}}{3b^2} - \frac{16a\sqrt{bx^3 + ax^2}}{3b^3x}$$

command

```
integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16a^{\frac{3}{2}}\operatorname{sgn}(x)}{3b^3} - \frac{2a^2}{\sqrt{bx+a}b^3\operatorname{sgn}(x)} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6 - 6\sqrt{bx+a}ab^6\right)}{3b^9\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^5}{(bx^3 + ax^2)^{\frac{3}{2}}} dx$$

### 11.14 Problem number 264

$$\int \frac{x^2}{(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{a^{\frac{3}{2}}} + \frac{2x}{a\sqrt{bx^3 + ax^2}}$$

command

```
integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\left(\sqrt{a}\operatorname{arctan}\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\right)\operatorname{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}} + \frac{2\operatorname{arctan}\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}(x)} + \frac{2}{\sqrt{bx+a}a\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 11.15 Problem number 265

$$\int \frac{x}{(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3b \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{a^{\frac{5}{2}}} + \frac{2}{a\sqrt{bx^3 + ax^2}} - \frac{3\sqrt{bx^3 + ax^2}}{a^2x^2}$$

command

```
integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3b \operatorname{arctan}\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a^2 \operatorname{sgn}(x)} - \frac{3(bx+a)b - 2ab}{\left((bx+a)^{\frac{3}{2}} - \sqrt{bx+a} a\right) a^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 11.16 Problem number 266

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{15b^2 \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{4a^{\frac{7}{2}}} + \frac{2}{ax\sqrt{bx^3 + ax^2}} - \frac{5\sqrt{bx^3 + ax^2}}{2a^2x^3} + \frac{15b\sqrt{bx^3 + ax^2}}{4a^3x^2}$$

command

```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15b^2 \operatorname{arctan}\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a} a^3 \operatorname{sgn}(x)} + \frac{2b^2}{\sqrt{bx+a} a^3 \operatorname{sgn}(x)} + \frac{7(bx+a)^{\frac{3}{2}}b^2 - 9\sqrt{bx+a} ab^2}{4a^3b^2x^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 11.17 Problem number 267

$$\int \frac{1}{x(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$\frac{35b^3 \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{8a^{\frac{9}{2}}} + \frac{2}{ax^2\sqrt{bx^3 + ax^2}} - \frac{7\sqrt{bx^3 + ax^2}}{3a^2x^4} + \frac{35b\sqrt{bx^3 + ax^2}}{12a^3x^3} - \frac{35b^2\sqrt{bx^3 + ax^2}}{8a^4x^2}$$

command

```
integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35b^3 \operatorname{arctan}\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^4\operatorname{sgn}(x)} - \frac{2b^3}{\sqrt{bx+a}a^4\operatorname{sgn}(x)} - \frac{57(bx+a)^{\frac{5}{2}}b^3 - 136(bx+a)^{\frac{3}{2}}ab^3 + 87\sqrt{bx+a}a^2b^3}{24a^4b^3x^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}}x} dx$$

## 11.18 Problem number 268

$$\int \frac{1}{x^2(ax^2 + bx^3)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{315b^4 \operatorname{arctanh}\left(\frac{x\sqrt{a}}{\sqrt{bx^3 + ax^2}}\right)}{64a^{\frac{11}{2}}} + \frac{2}{ax^3\sqrt{bx^3 + ax^2}} - \frac{9\sqrt{bx^3 + ax^2}}{4a^2x^5} + \frac{21b\sqrt{bx^3 + ax^2}}{8a^3x^4} - \frac{105b^2\sqrt{bx^3 + ax^2}}{32a^4x^3} + \frac{315b^3\sqrt{bx^3 + ax^2}}{64a^5x^2}$$

command

`integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{315 b^4 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64 \sqrt{-a} a^5 \operatorname{sgn}(x)} + \frac{2 b^4}{\sqrt{bx+a} a^5 \operatorname{sgn}(x)} + \frac{187 (bx+a)^{\frac{7}{2}} b^4 - 643 (bx+a)^{\frac{5}{2}} a b^4 + 765 (bx+a)^{\frac{3}{2}} a^2 b^4 - 325 \sqrt{bx+a} a^3 b^4}{64 a^5 b^4 x^4 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}} x^2} dx$$

### 11.19 Problem number 284

$$\int \frac{x^9}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{16a^2 \sqrt{bx^5 + ax^2}}{45b^3x} - \frac{8ax^2 \sqrt{bx^5 + ax^2}}{45b^2} + \frac{2x^5 \sqrt{bx^5 + ax^2}}{15b}$$

command

`integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{16 a^{\frac{5}{2}} \operatorname{sgn}(x)}{45 b^3} + \frac{2 \sqrt{bx^3+a} a^2}{3 b^3 \operatorname{sgn}(x)} + \frac{2 \left(3 (bx^3+a)^{\frac{5}{2}} - 10 (bx^3+a)^{\frac{3}{2}} a\right)}{45 b^3 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^9}{\sqrt{bx^5 + ax^2}} dx$$

## 11.20 Problem number 285

$$\int \frac{x^6}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$-\frac{4a\sqrt{bx^5 + ax^2}}{9b^2x} + \frac{2x^2\sqrt{bx^5 + ax^2}}{9b}$$

command

`integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^{\frac{3}{2}}\operatorname{sgn}(x)}{9b^2} + \frac{2(bx^3 + a)^{\frac{3}{2}}}{9b^2\operatorname{sgn}(x)} - \frac{2\sqrt{bx^3 + a}a}{3b^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^6}{\sqrt{bx^5 + ax^2}} dx$$

## 11.21 Problem number 286

$$\int \frac{x^3}{\sqrt{ax^2 + bx^5}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{bx^5 + ax^2}}{3bx}$$

command

`integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\sqrt{a}\operatorname{sgn}(x)}{3b} + \frac{2\sqrt{bx^3 + a}}{3b\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\sqrt{bx^5 + ax^2}} dx$$



## 11.22 Problem number 310

$$\int \frac{x^4}{\sqrt{ax^3 + bx^4}} dx$$

Optimal antiderivative

$$-\frac{5a^3 \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{bx^4 + x^3a}}\right)}{8b^{\frac{7}{2}}} - \frac{5a\sqrt{bx^4 + x^3a}}{12b^2} + \frac{5a^2\sqrt{bx^4 + x^3a}}{8b^3x} + \frac{x\sqrt{bx^4 + x^3a}}{3b}$$

command

```
integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{24} \sqrt{bx^2 + ax} \left( 2x \left( \frac{4x}{b\operatorname{sgn}(x)} - \frac{5a}{b^2\operatorname{sgn}(x)} \right) + \frac{15a^2}{b^3\operatorname{sgn}(x)} \right) - \frac{5a^3 \log(|a|)\operatorname{sgn}(x)}{16b^{\frac{7}{2}}} + \frac{5a^3 \log\left(\left| -2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a \right|\right)}{16b^{\frac{7}{2}}\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\sqrt{bx^4 + ax^3}} dx$$

## 11.23 Problem number 311

$$\int \frac{x^3}{\sqrt{ax^3 + bx^4}} dx$$

Optimal antiderivative

$$\frac{3a^2 \operatorname{arctanh}\left(\frac{x^2\sqrt{b}}{\sqrt{bx^4 + x^3a}}\right)}{4b^{\frac{5}{2}}} + \frac{\sqrt{bx^4 + x^3a}}{2b} - \frac{3a\sqrt{bx^4 + x^3a}}{4b^2x}$$

command

```
integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{1}{4} \sqrt{bx^2 + ax} \left( \frac{2x}{b \operatorname{sgn}(x)} - \frac{3a}{b^2 \operatorname{sgn}(x)} \right) + \frac{3a^2 \log(|a|) \operatorname{sgn}(x)}{8b^{\frac{5}{2}}}}{3a^2 \log \left( \left| -2 \left( \sqrt{b} x - \sqrt{bx^2 + ax} \right) \sqrt{b} - a \right| \right)} - \frac{1}{8b^{\frac{5}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\sqrt{bx^4 + ax^3}} dx$$

## 12 Test file number 31

Test folder name:

test\_cases/1\_Algebraic\_functions/1.1\_Binomial\_products/1.1.4\_Improper/31\_1.1.4.3-e\_x-  
 $\hat{m}-a_x^{\hat{j}}+b_x^{\hat{k}}-\hat{p}-c+d_x^{\hat{n}}-\hat{q}$

### 12.1 Problem number 139

$$\int \frac{x^6 (A + Bx^2)}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8b^2(-7Ac + 6bB) \sqrt{cx^4 + bx^2}}{105c^4x} + \frac{4b(-7Ac + 6bB)x \sqrt{cx^4 + bx^2}}{105c^3} \\ & -\frac{(-7Ac + 6bB)x^3 \sqrt{cx^4 + bx^2}}{35c^2} + \frac{Bx^5 \sqrt{cx^4 + bx^2}}{7c} \end{aligned}$$

command

`integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{8 \left( 6Bb^{\frac{7}{2}} - 7Ab^{\frac{5}{2}}c \right) \operatorname{sgn}(x)}{105c^4} - \frac{(Bb^3 - Ab^2c) \sqrt{cx^2 + b}}{c^4 \operatorname{sgn}(x)} \\ & + \frac{15 (cx^2 + b)^{\frac{7}{2}} B - 63 (cx^2 + b)^{\frac{5}{2}} Bb + 105 (cx^2 + b)^{\frac{3}{2}} Bb^2 + 21 (cx^2 + b)^{\frac{5}{2}} Ac - 70 (cx^2 + b)^{\frac{3}{2}} Abc}{105c^4 \operatorname{sgn}(x)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(Bx^2 + A)x^6}{\sqrt{cx^4 + bx^2}} dx$$

## 12.2 Problem number 140

$$\int \frac{x^4(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$\frac{2b(-5Ac+4bB)\sqrt{cx^4+bx^2}}{15c^3x} - \frac{(-5Ac+4bB)x\sqrt{cx^4+bx^2}}{15c^2} + \frac{Bx^3\sqrt{cx^4+bx^2}}{5c}$$

command

`integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\left(4Bb^{\frac{5}{2}}-5Ab^{\frac{3}{2}}c\right)\operatorname{sgn}(x)}{15c^3} + \frac{(Bb^2-Abc)\sqrt{cx^2+b}}{c^3\operatorname{sgn}(x)} \\ + \frac{3(cx^2+b)^{\frac{5}{2}}B-10(cx^2+b)^{\frac{3}{2}}Bb+5(cx^2+b)^{\frac{3}{2}}Ac}{15c^3\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(Bx^2+A)x^4}{\sqrt{cx^4+bx^2}} dx$$

## 12.3 Problem number 141

$$\int \frac{x^2(A+Bx^2)}{\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$-\frac{(-3Ac+2bB)\sqrt{cx^4+bx^2}}{3c^2x} + \frac{Bx\sqrt{cx^4+bx^2}}{3c}$$

command

`integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2Bb^{\frac{3}{2}}-3A\sqrt{b}c\right)\operatorname{sgn}(x)}{3c^2} + \frac{(cx^2+b)^{\frac{3}{2}}B}{3c^2\operatorname{sgn}(x)} - \frac{\sqrt{cx^2+b}(Bb-Ac)}{c^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(Bx^2+A)x^2}{\sqrt{cx^4+bx^2}} dx$$

## 12.4 Problem number 143

$$\int \frac{A + Bx^2}{x^2 \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{(-Ac + 2bB) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{2b^{\frac{3}{2}}} - \frac{A\sqrt{cx^4 + bx^2}}{2bx^3}$$

command

```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2Bbc - Ac^2) \operatorname{arctan}\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right) - \frac{\sqrt{cx^2 + b} Ac}{bx^2}}{\sqrt{-b} b} \frac{1}{2 \operatorname{csgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 12.5 Problem number 144

$$\int \frac{A + Bx^2}{x^4 \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{c(-3Ac + 4bB) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{8b^{\frac{5}{2}}} - \frac{A\sqrt{cx^4 + bx^2}}{4bx^5} - \frac{(-3Ac + 4bB) \sqrt{cx^4 + bx^2}}{8b^2x^3}$$

command

```
integrate((B*x^2+A)/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4Bbc^2 - 3Ac^3) \operatorname{arctan}\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right) + \frac{4(cx^2 + b)^{\frac{3}{2}} Bbc^2 - 4\sqrt{cx^2 + b} Bb^2c^2 - 3(cx^2 + b)^{\frac{3}{2}} Ac^3 + 5\sqrt{cx^2 + b} Abc^3}{b^2c^2x^4}}{\sqrt{-b} b^2} \frac{1}{8 \operatorname{csgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 12.6 Problem number 148

$$\int \frac{x^3(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{B \operatorname{arctanh}\left(\frac{x^2\sqrt{c}}{\sqrt{cx^4 + bx^2}}\right)}{c^{3/2}} - \frac{(-Ac + bB)x^2}{bc\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{B \log(|b|) \operatorname{sgn}(x)}{2c^{3/2}} - \frac{(Bb \operatorname{sgn}(x) - Ac \operatorname{sgn}(x))x}{\sqrt{cx^2 + b} bc} - \frac{B \log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right)}{c^{3/2} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 12.7 Problem number 150

$$\int \frac{A + Bx^2}{x(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{A}{3bx^2\sqrt{cx^4 + bx^2}} - \frac{(-4Ac + 3bB)(2cx^2 + b)}{3b^3\sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bbc - Ac^2)x}{\sqrt{cx^2 + b} b^3 \operatorname{sgn}(x)} + \frac{2\left(3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 Bb\sqrt{c} - 3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 Ac^{3/2} - 6\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 Bb^2\sqrt{c} + 12\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 b^2\sqrt{c}\right)}{3\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3 b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{3/2} x} dx$$

## 12.8 Problem number 151

$$\int \frac{A + Bx^2}{x^3 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{A}{5bx^4 \sqrt{cx^4 + bx^2}} + \frac{6Ac - 5bB}{15b^2x^2 \sqrt{cx^4 + bx^2}} + \frac{4c(-6Ac + 5bB)(2cx^2 + b)}{15b^4 \sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bbc^2 - Ac^3)x}{\sqrt{cx^2 + b} b^4 \operatorname{sgn}(x)} - \frac{2 \left( 15 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^8 Bbc^{\frac{3}{2}} - 15 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^8 Ac^{\frac{5}{2}} - 90 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^6 Bb^2c^{\frac{3}{2}} + 90 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^6 Bbc^{\frac{3}{2}} \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}} x^3} dx$$

## 12.9 Problem number 152

$$\int \frac{A + Bx^2}{x^5 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{A}{7bx^6 \sqrt{cx^4 + bx^2}} + \frac{8Ac - 7bB}{35b^2x^4 \sqrt{cx^4 + bx^2}} + \frac{2c(-8Ac + 7bB)}{35b^3x^2 \sqrt{cx^4 + bx^2}} - \frac{8c^2(-8Ac + 7bB)(2cx^2 + b)}{35b^5 \sqrt{cx^4 + bx^2}}$$

command

```
integrate((B*x^2+A)/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bbc^3 - Ac^4)x}{\sqrt{cx^2 + b} b^5 \operatorname{sgn}(x)} + \frac{2 \left( 35 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{12} Bbc^{\frac{5}{2}} - 35 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{12} Ac^{\frac{7}{2}} - 280 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{10} Bb^2 c^{\frac{5}{2}} + 280 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{10} Ac^{\frac{7}{2}} \right)}{15c^{20} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}} x^5} dx$$

### 12.10 Problem number 153

$$\int \frac{x^8(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-Ac + bB)x^7}{bc\sqrt{cx^4 + bx^2}} + \frac{8b(-5Ac + 6bB)\sqrt{cx^4 + bx^2}}{15c^4x} \\ & -\frac{4(-5Ac + 6bB)x\sqrt{cx^4 + bx^2}}{15c^3} + \frac{(-5Ac + 6bB)x^3\sqrt{cx^4 + bx^2}}{5bc^2} \end{aligned}$$

command

`integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{8(6Bb^3 - 5Ab^2c)\operatorname{sgn}(x)}{15\sqrt{b}c^4} + \frac{Bb^3 - Ab^2c}{\sqrt{cx^2 + b}c^4\operatorname{sgn}(x)} \\ & + \frac{3(cx^2 + b)^{\frac{5}{2}}Bc^{16} - 15(cx^2 + b)^{\frac{3}{2}}Bbc^{16} + 45\sqrt{cx^2 + b}Bb^2c^{16} + 5(cx^2 + b)^{\frac{3}{2}}Ac^{17} - 30\sqrt{cx^2 + b}Abc^{17}}{15c^{20}\operatorname{sgn}(x)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(Bx^2 + A)x^8}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

### 12.11 Problem number 154

$$\int \frac{x^6 (A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(-Ac + bB)x^5}{bc\sqrt{cx^4 + bx^2}} - \frac{2(-3Ac + 4bB)\sqrt{cx^4 + bx^2}}{3c^3x} + \frac{(-3Ac + 4bB)x\sqrt{cx^4 + bx^2}}{3bc^2}$$

command

```
integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(4Bb^2 - 3Abc)\operatorname{sgn}(x)}{3\sqrt{b}c^3} - \frac{Bb^2 - Abc}{\sqrt{cx^2 + b}c^3\operatorname{sgn}(x)} + \frac{(cx^2 + b)^{\frac{3}{2}}Bc^6 - 6\sqrt{cx^2 + b}Bbc^6 + 3\sqrt{cx^2 + b}Ac^7}{3c^9\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(Bx^2 + A)x^6}{(cx^4 + bx^2)^{\frac{3}{2}}} dx$$

### 12.12 Problem number 156

$$\int \frac{x^2(A + Bx^2)}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{A \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{b^{\frac{3}{2}}} - \frac{(-Ac + bB)x}{bc\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{A \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} b \operatorname{sgn}(x)} - \frac{\left(A\sqrt{b} c \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) - B\sqrt{-b} b + A\sqrt{-b} c\right) \operatorname{sgn}(x)}{\sqrt{-b} b^{\frac{3}{2}} c} - \frac{Bb - Ac}{\sqrt{cx^2+b} b c \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 12.13 Problem number 157

$$\int \frac{A + Bx^2}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(-3Ac + 2bB) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{2b^{\frac{5}{2}}} - \frac{B}{3cx\sqrt{cx^4 + bx^2}} + \frac{3Ac - 2bB}{3bcx\sqrt{cx^4 + bx^2}} + \frac{(-3Ac + 2bB)\sqrt{cx^4 + bx^2}}{2b^2cx^3}$$

command

`integrate((B*x^2+A)/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2Bb - 3Ac) \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b} b^2 \operatorname{sgn}(x)} + \frac{2(cx^2+b)Bb - 2Bb^2 - 3(cx^2+b)Ac + 2Abc}{2\left((cx^2+b)^{\frac{3}{2}} - \sqrt{cx^2+b} b\right) b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 12.14 Problem number 158

$$\int \frac{A + Bx^2}{x^2 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3c(-5Ac + 4bB) \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{8b^{\frac{7}{2}}} - \frac{A}{4bx^3\sqrt{cx^4 + bx^2}} + \frac{-5Ac + 4bB}{4b^2x\sqrt{cx^4 + bx^2}} - \frac{3(-5Ac + 4bB)\sqrt{cx^4 + bx^2}}{8b^3x^3}$$

command

```
integrate((B*x^2+A)/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3(4Bbc - 5Ac^2) \arctan\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^3\operatorname{sgn}(x)} - \frac{Bbc - Ac^2}{\sqrt{cx^2 + b}b^3\operatorname{sgn}(x)} - \frac{4(cx^2 + b)^{\frac{3}{2}}Bbc - 4\sqrt{cx^2 + b}Bb^2c - 7(cx^2 + b)^{\frac{3}{2}}Ac^2 + 9\sqrt{cx^2 + b}Abc^2}{8b^3c^2x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{Bx^2 + A}{(cx^4 + bx^2)^{\frac{3}{2}}x^2} dx$$

## 13 Test file number 32

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/32_1.2.1.1-a+b_x+c_x^2-^p
```

### 13.1 Problem number 17

$$\int \frac{1}{\sqrt{3ix + 4x^2}} dx$$

Optimal antiderivative

$$-\frac{i \arcsin\left(-1 + \frac{8ix}{3}\right)}{2}$$

command

```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{32} \sqrt{8x^2 + 2\sqrt{16x^2 + 9}} x (8x + 3i) \left( \frac{3ix}{4x^2 + \sqrt{16x^4 + 9x^2}} + 1 \right) \\ & - \frac{9}{64} \log \left( 2 \sqrt{8x^2 + 2\sqrt{16x^2 + 9}} x \left( \frac{3ix}{4x^2 + \sqrt{16x^4 + 9x^2}} + 1 \right) - 8x - 3i \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 13.2 Problem number 18

$$\int \frac{1}{(3ix + 4x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2i}{3} + \frac{16x}{9}}{\sqrt{4x^2 + 3ix}}$$

command

```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{8x^2 + 2\sqrt{16x^2 + 9}} x (8x + 3i) \left( \frac{3ix}{4x^2 + \sqrt{16x^4 + 9x^2}} + 1 \right)}{9(4x^2 + 3ix)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 13.3 Problem number 19

$$\int \frac{1}{(3ix + 4x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2i}{9} + \frac{16x}{27}}{(4x^2 + 3ix)^{3/2}} + \frac{\frac{64i}{81} + \frac{512x}{243}}{\sqrt{4x^2 + 3ix}}$$

command

```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(8(16(8x + 9i)x - 27)x + 27i)\sqrt{8x^2 + 2\sqrt{16x^2 + 9}}x \left( \frac{3ix}{4x^2 + \sqrt{16x^4 + 9x^2}} + 1 \right)}{243(4x^2 + 3ix)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 13.4 Problem number 20

$$\int \frac{1}{(3ix + 4x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\frac{2i}{15} + \frac{16x}{45}}{(4x^2 + 3ix)^{5/2}} + \frac{\frac{128i}{405} + \frac{1024x}{1215}}{(4x^2 + 3ix)^{3/2}} + \frac{\frac{4096i}{3645} + \frac{32768x}{10935}}{\sqrt{4x^2 + 3ix}}$$

command

```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(8(32(8(16(8x + 15i)x - 135)x - 135i)x - 405)x + 729i)\sqrt{8x^2 + 2\sqrt{16x^2 + 9}}x \left( \frac{3ix}{4x^2 + \sqrt{16x^4 + 9x^2}} + 1 \right)}{10935(4x^2 + 3ix)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 13.5 Problem number 67

$$\int \frac{1}{(4 + 12x + 9x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{6(2+3x)\sqrt{(2+3x)^2}}$$

command

```
integrate(1/(9*x^2+12*x+4)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6(3x+2)^2 \operatorname{sgn}(3x+2)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14 Test file number 33

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/33\_1.2.1.2-d+e\_x-^m-a+b\_x+c\_x^2-^p

### 14.1 Problem number 190

$$\int \frac{x^4}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3}{b^5 \sqrt{(bx+a)^2}} - \frac{a^4}{2b^5 (bx+a) \sqrt{(bx+a)^2}} - \frac{3ax(bx+a)}{b^4 \sqrt{(bx+a)^2}} \\ & + \frac{x^2(bx+a)}{2b^3 \sqrt{(bx+a)^2}} + \frac{6a^2(bx+a) \ln(bx+a)}{b^5 \sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6 a^2 \log (|b x+a|)}{b^5 \operatorname{sgn}(b x+a)}+\frac{b^3 x^2 \operatorname{sgn}(b x+a)-6 a b^2 x \operatorname{sgn}(b x+a)}{2 b^6}+\frac{8 a^3 b x+7 a^4}{2(b x+a)^2 b^5 \operatorname{sgn}(b x+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.2 Problem number 191

$$\int \frac{x^3}{\left(a^2+2 a b x+b^2 x^2\right)^{3 / 2}} d x$$

Optimal antiderivative

$$-\frac{3 a^2}{b^4 \sqrt{(b x+a)^2}}+\frac{a^3}{2 b^4(b x+a) \sqrt{(b x+a)^2}}+\frac{x(b x+a)}{b^3 \sqrt{(b x+a)^2}}-\frac{3 a(b x+a) \ln (b x+a)}{b^4 \sqrt{(b x+a)^2}}$$

command

`integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{b^3 \operatorname{sgn}(b x+a)}-\frac{3 a \log (|b x+a|)}{b^4 \operatorname{sgn}(b x+a)}-\frac{6 a^2 b x+5 a^3}{2(b x+a)^2 b^4 \operatorname{sgn}(b x+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.3 Problem number 192

$$\int \frac{x^2}{\left(a^2+2 a b x+b^2 x^2\right)^{3 / 2}} d x$$

Optimal antiderivative

$$\frac{2 a}{b^3 \sqrt{(b x+a)^2}}-\frac{a^2}{2 b^3(b x+a) \sqrt{(b x+a)^2}}+\frac{(b x+a) \ln (b x+a)}{b^3 \sqrt{(b x+a)^2}}$$

command

`integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|bx + a|)}{b^3 \operatorname{sgn}(bx + a)} + \frac{4ax + \frac{3a^2}{b}}{2(bx + a)^2 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.4 Problem number 193

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{b^2 \sqrt{(bx + a)^2}} + \frac{a}{2b^2 (bx + a) \sqrt{(bx + a)^2}}$$

command

`integrate(x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2bx + a}{2(bx + a)^2 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.5 Problem number 194

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2b(bx + a) \sqrt{(bx + a)^2}}$$

command

`integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2(bx+a)^2 \operatorname{bsgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.6 Problem number 195

$$\int \frac{1}{x(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{a^2 \sqrt{(bx+a)^2}} + \frac{1}{2a(bx+a) \sqrt{(bx+a)^2}} + \frac{(bx+a) \ln(x)}{a^3 \sqrt{(bx+a)^2}} - \frac{(bx+a) \ln(bx+a)}{a^3 \sqrt{(bx+a)^2}}$$

command

`integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log(|bx+a|)}{a^3 \operatorname{sgn}(bx+a)} + \frac{\log(|x|)}{a^3 \operatorname{sgn}(bx+a)} + \frac{2abx + 3a^2}{2(bx+a)^2 a^3 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.7 Problem number 196

$$\int \frac{1}{x^2(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2b}{a^3 \sqrt{(bx+a)^2}} - \frac{b}{2a^2(bx+a) \sqrt{(bx+a)^2}} + \frac{-bx-a}{a^3 x \sqrt{(bx+a)^2}} - \frac{3b(bx+a) \ln(x)}{a^4 \sqrt{(bx+a)^2}} + \frac{3b(bx+a) \ln(bx+a)}{a^4 \sqrt{(bx+a)^2}}$$



command

```
integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3b \log(|bx+a|)}{a^4 \operatorname{sgn}(bx+a)} - \frac{3b \log(|x|)}{a^4 \operatorname{sgn}(bx+a)} - \frac{6ab^2x^2 + 9a^2bx + 2a^3}{2(bx+a)^2 a^4 x \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.8 Problem number 197

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3b^2}{a^4 \sqrt{(bx+a)^2}} + \frac{b^2}{2a^3 (bx+a) \sqrt{(bx+a)^2}} + \frac{-bx-a}{2a^3 x^2 \sqrt{(bx+a)^2}} \\ & + \frac{3b(bx+a)}{a^4 x \sqrt{(bx+a)^2}} + \frac{6b^2(bx+a) \ln(x)}{a^5 \sqrt{(bx+a)^2}} - \frac{6b^2(bx+a) \ln(bx+a)}{a^5 \sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6b^2 \log(|bx+a|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{6b^2 \log(|x|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{12b^3x^3 + 18ab^2x^2 + 4a^2bx - a^3}{2(bx^2+ax)^2 a^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.9 Problem number 198

$$\int \frac{x^6}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20a^3}{b^7 \sqrt{(bx+a)^2}} - \frac{a^6}{4b^7 (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{2a^5}{b^7 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & - \frac{15a^4}{2b^7 (bx+a) \sqrt{(bx+a)^2}} - \frac{5ax(bx+a)}{b^6 \sqrt{(bx+a)^2}} + \frac{x^2(bx+a)}{2b^5 \sqrt{(bx+a)^2}} + \frac{15a^2(bx+a) \ln(bx+a)}{b^7 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(x^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{15a^2 \log(|bx+a|)}{b^7 \operatorname{sgn}(bx+a)} + \frac{b^5 x^2 \operatorname{sgn}(bx+a) - 10ab^4 x \operatorname{sgn}(bx+a)}{2b^{10}} \\ & + \frac{80a^3 b^3 x^3 + 210a^4 b^2 x^2 + 188a^5 bx + 57a^6}{4(bx+a)^4 b^7 \operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.10 Problem number 199

$$\int \frac{x^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10a^2}{b^6 \sqrt{(bx+a)^2}} + \frac{a^5}{4b^6 (bx+a)^3 \sqrt{(bx+a)^2}} - \frac{5a^4}{3b^6 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & + \frac{5a^3}{b^6 (bx+a) \sqrt{(bx+a)^2}} + \frac{x(bx+a)}{b^5 \sqrt{(bx+a)^2}} - \frac{5a(bx+a) \ln(bx+a)}{b^6 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(x^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{b^5 \operatorname{sgn}(bx+a)} - \frac{5a \log(|bx+a|)}{b^6 \operatorname{sgn}(bx+a)} - \frac{120a^2 b^3 x^3 + 300a^3 b^2 x^2 + 260a^4 bx + 77a^5}{12(bx+a)^4 b^6 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.11 Problem number 200

$$\int \frac{x^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{4a}{b^5 \sqrt{(bx+a)^2}} - \frac{a^4}{4b^5 (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{4a^3}{3b^5 (bx+a)^2 \sqrt{(bx+a)^2}} - \frac{3a^2}{b^5 (bx+a) \sqrt{(bx+a)^2}} + \frac{(bx+a) \ln(bx+a)}{b^5 \sqrt{(bx+a)^2}}$$

command

```
integrate(x^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)} + \frac{48ab^2x^3 + 108a^2bx^2 + 88a^3x + \frac{25a^4}{b}}{12(bx+a)^4 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.12 Problem number 201

$$\int \frac{x^3}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x^4}{4a(bx+a)^3 \sqrt{(bx+a)^2}}$$

command

```
integrate(x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4b^3x^3 + 6ab^2x^2 + 4a^2bx + a^3}{4(bx+a)^4 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.13 Problem number 202

$$\int \frac{x^2}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a^2}{4b^3 (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{2a}{3b^3 (bx+a)^2 \sqrt{(bx+a)^2}} - \frac{1}{2b^3 (bx+a) \sqrt{(bx+a)^2}}$$

command

```
integrate(x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6b^2x^2 + 4abx + a^2}{12(bx+a)^4 b^3 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.14 Problem number 203

$$\int \frac{x}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{3b^2 (b^2x^2 + 2abx + a^2)^{3/2}} + \frac{a}{4b^2 (bx+a) (b^2x^2 + 2abx + a^2)^{3/2}}$$

command

```
integrate(x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx+a}{12(bx+a)^4 b^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.15 Problem number 204

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4b(bx+a)(b^2x^2+2abx+a^2)^{3/2}}$$

command

`integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4(bx+a)^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.16 Problem number 205

$$\int \frac{1}{x(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{a^4 \sqrt{(bx+a)^2}} + \frac{1}{4a(bx+a)^3 \sqrt{(bx+a)^2}} + \frac{1}{3a^2(bx+a)^2 \sqrt{(bx+a)^2}} \\ & + \frac{1}{2a^3(bx+a) \sqrt{(bx+a)^2}} + \frac{(bx+a) \ln(x)}{a^5 \sqrt{(bx+a)^2}} - \frac{(bx+a) \ln(bx+a)}{a^5 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(1/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log(|bx+a|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{\log(|x|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{12ab^3x^3 + 42a^2b^2x^2 + 52a^3bx + 25a^4}{12(bx+a)^4 a^5 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.17 Problem number 206

$$\int \frac{1}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4b}{a^5 \sqrt{(bx+a)^2}} - \frac{b}{4a^2 (bx+a)^3 \sqrt{(bx+a)^2}} - \frac{2b}{3a^3 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & - \frac{3b}{2a^4 (bx+a) \sqrt{(bx+a)^2}} + \frac{-bx-a}{a^5 x \sqrt{(bx+a)^2}} - \frac{5b(bx+a) \ln(x)}{a^6 \sqrt{(bx+a)^2}} + \frac{5b(bx+a) \ln(bx+a)}{a^6 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(1/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5b \log(|bx+a|)}{a^6 \operatorname{sgn}(bx+a)} - \frac{5b \log(|x|)}{a^6 \operatorname{sgn}(bx+a)} - \frac{60ab^4x^4 + 210a^2b^3x^3 + 260a^3b^2x^2 + 125a^4bx + 12a^5}{12(bx+a)^4 a^6 x \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.18 Problem number 207

$$\int \frac{1}{x^3 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10b^2}{a^6 \sqrt{(bx+a)^2}} + \frac{b^2}{4a^3 (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{b^2}{a^4 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & + \frac{3b^2}{a^5 (bx+a) \sqrt{(bx+a)^2}} + \frac{-bx-a}{2a^5 x^2 \sqrt{(bx+a)^2}} + \frac{5b(bx+a)}{a^6 x \sqrt{(bx+a)^2}} \\ & + \frac{15b^2(bx+a) \ln(x)}{a^7 \sqrt{(bx+a)^2}} - \frac{15b^2(bx+a) \ln(bx+a)}{a^7 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(1/x^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15b^2 \log(|bx+a|)}{a^7 \operatorname{sgn}(bx+a)} + \frac{15b^2 \log(|x|)}{a^7 \operatorname{sgn}(bx+a)} + \frac{60ab^5x^5 + 210a^2b^4x^4 + 260a^3b^3x^3 + 125a^4b^2x^2 + 12a^5bx - 2a^6}{4(bx+a)^4 a^7 x^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.19 Problem number 212

$$\int \frac{x}{(9+12x+4x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4\sqrt{(3+2x)^2}} + \frac{3}{8(3+2x)\sqrt{(3+2x)^2}}$$

command

`integrate(x/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4x+3}{8(2x+3)^2 \operatorname{sgn}(2x+3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.20 Problem number 213

$$\int \frac{x}{(9+12x+4x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{12(4x^2+12x+9)^{3/2}} + \frac{3}{16(3+2x)(4x^2+12x+9)^{3/2}}$$

command

`integrate(x/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8x+3}{48(2x+3)^4 \operatorname{sgn}(2x+3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.21 Problem number 214

$$\int \frac{x}{(9+12x+4x^2)^{7/2}} dx$$

Optimal antiderivative

$$-\frac{1}{20(4x^2+12x+9)^{5/2}} + \frac{1}{8(3+2x)(4x^2+12x+9)^{5/2}}$$

command

`integrate(x/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4x+1}{40(2x+3)^6 \operatorname{sgn}(2x+3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.22 Problem number 217

$$\int \frac{x}{\sqrt{-4+12x-9x^2}} dx$$

Optimal antiderivative

$$-\frac{2(2-3x) \ln(2-3x)}{9\sqrt{-(-2+3x)^2}} - \frac{\sqrt{-(-2+3x)^2}}{9}$$

command



`integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i x}{3 \operatorname{sgn}(-3 x+2)} + \frac{2 i \log (|3 x-2|)}{9 \operatorname{sgn}(-3 x+2)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

### 14.23 Problem number 218

$$\int \frac{x}{\sqrt{-4-12x-9x^2}} dx$$

Optimal antiderivative

$$-\frac{2(2+3x) \ln (2+3x)}{9 \sqrt{-(2+3x)^2}} - \frac{\sqrt{-(2+3x)^2}}{9}$$

command

`integrate(x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i x}{3 \operatorname{sgn}(-3 x-2)} - \frac{2 i \log (|3 x+2|)}{9 \operatorname{sgn}(-3 x-2)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

### 14.24 Problem number 618

$$\int \frac{(d+ex)^{5/2}}{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2e(ex+d)^{\frac{3}{2}}}{3c} + \frac{4de\sqrt{ex+d}}{c} \\
& e \operatorname{arctanh} \left( \frac{-c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d} + \sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}} \right) \left( 2c^{\frac{3}{2}}d^3 + 2ade^2\sqrt{c} - (-ae^2 + 3cd^2)\sqrt{ae^2 + cd^2} \right) \\
& - \frac{2c^{\frac{7}{4}}\sqrt{ae^2 + cd^2}\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}} \\
& e \operatorname{arctanh} \left( \frac{c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d} + \sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}} \right) \left( 2c^{\frac{3}{2}}d^3 + 2ade^2\sqrt{c} - (-ae^2 + 3cd^2)\sqrt{ae^2 + cd^2} \right) \\
& + \frac{2c^{\frac{7}{4}}\sqrt{ae^2 + cd^2}\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2 + cd^2}}} \\
& e \ln \left( (ex+d)\sqrt{c} + \sqrt{ae^2 + cd^2} - c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d}\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}} \right) \left( 2c^{\frac{3}{2}}d^3 + 2ade^2\sqrt{c} + (-ae^2 + 3cd^2)\sqrt{ae^2 + cd^2} \right) \\
& + \frac{4c^{\frac{7}{4}}\sqrt{ae^2 + cd^2}\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}} \\
& e \ln \left( (ex+d)\sqrt{c} + \sqrt{ae^2 + cd^2} + c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d}\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}} \right) \left( 2c^{\frac{3}{2}}d^3 + 2ade^2\sqrt{c} + (-ae^2 + 3cd^2)\sqrt{ae^2 + cd^2} \right) \\
& - \frac{4c^{\frac{7}{4}}\sqrt{ae^2 + cd^2}\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}}{\sqrt{d\sqrt{c} + \sqrt{ae^2 + cd^2}}}
\end{aligned}$$

command

```
integrate((e*x+d)^(5/2)/(c*x^2+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{(c^4d^4 - 3ac^3d^2e^2 + (3acd^2e^2 - a^2e^4)c^2 + 2(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3)|c|) \operatorname{arctan} \left( \frac{\sqrt{xe+d}}{\sqrt{-\frac{c^4d + \sqrt{c^8d^2 - (c^4d^2 + a^2e^2)}}{c^4}}} \right)}{(ac^3e + \sqrt{-ac}c^3d)\sqrt{-c^2d - \sqrt{-ac}ce}} \\
& + \frac{(c^4d^4 - 3ac^3d^2e^2 + (3acd^2e^2 - a^2e^4)c^2 - 2(\sqrt{-ac}c^2d^3e + \sqrt{-ac}acde^3)|c|) \operatorname{arctan} \left( \frac{\sqrt{xe+d}}{\sqrt{-\frac{c^4d - \sqrt{c^8d^2 - (c^4d^2 + a^2e^2)}}{c^4}}} \right)}{(ac^3e - \sqrt{-ac}c^3d)\sqrt{-c^2d + \sqrt{-ac}ce}} \\
& + \frac{2\left((xe+d)^{\frac{3}{2}}c^2e + 6\sqrt{xe+d}c^2de\right)}{3c^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.25 Problem number 619

$$\int \frac{(d+ex)^{3/2}}{a+cx^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e\sqrt{ex+d}}{c} \\ & - \frac{e \operatorname{arctanh}\left(\frac{-c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d} + \sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2+cd^2}}}\right) (cd^2 + ae^2 - 2d\sqrt{c}\sqrt{ae^2+cd^2})\sqrt{2}}{2c^{\frac{5}{4}}\sqrt{ae^2+cd^2}\sqrt{d\sqrt{c} - \sqrt{ae^2+cd^2}}} \\ & + \frac{e \operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d} + \sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}}{\sqrt{d\sqrt{c} - \sqrt{ae^2+cd^2}}}\right) (cd^2 + ae^2 - 2d\sqrt{c}\sqrt{ae^2+cd^2})\sqrt{2}}{2c^{\frac{5}{4}}\sqrt{ae^2+cd^2}\sqrt{d\sqrt{c} - \sqrt{ae^2+cd^2}}} \\ & + \frac{e \ln\left((ex+d)\sqrt{c} + \sqrt{ae^2+cd^2} - c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d}\sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}\right) (cd^2 + ae^2 + 2d\sqrt{c}\sqrt{ae^2+cd^2})}{4c^{\frac{5}{4}}\sqrt{ae^2+cd^2}\sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}} \\ & + \frac{e \ln\left((ex+d)\sqrt{c} + \sqrt{ae^2+cd^2} + c^{\frac{1}{4}}\sqrt{2}\sqrt{ex+d}\sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}\right) (cd^2 + ae^2 + 2d\sqrt{c}\sqrt{ae^2+cd^2})}{4c^{\frac{5}{4}}\sqrt{ae^2+cd^2}\sqrt{d\sqrt{c} + \sqrt{ae^2+cd^2}}} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)/(c*x^2+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(c^3d^3 + ac^2de^2 + (\sqrt{-ac}cd^2e + \sqrt{-ac}ae^3)|c|) \operatorname{arctan}\left(\frac{\sqrt{xe+d}}{\sqrt{-\frac{c^2d + \sqrt{c^4d^2 - (c^2d^2 + ace^2)c^2}}{c^2}}}\right)}{(ac^2e + \sqrt{-ac}c^2d)\sqrt{-c^2d - \sqrt{-ac}ce}} \\ & + \frac{(c^3d^3 + ac^2de^2 - (\sqrt{-ac}cd^2e + \sqrt{-ac}ae^3)|c|) \operatorname{arctan}\left(\frac{\sqrt{xe+d}}{\sqrt{-\frac{c^2d - \sqrt{c^4d^2 - (c^2d^2 + ace^2)c^2}}{c^2}}}\right)}{(ac^2e - \sqrt{-ac}c^2d)\sqrt{-c^2d + \sqrt{-ac}ce}} \\ & + \frac{2\sqrt{xe+d}e}{c} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.26 Problem number 803

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5d^3 x (-e^2 x^2 + d^2)^{\frac{3}{2}}}{24} + \frac{dx (-e^2 x^2 + d^2)^{\frac{5}{2}}}{6} + \frac{(-e^2 x^2 + d^2)^{\frac{7}{2}}}{7e} \\ & + \frac{5d^7 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{16e} + \frac{5d^5 x \sqrt{-e^2 x^2 + d^2}}{16} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5}{16} d^7 \arcsin\left(\frac{xe}{d}\right) e^{(-1) \operatorname{sgn}(d)} \\ & + \frac{1}{336} \left( 48 d^6 e^{(-1)} + (231 d^5 - 2 (72 d^4 e + (91 d^3 e^2 - 4 (18 d^2 e^3 - (6 x e^5 - 7 d e^4) x) x) x) x) \right) \sqrt{-x^2 e^2 + d^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.27 Problem number 804

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7d^2 x (-e^2 x^2 + d^2)^{\frac{3}{2}}}{24} + \frac{7d (-e^2 x^2 + d^2)^{\frac{5}{2}}}{30e} + \frac{(-ex + d) (-e^2 x^2 + d^2)^{\frac{5}{2}}}{6e} \\ & + \frac{7d^6 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{16e} + \frac{7d^4 x \sqrt{-e^2 x^2 + d^2}}{16} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 6720 d^7 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^7 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \frac{\left( 105 d^7 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{11}{2}} e^7 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + 595 d^7 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{9}{2}} e^7 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 1686 d^7 \right)}{1} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.28 Problem number 805

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7d^2(-e^2x^2+d^2)^{\frac{3}{2}}}{12e} + \frac{7d(-ex+d)(-e^2x^2+d^2)^{\frac{3}{2}}}{20e} + \frac{(-ex+d)^2(-e^2x^2+d^2)^{\frac{3}{2}}}{5e} \\ & + \frac{7d^5 \arctan \left( \frac{ex}{\sqrt{-e^2x^2+d^2}} \right)}{8e} + \frac{7d^3x\sqrt{-e^2x^2+d^2}}{8} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{7}{8} d^5 \arcsin \left( \frac{xe}{d} \right) e^{(-1) \operatorname{sgn}(d)} \\ & + \frac{1}{120} \left( 136 d^4 e^{(-1)} + (15 d^3 - 2(56 d^2 e + 3(4 x e^3 - 15 d e^2) x) x) \right) \sqrt{-x^2 e^2 + d^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.29 Problem number 806

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35d(-e^2x^2 + d^2)^{\frac{3}{2}}}{12e} + \frac{7(-ex + d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{4e} + \frac{2(-e^2x^2 + d^2)^{\frac{7}{2}}}{e(ex + d)^3} \\ & + \frac{35d^4 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{8e} + \frac{35d^2x\sqrt{-e^2x^2 + d^2}}{8} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35}{8} d^4 \arcsin\left(\frac{xe}{d}\right) e^{(-1)\operatorname{sgn}(d)} + \frac{1}{24} \left(160 d^3 e^{(-1)} - (81 d^2 + 2(3xe^2 - 16de)x)x\right) \sqrt{-x^2e^2 + d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.30 Problem number 807

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{35(-e^2x^2 + d^2)^{\frac{3}{2}}}{3e} - \frac{14(-e^2x^2 + d^2)^{\frac{5}{2}}}{e(ex + d)^2} - \frac{2(-e^2x^2 + d^2)^{\frac{7}{2}}}{e(ex + d)^4} \\ & - \frac{35d^3 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2e} - \frac{35dx\sqrt{-e^2x^2 + d^2}}{2} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 840 d^4 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^4 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 384 d^4 \sqrt{\frac{2d}{xe+d} - 1} e^4 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - \left( 87 d^4 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} e^4 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \dots \right)}{24 d}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.31 Problem number 808

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\frac{35(-e^2 x^2 + d^2)^{\frac{3}{2}}}{6e(ex + d)} + \frac{14(-e^2 x^2 + d^2)^{\frac{5}{2}}}{3e(ex + d)^3} - \frac{2(-e^2 x^2 + d^2)^{\frac{7}{2}}}{3e(ex + d)^5} + \frac{35d^2 \arctan \left( \frac{ex}{\sqrt{-e^2 x^2 + d^2}} \right)}{2e} + \frac{35d \sqrt{-e^2 x^2 + d^2}}{2e}$$

command

`integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{35}{2} d^2 \arcsin \left( \frac{xe}{d} \right) e^{(-1)} \operatorname{sgn}(d) + \frac{1}{2} \sqrt{-x^2 e^2 + d^2} (12 d e^{(-1)} - x)}{32 \left( \frac{9 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^2 e^{(-2)}}{x} + \frac{3 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^2 e^{(-4)}}{x^2} + 4 d^2 \right) e^{(-1)} - \frac{3 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^3}{}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.32 Problem number 809

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{14(-e^2 x^2 + d^2)^{\frac{3}{2}}}{3e(ex + d)^2} + \frac{14(-e^2 x^2 + d^2)^{\frac{5}{2}}}{15e(ex + d)^4} - \frac{2(-e^2 x^2 + d^2)^{\frac{7}{2}}}{5e(ex + d)^6} \\ & - \frac{7d \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{e} - \frac{7\sqrt{-e^2 x^2 + d^2}}{e} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -7d \arcsin\left(\frac{xe}{d}\right) e^{(-1)} \operatorname{sgn}(d) - \sqrt{-x^2 e^2 + d^2} e^{(-1)} \\ & 16 \left( \frac{80 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) de^{(-2)}}{x} + \frac{130 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 de^{(-4)}}{x^2} + \frac{60 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 de^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 de^{(-8)}}{x^4} \right) \\ & + \frac{15 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}{1} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.33 Problem number 810

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-e^2 x^2 + d^2)^{\frac{3}{2}}}{3e(ex + d)^3} + \frac{2(-e^2 x^2 + d^2)^{\frac{5}{2}}}{5e(ex + d)^5} - \frac{2(-e^2 x^2 + d^2)^{\frac{7}{2}}}{7e(ex + d)^7} \\ & + \frac{\arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{e} + \frac{2\sqrt{-e^2 x^2 + d^2}}{e(ex + d)} \end{aligned}$$



command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\arcsin\left(\frac{xe}{d}\right)e^{(-1)}\operatorname{sgn}(d)}{16\left(\frac{133\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x}+\frac{294\left(de+\sqrt{-x^2e^2+d^2}\right)^2e^{(-4)}}{x^2}+\frac{490\left(de+\sqrt{-x^2e^2+d^2}\right)^3e^{(-6)}}{x^3}+\frac{175\left(de+\sqrt{-x^2e^2+d^2}\right)^4e^{(-8)}}{x^4}\right)}{105\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x}+1\right)^7}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.34 Problem number 811

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^9} dx$$

Optimal antiderivative

$$-\frac{(-e^2x^2 + d^2)^{\frac{9}{2}}}{9de(ex + d)^9}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\frac{36\left(de+\sqrt{-x^2e^2+d^2}\right)^2e^{(-4)}}{x^2}+\frac{126\left(de+\sqrt{-x^2e^2+d^2}\right)^4e^{(-8)}}{x^4}+\frac{84\left(de+\sqrt{-x^2e^2+d^2}\right)^6e^{(-12)}}{x^6}+\frac{9\left(de+\sqrt{-x^2e^2+d^2}\right)^8e^{(-16)}}{x^8}\right)}{9d\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x}+1\right)^9}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.35 Problem number 812

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^{10}} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{\frac{9}{2}}}{11de (ex + d)^{10}} - \frac{(-e^2 x^2 + d^2)^{\frac{9}{2}}}{99d^2 e (ex + d)^9}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{11 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{451 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{396 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{2376 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.36 Problem number 813

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^{11}} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{\frac{9}{2}}}{13de (ex + d)^{11}} - \frac{2(-e^2 x^2 + d^2)^{\frac{9}{2}}}{143d^2 e (ex + d)^{10}} - \frac{2(-e^2 x^2 + d^2)^{\frac{9}{2}}}{1287d^3 e (ex + d)^9}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{260 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{6708 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{11726 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{52481 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.37 Problem number 814

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^{12}} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{9/2}}{15de (ex + d)^{12}} - \frac{(-e^2 x^2 + d^2)^{9/2}}{65d^2 e (ex + d)^{11}} - \frac{2(-e^2 x^2 + d^2)^{9/2}}{715d^3 e (ex + d)^{10}} - \frac{2(-e^2 x^2 + d^2)^{9/2}}{6435d^4 e (ex + d)^9}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{1785 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{38235 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{99190 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{426270 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.38 Problem number 815

$$\int \frac{(d^2 - e^2 x^2)^{7/2}}{(d + ex)^{13}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-e^2 x^2 + d^2)^{\frac{9}{2}}}{17de (ex + d)^{13}} - \frac{4(-e^2 x^2 + d^2)^{\frac{9}{2}}}{255d^2 e (ex + d)^{12}} - \frac{4(-e^2 x^2 + d^2)^{\frac{9}{2}}}{1105d^3 e (ex + d)^{11}} \\ & - \frac{8(-e^2 x^2 + d^2)^{\frac{9}{2}}}{12155d^4 e (ex + d)^{10}} - \frac{8(-e^2 x^2 + d^2)^{\frac{9}{2}}}{109395d^5 e (ex + d)^9} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^13,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{37264 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{735692 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{2511580 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{11197}{x^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.39 Problem number 831

$$\int \frac{1}{(d + ex) \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2 x^2 + d^2}}{de (ex + d)}$$

command

```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2e^{(-1)}}{d \left( \frac{\left( \frac{de + \sqrt{-x^2e^2 + d^2}}{e} \right) e^{(-2)}}{x} + 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 14.40 Problem number 832

$$\int \frac{1}{(d+ex)^2 \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2x^2 + d^2}}{3de(ex+d)^2} - \frac{\sqrt{-e^2x^2 + d^2}}{3d^2e(ex+d)}$$

command

`integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{ie^{(-1)} \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{3d^2} - \frac{\left(\left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} + 3\sqrt{\frac{2d}{xe+d} - 1}\right)e^{(-1)}}{6d^2 \operatorname{sgn}\left(\frac{1}{xe+d}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.41 Problem number 833

$$\int \frac{1}{(d+ex)^3 \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2x^2 + d^2}}{5de(ex+d)^3} - \frac{2\sqrt{-e^2x^2 + d^2}}{15d^2e(ex+d)^2} - \frac{2\sqrt{-e^2x^2 + d^2}}{15d^3e(ex+d)}$$

command

`integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{20 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{40 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{30 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right) \\ \frac{1}{15 d^3 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 14.42 Problem number 834

$$\int \frac{1}{(d+ex)^4 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2 x^2 + d^2}}{7de (ex + d)^4} - \frac{3\sqrt{-e^2 x^2 + d^2}}{35d^2 e (ex + d)^3} - \frac{2\sqrt{-e^2 x^2 + d^2}}{35d^3 e (ex + d)^2} - \frac{2\sqrt{-e^2 x^2 + d^2}}{35d^4 e (ex + d)}$$

command

`integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{49 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{147 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{210 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{210 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right) \\ \frac{1}{35 d^4 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.43 Problem number 835

$$\int \frac{1}{(d+ex)^5 \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2x^2 + d^2}}{9de(ex+d)^5} - \frac{4\sqrt{-e^2x^2 + d^2}}{63d^2e(ex+d)^4} - \frac{4\sqrt{-e^2x^2 + d^2}}{105d^3e(ex+d)^3} - \frac{8\sqrt{-e^2x^2 + d^2}}{315d^4e(ex+d)^2} - \frac{8\sqrt{-e^2x^2 + d^2}}{315d^5e(ex+d)}$$

command

`integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{5040} \left( -\frac{128i \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^5} + \frac{35 \left(\frac{2d}{xe+d} - 1\right)^{\frac{9}{2}} + 180 \left(\frac{2d}{xe+d} - 1\right)^{\frac{7}{2}} + 378 \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} + 420 \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} + 315 \sqrt{\frac{2d}{xe+d} - 1}}{d^5 \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.44 Problem number 843

$$\int \frac{1}{(d+ex)^2 (d^2 - e^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{4x}{21d^4(-e^2x^2 + d^2)^{\frac{3}{2}}} - \frac{1}{7de(ex+d)^2(-e^2x^2 + d^2)^{\frac{3}{2}}} - \frac{1}{7d^2e(ex+d)(-e^2x^2 + d^2)^{\frac{3}{2}}} + \frac{8x}{21d^6\sqrt{-e^2x^2 + d^2}}$$

command

`integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{672} \left( \left( \frac{14 \left(\frac{15d}{xe+d} - 7\right) e^{-5}}{d^6 \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{\left(3d^{36} \left(\frac{2d}{xe+d} - 1\right)^{\frac{7}{2}} e^{30} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^6 + 21d^{36} \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} e^{30} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^6 + 70d^{36} \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} e^{30} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^6 + 35d^{36} \left(\frac{2d}{xe+d} - 1\right)^{\frac{1}{2}} e^{30} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^6 + 7d^{36} e^{30} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^6}{d^{42} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.45 Problem number 856

$$\int \frac{1}{(d+ex)^2 (d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2x}{15d^4 (-e^2x^2 + d^2)^{5/2}} - \frac{1}{9de (ex + d)^2 (-e^2x^2 + d^2)^{5/2}} - \frac{1}{9d^2e (ex + d) (-e^2x^2 + d^2)^{5/2}} + \frac{8x}{45d^6 (-e^2x^2 + d^2)^{3/2}} + \frac{16x}{45d^8 \sqrt{-e^2x^2 + d^2}}$$

command

`integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{5760} \left( \left( \frac{3 \left( 315 \left( \frac{2d}{xe+d} - 1 \right)^2 + \frac{70d}{xe+d} - 32 \right) e^{(-7)}}{d^8 \left( \frac{2d}{xe+d} - 1 \right)^{5/2} \operatorname{sgn} \left( \frac{1}{xe+d} \right)} - \frac{\left( 5 d^{64} \left( \frac{2d}{xe+d} - 1 \right)^{9/2} e^{56} \operatorname{sgn} \left( \frac{1}{xe+d} \right)^8 + 45 d^{64} \left( \frac{2d}{xe+d} - 1 \right)^{7/2} e^{56} \operatorname{sgn} \left( \frac{1}{xe+d} \right)^7}{d^8 \left( \frac{2d}{xe+d} - 1 \right)^{5/2} \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.46 Problem number 859

$$\int \frac{1}{(d+ex)^5 (d^2 - e^2x^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{32x}{715d^7 (-e^2x^2 + d^2)^{5/2}} - \frac{1}{15de (ex + d)^5 (-e^2x^2 + d^2)^{5/2}} - \frac{2}{39d^2e (ex + d)^4 (-e^2x^2 + d^2)^{5/2}} - \frac{143d^3e (ex + d)^3 (-e^2x^2 + d^2)^{5/2}}{16} - \frac{429d^4e (ex + d)^2 (-e^2x^2 + d^2)^{5/2}}{128x} - \frac{256x}{429d^5e (ex + d) (-e^2x^2 + d^2)^{5/2}} + \frac{2145d^9 (-e^2x^2 + d^2)^{3/2}}{2145d^{11} \sqrt{-e^2x^2 + d^2}}$$

command

`integrate(1/(e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")`



Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2196480} \left( \frac{143 \left( 675 \left( \frac{2d}{xe+d} - 1 \right)^2 + \frac{100d}{xe+d} - 47 \right) e^{(-10)}}{d^{11} \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} \operatorname{sgn} \left( \frac{1}{xe+d} \right)} - \frac{\left( 143 d^{154} \left( \frac{2d}{xe+d} - 1 \right)^{\frac{15}{2}} e^{140} \operatorname{sgn} \left( \frac{1}{xe+d} \right)^{14} + 1650 d^{154} \left( \frac{1}{xe+d} \right)^{14} \right)}{d^{11} \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.47 Problem number 862

$$\int (d + ex)^{5/2} \sqrt{cd^2 - ce^2x^2} dx$$

Optimal antiderivative

$$\frac{256d^3(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{315ce(ex + d)^{\frac{3}{2}}} - \frac{2(ex + d)^{\frac{3}{2}}(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{9ce} - \frac{64d^2(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{105ce\sqrt{ex + d}} - \frac{8d(-ce^2x^2 + cd^2)^{\frac{3}{2}}\sqrt{ex + d}}{21ce}$$

command

`integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{315} \left( 105 \left( 2\sqrt{2}\sqrt{cd}d - \frac{(-(xe+d)c + 2cd)^{\frac{3}{2}}}{c} \right) d^3 - 63 \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c + 2cd)^{\frac{3}{2}}cd - 3((xe+d)c + 2cd)^{\frac{3}{2}}}{c^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-ce^2x^2 + cd^2} (ex + d)^{\frac{5}{2}} dx$$

## 14.48 Problem number 863

$$\int (d+ex)^{3/2} \sqrt{cd^2 - ce^2x^2} dx$$

Optimal antiderivative

$$-\frac{64d^2(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{105ce(ex+d)^{\frac{3}{2}}} - \frac{16d(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{35ce\sqrt{ex+d}} - \frac{2(-ce^2x^2 + cd^2)^{\frac{3}{2}}\sqrt{ex+d}}{7ce}$$

command

`integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( 35 \left( 2\sqrt{2}\sqrt{cd}d - \frac{(-(xe+d)c+2cd)^{\frac{3}{2}}}{c} \right) d^2 - 14 \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c+2cd)^{\frac{3}{2}}cd - 3((xe+d)c - 2cd)}{c^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-ce^2x^2 + cd^2} (ex+d)^{\frac{3}{2}} dx$$

## 14.49 Problem number 864

$$\int \sqrt{d+ex} \sqrt{cd^2 - ce^2x^2} dx$$

Optimal antiderivative

$$-\frac{8d(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{15ce(ex+d)^{\frac{3}{2}}} - \frac{2(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{5ce\sqrt{ex+d}}$$

command

`integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( 2\sqrt{2}\sqrt{cd}d^2 - 5 \left( 2\sqrt{2}\sqrt{cd}d - \frac{(-(xe+d)c+2cd)^{\frac{3}{2}}}{c} \right) d + \frac{5(-(xe+d)c+2cd)^{\frac{3}{2}}cd - 3((xe+d)c - 2cd)}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-ce^2x^2 + cd^2} \sqrt{ex+d} dx$$

## 14.50 Problem number 865

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{2(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{3ce(ex+d)^{\frac{3}{2}}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( 2\sqrt{2}\sqrt{cd}d - \frac{-(xe+d)c + 2cd)^{\frac{3}{2}}}{c} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 + cd^2}}{\sqrt{ex+d}} dx$$

## 14.51 Problem number 866

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh} \left( \frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex+d}} \right) \sqrt{2} \sqrt{c} \sqrt{d}}{e} + \frac{2\sqrt{-ce^2x^2 + cd^2}}{e\sqrt{ex+d}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\sqrt{2} d \arctan \left( \frac{\sqrt{2} \sqrt{-(xe+d)c + 2cd}}{2\sqrt{-cd}} \right)}{\sqrt{-cd}} + \frac{\sqrt{-(xe+d)c + 2cd}}{c} \right) ce^{(-1)}$$

$$-\frac{2 \left( \sqrt{2} cd \arctan \left( \frac{\sqrt{cd}}{\sqrt{-cd}} \right) + \sqrt{2} \sqrt{cd} \sqrt{-cd} \right) e^{(-1)}}{\sqrt{-cd}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 + cd^2}}{(ex + d)^{\frac{3}{2}}} dx$$

#### 14.52 Problem number 867

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex + d}}\right) \sqrt{c} \sqrt{2}}{2e\sqrt{d}} - \frac{\sqrt{-ce^2x^2 + cd^2}}{e(ex + d)^{\frac{3}{2}}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} \left( \frac{\sqrt{2} c \arctan\left(\frac{\sqrt{2} \sqrt{-(xe + d)c + 2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{2\sqrt{-(xe + d)c + 2cd}}{xe + d} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 + cd^2}}{(ex + d)^{\frac{5}{2}}} dx$$

#### 14.53 Problem number 868

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex + d}}\right) \sqrt{c} \sqrt{2}}{16d^{\frac{3}{2}}e} - \frac{\sqrt{-ce^2x^2 + cd^2}}{2e(ex + d)^{\frac{5}{2}}} + \frac{\sqrt{-ce^2x^2 + cd^2}}{8de(ex + d)^{\frac{3}{2}}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{16} \left( \frac{\sqrt{2} c \arctan \left( \frac{\sqrt{2} \sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}} \right)}{\sqrt{-cd} d} + \frac{2 \left( 2 \sqrt{-(xe+d)c+2cd} c^2 d + (-(xe+d)c+2cd)^{\frac{3}{2}} c \right)}{(xe+d)^2 c^2 d} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{7}{2}}} dx$$

#### 14.54 Problem number 869

$$\int (d+ex)^{5/2} (cd^2-ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4096d^4(-ce^2x^2+cd^2)^{\frac{5}{2}}}{15015ce(ex+d)^{\frac{5}{2}}} - \frac{1024d^3(-ce^2x^2+cd^2)^{\frac{5}{2}}}{3003ce(ex+d)^{\frac{3}{2}}} - \frac{2(ex+d)^{\frac{3}{2}}(-ce^2x^2+cd^2)^{\frac{5}{2}}}{13ce} \\ & - \frac{128d^2(-ce^2x^2+cd^2)^{\frac{5}{2}}}{429ce\sqrt{ex+d}} - \frac{32d(-ce^2x^2+cd^2)^{\frac{5}{2}}\sqrt{ex+d}}{143ce} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{45045} \left( 15015 \left( 2\sqrt{2}\sqrt{cd}d - \frac{(-(xe+d)c+2cd)^{\frac{3}{2}}}{c} \right) cd^5 - 9009 \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c+2cd)^{\frac{3}{2}}cd - 3}{c} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2+cd^2)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}} dx$$

## 14.55 Problem number 870

$$\int (d+ex)^{3/2} (cd^2 - ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{256d^3(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{1155ce(ex+d)^{\frac{5}{2}}} - \frac{64d^2(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{231ce(ex+d)^{\frac{3}{2}}} \\ & - \frac{8d(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{33ce\sqrt{ex+d}} - \frac{2(-ce^2x^2 + cd^2)^{\frac{5}{2}}\sqrt{ex+d}}{11ce} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3465} \left( 1155 \left( 2\sqrt{2}\sqrt{cd}d - \frac{-(xe+d)c + 2cd}{c} \right)^{\frac{3}{2}} \right) cd^4 - 462 \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c + 2cd)^{\frac{3}{2}}cd - 3((xe+d)c + 2cd)^{\frac{3}{2}}}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 + cd^2)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}} dx$$

## 14.56 Problem number 871

$$\int \sqrt{d+ex} (cd^2 - ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{64d^2(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{315ce(ex+d)^{\frac{5}{2}}} - \frac{16d(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{63ce(ex+d)^{\frac{3}{2}}} - \frac{2(-ce^2x^2 + cd^2)^{\frac{5}{2}}}{9ce\sqrt{ex+d}} \end{aligned}$$

command

`integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{315} \left( 105 \left( 2\sqrt{2}\sqrt{cd}d - \frac{-(xe+d)c + 2cd}{c} \right)^{\frac{3}{2}} \right) cd^3 - 21 \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c + 2cd)^{\frac{3}{2}}cd - 3((xe+d)c + 2cd)^{\frac{3}{2}}}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 + cd^2)^{\frac{3}{2}}\sqrt{ex+d} dx$$

## 14.57 Problem number 872

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\frac{8d(-ce^2x^2 + cd^2)^{5/2}}{35ce(ex+d)^{5/2}} - \frac{2(-ce^2x^2 + cd^2)^{5/2}}{7ce(ex+d)^{3/2}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( 35 \left( 2\sqrt{2}\sqrt{cd}d - \frac{-(xe+d)c + 2cd}{c} \right)^{3/2} \right) cd^2 - \left( 22\sqrt{2}\sqrt{cd}d^3e^{(-2)} - \frac{(35(-(xe+d)c + 2cd)^{3/2}c^2d^2 - 42)}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 + cd^2)^{3/2}}{\sqrt{ex+d}} dx$$

## 14.58 Problem number 873

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-ce^2x^2 + cd^2)^{5/2}}{5ce(ex+d)^{5/2}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{15} \left( 5 \left( 2\sqrt{2}\sqrt{cd}d - \frac{-(xe+d)c + 2cd}{c} \right)^{3/2} \right) cd + \left( 2\sqrt{2}\sqrt{cd}d^2 + \frac{5(-(xe+d)c + 2cd)^{3/2}cd - 3((xe+d)c - 2)}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 + cd^2)^{3/2}}{(ex+d)^{3/2}} dx$$

## 14.59 Problem number 874

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{3e(ex + d)^{\frac{3}{2}}} - \frac{4c^{\frac{3}{2}}d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex + d}}\right) \sqrt{2}}{e} + \frac{4cd\sqrt{-ce^2x^2 + cd^2}}{e\sqrt{ex + d}}$$

command

```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{6\sqrt{2}cd^2 \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{6\sqrt{-(xe+d)c+2cd}c^3d + (-(xe+d)c+2cd)^{\frac{3}{2}}c^2}{c^3} \right) ce^{(-1)}$$

$$- \frac{4\sqrt{2}\left(3c^2d^2 \arctan\left(\frac{\sqrt{cd}}{\sqrt{-cd}}\right) + 4\sqrt{cd}\sqrt{-cd}cd\right)e^{(-1)}}{3\sqrt{-cd}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.60 Problem number 875

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$-\frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{e(ex + d)^{\frac{5}{2}}} + \frac{3c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex + d}}\right) \sqrt{2} \sqrt{d}}{e} - \frac{3c\sqrt{-ce^2x^2 + cd^2}}{e\sqrt{ex + d}}$$

command

```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$- \left( \frac{3\sqrt{2}c^2d \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + 2\sqrt{-(xe+d)c+2cd}c + \frac{2\sqrt{-(xe+d)c+2cd}cd}{xe+d} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.61 Problem number 876

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{9/2}} dx$$

Optimal antiderivative

$$-\frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{2e(ex+d)^{\frac{7}{2}}} - \frac{3c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{8e\sqrt{d}} + \frac{3c\sqrt{-ce^2x^2 + cd^2}}{4e(ex+d)^{\frac{3}{2}}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{8} \left( \frac{3\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{2\left(6\sqrt{-(xe+d)c+2cd}c^3d - 5(-(xe+d)c+2cd)^{\frac{3}{2}}c^2\right)}{(xe+d)^2c^2} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.62 Problem number 877

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}} dx$$

Optimal antiderivative

$$-\frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{3e(ex+d)^{\frac{9}{2}}} - \frac{c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{32d^{\frac{3}{2}}e} + \frac{c\sqrt{-ce^2x^2 + cd^2}}{4e(ex+d)^{\frac{5}{2}}} - \frac{c\sqrt{-ce^2x^2 + cd^2}}{16de(ex+d)^{\frac{3}{2}}}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{96} \left( \frac{3\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}d} + \frac{2\left(12\sqrt{-(xe+d)c+2cd}c^4d^2 - 16(-(xe+d)c+2cd)^{\frac{3}{2}}c^3d\right)}{(xe+d)^3c^3d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.63 Problem number 878

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{4e(ex+d)^{\frac{11}{2}}} - \frac{3c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{512d^{\frac{5}{2}}e} \\ & + \frac{c\sqrt{-ce^2x^2 + cd^2}}{8e(ex+d)^{\frac{7}{2}}} - \frac{c\sqrt{-ce^2x^2 + cd^2}}{64de(ex+d)^{\frac{5}{2}}} - \frac{3c\sqrt{-ce^2x^2 + cd^2}}{256d^2e(ex+d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{512} \left( \frac{3\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}d^2} + \frac{2\left(24\sqrt{-(xe+d)c+2cd}c^5d^3 - 44(-(xe+d)c+2cd)^{\frac{3}{2}}c^4d\right)}{(xe+d)^3c^3d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.64 Problem number 879

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{24d(ex+d)^{\frac{3}{2}}\sqrt{-ce^2x^2+cd^2}}{35ce} - \frac{2(ex+d)^{\frac{5}{2}}\sqrt{-ce^2x^2+cd^2}}{7ce} \\ & -\frac{256d^3\sqrt{-ce^2x^2+cd^2}}{35ce\sqrt{ex+d}} - \frac{64d^2\sqrt{ex+d}\sqrt{-ce^2x^2+cd^2}}{35ce} \end{aligned}$$

command

`integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{35} \left( \frac{128\sqrt{2}\sqrt{cd}d^3}{c} - \frac{280\sqrt{-(xe+d)c+2cd}d^3}{c} + \frac{140(-(xe+d)c+2cd)^{\frac{3}{2}}c^2d^2 - 42((xe+d)c-2cd)^2\sqrt{-(xe+d)c+2cd}}{c^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{-ce^2x^2+cd^2}} dx$$

## 14.65 Problem number 880

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ex+d)^{\frac{3}{2}}\sqrt{-ce^2x^2+cd^2}}{5ce} - \frac{64d^2\sqrt{-ce^2x^2+cd^2}}{15ce\sqrt{ex+d}} - \frac{16d\sqrt{ex+d}\sqrt{-ce^2x^2+cd^2}}{15ce} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{15} \left( \frac{32\sqrt{2}\sqrt{cd}d^2}{c} - \frac{60\sqrt{-(xe+d)c+2cd}d^2}{c} + \frac{20(-(xe+d)c+2cd)^{\frac{3}{2}}cd - 3((xe+d)c-2cd)^2\sqrt{-(xe+d)c+2cd}}{c^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{-ce^2x^2+cd^2}} dx$$

### 14.66 Problem number 881

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{8d\sqrt{-ce^2x^2 + cd^2}}{3ce\sqrt{ex+d}} - \frac{2\sqrt{ex+d}\sqrt{-ce^2x^2 + cd^2}}{3ce}$$

command

```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{4\sqrt{2}\sqrt{cd}d}{c} - \frac{6\sqrt{-(xe+d)c+2cd}d}{c} + \frac{(-(xe+d)c+2cd)^{3/2}}{c^2} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{3/2}}{\sqrt{-ce^2x^2 + cd^2}} dx$$

### 14.67 Problem number 882

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{-ce^2x^2 + cd^2}}{ce\sqrt{ex+d}}$$

command

```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\sqrt{2}\sqrt{cd}}{c} - \frac{\sqrt{-(xe+d)c+2cd}}{c} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}}{\sqrt{-ce^2x^2 + cd^2}} dx$$

## 14.68 Problem number 883

$$\int \frac{1}{\sqrt{d+ex} \sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex+d}}\right) \sqrt{2}}{e\sqrt{c} \sqrt{d}}$$

command

`integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-(xe+d)c + 2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{cd}}{\sqrt{-cd}}\right)}{\sqrt{-cd}} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-ce^2x^2 + cd^2} \sqrt{ex+d}} dx$$

## 14.69 Problem number 884

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{cd^2 - ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2 + cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex+d}}\right) \sqrt{2}}{4d^{3/2} e \sqrt{c}} - \frac{\sqrt{-ce^2x^2 + cd^2}}{2cde (ex+d)^{3/2}}$$

command

`integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{\sqrt{2} c \arctan\left(\frac{\sqrt{2} \sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd} d} - \frac{2\sqrt{-(xe+d)c+2cd}}{(xe+d)d} \right) e^{(-1)}}{4c}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-ce^2x^2+cd^2} (ex+d)^{\frac{3}{2}}} dx$$

**14.70 Problem number 885**

$$\int \frac{1}{(d+ex)^{5/2} \sqrt{cd^2-ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2+cd^2} \sqrt{2}}{2\sqrt{c} \sqrt{d} \sqrt{ex+d}}\right) \sqrt{2}}{32d^{\frac{5}{2}} e \sqrt{c}} - \frac{\sqrt{-ce^2x^2+cd^2}}{4cde (ex+d)^{\frac{5}{2}}} - \frac{3\sqrt{-ce^2x^2+cd^2}}{16cd^2e (ex+d)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{3\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd} d^2} - \frac{2\left(10\sqrt{-(xe+d)c+2cd} c^2 d - 3(-(xe+d)c+2cd)^{\frac{3}{2}} c\right)}{(xe+d)^2 c^2 d^2} \right) e^{(-1)}}{32c}$$

Giac 1.7.0 via sagemath 9.3 output

`sage_0x`

## 14.71 Problem number 886

$$\int \frac{(d+ex)^{9/2}}{(cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{64d^2(ex+d)^{\frac{3}{2}}}{5ce\sqrt{-ce^2x^2+cd^2}} - \frac{8d(ex+d)^{\frac{5}{2}}}{5ce\sqrt{-ce^2x^2+cd^2}} - \frac{2(ex+d)^{\frac{7}{2}}}{5ce\sqrt{-ce^2x^2+cd^2}} + \frac{256d^3\sqrt{ex+d}}{5ce\sqrt{-ce^2x^2+cd^2}}$$

command

`integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{128\sqrt{2}d^3e^{(-1)}}{5\sqrt{cd}c} + \frac{16d^3e^{(-1)}}{\sqrt{-(xe+d)c+2cd}c}$$

$$+ \frac{2\left(60\sqrt{-(xe+d)c+2cd}c^{18}d^2e^4 - 10(-(xe+d)c+2cd)^{\frac{3}{2}}c^{17}de^4 + ((xe+d)c-2cd)^2\sqrt{-(xe+d)c+2cd}c^{16}\right)}{5c^{20}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.72 Problem number 887

$$\int \frac{(d+ex)^{7/2}}{(cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{16d(ex+d)^{\frac{3}{2}}}{3ce\sqrt{-ce^2x^2+cd^2}} - \frac{2(ex+d)^{\frac{5}{2}}}{3ce\sqrt{-ce^2x^2+cd^2}} + \frac{64d^2\sqrt{ex+d}}{3ce\sqrt{-ce^2x^2+cd^2}}$$

command

`integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{32\sqrt{2}d^2e^{(-1)}}{3\sqrt{cd}c} + \frac{8d^2e^{(-1)}}{\sqrt{-(xe+d)c+2cd}c}$$

$$+ \frac{2\left(12\sqrt{-(xe+d)c+2cd}c^7de^2 - (-(xe+d)c+2cd)^{\frac{3}{2}}c^6e^2\right)e^{(-3)}}{3c^9}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.73 Problem number 888

$$\int \frac{(d+ex)^{5/2}}{(cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex+d)^{3/2}}{ce\sqrt{-ce^2x^2+cd^2}} + \frac{8d\sqrt{ex+d}}{ce\sqrt{-ce^2x^2+cd^2}}$$

command

`integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4\sqrt{2}de^{(-1)}}{\sqrt{cd}c} + \frac{2\left(\frac{2de^{(-1)}}{\sqrt{-(xe+d)c+2cd}} + \frac{\sqrt{-(xe+d)c+2cd}e^{(-1)}}{c}\right)}{c}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.74 Problem number 889

$$\int \frac{(d+ex)^{3/2}}{(cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ex+d}}{ce\sqrt{-ce^2x^2+cd^2}}$$

command

`integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{2}e^{(-1)}}{\sqrt{cd}c} + \frac{2e^{(-1)}}{\sqrt{-(xe+d)c+2cd}c}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 14.75 Problem number 890

$$\int \frac{\sqrt{d+ex}}{(cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2+cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{2c^{\frac{3}{2}}d^{\frac{3}{2}}e} + \frac{\sqrt{ex+d}}{cde\sqrt{-ce^2x^2+cd^2}}$$

command

```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right) e^{(-1)}}{2\sqrt{-cd} cd} - \frac{\sqrt{2} \left(\sqrt{cd} \arctan\left(\frac{\sqrt{cd}}{\sqrt{-cd}}\right) + \sqrt{-cd}\right) e^{(-1)}}{2\sqrt{cd} \sqrt{-cd} cd} + \frac{e^{(-1)}}{\sqrt{-(xe+d)c+2cd} cd}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.76 Problem number 891

$$\int \frac{1}{\sqrt{d+ex} (cd^2 - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2+cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{8c^{\frac{3}{2}}d^{\frac{5}{2}}e} - \frac{1}{2cde\sqrt{ex+d}\sqrt{-ce^2x^2+cd^2}} + \frac{3\sqrt{ex+d}}{4cd^2e\sqrt{-ce^2x^2+cd^2}}$$

command

```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)e^{(-1)}}{8\sqrt{-cd}cd^2} + \frac{(3(xe+d)c-2cd)e^{(-1)}}{4\left(2\sqrt{-(xe+d)c+2cd}cd - (-(xe+d)c+2cd)^{\frac{3}{2}}\right)cd^2}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.77 Problem number 892

$$\int \frac{1}{(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{15 \operatorname{arctanh}\left(\frac{\sqrt{-ce^2x^2+cd^2}\sqrt{2}}{2\sqrt{c}\sqrt{d}\sqrt{ex+d}}\right)\sqrt{2}}{64c^{\frac{3}{2}}d^{\frac{7}{2}}e} - \frac{1}{4cde(ex+d)^{\frac{3}{2}}\sqrt{-ce^2x^2+cd^2}} \\ & - \frac{5}{16cd^2e\sqrt{ex+d}\sqrt{-ce^2x^2+cd^2}} + \frac{15\sqrt{ex+d}}{32cd^3e\sqrt{-ce^2x^2+cd^2}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{15\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{-(xe+d)c+2cd}}{2\sqrt{-cd}}\right)e^{(-1)}}{64\sqrt{-cd}cd^3} + \frac{e^{(-1)}}{4\sqrt{-(xe+d)c+2cd}cd^3} \\ & - \frac{\left(18\sqrt{-(xe+d)c+2cd}cd - 7(-(xe+d)c+2cd)^{\frac{3}{2}}\right)e^{(-1)}}{32(xe+d)^2c^3d^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.78 Problem number 895

$$\int (2 + ex)^{3/2} \sqrt{12 - 3e^2x^2} dx$$

Optimal antiderivative

$$-\frac{32(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{3e} + \frac{16(-ex + 2)^{\frac{5}{2}} \sqrt{3}}{5e} - \frac{2(-ex + 2)^{\frac{7}{2}} \sqrt{3}}{7e}$$

command

`integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \sqrt{3} \left( 84(xe - 2)^2 \sqrt{-xe + 2} + \left( (15(xe - 2)^3 \sqrt{-xe + 2} + 84(xe - 2)^2 \sqrt{-xe + 2} - 140(-xe + 2)^{\frac{3}{2}} \right) e^{(-2)} + \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-3e^2x^2 + 12} (ex + 2)^{\frac{3}{2}} dx$$

## 14.79 Problem number 897

$$\int \frac{\sqrt{12 - 3e^2x^2}}{\sqrt{2 + ex}} dx$$

Optimal antiderivative

$$-\frac{2(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{3e}$$

command

`integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{3} \sqrt{3} \left( (-xe + 2)^{\frac{3}{2}} - 8 \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-3e^2x^2 + 12}}{\sqrt{ex + 2}} dx$$

## 14.80 Problem number 899

$$\int \frac{\sqrt{12 - 3e^2x^2}}{(2 + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ex+2}}{2}\right)\sqrt{3}}{2e} - \frac{\sqrt{3}\sqrt{-ex+2}}{e(ex+2)}$$

command

`integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4}\sqrt{3}\left(\frac{4\sqrt{-xe+2}}{xe+2} - \log(\sqrt{-xe+2}+2) + \log(-\sqrt{-xe+2}+2)\right)e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-3e^2x^2+12}}{(ex+2)^{5/2}} dx$$

## 14.81 Problem number 902

$$\int (2 + ex)^{3/2} (12 - 3e^2x^2)^{3/2} dx$$

Optimal antiderivative

$$-\frac{384(-ex+2)^{5/2}\sqrt{3}}{5e} + \frac{288(-ex+2)^{7/2}\sqrt{3}}{7e} - \frac{8(-ex+2)^{9/2}\sqrt{3}}{e} + \frac{6(-ex+2)^{11/2}\sqrt{3}}{11e}$$

command

`integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{1155}\sqrt{3}\left(11088(xe-2)^2\sqrt{-xe+2} - \left(\left(315(xe-2)^5\sqrt{-xe+2} + 3080(xe-2)^4\sqrt{-xe+2} + 11880(xe-2)^3\sqrt{-xe+2} + 11880(xe-2)^2\sqrt{-xe+2} + 3080(xe-2)\sqrt{-xe+2} + 1188\right)\sqrt{-xe+2}\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (-3e^2x^2+12)^{3/2}(ex+2)^{3/2} dx$$

### 14.82 Problem number 904

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{\sqrt{2 + ex}} dx$$

Optimal antiderivative

$$-\frac{24(-ex + 2)^{\frac{5}{2}} \sqrt{3}}{5e} + \frac{6(-ex + 2)^{\frac{7}{2}} \sqrt{3}}{7e}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{35} \sqrt{3} \left( \left( \left( 15(xe - 2)^3 \sqrt{-xe + 2} + 84(xe - 2)^2 \sqrt{-xe + 2} - 140(-xe + 2)^{\frac{3}{2}} \right) e^{(-2)} + 352 e^{(-2)} \right) e^2 + 140(-xe + 2) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-3e^2x^2 + 12)^{\frac{3}{2}}}{\sqrt{ex + 2}} dx$$

### 14.83 Problem number 905

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{6(-ex + 2)^{\frac{5}{2}} \sqrt{3}}{5e}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6}{5} \sqrt{3} \left( (xe - 2)^2 \sqrt{-xe + 2} - 32 \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.84 Problem number 906

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{e} - \frac{48 \operatorname{arctanh}\left(\frac{\sqrt{-ex + 2}}{2}\right) \sqrt{3}}{e} + \frac{24\sqrt{3} \sqrt{-ex + 2}}{e}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{3} \left( (-xe + 2)^{\frac{3}{2}} + 12\sqrt{-xe + 2} - 12 \log(\sqrt{-xe + 2} + 2) + 12 \log(-\sqrt{-xe + 2} + 2) \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

## 14.85 Problem number 907

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{7/2}} dx$$

Optimal antiderivative

$$-\frac{3(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{e(ex + 2)} + \frac{18 \operatorname{arctanh}\left(\frac{\sqrt{-ex + 2}}{2}\right) \sqrt{3}}{e} - \frac{9\sqrt{3} \sqrt{-ex + 2}}{e}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-3\sqrt{3} \left( 2\sqrt{-xe + 2} + \frac{4\sqrt{-xe + 2}}{xe + 2} - 3 \log(\sqrt{-xe + 2} + 2) + 3 \log(-\sqrt{-xe + 2} + 2) \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.86 Problem number 908

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{9/2}} dx$$

Optimal antiderivative

$$-\frac{3(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{2e(ex + 2)^2} - \frac{9 \operatorname{arctanh}\left(\frac{\sqrt{-ex + 2}}{2}\right) \sqrt{3}}{8e} + \frac{9\sqrt{3} \sqrt{-ex + 2}}{4e(ex + 2)}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3}{16} \sqrt{3} \left( \frac{4 \left( 5(-xe + 2)^{\frac{3}{2}} - 12 \sqrt{-xe + 2} \right)}{(xe + 2)^2} + 3 \log(\sqrt{-xe + 2} + 2) - 3 \log(-\sqrt{-xe + 2} + 2) \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

## 14.87 Problem number 909

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{11/2}} dx$$

Optimal antiderivative

$$-\frac{(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{e(ex + 2)^3} - \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{-ex + 2}}{2}\right) \sqrt{3}}{64e} + \frac{3\sqrt{3} \sqrt{-ex + 2}}{4e(ex + 2)^2} - \frac{3\sqrt{3} \sqrt{-ex + 2}}{32e(ex + 2)}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{128} \sqrt{3} \left( \frac{4 \left( 3(xe - 2)^2 \sqrt{-xe + 2} + 32(-xe + 2)^{\frac{3}{2}} - 48 \sqrt{-xe + 2} \right)}{(xe + 2)^3} + 3 \log(\sqrt{-xe + 2} + 2) - 3 \log(-\sqrt{-xe + 2} + 2) \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.88 Problem number 910

$$\int \frac{(12 - 3e^2x^2)^{3/2}}{(2 + ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(-ex + 2)^{\frac{3}{2}} \sqrt{3}}{4e(ex + 2)^4} - \frac{9 \operatorname{arctanh}\left(\frac{\sqrt{-ex + 2}}{2}\right) \sqrt{3}}{2048e} \\ & + \frac{3\sqrt{3} \sqrt{-ex + 2}}{8e(ex + 2)^3} - \frac{3\sqrt{3} \sqrt{-ex + 2}}{128e(ex + 2)^2} - \frac{9\sqrt{3} \sqrt{-ex + 2}}{1024e(ex + 2)} \end{aligned}$$

command

```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3}{4096} \sqrt{3} \left( \frac{4 \left( 3(xe - 2)^3 \sqrt{-xe + 2} + 44(xe - 2)^2 \sqrt{-xe + 2} + 176(-xe + 2)^{\frac{3}{2}} - 192 \sqrt{-xe + 2} \right)}{(xe + 2)^4} \right) + 3 \log(\sqrt{\dots})$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.89 Problem number 914

$$\int \frac{\sqrt{2 + ex}}{\sqrt{12 - 3e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{3} \sqrt{-ex + 2}}{3e}$$

command

```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{3} \sqrt{3} (\sqrt{-xe + 2} - 2) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex + 2}}{\sqrt{-3e^2x^2 + 12}} dx$$



## 14.90 Problem number 915

$$\int \frac{1}{\sqrt{2+ex} \sqrt{12-3e^2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ex+2}}{2}\right) \sqrt{3}}{3e}$$

command

```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6} \sqrt{3} (\log(\sqrt{-xe+2} + 2) - \log(-\sqrt{-xe+2} + 2)) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-3e^2x^2+12} \sqrt{ex+2}} dx$$

## 14.91 Problem number 917

$$\int \frac{1}{(2+ex)^{5/2} \sqrt{12-3e^2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ex+2}}{2}\right) \sqrt{3}}{128e} - \frac{\sqrt{3} \sqrt{-ex+2}}{24e (ex+2)^2} - \frac{\sqrt{3} \sqrt{-ex+2}}{64e (ex+2)}$$

command

```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{768} \sqrt{3} \left( \frac{4 \left( 3(-xe+2)^{\frac{3}{2}} - 20 \sqrt{-xe+2} \right)}{(xe+2)^2} - 3 \log(\sqrt{-xe+2} + 2) + 3 \log(-\sqrt{-xe+2} + 2) \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.92 Problem number 919

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{8(-ex+2)^{\frac{3}{2}}\sqrt{3}}{9e} + \frac{2(-ex+2)^{\frac{5}{2}}\sqrt{3}}{45e} + \frac{128\sqrt{3}}{9e\sqrt{-ex+2}} + \frac{32\sqrt{3}\sqrt{-ex+2}}{3e}$$

command

```
integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{45}\sqrt{3}\left((xe-2)^2\sqrt{-xe+2}e^4 - 20(-xe+2)^{\frac{3}{2}}e^4 + 240\sqrt{-xe+2}e^4\right)e^{(-5)} \\ - \frac{1024}{45}\sqrt{3}e^{(-1)} + \frac{128\sqrt{3}e^{(-1)}}{9\sqrt{-xe+2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 14.93 Problem number 921

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{8\sqrt{3}}{9e\sqrt{-ex+2}} + \frac{2\sqrt{3}\sqrt{-ex+2}}{9e}$$

command

```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{9}\sqrt{3}\sqrt{-xe+2}e^{(-1)} - \frac{8}{9}\sqrt{3}e^{(-1)} + \frac{8\sqrt{3}e^{(-1)}}{9\sqrt{-xe+2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.94 Problem number 922

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{3}}{9e\sqrt{-ex+2}}$$

command

```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{9}\sqrt{3}e^{(-1)} + \frac{2\sqrt{3}e^{(-1)}}{9\sqrt{-xe+2}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.95 Problem number 923

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ex+2}}{2}\right)\sqrt{3}}{36e} + \frac{\sqrt{3}}{18e\sqrt{-ex+2}}$$

command

```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{72}\sqrt{3}e^{(-1)}\log(\sqrt{-xe+2}+2) + \frac{1}{72}\sqrt{3}e^{(-1)}\log(-\sqrt{-xe+2}+2) + \frac{\sqrt{3}e^{(-1)}}{18\sqrt{-xe+2}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.96 Problem number 924

$$\int \frac{1}{\sqrt{2+ex} (12-3e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-ex+2}}{2}\right)\sqrt{3}}{96e} + \frac{\sqrt{3}}{48e\sqrt{-ex+2}} - \frac{\sqrt{3}}{36e(ex+2)\sqrt{-ex+2}}$$

command

`integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{192}\sqrt{3}e^{(-1)}\log(\sqrt{-xe+2}+2) + \frac{1}{192}\sqrt{3}e^{(-1)}\log(-\sqrt{-xe+2}+2) - \frac{\sqrt{3}(3xe+2)e^{(-1)}}{144\left((-xe+2)^{\frac{3}{2}}-4\sqrt{-xe+2}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.97 Problem number 938

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5\sqrt[3]{4}(-ex+2)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}}{6e} - \frac{\sqrt[3]{4}(-ex+2)^{\frac{3}{4}}(ex+2)^{\frac{5}{4}}}{6e} \\ & -\frac{5\operatorname{arctan}\left(-1+\frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}}\right)\sqrt{2}\sqrt[3]{4}}{6e} - \frac{5\operatorname{arctan}\left(1+\frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}}\right)\sqrt{2}\sqrt[3]{4}}{6e} \\ & -\frac{5\ln\left(\sqrt{3}-\frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}}+\frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}}\right)\sqrt[3]{4}\sqrt{2}}{12e} \\ & +\frac{5\ln\left(\sqrt{3}+\frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}}+\frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}}\right)\sqrt[3]{4}\sqrt{2}}{12e} \end{aligned}$$

command

```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{24} \cdot 3^{\frac{3}{4}} \left( (xe+2)^2 \left( 5 \left( \frac{4}{xe+2} - 1 \right)^{\frac{7}{4}} + 9 \left( \frac{4}{xe+2} - 1 \right)^{\frac{3}{4}} \right) + 20 \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} \left( \sqrt{2} + 2 \left( \frac{4}{xe+2} - 1 \right)^{\frac{1}{4}} \right) \right) \right) + 2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+2)^{\frac{3}{2}}}{(-3e^2x^2+12)^{\frac{1}{4}}} dx$$

### 14.98 Problem number 939

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3^{\frac{3}{4}}(-ex+2)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}}{3e} \ln \left( \sqrt{3} - \frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}} + \frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}} \right) \frac{3^{\frac{3}{4}}\sqrt{2}}{6e} \\ & + \frac{\ln \left( \sqrt{3} + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}} + \frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}} \right) 3^{\frac{3}{4}}\sqrt{2}}{6e} \\ & - \frac{\arctan \left( -1 + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}} \right) \sqrt{2} 3^{\frac{3}{4}}}{3e} - \frac{\arctan \left( 1 + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}} \right) \sqrt{2} 3^{\frac{3}{4}}}{3e} \end{aligned}$$

command

```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6} \cdot 3^{\frac{3}{4}} \left( 2(xe+2) \left( \frac{4}{xe+2} - 1 \right)^{\frac{3}{4}} + 2 \sqrt{2} \arctan \left( \frac{1}{2} \sqrt{2} \left( \sqrt{2} + 2 \left( \frac{4}{xe+2} - 1 \right)^{\frac{1}{4}} \right) \right) \right) + 2 \sqrt{2} \arctan \left( -\frac{1}{2} \sqrt{2} \left( \sqrt{2} + 2 \left( \frac{4}{xe+2} - 1 \right)^{\frac{1}{4}} \right) \right) + 2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+2}}{(-3e^2x^2+12)^{\frac{1}{4}}} dx$$

## 14.99 Problem number 940

$$\int \frac{1}{\sqrt{2+ex} \sqrt[4]{12-3e^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln\left(\sqrt{3} - \frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}} + \frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}}\right) 3^{\frac{3}{4}}\sqrt{2}}{6e} \\ & + \frac{\ln\left(\sqrt{3} + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{6}}{(ex+2)^{\frac{1}{4}}} + \frac{\sqrt{3}\sqrt{-ex+2}}{\sqrt{ex+2}}\right) 3^{\frac{3}{4}}\sqrt{2}}{6e} \\ & - \frac{\arctan\left(-1 + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}}\right) \sqrt{2} 3^{\frac{3}{4}}}{3e} - \frac{\arctan\left(1 + \frac{(-ex+2)^{\frac{1}{4}}\sqrt{2}}{(ex+2)^{\frac{1}{4}}}\right) \sqrt{2} 3^{\frac{3}{4}}}{3e} \end{aligned}$$

command

`integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{6} \\ & \cdot 3^{\frac{3}{4}} \left( 2\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\left(\frac{4}{xe+2} - 1\right)^{\frac{1}{4}}\right)\right) + 2\sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\left(\frac{4}{xe+2} - 1\right)^{\frac{1}{4}}\right)\right) \right) - \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(-3e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}} dx$$

## 14.100 Problem number 942

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{(-e^2x^2+4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{21e(ex+2)^{\frac{5}{2}}} - \frac{(-e^2x^2+4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{63e(ex+2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{252} \cdot 3^{\frac{3}{4}} \left( 3 \left( \frac{4}{xe+2} - 1 \right)^{\frac{7}{4}} + 7 \left( \frac{4}{xe+2} - 1 \right)^{\frac{3}{4}} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(-3e^2x^2 + 12)^{\frac{1}{4}}(ex + 2)^{\frac{5}{2}}} dx$$

**14.101 Problem number 944**

$$\int \frac{1}{(2 + ex)^{9/2} \sqrt[4]{12 - 3e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{(-e^2x^2 + 4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{45e(ex + 2)^{\frac{9}{2}}} - \frac{(-e^2x^2 + 4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{165e(ex + 2)^{\frac{7}{2}}} - \frac{2(-e^2x^2 + 4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{1155e(ex + 2)^{\frac{5}{2}}} - \frac{2(-e^2x^2 + 4)^{\frac{3}{4}} 3^{\frac{3}{4}}}{3465e(ex + 2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{221760} \cdot 3^{\frac{3}{4}} \left( 77 \left( \frac{4}{xe+2} - 1 \right)^{\frac{15}{4}} + 315 \left( \frac{4}{xe+2} - 1 \right)^{\frac{11}{4}} + 495 \left( \frac{4}{xe+2} - 1 \right)^{\frac{7}{4}} + 385 \left( \frac{4}{xe+2} - 1 \right)^{\frac{3}{4}} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(-3e^2x^2 + 12)^{\frac{1}{4}}(ex + 2)^{\frac{9}{2}}} dx$$

### 14.102 Problem number 998

$$\int \frac{(d + ex)^5}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^4}{4ce}$$

command

```
integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^4e^3 + 4dx^3e^2 + 6d^2x^2e + 4d^3x}{4c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.103 Problem number 999

$$\int \frac{(d + ex)^4}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^3}{3ce}$$

command

```
integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3e^2 + 3dx^2e + 3d^2x}{3c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 14.104 Problem number 1000

$$\int \frac{(d + ex)^3}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^2}{2ce}$$

command

```
integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2e + 2 dx}{2c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.105 Problem number 1001

$$\int \frac{(d + ex)^2}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{x}{c}$$

command

```
integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.106 Problem number 1002

$$\int \frac{d + ex}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{\ln(ex + d)}{ce}$$

command

```
integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{(-1)} \log(|cd^2 + (x^2e + 2dx)ce|)}{2c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.107 Problem number 1003

$$\int \frac{1}{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$-\frac{1}{ce(ex + d)}$$

command

```
integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{(xe + d)c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.108 Problem number 1004

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)} dx$$

Optimal antiderivative

$$-\frac{1}{2ce(ex+d)^2}$$

command

```
integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{2(xe+d)^2c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.109 Problem number 1005

$$\int \frac{1}{(d+ex)^2(cd^2+2cdex+ce^2x^2)} dx$$

Optimal antiderivative

$$-\frac{1}{3ce(ex+d)^3}$$

command

```
integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{3(xe+d)^3c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.110 Problem number 1006

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)} dx$$

Optimal antiderivative

$$-\frac{1}{4ce(ex+d)^4}$$

command

```
integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(xe+d)^4c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.111 Problem number 1007

$$\int \frac{(d+ex)^7}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$\frac{(ex+d)^4}{4c^2e}$$

command

```
integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^4e^3 + 4dx^3e^2 + 6d^2x^2e + 4d^3x}{4c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.112 Problem number 1008

$$\int \frac{(d + ex)^6}{(cd^2 + 2cde x + ce^2 x^2)^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^3}{3c^2e}$$

command

```
integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3 e^2 + 3 dx^2 e + 3 d^2 x}{3 c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.113 Problem number 1009

$$\int \frac{(d + ex)^5}{(cd^2 + 2cde x + ce^2 x^2)^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^2}{2c^2e}$$

command

```
integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2 e + 2 dx}{2 c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.114 Problem number 1010

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$\frac{x}{c^2}$$

command

```
integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.115 Problem number 1011

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$\frac{\ln(ex + d)}{c^2e}$$

command

```
integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{d^2e^{(-1)}}{2(cd^2 + (x^2e + 2dx)ce)c} + \frac{d^2e^{(-1)}}{cd^2+(x^2e+2dx)ce} - \frac{e^{(-1)} \log\left(\frac{|cd^2+(x^2e+2dx)ce|e^{(-1)}}{2(cd^2+(x^2e+2dx)ce)^2|c|}\right)}{2c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.116 Problem number 1012**

$$\int \frac{(d + ex)^2}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{c^2e(ex + d)}$$

command

```
integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{(xe + d)c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.117 Problem number 1013**

$$\int \frac{d + ex}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{2c^2e(ex + d)^2}$$

command

```
integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{2(cd^2 + (x^2e + 2dx)ce)c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.118 Problem number 1014

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{3c^2e(ex+d)^3}$$

command

```
integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{3(xe+d)^3c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.119 Problem number 1015

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{4c^2e(ex+d)^4}$$

command

```
integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(xe+d)^4c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 14.120 Problem number 1016

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{5c^2e(ex+d)^5}$$

command

```
integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{5(xe+d)^5c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.121 Problem number 1017

$$\int \frac{(d+ex)^9}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$\frac{(ex+d)^4}{4c^3e}$$

command

```
integrate((e*x+d)^9/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^4e^3 + 4dx^3e^2 + 6d^2x^2e + 4d^3x}{4c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.122 Problem number 1018**

$$\int \frac{(d + ex)^8}{(cd^2 + 2cde x + ce^2 x^2)^3} dx$$

Optimal antiderivative

$$\frac{(ex + d)^3}{3c^3 e}$$

command

```
integrate((e*x+d)^8/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3 e^2 + 3 dx^2 e + 3 d^2 x}{3 c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.123 Problem number 1019**

$$\int \frac{(d + ex)^7}{(cd^2 + 2cde x + ce^2 x^2)^3} dx$$

Optimal antiderivative

$$\frac{(ex + d)^2}{2c^3 e}$$

command

```
integrate((e*x+d)^7/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2 e + 2 dx}{2 c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.124 Problem number 1020

$$\int \frac{(d + ex)^6}{(cd^2 + 2cde x + ce^2 x^2)^3} dx$$

Optimal antiderivative

$$\frac{x}{c^3}$$

command

```
integrate((e*x+d)^6/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.125 Problem number 1021

$$\int \frac{(d + ex)^5}{(cd^2 + 2cde x + ce^2 x^2)^3} dx$$

Optimal antiderivative

$$\frac{\ln(ex + d)}{c^3 e}$$

command

```
integrate((e*x+d)^5/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{(-1)} \log(|xe + d|)}{c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.126 Problem number 1022

$$\int \frac{(d + ex)^4}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{c^3 e (ex + d)}$$

command

```
integrate((e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{(xe + d)c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.127 Problem number 1023

$$\int \frac{(d + ex)^3}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{2c^3 e (ex + d)^2}$$

command

```
integrate((e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{2(d^2 + (x^2e + 2dx)e)c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.128 Problem number 1024

$$\int \frac{(d+ex)^2}{(cd^2+2cdex+ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{3c^3e(ex+d)^3}$$

command

```
integrate((e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{3(xe+d)^3c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.129 Problem number 1025

$$\int \frac{d+ex}{(cd^2+2cdex+ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{4c^3e(ex+d)^4}$$

command

```
integrate((e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(cd^2+(x^2e+2dx)ce)^2c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.130 Problem number 1026

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{5c^3e(ex+d)^5}$$

command

```
integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{5(xe+d)^5c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.131 Problem number 1027

$$\int \frac{1}{(d+ex)(cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{6c^3e(ex+d)^6}$$

command

```
integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{6(xe+d)^6c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.132 Problem number 1028

$$\int \frac{1}{(d+ex)^2 (cd^2 + 2cdex + ce^2x^2)^3} dx$$

Optimal antiderivative

$$-\frac{1}{7c^3e(ex+d)^7}$$

command

```
integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{7(xe+d)^7c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.133 Problem number 1033

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{d+ex} dx$$

Optimal antiderivative

$$\frac{\sqrt{ce^2x^2 + 2cdex + cd^2}}{e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( de^{(-1)} \operatorname{sgn}(xe+d) + x \operatorname{sgn}(xe+d) \right) \sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.134 Problem number 1035

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$-\frac{c}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{c} e^{(-1)\text{sgn}(xe + d)}}{xe + d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.135 Problem number 1036

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{c}{2e(ex + d)\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{c} e^{(-1)\text{sgn}(xe + d)}}{2(xe + d)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 14.136 Problem number 1037

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$-\frac{c^2}{3e(c e^2 x^2 + 2cdex + c d^2)^{\frac{3}{2}}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{c} e^{(-1)} \operatorname{sgn}(xe + d)}{3(xe + d)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.137 Problem number 1038

$$\int \frac{\sqrt{cd^2 + 2cdex + ce^2x^2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$-\frac{c^2}{4e(ex + d)(c e^2 x^2 + 2cdex + c d^2)^{\frac{3}{2}}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{c} e^{(-1)} \operatorname{sgn}(xe + d)}{4(xe + d)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.138 Problem number 1043**

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{(ce^2x^2 + 2cdex + cd^2)^{3/2}}{3e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} (cx^3e^2 + 3cdx^2e + 3cd^2x) \sqrt{c} \operatorname{sgn}(xe + d)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**14.139 Problem number 1044**

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\frac{c(ex + d) \sqrt{ce^2x^2 + 2cdex + cd^2}}{2e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} (cx^2e \operatorname{sgn}(xe + d) + 2cdx \operatorname{sgn}(xe + d)) \sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.140 Problem number 1045

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\frac{c\sqrt{ce^2x^2 + 2cdex + cd^2}}{e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left(cde^{(-1)}\operatorname{sgn}(xe + d) + cx\operatorname{sgn}(xe + d)\right)\sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.141 Problem number 1046

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\frac{c^2(ex + d) \ln(ex + d)}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$c^{\frac{3}{2}}e^{(-1)}\log(|xe + d|)\operatorname{sgn}(xe + d)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.142 Problem number 1047

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$-\frac{c^2}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^{\frac{3}{2}}e^{(-1)}\operatorname{sgn}(xe + d)}{xe + d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.143 Problem number 1048

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$-\frac{c^2}{2e(ex + d)\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^{\frac{3}{2}}e^{(-1)}\operatorname{sgn}(xe + d)}{2(xe + d)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.144 Problem number 1049

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{3/2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$-\frac{c^3}{3e(c e^2 x^2 + 2cdex + c d^2)^{\frac{3}{2}}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^{\frac{3}{2}}e^{(-1)}\operatorname{sgn}(xe+d)}{3(xe+d)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.145 Problem number 1054

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{(c e^2 x^2 + 2cdex + c d^2)^{\frac{5}{2}}}{5e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{5} (c^2 x^5 e^4 + 5 c^2 d x^4 e^3 + 10 c^2 d^2 x^3 e^2 + 10 c^2 d^3 x^2 e + 5 c^2 d^4 x) \sqrt{c} \operatorname{sgn}(xe + d)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.146 Problem number 1055

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\frac{c(ex + d)(ce^2x^2 + 2cdex + cd^2)^{\frac{3}{2}}}{4e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} (c^2x^4e^3\operatorname{sgn}(xe + d) + 4c^2dx^3e^2\operatorname{sgn}(xe + d) + 6c^2d^2x^2e\operatorname{sgn}(xe + d) + 4c^2d^3x\operatorname{sgn}(xe + d))\sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.147 Problem number 1056

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\frac{c(ce^2x^2 + 2cdex + cd^2)^{\frac{3}{2}}}{3e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} (c^2x^3e^2\operatorname{sgn}(xe + d) + 3c^2dx^2e\operatorname{sgn}(xe + d) + 3c^2d^2x\operatorname{sgn}(xe + d))\sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.148 Problem number 1057

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\frac{c^2(ex + d) \sqrt{ce^2x^2 + 2cdex + cd^2}}{2e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} (c^2x^2 \operatorname{esgn}(xe + d) + 2c^2dx \operatorname{sgn}(xe + d)) \sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.149 Problem number 1058

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\frac{c^2 \sqrt{ce^2x^2 + 2cdex + cd^2}}{e}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( c^2de^{(-1)} \operatorname{sgn}(xe + d) + c^2x \operatorname{sgn}(xe + d) \right) \sqrt{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.150 Problem number 1059

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\frac{c^3(ex + d) \ln(ex + d)}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$c^{\frac{5}{2}}e^{(-1)} \log(|xe + d|) \operatorname{sgn}(xe + d)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.151 Problem number 1060

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$-\frac{c^3}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^{\frac{5}{2}}e^{(-1)}\operatorname{sgn}(xe + d)}{xe + d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 14.152 Problem number 1061

$$\int \frac{(cd^2 + 2cdex + ce^2x^2)^{5/2}}{(d + ex)^8} dx$$

Optimal antiderivative

$$-\frac{c^3}{2e(ex + d)\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^{\frac{5}{2}}e^{(-1)}\operatorname{sgn}(xe + d)}{2(xe + d)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.153 Problem number 1067

$$\int \frac{1}{(d + ex)\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{1}{e\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{(xe + d)\sqrt{c}\operatorname{sgn}(xe + d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.154 Problem number 1068

$$\int \frac{1}{(d+ex)^2 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{1}{2e(ex+d)\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

`integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{2(xe+d)^2\sqrt{c}\operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.155 Problem number 1069

$$\int \frac{1}{(d+ex)^3 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{c}{3e(c e^2 x^2 + 2 c d e x + c d^2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{3(xe+d)^3\sqrt{c}\operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.156 Problem number 1070

$$\int \frac{1}{(d+ex)^4 \sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{c}{4e(ex+d)(ce^2x^2 + 2cdex + cd^2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)^4/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(xe+d)^4 \sqrt{c} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.157 Problem number 1075

$$\int \frac{1}{(cd^2 + 2cdex + ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2ce(ex+d)\sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

`integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{2(xe+d)^2 c^{\frac{3}{2}} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.158 Problem number 1076

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{3e(c e^2 x^2 + 2cdex + c d^2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{3(xe+d)^3 c^{\frac{3}{2}} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.159 Problem number 1077

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4e(ex+d)(c e^2 x^2 + 2cdex + c d^2)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(xe+d)^4 c^{\frac{3}{2}} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.160 Problem number 1078

$$\int \frac{1}{(d+ex)^3 (cd^2+2cdex+ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c}{5e(c e^2 x^2 + 2cdex + c d^2)^{5/2}}$$

command

```
integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{5(xe+d)^5 c^{3/2} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.161 Problem number 1085

$$\int \frac{1}{(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4ce(ex+d)(ce^2x^2+2cdex+cd^2)^{3/2}}$$

command

```
integrate(1/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{4(xe+d)^4 c^{5/2} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.162 Problem number 1086

$$\int \frac{1}{(d+ex)(cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{5e(c e^2 x^2 + 2cdex + c d^2)^{5/2}}$$

command

```
integrate(1/(e*x+d)/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{5(xe+d)^5 c^{5/2} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.163 Problem number 1087

$$\int \frac{1}{(d+ex)^2 (cd^2+2cdex+ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{6e(ex+d)(ce^2x^2+2cdex+cd^2)^{5/2}}$$

command

```
integrate(1/(e*x+d)^2/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{6(xe+d)^6 c^{5/2} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.164 Problem number 1088

$$\int \frac{1}{(d+ex)^3 (cd^2 + 2cdex + ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{c}{7e(c e^2 x^2 + 2cdex + c d^2)^{7/2}}$$

command

```
integrate(1/(e*x+d)^3/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{(-1)}}{7(xe+d)^7 c^{5/2} \operatorname{sgn}(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.165 Problem number 1094

$$\int (d+ex)^m (cd^2 + 2cdex + ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{(ex+d)^{1+m} (ce^2x^2 + 2cdex + cd^2)^{3/2}}{e(4+m)}$$

command

```
integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(cx^3 e^{(m \log(xe+d) + \log(xe+d) + 3)} \operatorname{sgn}(xe+d) + 3cdx^2 e^{(m \log(xe+d) + \log(xe+d) + 2)} \operatorname{sgn}(xe+d) + 3cd^2 x e^{(m \log(xe+d) + \log(xe+d) + 1)} \operatorname{sgn}(xe+d))}{me + 4e}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (ce^2x^2 + 2cdex + cd^2)^{3/2} (ex+d)^m dx$$

## 14.166 Problem number 1095

$$\int (d + ex)^m \sqrt{cd^2 + 2cdex + ce^2x^2} dx$$

Optimal antiderivative

$$\frac{(ex + d)^{1+m} \sqrt{ce^2x^2 + 2cdex + cd^2}}{e(2 + m)}$$

command

```
integrate((e*x+d)^m*(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(xe + d)^{m+2} \sqrt{c} e^{(-1)} \operatorname{sgn}(xe + d)}{m + 2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{ce^2x^2 + 2cdex + cd^2} (ex + d)^m dx$$

## 14.167 Problem number 1096

$$\int \frac{(d + ex)^m}{\sqrt{cd^2 + 2cdex + ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{(ex + d)^{1+m}}{em \sqrt{ce^2x^2 + 2cdex + cd^2}}$$

command

```
integrate((e*x+d)^m/(c*e^2*x^2+2*c*d*e*x+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(xe + d)^m e^{(-1)}}{\sqrt{c} m \operatorname{sgn}(xe + d)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex + d)^m}{\sqrt{ce^2x^2 + 2cdex + cd^2}} dx$$



## 14.168 Problem number 1596

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e(-ae+bd)^3}{b^5\sqrt{(bx+a)^2}} - \frac{(-ae+bd)^4}{2b^5(bx+a)\sqrt{(bx+a)^2}} + \frac{e^3(-3ae+4bd)x(bx+a)}{b^4\sqrt{(bx+a)^2}} \\ & + \frac{e^4x^2(bx+a)}{2b^3\sqrt{(bx+a)^2}} + \frac{6e^2(-ae+bd)^2(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{6(b^2d^2e^2 - 2abde^3 + a^2e^4) \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)} \\ & + \frac{b^3x^2e^4 \operatorname{sgn}(bx+a) + 8b^3dx e^3 \operatorname{sgn}(bx+a) - 6ab^2x e^4 \operatorname{sgn}(bx+a)}{2b^6} \\ & - \frac{b^4d^4 + 4ab^3d^3e - 18a^2b^2d^2e^2 + 20a^3bde^3 - 7a^4e^4 + 8(b^4d^3e - 3ab^3d^2e^2 + 3a^2b^2de^3 - a^3be^4)x}{2(bx+a)^2 b^5 \operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.169 Problem number 1597

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3e(-ae+bd)^2}{b^4\sqrt{(bx+a)^2}} - \frac{(-ae+bd)^3}{2b^4(bx+a)\sqrt{(bx+a)^2}} + \frac{e^3x(bx+a)}{b^3\sqrt{(bx+a)^2}} + \frac{3e^2(-ae+bd)(bx+a)\ln(bx+a)}{b^4\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^3}{b^3 \operatorname{sgn}(bx+a)} + \frac{3(bde^2 - ae^3) \log(|bx+a|)}{b^4 \operatorname{sgn}(bx+a)} - \frac{b^3 d^3 + 3ab^2 d^2 e - 9a^2 bde^2 + 5a^3 e^3 + 6(b^3 d^2 e - 2ab^2 de^2 + a^2 be^3)x}{2(bx+a)^2 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage*<sub>0</sub>*x*

#### 14.170 Problem number 1598

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2e(-ae+bd)}{b^3 \sqrt{(bx+a)^2}} - \frac{(-ae+bd)^2}{2b^3 (bx+a) \sqrt{(bx+a)^2}} + \frac{e^2 (bx+a) \ln(bx+a)}{b^3 \sqrt{(bx+a)^2}}$$

command

`integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^2 \log(|bx+a|)}{b^3 \operatorname{sgn}(bx+a)} - \frac{4(bde - ae^2)x + \frac{b^2 d^2 + 2abde - 3a^2 e^2}{b}}{2(bx+a)^2 b^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage*<sub>0</sub>*x*

#### 14.171 Problem number 1599

$$\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{e}{b^2 \sqrt{(bx+a)^2}} + \frac{ae-bd}{2b^2 (bx+a) \sqrt{(bx+a)^2}}$$

command

`integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2bx + bd + ae}{2(bx + a)^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.172 Problem number 1600

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{2b(bx + a) \sqrt{(bx + a)^2}}$$

command

`integrate(1/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2(bx + a)^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.173 Problem number 1601

$$\int \frac{1}{(d + ex)(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e}{(-ae + bd)^2 \sqrt{(bx + a)^2}} - \frac{1}{2(-ae + bd)(bx + a) \sqrt{(bx + a)^2}} \\ & + \frac{e^2(bx + a) \ln(bx + a)}{(-ae + bd)^3 \sqrt{(bx + a)^2}} - \frac{e^2(bx + a) \ln(ex + d)}{(-ae + bd)^3 \sqrt{(bx + a)^2}} \end{aligned}$$

command

`integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{be^2 \log(|bx + a|)}{b^4 d^3 \operatorname{sgn}(bx + a) - 3ab^3 d^2 e \operatorname{sgn}(bx + a) + 3a^2 b^2 d e^2 \operatorname{sgn}(bx + a) - a^3 b e^3 \operatorname{sgn}(bx + a)} - \frac{e^3 \log(|xe + d|)}{b^3 d^3 e \operatorname{sgn}(bx + a) - 3ab^2 d^2 e^2 \operatorname{sgn}(bx + a) + 3a^2 b d e^3 \operatorname{sgn}(bx + a) - a^3 e^4 \operatorname{sgn}(bx + a)} - \frac{b^2 d^2 - 4abde + 3a^2 e^2 - 2(b^2 de - abe^2)x}{2(bd - ae)^3 (bx + a)^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.174 Problem number 1603

$$\int \frac{1}{(d + ex)^3 (a^2 + 2abx + b^2 x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3b^2 e}{(-ae + bd)^4 \sqrt{(bx + a)^2}} - \frac{b^2}{2(-ae + bd)^3 (bx + a) \sqrt{(bx + a)^2}} + \frac{e^2 (bx + a)}{2(-ae + bd)^3 (ex + d)^2 \sqrt{(bx + a)^2}} + \frac{3b e^2 (bx + a)}{(-ae + bd)^4 (ex + d) \sqrt{(bx + a)^2}} + \frac{6b^2 e^2 (bx + a) \ln(bx + a)}{(-ae + bd)^5 \sqrt{(bx + a)^2}} - \frac{6b^2 e^2 (bx + a) \ln(ex + d)}{(-ae + bd)^5 \sqrt{(bx + a)^2}}$$

command

`integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6b^3 e^2 \log(|bx + a|)}{b^6 d^5 \operatorname{sgn}(bx + a) - 5ab^5 d^4 e \operatorname{sgn}(bx + a) + 10a^2 b^4 d^3 e^2 \operatorname{sgn}(bx + a) - 10a^3 b^3 d^2 e^3 \operatorname{sgn}(bx + a) + 5a^4 b^2 d e^4 \operatorname{sgn}(bx + a)} - \frac{6b^2 e^3 \log(|xe + d|)}{b^5 d^5 e \operatorname{sgn}(bx + a) - 5ab^4 d^4 e^2 \operatorname{sgn}(bx + a) + 10a^2 b^3 d^3 e^3 \operatorname{sgn}(bx + a) - 10a^3 b^2 d^2 e^4 \operatorname{sgn}(bx + a) + 5a^4 b d e^5 \operatorname{sgn}(bx + a)} + \frac{12b^3 x^3 e^3 + 18b^3 d x^2 e^2 + 4b^3 d^2 x e - b^3 d^3 + 18ab^2 x^2 e^3 + 28ab^2 d x e^2 + 7ab^2 d^2 e + 4a^2 b x e^3 + 7a^2 b d e^3}{2(b^4 d^4 \operatorname{sgn}(bx + a) - 4ab^3 d^3 e \operatorname{sgn}(bx + a) + 6a^2 b^2 d^2 e^2 \operatorname{sgn}(bx + a) - 4a^3 b d e^3 \operatorname{sgn}(bx + a) + a^4 e^4 \operatorname{sgn}(bx + a)) (bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.175 Problem number 1604

$$\int \frac{(d+ex)^6}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{20e^3(-ae+bd)^3}{b^7\sqrt{(bx+a)^2}} - \frac{(-ae+bd)^6}{4b^7(bx+a)^3\sqrt{(bx+a)^2}} - \frac{2e(-ae+bd)^5}{b^7(bx+a)^2\sqrt{(bx+a)^2}} \\ & - \frac{15e^2(-ae+bd)^4}{2b^7(bx+a)\sqrt{(bx+a)^2}} + \frac{e^5(-5ae+6bd)x(bx+a)}{b^6\sqrt{(bx+a)^2}} \\ & + \frac{e^6x^2(bx+a)}{2b^5\sqrt{(bx+a)^2}} + \frac{15e^4(-ae+bd)^2(bx+a)\ln(bx+a)}{b^7\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((e*x+d)^6/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{15(b^2d^2e^4 - 2abde^5 + a^2e^6) \log(|bx+a|)}{b^7 \operatorname{sgn}(bx+a)} \\ & + \frac{b^5x^2e^6 \operatorname{sgn}(bx+a) + 12b^5dx e^5 \operatorname{sgn}(bx+a) - 10ab^4x e^6 \operatorname{sgn}(bx+a)}{2b^{10}} \\ & \frac{b^6d^6 + 2ab^5d^5e + 5a^2b^4d^4e^2 + 20a^3b^3d^3e^3 - 125a^4b^2d^2e^4 + 154a^5bde^5 - 57a^6e^6 + 80(b^6d^3e^3 - 3ab^5d^2e^4 + 3a^2b^4d^2e^4)}{b^7\sqrt{(bx+a)^2}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.176 Problem number 1605

$$\int \frac{(d+ex)^5}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{10e^3(-ae+bd)^2}{b^6\sqrt{(bx+a)^2}} - \frac{(-ae+bd)^5}{4b^6(bx+a)^3\sqrt{(bx+a)^2}} - \frac{5e(-ae+bd)^4}{3b^6(bx+a)^2\sqrt{(bx+a)^2}} \\ & - \frac{5e^2(-ae+bd)^3}{b^6(bx+a)\sqrt{(bx+a)^2}} + \frac{e^5x(bx+a)}{b^5\sqrt{(bx+a)^2}} + \frac{5e^4(-ae+bd)(bx+a)\ln(bx+a)}{b^6\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^5}{b^5 \operatorname{sgn}(bx+a)} + \frac{5(bde^4 - ae^5) \log(|bx+a|)}{b^6 \operatorname{sgn}(bx+a)}$$


---


$$\frac{3b^5d^5 + 5ab^4d^4e + 10a^2b^3d^3e^2 + 30a^3b^2d^2e^3 - 125a^4bde^4 + 77a^5e^5 + 120(b^5d^2e^3 - 2ab^4de^4 + a^2b^3e^5)x^3 + 60}{12(bx+a)^4b^6 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 14.177 Problem number 1606

$$\int \frac{(d+ex)^4}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4e^3(-ae+bd)}{b^5\sqrt{(bx+a)^2}} - \frac{(-ae+bd)^4}{4b^5(bx+a)^3\sqrt{(bx+a)^2}} - \frac{4e(-ae+bd)^3}{3b^5(bx+a)^2\sqrt{(bx+a)^2}} \\ & - \frac{3e^2(-ae+bd)^2}{b^5(bx+a)\sqrt{(bx+a)^2}} + \frac{e^4(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^4 \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)}$$


---


$$\frac{48(b^3de^3 - ab^2e^4)x^3 + 36(b^3d^2e^2 + 2ab^2de^3 - 3a^2be^4)x^2 + 8(2b^3d^3e + 3ab^2d^2e^2 + 6a^2bde^3 - 11a^3e^4)x + 3b^4d^4e^4}{12(bx+a)^4b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.178 Problem number 1607

$$\int \frac{(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(ex+d)^4}{4(-ae+bd)(bx+a)^3\sqrt{(bx+a)^2}}$$

command

`integrate((e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4b^3x^3e^3 + 6b^3dx^2e^2 + 4b^3d^2xe + b^3d^3 + 6ab^2x^2e^3 + 4ab^2dxe^2 + ab^2d^2e + 4a^2bx^3e^3 + a^2bde^2 + a^3e^3}{4(bx+a)^4b^4\text{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.179 Problem number 1608

$$\int \frac{(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(-ae+bd)^2}{4b^3(bx+a)^3\sqrt{(bx+a)^2}} - \frac{2e(-ae+bd)}{3b^3(bx+a)^2\sqrt{(bx+a)^2}} - \frac{e^2}{2b^3(bx+a)\sqrt{(bx+a)^2}}$$

command

`integrate((e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6b^2x^2e^2 + 8b^2dxe + 3b^2d^2 + 4abxe^2 + 2abde + a^2e^2}{12(bx+a)^4b^3\text{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.180 Problem number 1609

$$\int \frac{d + ex}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{e}{3b^2 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} + \frac{ae - bd}{4b^2 (bx + a) (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}}$$

command

`integrate((e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx + 3bd + ae}{12(bx + a)^4 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.181 Problem number 1610

$$\int \frac{1}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{4b (bx + a) (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}}$$

command

`integrate(1/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4(bx + a)^4 b \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 14.182 Problem number 1611

$$\int \frac{1}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^3}{(-ae+bd)^4 \sqrt{(bx+a)^2}} - \frac{1}{4(-ae+bd)(bx+a)^3 \sqrt{(bx+a)^2}} \\ & + \frac{e}{3(-ae+bd)^2 (bx+a)^2 \sqrt{(bx+a)^2}} - \frac{e^2}{2(-ae+bd)^3 (bx+a) \sqrt{(bx+a)^2}} \\ & + \frac{e^4 (bx+a) \ln(bx+a)}{(-ae+bd)^5 \sqrt{(bx+a)^2}} - \frac{e^4 (bx+a) \ln(ex+d)}{(-ae+bd)^5 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(1/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^6 d^5 \operatorname{sgn}(bx+a) - 5 ab^5 d^4 e \operatorname{sgn}(bx+a) + 10 a^2 b^4 d^3 e^2 \operatorname{sgn}(bx+a) - 10 a^3 b^3 d^2 e^3 \operatorname{sgn}(bx+a) + 5 a^4 b^2 d e^4 \operatorname{sgn}(bx+a) + b e^4 \log(|bx+a|)}{e^5 \log(|xe+d|)}$$


---


$$\frac{b^5 d^5 e \operatorname{sgn}(bx+a) - 5 ab^4 d^4 e^2 \operatorname{sgn}(bx+a) + 10 a^2 b^3 d^3 e^3 \operatorname{sgn}(bx+a) - 10 a^3 b^2 d^2 e^4 \operatorname{sgn}(bx+a) + 5 a^4 b d e^5 \operatorname{sgn}(bx+a) + 3 b^4 d^4 - 16 ab^3 d^3 e + 36 a^2 b^2 d^2 e^2 - 48 a^3 b d e^3 + 25 a^4 e^4 - 12 (b^4 d e^3 - ab^3 e^4) x^3 + 6 (b^4 d^2 e^2 - 8 ab^3 d e^3 + 7 a^2 b^2 e^4)}{12 (bd - ae)^5 (bx+a)^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.183 Problem number 1613

$$\int \frac{1}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{10b^2e^3}{(-ae+bd)^6 \sqrt{(bx+a)^2}} - \frac{b^2}{4(-ae+bd)^3 (bx+a)^3 \sqrt{(bx+a)^2}} \\ & + \frac{b^2e}{(-ae+bd)^4 (bx+a)^2 \sqrt{(bx+a)^2}} - \frac{3b^2e^2}{(-ae+bd)^5 (bx+a) \sqrt{(bx+a)^2}} \\ & + \frac{e^4 (bx+a)}{2(-ae+bd)^5 (ex+d)^2 \sqrt{(bx+a)^2}} + \frac{5b e^4 (bx+a)}{(-ae+bd)^6 (ex+d) \sqrt{(bx+a)^2}} \\ & + \frac{15b^2 e^4 (bx+a) \ln(bx+a)}{(-ae+bd)^7 \sqrt{(bx+a)^2}} - \frac{15b^2 e^4 (bx+a) \ln(ex+d)}{(-ae+bd)^7 \sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 b^3 e^4 \log(|bx + a|)}{b^8 d^7 \operatorname{sgn}(bx + a) - 7 ab^7 d^6 \operatorname{sgn}(bx + a) + 21 a^2 b^6 d^5 e^2 \operatorname{sgn}(bx + a) - 35 a^3 b^5 d^4 e^3 \operatorname{sgn}(bx + a) + 35 a^4 b^4 d^3 e^4 \operatorname{sgn}(bx + a) - 15 b^2 e^5 \log(|xe + d|)}$$

$$\frac{b^7 d^7 \operatorname{sgn}(bx + a) - 7 ab^6 d^6 e^2 \operatorname{sgn}(bx + a) + 21 a^2 b^5 d^5 e^3 \operatorname{sgn}(bx + a) - 35 a^3 b^4 d^4 e^4 \operatorname{sgn}(bx + a) + 35 a^4 b^3 d^3 e^5 \operatorname{sgn}(bx + a) - b^6 d^6 - 8 ab^5 d^5 e + 30 a^2 b^4 d^4 e^2 - 80 a^3 b^3 d^3 e^3 + 35 a^4 b^2 d^2 e^4 + 24 a^5 b d e^5 - 2 a^6 e^6 - 60 (b^6 d e^5 - ab^5 e^6) x^5 - 30 (3 b^6 d^2 e^5 - 5 ab^5 d e^6) x^4 - 30 (3 b^6 d^3 e^5 - 5 ab^5 d^2 e^6) x^3 - 30 (3 b^6 d^4 e^5 - 5 ab^5 d^3 e^6) x^2 - 30 (3 b^6 d^5 e^5 - 5 ab^5 d^4 e^6) x - 30 (3 b^6 d^6 e^5 - 5 ab^5 d^5 e^6)}{b^8 d^7 \operatorname{sgn}(bx + a) - 7 ab^7 d^6 \operatorname{sgn}(bx + a) + 21 a^2 b^6 d^5 e^2 \operatorname{sgn}(bx + a) - 35 a^3 b^5 d^4 e^3 \operatorname{sgn}(bx + a) + 35 a^4 b^4 d^3 e^4 \operatorname{sgn}(bx + a) - 15 b^2 e^5 \log(|xe + d|)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.184 Problem number 1618

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{e}{4\sqrt{(3+2x)^2}} + \frac{-2d+3e}{8(3+2x)\sqrt{(3+2x)^2}}$$

command

```
integrate((e*x+d)/(4*x^2+12*x+9)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4xe + 2d + 3e}{8(2x + 3)^2 \operatorname{sgn}(2x + 3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.185 Problem number 1619

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{e}{12(4x^2 + 12x + 9)^{\frac{3}{2}}} + \frac{-2d + 3e}{16(3 + 2x)(4x^2 + 12x + 9)^{\frac{3}{2}}}$$

command

`integrate((e*x+d)/(4*x^2+12*x+9)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8xe + 6d + 3e}{48(2x + 3)^4 \operatorname{sgn}(2x + 3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.186 Problem number 1620

$$\int \frac{d + ex}{(9 + 12x + 4x^2)^{7/2}} dx$$

Optimal antiderivative

$$-\frac{e}{20(4x^2 + 12x + 9)^{\frac{5}{2}}} + \frac{-2d + 3e}{24(3 + 2x)(4x^2 + 12x + 9)^{\frac{5}{2}}}$$

command

`integrate((e*x+d)/(4*x^2+12*x+9)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{12xe + 10d + 3e}{120(2x + 3)^6 \operatorname{sgn}(2x + 3)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.187 Problem number 1871

$$\int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)} dx$$

Optimal antiderivative

$$\frac{1}{(-ae^2+cd^2)(ex+d)} + \frac{cd \ln(cdx+ae)}{(-ae^2+cd^2)^2} - \frac{cd \ln(ex+d)}{(-ae^2+cd^2)^2}$$

command

`integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 d^2 \log(|cdx+ae|)}{c^3 d^5 - 2ac^2 d^3 e^2 + a^2 cde^4} - \frac{cde \log(|xe+d|)}{c^2 d^4 e - 2acd^2 e^3 + a^2 e^5} + \frac{1}{(cd^2 - ae^2)(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.188 Problem number 1872

$$\int \frac{1}{(d+ex)^2 (ade+(cd^2+ae^2)x+cdex^2)} dx$$

Optimal antiderivative

$$\frac{1}{2(-ae^2+cd^2)(ex+d)^2} + \frac{cd}{(-ae^2+cd^2)^2(ex+d)} + \frac{c^2 d^2 \ln(cdx+ae)}{(-ae^2+cd^2)^3} - \frac{c^2 d^2 \ln(ex+d)}{(-ae^2+cd^2)^3}$$

command

`integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 d^2 e \log\left(-cd + \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)}{c^3 d^6 e - 3ac^2 d^4 e^3 + 3a^2 cd^2 e^5 - a^3 e^7} + \frac{\frac{2cde^2}{xe+d} + \frac{cd^2 e^2}{(xe+d)^2} - \frac{ae^4}{(xe+d)^2}}{2(c^2 d^4 e^2 - 2acd^2 e^4 + a^2 e^6)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.189 Problem number 1873

$$\int \frac{1}{(d+ex)^3 (ade + (cd^2 + ae^2)x + cdex^2)} dx$$

Optimal antiderivative

$$\frac{1}{3(-ae^2 + cd^2)(ex + d)^3} + \frac{cd}{2(-ae^2 + cd^2)^2(ex + d)^2} + \frac{c^2d^2}{(-ae^2 + cd^2)^3(ex + d)} + \frac{c^3d^3 \ln(cdx + ae)}{(-ae^2 + cd^2)^4} - \frac{c^3d^3 \ln(ex + d)}{(-ae^2 + cd^2)^4}$$

command

`integrate(1/(e*x+d)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^4d^4 \log(|cdx + ae|)}{c^5d^9 - 4ac^4d^7e^2 + 6a^2c^3d^5e^4 - 4a^3c^2d^3e^6 + a^4cde^8} - \frac{c^3d^3e \log(|xe + d|)}{c^4d^8e - 4ac^3d^6e^3 + 6a^2c^2d^4e^5 - 4a^3cd^2e^7 + a^4e^9} + \frac{11c^3d^6 - 18ac^2d^4e^2 + 9a^2cd^2e^4 - 2a^3e^6 + 6(c^3d^4e^2 - ac^2d^2e^4)x^2 + 3(5c^3d^5e - 6ac^2d^3e^3 + a^2cde^5)x}{6(cd^2 - ae^2)^4(xe + d)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.190 Problem number 1883

$$\int \frac{1}{(d+ex)(ade + (cd^2 + ae^2)x + cdex^2)^2} dx$$

Optimal antiderivative

$$-\frac{c^2d^2}{(-ae^2 + cd^2)^3(cdx + ae)} - \frac{e}{2(-ae^2 + cd^2)^2(ex + d)^2} - \frac{2cde}{(-ae^2 + cd^2)^3(ex + d)} - \frac{3c^2d^2e \ln(cdx + ae)}{(-ae^2 + cd^2)^4} + \frac{3c^2d^2e \ln(ex + d)}{(-ae^2 + cd^2)^4}$$

command

`integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3c^3d^3e \log(|cdx + ae|)}{c^5d^9 - 4ac^4d^7e^2 + 6a^2c^3d^5e^4 - 4a^3c^2d^3e^6 + a^4cde^8} + \frac{3c^2d^2e^2 \log(|xe + d|)}{c^4d^8e - 4ac^3d^6e^3 + 6a^2c^2d^4e^5 - 4a^3cd^2e^7 + a^4e^9} \\ \frac{2c^3d^6 + 3ac^2d^4e^2 - 6a^2cd^2e^4 + a^3e^6 + 6(c^3d^4e^2 - ac^2d^2e^4)x^2 + 3(3c^3d^5e - 2ac^2d^3e^3 - a^2cde^5)x}{2(cd^2 - ae^2)^4(cdx + ae)(xe + d)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 14.191 Problem number 1884

$$\int \frac{1}{(d + ex)^2 (ade + (cd^2 + ae^2)x + cdex^2)^2} dx$$

Optimal antiderivative

$$-\frac{c^3d^3}{(-ae^2 + cd^2)^4(cdx + ae)} - \frac{e}{3(-ae^2 + cd^2)^2(ex + d)^3} - \frac{cde}{(-ae^2 + cd^2)^3(ex + d)^2} \\ - \frac{3c^2d^2e}{(-ae^2 + cd^2)^4(ex + d)} - \frac{4c^3d^3e \ln(cdx + ae)}{(-ae^2 + cd^2)^5} + \frac{4c^3d^3e \ln(ex + d)}{(-ae^2 + cd^2)^5}$$

command

`integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4c^3d^3e^2 \log\left(-cd + \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)}{c^5d^{10}e - 5ac^4d^8e^3 + 10a^2c^3d^6e^5 - 10a^3c^2d^4e^7 + 5a^4cd^2e^9 - a^5e^{11}} \\ - \frac{c^4d^4e}{(cd^2 - ae^2)^5 \left(cd - \frac{cd^2}{xe+d} + \frac{ae^2}{xe+d}\right)} \\ - \frac{\frac{9c^4d^6e^7}{xe+d} + \frac{3c^4d^7e^7}{(xe+d)^2} + \frac{c^4d^8e^7}{(xe+d)^3} - \frac{18ac^3d^4e^9}{xe+d} - \frac{9ac^3d^5e^9}{(xe+d)^2} - \frac{4ac^3d^6e^9}{(xe+d)^3} + \frac{9a^2c^2d^2e^{11}}{xe+d} + \frac{9a^2c^2d^3e^{11}}{(xe+d)^2} + \frac{6a^2c^2d^4e^{11}}{(xe+d)^3} - \frac{3a^3cde^{13}}{(xe+d)^2} - 4}{3(c^6d^{12}e^6 - 6ac^5d^{10}e^8 + 15a^2c^4d^8e^{10} - 20a^3c^3d^6e^{12} + 15a^4c^2d^4e^{14} - 6a^5cd^2e^{16} + a^6e^{18})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.192 Problem number 1895

$$\int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cde x^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c^3 d^3}{2(-ae^2+cd^2)^4(cdx+ae)^2} + \frac{4c^3 d^3 e}{(-ae^2+cd^2)^5(cdx+ae)} \\ & + \frac{e^2}{3(-ae^2+cd^2)^3(ex+d)^3} + \frac{3cd e^2}{2(-ae^2+cd^2)^4(ex+d)^2} \\ & + \frac{6c^2 d^2 e^2}{(-ae^2+cd^2)^5(ex+d)} + \frac{10c^3 d^3 e^2 \ln(cdx+ae)}{(-ae^2+cd^2)^6} - \frac{10c^3 d^3 e^2 \ln(ex+d)}{(-ae^2+cd^2)^6} \end{aligned}$$

command

`integrate(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10c^4 d^4 e^2 \log(|cdx+ae|)}{c^7 d^{13} - 6ac^6 d^{11} e^2 + 15a^2 c^5 d^9 e^4 - 20a^3 c^4 d^7 e^6 + 15a^4 c^3 d^5 e^8 - 6a^5 c^2 d^3 e^{10} + a^6 c d e^{12}} - \frac{10c^3 d^3 e^3 \log(|xe+d|)}{c^6 d^{12} e - 6ac^5 d^{10} e^3 + 15a^2 c^4 d^8 e^5 - 20a^3 c^3 d^6 e^7 + 15a^4 c^2 d^4 e^9 - 6a^5 c d^2 e^{11} + a^6 e^{13}} - \frac{3c^5 d^{10} - 30ac^4 d^8 e^2 - 20a^2 c^3 d^6 e^4 + 60a^3 c^2 d^4 e^6 - 15a^4 c d^2 e^8 + 2a^5 e^{10} - 60(c^5 d^6 e^4 - ac^4 d^4 e^6)x^4 - 30(5c^5 d^7 e^5 - 15c^4 d^5 e^7)x^3 - 30(5c^5 d^7 e^5 - 15c^4 d^5 e^7)x^2 - 30(5c^5 d^7 e^5 - 15c^4 d^5 e^7)x - 30(5c^5 d^7 e^5 - 15c^4 d^5 e^7)}{c^7 d^{13} - 6ac^6 d^{11} e^2 + 15a^2 c^5 d^9 e^4 - 20a^3 c^4 d^7 e^6 + 15a^4 c^3 d^5 e^8 - 6a^5 c^2 d^3 e^{10} + a^6 c d e^{12}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.193 Problem number 1896

$$\int \frac{(d+ex)^{10}}{(ade+(cd^2+ae^2)x+cde x^2)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^4(10a^2e^4 - 24acd^2e^2 + 15c^2d^4)x}{c^6d^6} + \frac{e^5(-2ae^2 + 3cd^2)x^2}{c^5d^5} + \frac{e^6x^3}{3c^4d^4} - \frac{(-ae^2 + cd^2)^6}{3c^7d^7(cdx+ae)^3} \\ & - \frac{3e(-ae^2 + cd^2)^5}{c^7d^7(cdx+ae)^2} - \frac{15e^2(-ae^2 + cd^2)^4}{c^7d^7(cdx+ae)} + \frac{20e^3(-ae^2 + cd^2)^3 \ln(cdx+ae)}{c^7d^7} \end{aligned}$$

command

`integrate((e*x+d)^10/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{20(c^3d^6e^3 - 3ac^2d^4e^5 + 3a^2cd^2e^7 - a^3e^9) \log(|cdx + ae|)}{c^7d^7} \\ - \frac{c^6d^{12} + 3ac^5d^{10}e^2 + 15a^2c^4d^8e^4 - 110a^3c^3d^6e^6 + 195a^4c^2d^4e^8 - 141a^5cd^2e^{10} + 37a^6e^{12} + 45(c^6d^{10}e^2 - 4ac^5d^8e^4)}{3c^{12}d^{12}} \\ + \frac{c^8d^8x^3e^6 + 9c^8d^9x^2e^5 + 45c^8d^{10}xe^4 - 6ac^7d^7x^2e^7 - 72ac^7d^8xe^6 + 30a^2c^6d^6xe^8}{3c^{12}d^{12}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

#### 14.194 Problem number 1897

$$\int \frac{(d+ex)^9}{(ade + (cd^2 + ae^2)x + cdx^2)^4} dx$$

Optimal antiderivative

$$\frac{e^4(-4ae^2 + 5cd^2)x}{c^5d^5} + \frac{e^5x^2}{2c^4d^4} - \frac{(-ae^2 + cd^2)^5}{3c^6d^6(cdx + ae)^3} - \frac{5e(-ae^2 + cd^2)^4}{2c^6d^6(cdx + ae)^2} \\ - \frac{10e^2(-ae^2 + cd^2)^3}{c^6d^6(cdx + ae)} + \frac{10e^3(-ae^2 + cd^2)^2 \ln(cdx + ae)}{c^6d^6}$$

command

`integrate((e*x+d)^9/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10(c^2d^4e^3 - 2acd^2e^5 + a^2e^7) \log(|cdx + ae|)}{c^6d^6} \\ - \frac{2c^5d^{10} + 5ac^4d^8e^2 + 20a^2c^3d^6e^4 - 110a^3c^2d^4e^6 + 130a^4cd^2e^8 - 47a^5e^{10} + 60(c^5d^8e^2 - 3ac^4d^6e^4 + 3a^2c^3d^4e^6)}{6(cdx + ae)^3c^6d^6} \\ + \frac{c^4d^4x^2e^5 + 10c^4d^5xe^4 - 8ac^3d^3xe^6}{2c^8d^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 14.195 Problem number 1898

$$\int \frac{(d+ex)^8}{(ade+(cd^2+ae^2)x+cde^2x^2)^4} dx$$

Optimal antiderivative

$$\frac{e^4 x}{c^4 d^4} - \frac{(-ae^2+cd^2)^4}{3c^5 d^5 (cdx+ae)^3} - \frac{2e(-ae^2+cd^2)^3}{c^5 d^5 (cdx+ae)^2} - \frac{6e^2(-ae^2+cd^2)^2}{c^5 d^5 (cdx+ae)} + \frac{4e^3(-ae^2+cd^2) \ln(cdx+ae)}{c^5 d^5}$$

command

`integrate((e*x+d)^8/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^4}{c^4 d^4} + \frac{4(cd^2e^3 - ae^5) \log(|cdx + ae|)}{c^5 d^5}$$


---


$$\frac{c^4 d^8 + 2ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 22a^3 c d^2 e^6 + 13a^4 e^8 + 18(c^4 d^6 e^2 - 2ac^3 d^4 e^4 + a^2 c^2 d^2 e^6)x^2 + 6(c^4 d^7 e + 3ac^3 d^5 e^3)}{3(cdx + ae)^3 c^5 d^5}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.196 Problem number 1899

$$\int \frac{(d+ex)^7}{(ade+(cd^2+ae^2)x+cde^2x^2)^4} dx$$

Optimal antiderivative

$$-\frac{(-ae^2+cd^2)^3}{3c^4 d^4 (cdx+ae)^3} - \frac{3e(-ae^2+cd^2)^2}{2c^4 d^4 (cdx+ae)^2} - \frac{3e^2(-ae^2+cd^2)}{c^4 d^4 (cdx+ae)} + \frac{e^3 \ln(cdx+ae)}{c^4 d^4}$$

command

`integrate((e*x+d)^7/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^3 \log(|cdx + ae|)}{c^4 d^4}$$


---


$$-\frac{18(c^2 d^3 e^2 - acde^4)x^2 + 9(c^2 d^4 e + 2acd^2 e^3 - 3a^2 e^5)x + \frac{2c^3 d^6 + 3ac^2 d^4 e^2 + 6a^2 c d^2 e^4 - 11a^3 e^6}{cd}}{6(cdx + ae)^3 c^3 d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.197 Problem number 1912

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(-ae^2 + cd^2) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{2e^{\frac{3}{2}}\sqrt{c}\sqrt{d}} \\ & + \frac{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{cdx^2e + cd^2x + axe^2 + ade} e^{(-1)}}{(cd^2 - ae^2)\sqrt{cd} e^{(-\frac{3}{2})} \log\left(\left|-\sqrt{cd} cd^2 e^{\frac{1}{2}} - 2\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd} ae^{\frac{5}{2}}\right|\right)} \\ & + \frac{\sqrt{cdx^2e + cd^2x + axe^2 + ade} e^{(-1)}}{2cd} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.198 Problem number 1913

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right) \sqrt{c}\sqrt{d}}{e^{\frac{3}{2}}} \\ & - \frac{2\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e(ex + d)} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-2 \left( \frac{cd \arctan \left( \frac{\sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}}}{\sqrt{-cde}} \right) e^{(-2) \operatorname{sgn} \left( \frac{1}{xe+d} \right)}}{\sqrt{-cde}} + \sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}} e^{(-3) \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right) -$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.199 Problem number 1916

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{7(-ae^2 + cd^2)(ex + d)^5} + \frac{8cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{35(-ae^2 + cd^2)^2(ex + d)^4} + \frac{16c^2d^2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{105(-ae^2 + cd^2)^3(ex + d)^3}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{105} \left( \frac{8\sqrt{cd}c^3d^3e^{\frac{1}{2}}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{c^3d^6e^3 - 3ac^2d^4e^5 + 3a^2cd^2e^7 - a^3e^9} + \frac{3\left(35\sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}}c^3d^3e^3 - 35\left(cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}\right)^{\frac{3}{2}}c^2d^2e^2 + \dots\right)}{c^3d^6e^6 - 3ac^2d^4e^8 + 3a^2c^2d^2e^{10} - a^3e^{12}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.200 Problem number 1923

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e} \\ & + \frac{(-ae^2 + cd^2)^3 \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{16c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{5}{2}}} \\ & + \frac{\left(\frac{a}{cd} - \frac{d}{e^2}\right)(2cdex + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{24} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4cdx + \frac{(c^3d^4e + 7ac^2d^2e^3)e^{(-2)}}{c^2d^2} \right) x - \frac{(3c^3d^5 - 8ac^2d^3e^2 - 3a^2cde^4)e^{(-2)}}{c^2d^2} \right. \\ & \left. - \frac{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)e^{(-\frac{5}{2})} \log\left(\left| -cd^2 - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \sqrt{cd}e^{\frac{1}{2}} \right.\right)}{16\sqrt{cd}cd} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.201 Problem number 1924

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{2e(ex + d)} \\ & + \frac{3(-ae^2 + cd^2)^2 \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{8e^{\frac{5}{2}}\sqrt{c}\sqrt{d}} \\ & + \frac{3\left(a - \frac{cd^2}{e^2}\right)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{4} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( \frac{3 \left( c^2 d^4 \operatorname{sgn}\left(\frac{1}{xe+d}\right) - 2acd^2 e^2 \operatorname{sgn}\left(\frac{1}{xe+d}\right) + a^2 e^4 \operatorname{sgn}\left(\frac{1}{xe+d}\right) \right) \arctan\left(\frac{\sqrt{cde - \frac{cd^2 e}{xe+d} + \frac{ae^3}{xe+d}}}{\sqrt{-cde}}\right) e^{(-3)}}{\sqrt{-cde}} \right) + \dots$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.202 Problem number 1925

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{e(ex+d)^2} \\ & -\frac{3(-ae^2 + cd^2) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right) \sqrt{c}\sqrt{d}}{2e^{\frac{5}{2}}} \\ & + \frac{3cd\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \sqrt{cdx^2e + cd^2x + axe^2 + ade} cde^{(-2)} + \frac{2(c^2d^4 - 2acd^2e^2 + a^2e^4)e^{(-2)}}{\sqrt{cd}de^{\frac{1}{2}} + \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)e} \\ & + \frac{3\left(\sqrt{cd}c^2d^3e^{\frac{1}{2}} - \sqrt{cd}acde^{\frac{5}{2}}\right)e^{(-3)} \log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)cde - \sqrt{cd}\right|\right)}{2cd} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.203 Problem number 1927

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{5(-ae^2 + cd^2)(ex + d)^5}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( \frac{3\sqrt{cd}c^2d^2e^{\frac{1}{2}}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{cd^2e^4 - ae^6} - \frac{\left(15\sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}}c^2d^2e^2 - 10\left(cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}\right)^{\frac{3}{2}}cde + 3\left(cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}\right)^{\frac{5}{2}}\right)}{c^2d^4e^4 - 2acd^2e^6 + a^2e^8} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.204 Problem number 1936

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{a}{cd} - \frac{d}{e^2}\right)(2cdex + ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{16} \\ & + \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{5e} \\ & - \frac{3(-ae^2 + cd^2)^5 \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{256c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \\ & + \frac{3(-ae^2 + cd^2)^3(2cdex + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{128c^2d^2e^3} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{640} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8c^2d^2xe + \frac{(11c^6d^7e^4 + 21ac^5d^5e^6)e^{(-4)}}{c^4d^4} \right) x + \frac{(c^6d^8e^3 + 64ac^5d^6e^5)}{c^4} \right) \right) \right. \\ \left. + \frac{3(c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10})e^{(-\frac{7}{2})} \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e} \right) \right. \right)}{256 \sqrt{cd} c^2d^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.205 Problem number 1937

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\frac{5 \left( a - \frac{cd^2}{e^2} \right) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{24} + \frac{(cdx + ae) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{4e} \\ - \frac{5(-ae^2 + cd^2)^4 \operatorname{arctanh} \left( \frac{2cdex + ae^2 + cd^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \right)}{128c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{7}{2}}} \\ + \frac{5(-ae^2 + cd^2)^2 (2cdex + ae^2 + cd^2) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{64cd e^3}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.206 Problem number 1938

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{3e^2} + \frac{2(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{e(ex + d)^2} \\ & - \frac{5(-ae^2 + cd^2)^3 \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{16e^{\frac{7}{2}}\sqrt{c}\sqrt{d}} \\ & + \frac{5(-ae^2 + cd^2)(2cde x + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{8e^3} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{24} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4c^2d^2xe^{(-1)} - \frac{(5c^4d^5e - 13ac^3d^3e^3)e^{(-3)}}{c^2d^2} \right) x + \frac{(15c^4d^6 - 40ac^3d^4e^2 + 33c^2d^5e^3 - 3cd^6e^4)}{c^2d^2} \right) \\ & + \frac{5(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)\sqrt{cd}e^{(-\frac{7}{2})} \log\left(\left| -\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) \right|\right)}{16cd} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.207 Problem number 1939

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{2e^2(ex + d)} - \frac{2(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{e(ex + d)^3} \\ & + \frac{15(-ae^2 + cd^2)^2 \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right) \sqrt{c}\sqrt{d}}{8e^{\frac{7}{2}}} \\ & + \frac{15cd\left(a - \frac{cd^2}{e^2}\right) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4e} \end{aligned}$$



command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{1}{4} \left( 2c^2d^2xe^{(-2)} - \frac{(7c^3d^4e^5 - 9ac^2d^2e^7)e^{(-8)}}{cd} \right) \sqrt{cdx^2e + cd^2x + axe^2 + ade} - \frac{2(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)e^{(-3)}}{\sqrt{cd}de^{\frac{1}{2}} + \left( \sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) e}{15 \left( \sqrt{cd}c^3d^5e^{\frac{1}{2}} - 2\sqrt{cd}ac^2d^3e^{\frac{5}{2}} + \sqrt{cd}a^2cde^{\frac{9}{2}} \right) e^{(-4)} \log \left( \left| -\sqrt{cd}cd^2e^{\frac{1}{2}} - 2 \left( \sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right)}{8cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.208 Problem number 1940

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\frac{10cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e^2(ex + d)^2} - \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{3e(ex + d)^4} - \frac{5c^{\frac{3}{2}}d^{\frac{3}{2}}(-ae^2 + cd^2) \operatorname{arctanh} \left( \frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \right)}{2e^{\frac{7}{2}}} + \frac{5c^2d^2\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e^3}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} \left( \frac{15 \left( c^3d^4 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - ac^2d^2e^2 \operatorname{sgn} \left( \frac{1}{xe+d} \right) \right) \operatorname{arctan} \left( \frac{\sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}}}{\sqrt{-cde}} \right) e^{(-4)}}{\sqrt{-cde}} + 2 \left( 6 \sqrt{cde - \frac{cd^2e}{xe+d}} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.209 Problem number 1951

$$\int \frac{1}{(d+ex)^2 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3(-ae^2 + cd^2)(ex + d)^2} + \frac{4cd\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3(-ae^2 + cd^2)^2(ex + d)}$$

command

`integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4\sqrt{cd}cde^{\frac{1}{2}}\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{3(c^2d^4e - 2acd^2e^3 + a^2e^5)} + \frac{2\left(3\sqrt{cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}}cde - \left(cde - \frac{cd^2e}{xe+d} + \frac{ae^3}{xe+d}\right)^{\frac{3}{2}}\right)}{3(cd^2e^2 - ae^4)\left(cd^2\operatorname{sgn}\left(\frac{1}{xe+d}\right) - ae^2\operatorname{sgn}\left(\frac{1}{xe+d}\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.210 Problem number 1961

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2}{5(-ae^2 + cd^2)(ex + d)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} + \frac{4cd}{5(-ae^2 + cd^2)^2(ex + d) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} - \frac{16c^2d^2(2cdex + ae^2 + cd^2)}{5(-ae^2 + cd^2)^4 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

command

`integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.211 Problem number 1972

$$\int \frac{1}{(d+ex)^2 (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2}{7(-ae^2 + cd^2)(ex+d)^2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}} \\ & + \frac{4cd}{7(-ae^2 + cd^2)^2(ex+d)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}} \\ & - \frac{32c^2d^2(2cdex + ae^2 + cd^2)}{21(-ae^2 + cd^2)^4(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}} \\ & + \frac{256c^3d^3e(2cdex + ae^2 + cd^2)}{21(-ae^2 + cd^2)^6\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.212 Problem number 2000

$$\int \frac{(d+ex)^{9/2}}{ade + (cd^2 + ae^2)x + cdex^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-ae^2 + cd^2)^2(ex+d)^{\frac{3}{2}}}{3c^3d^3} + \frac{2(-ae^2 + cd^2)(ex+d)^{\frac{5}{2}}}{5c^2d^2} + \frac{2(ex+d)^{\frac{7}{2}}}{7cd} \\ & - \frac{2(-ae^2 + cd^2)^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right)}{c^{\frac{9}{2}}d^{\frac{9}{2}}} + \frac{2(-ae^2 + cd^2)^3\sqrt{ex+d}}{c^4d^4} \end{aligned}$$

command

```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^4 d^8 - 4ac^3 d^6 e^2 + 6a^2 c^2 d^4 e^4 - 4a^3 c d^2 e^6 + a^4 e^8) \arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2 d^3 + acde^2}}\right)}{\sqrt{-c^2 d^3 + acde^2} c^4 d^4} + \frac{2\left(15(xe+d)^{\frac{7}{2}} c^6 d^6 + 21(xe+d)^{\frac{5}{2}} c^6 d^7 + 35(xe+d)^{\frac{3}{2}} c^6 d^8 + 105\sqrt{xe+d} c^6 d^9 - 21(xe+d)^{\frac{5}{2}} ac^5 d^5 e^2 - 70(xe+d)^{\frac{3}{2}} ac^5 d^5 e^2 - 70(xe+d)^{\frac{1}{2}} ac^5 d^5 e^2\right)}{105 c^5 d^5}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.213 Problem number 2001

$$\int \frac{(d+ex)^{7/2}}{ade + (cd^2 + ae^2)x + cdex^2} dx$$

Optimal antiderivative

$$\frac{2(-ae^2 + cd^2)(ex+d)^{\frac{3}{2}}}{3c^2 d^2} + \frac{2(ex+d)^{\frac{5}{2}}}{5cd} - \frac{2(-ae^2 + cd^2)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right)}{c^{\frac{7}{2}} d^{\frac{7}{2}}} + \frac{2(-ae^2 + cd^2)^2 \sqrt{ex+d}}{c^3 d^3}$$

command

`integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^3 d^6 - 3ac^2 d^4 e^2 + 3a^2 c d^2 e^4 - a^3 e^6) \arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2 d^3 + acde^2}}\right)}{\sqrt{-c^2 d^3 + acde^2} c^3 d^3} + \frac{2\left(3(xe+d)^{\frac{5}{2}} c^4 d^4 + 5(xe+d)^{\frac{3}{2}} c^4 d^5 + 15\sqrt{xe+d} c^4 d^6 - 5(xe+d)^{\frac{3}{2}} ac^3 d^3 e^2 - 30\sqrt{xe+d} ac^3 d^4 e^2 + 15\sqrt{xe+d} ac^3 d^5 e^2\right)}{15 c^5 d^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.214 Problem number 2002

$$\int \frac{(d+ex)^{5/2}}{ade + (cd^2 + ae^2)x + cdx^2} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}}{3cd} - \frac{2(-ae^2 + cd^2)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right)}{c^{\frac{5}{2}}d^{\frac{5}{2}}} + \frac{2(-ae^2 + cd^2)\sqrt{ex+d}}{c^2d^2}$$

command

`integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^2d^4 - 2acd^2e^2 + a^2e^4) \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3 + acde^2}}\right)}{\sqrt{-c^2d^3 + acde^2}c^2d^2} + \frac{2\left((xe+d)^{\frac{3}{2}}c^2d^2 + 3\sqrt{xe+d}c^2d^3 - 3\sqrt{xe+d}acde^2\right)}{3c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.215 Problem number 2003

$$\int \frac{(d+ex)^{3/2}}{ade + (cd^2 + ae^2)x + cdx^2} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right) \sqrt{-ae^2 + cd^2}}{c^{\frac{3}{2}}d^{\frac{3}{2}}} + \frac{2\sqrt{ex+d}}{cd}$$

command

`integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(cd^2 - ae^2) \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3 + acde^2}}\right)}{\sqrt{-c^2d^3 + acde^2}cd} + \frac{2\sqrt{xe+d}}{cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.216 Problem number 2005

$$\int \frac{1}{\sqrt{d+ex} (ade + (cd^2 + ae^2)x + cdex^2)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right) \sqrt{c} \sqrt{d}}{(-ae^2+cd^2)^{\frac{3}{2}}} + \frac{2}{(-ae^2+cd^2)\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2cd \operatorname{arctan}\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2} (cd^2-ae^2)} + \frac{2}{(cd^2-ae^2)\sqrt{xe+d}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.217 Problem number 2006

$$\int \frac{1}{(d+ex)^{3/2} (ade + (cd^2 + ae^2)x + cdex^2)} dx$$

Optimal antiderivative

$$\frac{2}{3(-ae^2+cd^2)(ex+d)^{\frac{3}{2}}} - \frac{2c^{\frac{3}{2}}d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{(-ae^2+cd^2)^{\frac{5}{2}}} + \frac{2cd}{(-ae^2+cd^2)^2 \sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2c^2d^2 \operatorname{arctan}\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^2d^4-2acd^2e^2+a^2e^4)\sqrt{-c^2d^3+acde^2}} + \frac{2(3(xe+d)cd+cd^2-ae^2)}{3(c^2d^4-2acd^2e^2+a^2e^4)(xe+d)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.218 Problem number 2007

$$\int \frac{1}{(d+ex)^{5/2} (ade + (cd^2 + ae^2)x + cdex^2)} dx$$

Optimal antiderivative

$$\frac{2}{5(-ae^2 + cd^2)(ex+d)^{5/2}} + \frac{2cd}{3(-ae^2 + cd^2)^2(ex+d)^{3/2}} - \frac{2c^{5/2}d^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right)}{(-ae^2 + cd^2)^{7/2}} + \frac{2c^2d^2}{(-ae^2 + cd^2)^3\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2c^3d^3 \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3 + acde^2}}\right)}{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)\sqrt{-c^2d^3 + acde^2}} + \frac{2\left(15(xe+d)^2c^2d^2 + 5(xe+d)c^2d^3 + 3c^2d^4 - 5(xe+d)acde^2 - 6acd^2e^2 + 3a^2e^4\right)}{15(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)(xe+d)^{5/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.219 Problem number 2008

$$\int \frac{1}{(d+ex)^{7/2} (ade + (cd^2 + ae^2)x + cdex^2)} dx$$

Optimal antiderivative

$$\frac{2}{7(-ae^2 + cd^2)(ex+d)^{7/2}} + \frac{2cd}{5(-ae^2 + cd^2)^2(ex+d)^{5/2}} + \frac{2c^2d^2}{3(-ae^2 + cd^2)^3(ex+d)^{3/2}} - \frac{2c^{7/2}d^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2 + cd^2}}\right)}{(-ae^2 + cd^2)^{9/2}} + \frac{2c^3d^3}{(-ae^2 + cd^2)^4\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2c^4d^4 \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{(c^4d^8 - 4ac^3d^6e^2 + 6a^2c^2d^4e^4 - 4a^3cd^2e^6 + a^4e^8)\sqrt{-c^2d^3+acde^2}} + \frac{2\left(105(xe+d)^3c^3d^3 + 35(xe+d)^2c^3d^4 + 21(xe+d)c^3d^5 + 15c^3d^6 - 35(xe+d)^2ac^2d^2e^2 - 42(xe+d)ac^2d^3e^2\right)}{105(c^4d^8 - 4ac^3d^6e^2 + 6a^2c^2d^4e^4 - 4a^3cd^2e^6 + a^4e^8)(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.220 Problem number 2009

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+c dex^2)^2} dx$$

Optimal antiderivative

$$\frac{3e(-ae^2+cd^2)^2(ex+d)^{3/2}}{c^4d^4} + \frac{9e(-ae^2+cd^2)(ex+d)^{5/2}}{5c^3d^3} + \frac{9e(ex+d)^{7/2}}{7c^2d^2} - \frac{(ex+d)^{9/2}}{cd(cdx+ae)}$$

$$- \frac{9e(-ae^2+cd^2)^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{c^{11/2}d^{11/2}} + \frac{9e(-ae^2+cd^2)^3\sqrt{ex+d}}{c^5d^5}$$

command

`integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9(c^4d^8e - 4ac^3d^6e^3 + 6a^2c^2d^4e^5 - 4a^3cd^2e^7 + a^4e^9) \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^5d^5} + \frac{\sqrt{xe+d}c^4d^8e - 4\sqrt{xe+d}ac^3d^6e^3 + 6\sqrt{xe+d}a^2c^2d^4e^5 - 4\sqrt{xe+d}a^3cd^2e^7 + \sqrt{xe+d}a^4e^9}{((xe+d)cd - cd^2 + ae^2)c^5d^5} + \frac{2\left(5(xe+d)^{7/2}c^{12}d^{12}e + 14(xe+d)^{5/2}c^{12}d^{13}e + 35(xe+d)^{3/2}c^{12}d^{14}e + 140\sqrt{xe+d}c^{12}d^{15}e - 14(xe+d)^{5/2}ac^{11}d^{11}\right)}{105(c^4d^8e - 4ac^3d^6e^3 + 6a^2c^2d^4e^5 - 4a^3cd^2e^7 + a^4e^9)(xe+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 14.221 Problem number 2010

$$\int \frac{(d+ex)^{11/2}}{(ade+(cd^2+ae^2)x+c dex^2)^2} dx$$

Optimal antiderivative

$$\frac{7e(-ae^2+cd^2)(ex+d)^{\frac{3}{2}}}{3c^3d^3} + \frac{7e(ex+d)^{\frac{5}{2}}}{5c^2d^2} - \frac{(ex+d)^{\frac{7}{2}}}{cd(cdx+ae)}$$

$$- \frac{7e(-ae^2+cd^2)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{c^{\frac{9}{2}}d^{\frac{9}{2}}} + \frac{7e(-ae^2+cd^2)^2\sqrt{ex+d}}{c^4d^4}$$

command

`integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{7(c^3d^6e - 3ac^2d^4e^3 + 3a^2cd^2e^5 - a^3e^7) \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{\sqrt{-c^2d^3+acde^2}c^4d^4}$$

$$- \frac{\sqrt{xe+d}c^3d^6e - 3\sqrt{xe+d}ac^2d^4e^3 + 3\sqrt{xe+d}a^2cd^2e^5 - \sqrt{xe+d}a^3e^7}{((xe+d)cd - cd^2 + ae^2)c^4d^4}$$

$$+ \frac{2\left(3(xe+d)^{\frac{5}{2}}c^8d^8e + 10(xe+d)^{\frac{3}{2}}c^8d^9e + 45\sqrt{xe+d}c^8d^{10}e - 10(xe+d)^{\frac{3}{2}}ac^7d^7e^3 - 90\sqrt{xe+d}ac^7d^8e^3 + 45\right)}{15c^{10}d^{10}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.222 Problem number 2011

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+c dex^2)^2} dx$$

Optimal antiderivative

$$\frac{5e(ex+d)^{\frac{3}{2}}}{3c^2d^2} - \frac{(ex+d)^{\frac{5}{2}}}{cd(cdx+ae)} - \frac{5e(-ae^2+cd^2)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{c^{\frac{7}{2}}d^{\frac{7}{2}}}$$

$$+ \frac{5e(-ae^2+cd^2)\sqrt{ex+d}}{c^3d^3}$$

command

```
integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5(c^2d^4e - 2acd^2e^3 + a^2e^5) \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3 + acde^2}}\right)}{\sqrt{-c^2d^3 + acde^2}c^3d^3} - \frac{\sqrt{xe+d}c^2d^4e - 2\sqrt{xe+d}acd^2e^3 + \sqrt{xe+d}a^2e^5}{((xe+d)cd - cd^2 + ae^2)c^3d^3} + \frac{2\left((xe+d)^{\frac{3}{2}}c^4d^4e + 6\sqrt{xe+d}c^4d^5e - 6\sqrt{xe+d}ac^3d^3e^3\right)}{3c^6d^6}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.223 Problem number 2012

$$\int \frac{(d+ex)^{7/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^2} dx$$

Optimal antiderivative

$$-\frac{(ex+d)^{\frac{3}{2}}}{cd(cdx+ae)} - \frac{3e \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right) \sqrt{-ae^2+cd^2}}{c^{\frac{5}{2}}d^{\frac{5}{2}}} + \frac{3e\sqrt{ex+d}}{c^2d^2}$$

command

```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(cd^2e - ae^3) \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3 + acde^2}}\right)}{\sqrt{-c^2d^3 + acde^2}c^2d^2} + \frac{2\sqrt{xe+d}e}{c^2d^2} - \frac{\sqrt{xe+d}cd^2e - \sqrt{xe+d}ae^3}{((xe+d)cd - cd^2 + ae^2)c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.224 Problem number 2013

$$\int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx$$

Optimal antiderivative

$$-\frac{e \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{c^{\frac{3}{2}} d^{\frac{3}{2}} \sqrt{-ae^2+cd^2}} - \frac{\sqrt{ex+d}}{cd(cd+ae)}$$

command

`integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right) e}{\sqrt{-c^2d^3+acde^2} cd} - \frac{\sqrt{xe+d} e}{((xe+d)cd - cd^2 + ae^2)cd}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.225 Problem number 2014

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^2} dx$$

Optimal antiderivative

$$-\frac{e \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{d} \sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{(-ae^2+cd^2)^{\frac{3}{2}} \sqrt{c} \sqrt{d}} - \frac{\sqrt{ex+d}}{(-ae^2+cd^2)(cd+ae)}$$

command

`integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right) e}{\sqrt{-c^2d^3+acde^2} (cd^2 - ae^2)} - \frac{\sqrt{xe+d} e}{((xe+d)cd - cd^2 + ae^2)(cd^2 - ae^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.226 Problem number 2015

$$\int \frac{\sqrt{d+ex}}{(ade + (cd^2 + ae^2)x + cdex^2)^2} dx$$

Optimal antiderivative

$$\frac{3e \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)\sqrt{c}\sqrt{d}}{(-ae^2+cd^2)^{\frac{5}{2}}} - \frac{3e}{(-ae^2+cd^2)^2\sqrt{ex+d}} - \frac{1}{(-ae^2+cd^2)(cdx+ae)\sqrt{ex+d}}$$

command

`integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3cd \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e}{(c^2d^4 - 2acd^2e^2 + a^2e^4)\sqrt{-c^2d^3 + acde^2}} - \frac{3(xe+d)cde - 2cd^2e + 2ae^3}{(c^2d^4 - 2acd^2e^2 + a^2e^4)\left((xe+d)^{\frac{3}{2}}cd - \sqrt{xe+d}cd^2 + \sqrt{xe+d}ae^2\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.227 Problem number 2016

$$\int \frac{1}{\sqrt{d+ex} (ade + (cd^2 + ae^2)x + cdex^2)^2} dx$$

Optimal antiderivative

$$\frac{5e}{3(-ae^2+cd^2)^2(ex+d)^{\frac{3}{2}}} - \frac{1}{(-ae^2+cd^2)(cdx+ae)(ex+d)^{\frac{3}{2}}} + \frac{5c^{\frac{3}{2}}d^{\frac{3}{2}}e \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{(-ae^2+cd^2)^{\frac{7}{2}}} - \frac{5cde}{(-ae^2+cd^2)^3\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5c^2d^2 \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e}{(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)\sqrt{-c^2d^3+acde^2}} - \frac{\sqrt{xe+d}c^2d^2e}{(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)((xe+d)cd-cd^2+ae^2)} - \frac{2(6(xe+d)cde+cd^2e-ae^3)}{3(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)(xe+d)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.228 Problem number 2017

$$\int \frac{1}{(d+ex)^{3/2}(ade+(cd^2+ae^2)x+cde^2)^2} dx$$

Optimal antiderivative

$$\frac{7e}{5(-ae^2+cd^2)^2(ex+d)^{\frac{5}{2}}} - \frac{1}{(-ae^2+cd^2)(cdx+ae)(ex+d)^{\frac{5}{2}}} - \frac{7cde}{3(-ae^2+cd^2)^3(ex+d)^{\frac{3}{2}}} + \frac{7c^{\frac{5}{2}}d^{\frac{5}{2}}e \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{(-ae^2+cd^2)^{\frac{9}{2}}} - \frac{7c^2d^2e}{(-ae^2+cd^2)^4\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{7c^3d^3 \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e}{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)\sqrt{-c^2d^3+acde^2}} - \frac{\sqrt{xe+d}c^3d^3e}{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)((xe+d)cd-cd^2+ae^2)} - \frac{2\left(45(xe+d)^2c^2d^2e+10(xe+d)c^2d^3e+3c^2d^4e-10(xe+d)acde^3-6acd^2e^3+3a^2e^5\right)}{15(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)(xe+d)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.229 Problem number 2018

$$\int \frac{(d+ex)^{15/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$\frac{21e^2(-ae^2+cd^2)(ex+d)^{\frac{3}{2}}}{4c^4d^4} + \frac{63e^2(ex+d)^{\frac{5}{2}}}{20c^3d^3} - \frac{9e(ex+d)^{\frac{7}{2}}}{4c^2d^2(cd x+ae)} - \frac{(ex+d)^{\frac{9}{2}}}{2cd(cd x+ae)^2}$$

$$- \frac{63e^2(-ae^2+cd^2)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4c^{\frac{11}{2}}d^{\frac{11}{2}}} + \frac{63e^2(-ae^2+cd^2)^2\sqrt{ex+d}}{4c^5d^5}$$

command

```
integrate((e*x+d)^(15/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{63(c^3d^6e^2 - 3ac^2d^4e^4 + 3a^2cd^2e^6 - a^3e^8) \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)}{4\sqrt{-c^2d^3+acde^2}c^5d^5}$$

$$- \frac{17(xe+d)^{\frac{3}{2}}c^4d^7e^2 - 15\sqrt{xe+d}c^4d^8e^2 - 51(xe+d)^{\frac{3}{2}}ac^3d^5e^4 + 60\sqrt{xe+d}ac^3d^6e^4 + 51(xe+d)^{\frac{3}{2}}a^2c^2d^3e^6 - 4((xe+d)cd - cd^2 + ae^2)^2c^5}{5c^{15}d^{15}}$$

$$+ \frac{2\left((xe+d)^{\frac{5}{2}}c^{12}d^{12}e^2 + 5(xe+d)^{\frac{3}{2}}c^{12}d^{13}e^2 + 30\sqrt{xe+d}c^{12}d^{14}e^2 - 5(xe+d)^{\frac{3}{2}}ac^{11}d^{11}e^4 - 60\sqrt{xe+d}ac^{11}d^{12}\right)}{5c^{15}d^{15}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.230 Problem number 2019

$$\int \frac{(d+ex)^{13/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$\frac{35e^2(ex+d)^{\frac{3}{2}}}{12c^3d^3} - \frac{7e(ex+d)^{\frac{5}{2}}}{4c^2d^2(cd x+ae)} - \frac{(ex+d)^{\frac{7}{2}}}{2cd(cd x+ae)^2}$$

$$- \frac{35e^2(-ae^2+cd^2)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4c^{\frac{9}{2}}d^{\frac{9}{2}}} + \frac{35e^2(-ae^2+cd^2)\sqrt{ex+d}}{4c^4d^4}$$

command

```
integrate((e*x+d)^(13/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35 (c^2 d^4 e^2 - 2 a c d^2 e^4 + a^2 e^6) \arctan\left(\frac{\sqrt{x e + d} c d}{\sqrt{-c^2 d^3 + a c d e^2}}\right)}{4 \sqrt{-c^2 d^3 + a c d e^2} c^4 d^4} - \frac{13 (x e + d)^{\frac{3}{2}} c^3 d^5 e^2 - 11 \sqrt{x e + d} c^3 d^6 e^2 - 26 (x e + d)^{\frac{3}{2}} a c^2 d^3 e^4 + 33 \sqrt{x e + d} a c^2 d^4 e^4 + 13 (x e + d)^{\frac{3}{2}} a^2 c d e^6 - 3 a^3 e^8}{4 ((x e + d) c d - c d^2 + a e^2)^2 c^4 d^4} + \frac{2 \left( (x e + d)^{\frac{3}{2}} c^6 d^6 e^2 + 9 \sqrt{x e + d} c^6 d^7 e^2 - 9 \sqrt{x e + d} a c^5 d^5 e^4 \right)}{3 c^9 d^9}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.231 Problem number 2020

$$\int \frac{(d + ex)^{11/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^3} dx$$

Optimal antiderivative

$$-\frac{5e(ex+d)^{\frac{3}{2}}}{4c^2d^2(cdx+ae)} - \frac{(ex+d)^{\frac{5}{2}}}{2cd(cdx+ae)^2} - \frac{15e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right) \sqrt{-ae^2+cd^2}}{4c^{\frac{7}{2}}d^{\frac{7}{2}}} + \frac{15e^2\sqrt{ex+d}}{4c^3d^3}$$

command

```
integrate((e*x+d)^(11/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 (c d^2 e^2 - a e^4) \arctan\left(\frac{\sqrt{x e + d} c d}{\sqrt{-c^2 d^3 + a c d e^2}}\right)}{4 \sqrt{-c^2 d^3 + a c d e^2} c^3 d^3} + \frac{2 \sqrt{x e + d} e^2}{c^3 d^3} - \frac{9 (x e + d)^{\frac{3}{2}} c^2 d^3 e^2 - 7 \sqrt{x e + d} c^2 d^4 e^2 - 9 (x e + d)^{\frac{3}{2}} a c d e^4 + 14 \sqrt{x e + d} a c d^2 e^4 - 7 \sqrt{x e + d} a^2 e^6}{4 ((x e + d) c d - c d^2 + a e^2)^2 c^3 d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.232 Problem number 2021

$$\int \frac{(d+ex)^{9/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$-\frac{(ex+d)^{\frac{3}{2}}}{2cd(cdx+ae)^2} - \frac{3e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4c^{\frac{5}{2}}d^{\frac{5}{2}}\sqrt{-ae^2+cd^2}} - \frac{3e\sqrt{ex+d}}{4c^2d^2(cdx+ae)}$$

command

`integrate((e*x+d)^(9/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right) e^2}{4 \sqrt{-c^2d^3+acde^2} c^2d^2} - \frac{5 (xe+d)^{\frac{3}{2}} cde^2 - 3 \sqrt{xe+d} cd^2e^2 + 3 \sqrt{xe+d} ae^4}{4 ((xe+d)cd - cd^2 + ae^2)^2 c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.233 Problem number 2022

$$\int \frac{(d+ex)^{7/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$\frac{e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4c^{\frac{3}{2}}d^{\frac{3}{2}}(-ae^2+cd^2)^{\frac{3}{2}}} - \frac{\sqrt{ex+d}}{2cd(cdx+ae)^2} - \frac{e\sqrt{ex+d}}{4cd(-ae^2+cd^2)(cdx+ae)}$$

command

`integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\arctan\left(\frac{\sqrt{xe+d} cd}{\sqrt{-c^2d^3+acde^2}}\right) e^2}{4(c^2d^3-acde^2)\sqrt{-c^2d^3+acde^2}} - \frac{(xe+d)^{\frac{3}{2}} cde^2 + \sqrt{xe+d} cd^2e^2 - \sqrt{xe+d} ae^4}{4(c^2d^3-acde^2)((xe+d)cd - cd^2 + ae^2)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 14.234 Problem number 2024

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{15e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)\sqrt{c}\sqrt{d}}{4(-ae^2+cd^2)^{\frac{7}{2}}} + \frac{15e^2}{4(-ae^2+cd^2)^3\sqrt{ex+d}} \\ & -\frac{1}{2(-ae^2+cd^2)(cdx+ae)^2\sqrt{ex+d}} + \frac{5e}{4(-ae^2+cd^2)^2(cdx+ae)\sqrt{ex+d}} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{15cd \operatorname{arctan}\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e^2}{4(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)\sqrt{-c^2d^3+acde^2}} \\ & + \frac{2e^2}{(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)\sqrt{xe+d}} \\ & + \frac{7(xe+d)^{\frac{3}{2}}c^2d^2e^2-9\sqrt{xe+d}c^2d^3e^2+9\sqrt{xe+d}acde^4}{4(c^3d^6-3ac^2d^4e^2+3a^2cd^2e^4-a^3e^6)((xe+d)cd-cd^2+ae^2)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.235 Problem number 2025

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cdex^2)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35e^2}{12(-ae^2+cd^2)^3(ex+d)^{\frac{3}{2}}} - \frac{1}{2(-ae^2+cd^2)(cdx+ae)^2(ex+d)^{\frac{3}{2}}} \\ & + \frac{7e}{4(-ae^2+cd^2)^2(cdx+ae)(ex+d)^{\frac{3}{2}}} \\ & - \frac{35c^{\frac{3}{2}}d^{\frac{3}{2}}e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4(-ae^2+cd^2)^{\frac{9}{2}}} + \frac{35cde^2}{4(-ae^2+cd^2)^4\sqrt{ex+d}} \end{aligned}$$

command

`integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35c^2d^2 \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e^2}{4(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)\sqrt{-c^2d^3+acde^2}} + \frac{2(9(xe+d)cde^2+cd^2e^2-ae^4)}{3(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)(xe+d)^{\frac{3}{2}}} + \frac{11(xe+d)^{\frac{3}{2}}c^3d^3e^2-13\sqrt{xe+d}c^3d^4e^2+13\sqrt{xe+d}ac^2d^2e^4}{4(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)((xe+d)cd-cd^2+ae^2)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 14.236 Problem number 2026

$$\int \frac{1}{\sqrt{d+ex} (ade+(cd^2+ae^2)x+c dex^2)^3} dx$$

Optimal antiderivative

$$\frac{63e^2}{20(-ae^2+cd^2)^3(ex+d)^{\frac{5}{2}}} - \frac{1}{2(-ae^2+cd^2)(cdx+ae)^2(ex+d)^{\frac{5}{2}}} + \frac{9e}{4(-ae^2+cd^2)^2(cdx+ae)(ex+d)^{\frac{5}{2}}} + \frac{21cde^2}{4(-ae^2+cd^2)^4(ex+d)^{\frac{3}{2}}} - \frac{63c^{\frac{5}{2}}d^{\frac{5}{2}}e^2 \operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{ex+d}}{\sqrt{-ae^2+cd^2}}\right)}{4(-ae^2+cd^2)^{\frac{11}{2}}} + \frac{63c^2d^2e^2}{4(-ae^2+cd^2)^5\sqrt{ex+d}}$$

command

`integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{63c^3d^3 \arctan\left(\frac{\sqrt{xe+d}cd}{\sqrt{-c^2d^3+acde^2}}\right)e^2}{4(c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4cd^2e^8-a^5e^{10})\sqrt{-c^2d^3+acde^2}} + \frac{15(xe+d)^{\frac{3}{2}}c^4d^4e^2-17\sqrt{xe+d}c^4d^5e^2+17\sqrt{xe+d}ac^3d^3e^4}{4(c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4cd^2e^8-a^5e^{10})((xe+d)cd-cd^2+ae^2)^2} + \frac{2(30(xe+d)^2c^2d^2e^2+5(xe+d)c^2d^3e^2+c^2d^4e^2-5(xe+d)acde^4-2acd^2e^4+a^2e^6)}{5(c^5d^{10}-5ac^4d^8e^2+10a^2c^3d^6e^4-10a^3c^2d^4e^6+5a^4cd^2e^8-a^5e^{10})(xe+d)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 14.237 Problem number 2027

$$\int (d + ex)^{7/2} \sqrt{ade + (cd^2 + ae^2)x + cdex^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256(-ae^2 + cd^2)^4 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3465c^5d^5 (ex + d)^{\frac{3}{2}}} \\ & + \frac{16(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}} (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{99c^2d^2} \\ & + \frac{2(ex + d)^{\frac{5}{2}} (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{11cd} \\ & + \frac{128(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{1155c^4d^4\sqrt{ex + d}} \\ & + \frac{32(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}} \sqrt{ex + d}}{231c^3d^3} \end{aligned}$$

command

```
integrate((e*x+d)^(7/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{c dex^2 + ade + (cd^2 + ae^2)x} (ex + d)^{\frac{7}{2}} dx$$

### 14.238 Problem number 2028

$$\int (d + ex)^{5/2} \sqrt{ade + (cd^2 + ae^2)x + cdex^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{315c^4 d^4 (ex + d)^{\frac{3}{2}}} \\ & + \frac{2(ex + d)^{\frac{3}{2}} (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{9cd} \\ & + \frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{105c^3 d^3 \sqrt{ex + d}} \\ & + \frac{4(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}} \sqrt{ex + d}}{21c^2 d^2} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{315} \left( 105 d^3 \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} - 63 d^2 \left( \frac{5}{\dots} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (ex + d)^{\frac{5}{2}} dx$$

### 14.239 Problem number 2029

$$\int (d + ex)^{3/2} \sqrt{ade + (cd^2 + ae^2)x + cde x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{105c^3 d^3 (ex + d)^{\frac{3}{2}}} \\ & + \frac{8(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{35c^2 d^2 \sqrt{ex + d}} \\ & + \frac{2(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}} \sqrt{ex + d}}{7cd} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( 35 d^2 \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} - 14 d \left( \frac{5((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (ex + d)^{\frac{3}{2}} dx$$

**14.240 Problem number 2030**

$$\int \sqrt{d + ex} \sqrt{ade + (cd^2 + ae^2)x + cdex^2} dx$$

Optimal antiderivative

$$\frac{4(-ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{15c^2d^2(ex + d)^{\frac{3}{2}}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{5cd\sqrt{ex + d}}$$

command

`integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{15} \left( 5 d \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} - \left( \frac{5((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d} dx$$

**14.241 Problem number 2031**

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3cd(ex + d)^{\frac{3}{2}}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-2)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}} dx$$

#### 14.242 Problem number 2032

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdx^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left( \frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}} \right) \sqrt{-ae^2 + cd^2}}{e^{\frac{3}{2}}} + \frac{2 \sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{e \sqrt{ex + d}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-2 \left( \frac{(cd^2 - ae^2) \arctan \left( \frac{\sqrt{(xe + d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}} \right)}{\sqrt{cd^2e - ae^3}} - \sqrt{(xe + d)cde - cd^2e + ae^3} e^{(-1)} - \frac{(cd^2 \arctan \left( \frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}} \right) - \sqrt{-cd^2e + ae^3})}{\sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{3}{2}}} dx$$

## 14.243 Problem number 2033

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{cd \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{e^{\frac{3}{2}} \sqrt{-ae^2 + cd^2}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{e(ex + d)^{\frac{3}{2}}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{c^2 d^2 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e - \frac{\sqrt{(xe+d)cde - cd^2e + ae^3} cd}{xe+d}}{\sqrt{cd^2e - ae^3}}\right) e^{(-2)}}{cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cde x^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{5}{2}}} dx$$

## 14.244 Problem number 2034

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{c^2 d^2 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{4e^{\frac{3}{2}} (-ae^2 + cd^2)^{\frac{3}{2}}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{2e(ex + d)^{\frac{5}{2}}} + \frac{cd \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4e(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{c^3 d^3 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{\sqrt{cd^2e - ae^3} (cd^2 - ae^2)} - \frac{(\sqrt{(xe+d)cde - cd^2e + ae^3} c^4 d^5 e^2 - \sqrt{(xe+d)cde - cd^2e + ae^3} ac^3)}{(cd^2 - ae^2)(xe+d)^2 c^2 d^2} \right) \frac{1}{4cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{7}{2}}} dx$$

**14.245 Problem number 2035**

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\frac{c^3 d^3 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{8e^{\frac{3}{2}} (-ae^2 + cd^2)^{\frac{5}{2}}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3e (ex + d)^{\frac{7}{2}}} + \frac{cd \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{12e (-ae^2 + cd^2) (ex + d)^{\frac{5}{2}}} + \frac{c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8e (-ae^2 + cd^2)^2 (ex + d)^{\frac{3}{2}}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3c^4 d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^2 d^4 - 2acd^2 e^2 + a^2 e^4) \sqrt{cd^2e - ae^3}} - \frac{(3 \sqrt{(xe+d)cde - cd^2e + ae^3} c^6 d^8 e^3 - 6 \sqrt{(xe+d)cde - cd^2e + ae^3} ac^3)}{(cd^2 - ae^2)(xe+d)^2 c^2 d^2} \right) \frac{1}{4cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)^{\frac{9}{2}}} dx$$



## 14.246 Problem number 2036

$$\int (d + ex)^{5/2} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256(-ae^2 + cd^2)^4 (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{15015c^5d^5 (ex + d)^{5/2}} \\ & + \frac{128(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{3003c^4d^4 (ex + d)^{3/2}} \\ & + \frac{2(ex + d)^{3/2} (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{13cd} \\ & + \frac{32(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{429c^3d^3 \sqrt{ex + d}} \\ & + \frac{16(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{5/2} \sqrt{ex + d}}{143c^2d^2} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (cdex^2 + ade + (cd^2 + ae^2)x)^{3/2} (ex + d)^{5/2} dx$$

## 14.247 Problem number 2037

$$\int (d + ex)^{3/2} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{1155c^4d^4 (ex + d)^{5/2}} \\ & + \frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{231c^3d^3 (ex + d)^{3/2}} \\ & + \frac{4(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{5/2}}{33c^2d^2 \sqrt{ex + d}} \\ & + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{5/2} \sqrt{ex + d}}{11cd} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(ex + d)^{\frac{3}{2}} dx$$

#### 14.248 Problem number 2038

$$\int \sqrt{d + ex} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2} dx$$

Optimal antiderivative

$$\frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{315c^3d^3 (ex + d)^{\frac{5}{2}}} + \frac{8(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{63c^2d^2 (ex + d)^{\frac{3}{2}}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{9cd\sqrt{ex + d}}$$

command

```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}\sqrt{ex + d} dx$$

#### 14.249 Problem number 2039

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{4(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{35c^2d^2 (ex + d)^{\frac{5}{2}}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{7cd (ex + d)^{\frac{3}{2}}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{105} \left( 7cd^2 \left( \frac{\left( 5((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}ae^3 - 3((xe+d)cde - cd^2e + ae^3)^{\frac{5}{2}} \right) e^{(-2)}}{c^2d^2} + \frac{3\sqrt{-cd^2e + ae^3} c^2d^4}{c^2d^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{\sqrt{ex + d}} dx$$

#### 14.250 Problem number 2040

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{5cd(ex + d)^{\frac{5}{2}}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( cd \left( \frac{\left( 5((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}ae^3 - 3((xe+d)cde - cd^2e + ae^3)^{\frac{5}{2}} \right) e^{(-2)}}{c^2d^2} + \frac{3\sqrt{-cd^2e + ae^3} c^2d^4}{c^2d^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}} dx$$

## 14.251 Problem number 2041

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e(ex + d)^{\frac{3}{2}}} \\ & + \frac{2(-ae^2 + cd^2)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{e^{\frac{5}{2}}} \\ & + \frac{2\left(a - \frac{cd^2}{e^2}\right) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{ex + d}} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{3} \left( 3 \sqrt{(xe + d)cde - cd^2e + ae^3} cd^2e^5 - 3 \sqrt{(xe + d)cde - cd^2e + ae^3} ae^7 - ((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^4 \right) e \\ & \frac{2 \left( 3c^2d^4 \arctan\left(\frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e - 6acd^2 \arctan\left(\frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e^3 - 4 \sqrt{cd^2e - ae^3} \sqrt{-cd^2e + ae^3} cd \right)}{3 \sqrt{cd^2e - ae^3}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.252 Problem number 2042

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{e(ex + d)^{\frac{5}{2}}} \\ & - \frac{3cd \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right) \sqrt{-ae^2 + cd^2}}{e^{\frac{5}{2}}} \\ & + \frac{3cd \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e^2 \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{2 \sqrt{(xe+d)cde - cd^2e + ae^3} c^2 d^2 - \frac{3(c^3 d^4 e - ac^2 d^2 e^3) \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}} + \frac{(\sqrt{(xe+d)cde - cd^2e + ae^3})^2}{cd}}{cd} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

#### 14.253 Problem number 2043

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{2e(ex+d)^{7/2}} + \frac{3c^2 d^2 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex+d}}\right)}{4e^{5/2} \sqrt{-ae^2 + cd^2}} \\ & -\frac{3cd \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4e^2 (ex+d)^{3/2}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3c^3 d^3 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e - \left(3 \sqrt{(xe+d)cde - cd^2e + ae^3} c^4 d^5 e^2 - 3 \sqrt{(xe+d)cde - cd^2e + ae^3} (xe+d)^2 c^2 d^2\right)}{4cd} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

## 14.254 Problem number 2044

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e(ex + d)^{\frac{9}{2}}} + \frac{c^3 d^3 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{8e^{\frac{5}{2}}(-ae^2 + cd^2)^{\frac{3}{2}}} \\ & - \frac{cd \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{4e^2(ex + d)^{\frac{5}{2}}} + \frac{c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8e^2(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3c^4 d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{\sqrt{cd^2e - ae^3} (cd^2 - ae^2)} - \frac{(3 \sqrt{(xe+d)cde - cd^2e + ae^3} c^6 d^8 e^3 - 6 \sqrt{(xe+d)cde - cd^2e + ae^3})}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

## 14.255 Problem number 2045

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{4e(ex + d)^{\frac{11}{2}}} + \frac{3c^4 d^4 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{64e^{\frac{5}{2}}(-ae^2 + cd^2)^{\frac{5}{2}}} \\ & - \frac{cd \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8e^2(ex + d)^{\frac{7}{2}}} + \frac{c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{32e^2(-ae^2 + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{3c^3 d^3 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{64e^2(-ae^2 + cd^2)^2(ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3c^5d^5 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^2d^4 - 2acd^2e^2 + a^2e^4)\sqrt{cd^2e - ae^3}} - \frac{(3\sqrt{(xe+d)cde - cd^2e + ae^3} c^8d^{11}e^4 - 9\sqrt{(xe+d)cde - cd^2e + ae^3})}{(c^2d^4 - 2acd^2e^2 + a^2e^4)\sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

#### 14.256 Problem number 2046

$$\int (d+ex)^{3/2} (ade + (cd^2 + ae^2)x + cdex^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256(-ae^2 + cd^2)^4 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{45045c^5d^5 (ex + d)^{7/2}} \\ & + \frac{128(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{6435c^4d^4 (ex + d)^{5/2}} \\ & + \frac{32(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{715c^3d^3 (ex + d)^{3/2}} \\ & + \frac{16(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{195c^2d^2 \sqrt{ex + d}} \\ & + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{7/2} \sqrt{ex + d}}{15cd} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.257 Problem number 2047

$$\int \sqrt{d+ex} (ade + (cd^2 + ae^2)x + cdex^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32(-ae^2 + cd^2)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{3003c^4d^4 (ex + d)^{7/2}} \\ & + \frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{429c^3d^3 (ex + d)^{5/2}} \\ & + \frac{12(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{143c^2d^2 (ex + d)^{3/2}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{13cd\sqrt{ex + d}} \end{aligned}$$

command

```
integrate((e*x+d)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (cdex^2 + ade + (cd^2 + ae^2)x)^{5/2} \sqrt{ex + d} dx$$

## 14.258 Problem number 2048

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16(-ae^2 + cd^2)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{693c^3d^3 (ex + d)^{7/2}} \\ & + \frac{8(-ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{99c^2d^2 (ex + d)^{5/2}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{7/2}}{11cd (ex + d)^{3/2}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{\sqrt{ex + d}} dx$$

#### 14.259 Problem number 2049

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4(-ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{7}{2}}}{63c^2d^2(ex + d)^{\frac{7}{2}}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{7}{2}}}{9cd(ex + d)^{\frac{5}{2}}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{3}{2}}} dx$$

#### 14.260 Problem number 2050

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{7}{2}}}{7cd(ex + d)^{\frac{7}{2}}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( c^2 d^2 \left( \frac{(15 \sqrt{-cd^2e + ae^3} c^3 d^6 - 3 \sqrt{-cd^2e + ae^3} ac^2 d^4 e^2 - 4 \sqrt{-cd^2e + ae^3} a^2 cd^2 e^4 - 8 \sqrt{-cd^2e + ae^3} a^3)}{c^3 d^3} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 14.261 Problem number 2051

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2\left(a - \frac{cd^2}{e^2}\right) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3(ex + d)^{\frac{3}{2}}} + \frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{5e(ex + d)^{\frac{5}{2}}} \\ & - \frac{2(-ae^2 + cd^2)^{\frac{5}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{e^{\frac{7}{2}}} \\ & + \frac{2(-ae^2 + cd^2)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e^3 \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2}{15} \left( (15 \sqrt{(xe + d)cde - cd^2e + ae^3} c^2 d^4 e^{14} - 30 \sqrt{(xe + d)cde - cd^2e + ae^3} acd^2 e^{16} - 5((xe + d)cde - cd^2e + ae^3)) \right. \\ & \left. + \frac{2\left(15c^3d^6 \arctan\left(\frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e - 45ac^2d^4 \arctan\left(\frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e^3 - 23\sqrt{cd^2e - ae^3} \sqrt{-cd^2e + ae^3}\right)}{c^3 d^3} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.262 Problem number 2052

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e^2(ex + d)^{\frac{3}{2}}} - \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{e(ex + d)^{\frac{7}{2}}} \\ & + \frac{5cd(-ae^2 + cd^2)^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{e^{\frac{7}{2}}} \\ & + \frac{5cd\left(a - \frac{cd^2}{e^2}\right) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{e\sqrt{ex + d}} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 2 \left( 6 \sqrt{(xe + d)cde - cd^2e + ae^3} c^3 d^4 e^3 - 6 \sqrt{(xe + d)cde - cd^2e + ae^3} ac^2 d^2 e^5 - ((xe + d)cde - cd^2e + ae^3) \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.263 Problem number 2053

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{4e^2(ex + d)^{\frac{5}{2}}} - \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{2e(ex + d)^{\frac{9}{2}}} \\ & - \frac{15c^2d^2 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right) \sqrt{-ae^2 + cd^2}}{4e^{\frac{7}{2}}} \\ & + \frac{15c^2d^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{4e^3 \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 8 \sqrt{(xe+d)cde - cd^2e + ae^3} c^3 d^3 - \frac{15 (c^4 d^5 e - ac^3 d^3 e^3) \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)}{\sqrt{cd^2e - ae^3}} + \frac{7 \sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.264 Problem number 2054

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5cd(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{12e^2(ex+d)^{\frac{7}{2}}} - \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{3e(ex+d)^{\frac{11}{2}}} \\ & + \frac{5c^3d^3 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex+d}}\right)}{8e^{\frac{7}{2}} \sqrt{-ae^2 + cd^2}} \\ & - \frac{5c^2d^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8e^3(ex+d)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{15 c^4 d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{\sqrt{cd^2e - ae^3}} - \frac{(15 \sqrt{(xe+d)cde - cd^2e + ae^3} c^6 d^8 e^3 - 30 \sqrt{(xe+d)cde - cd^2e + ae^3})}{\sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.265 Problem number 2055

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{24e^2(ex + d)^{\frac{9}{2}}} - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{4e(ex + d)^{\frac{13}{2}}} \\ & + \frac{5c^4d^4 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{64e^{\frac{7}{2}}(-ae^2 + cd^2)^{\frac{3}{2}}} \\ & - \frac{5c^2d^2 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{32e^3(ex + d)^{\frac{5}{2}}} + \frac{5c^3d^3 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{64e^3(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{15c^5d^5 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right)e}{\sqrt{cd^2e - ae^3}(cd^2 - ae^2)} - \frac{(15 \sqrt{(xe+d)cde - cd^2e + ae^3} c^8 d^{11} e^4 - 45 \sqrt{(xe+d)cde - cd^2e + ae^3})}{\sqrt{cd^2e - ae^3}(cd^2 - ae^2)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.266 Problem number 2056

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{8e^2(ex + d)^{\frac{11}{2}}} - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{5e(ex + d)^{\frac{15}{2}}} \\ & + \frac{3c^5d^5 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{128e^{\frac{7}{2}}(-ae^2 + cd^2)^{\frac{5}{2}}} \\ & - \frac{c^2d^2 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{16e^3(ex + d)^{\frac{7}{2}}} + \frac{c^3d^3 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{64e^3(-ae^2 + cd^2)(ex + d)^{\frac{5}{2}}} \\ & + \frac{3c^4d^4 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{128e^3(-ae^2 + cd^2)^2(ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(17/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{15 c^6 d^6 \arctan\left(\frac{\sqrt{(x e + d) c d e - c d^2 e + a e^3}}{\sqrt{c d^2 e - a e^3}}\right) e}{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) \sqrt{c d^2 e - a e^3}} - \frac{(15 \sqrt{(x e + d) c d e - c d^2 e + a e^3} c^{10} d^{14} e^5 - 60 \sqrt{(x e + d) c d e - c d^2 e - a e^3} c^9 d^{13} e^4 + \dots)}{(c^2 d^4 - 2 a c d^2 e^2 + a^2 e^4) \sqrt{c d^2 e - a e^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.267 Problem number 2057

$$\int \frac{(d + ex)^{7/2}}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^2d^2} \\ & + \frac{2(ex + d)^{\frac{5}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{7cd} \\ & + \frac{32(-ae^2 + cd^2)^3 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^4d^4\sqrt{ex + d}} \\ & + \frac{16(-ae^2 + cd^2)^2 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^3d^3} \end{aligned}$$

command

`integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6) \sqrt{(x e + d) c d e - c d^2 e + a e^3} e^{(-1)}}{c^4d^4} \\ & - \frac{32 \left( \sqrt{-cd^2e + ae^3} c^3d^6 - 3 \sqrt{-cd^2e + ae^3} ac^2d^4e^2 + 3 \sqrt{-cd^2e + ae^3} a^2cd^2e^4 - \sqrt{-cd^2e + ae^3} a^3e^6 \right) e^{(-1)}}{35c^4d^4} \\ & + \frac{2 \left( 35 \left( (x e + d) c d e - c d^2 e + a e^3 \right)^{\frac{3}{2}} c^2 d^4 e^2 - 70 \left( (x e + d) c d e - c d^2 e + a e^3 \right)^{\frac{3}{2}} a c d^2 e^4 + 21 \left( (x e + d) c d e - c d^2 e + a e^3 \right)^{\frac{3}{2}} a^2 c d^2 e^4 - 7 \left( (x e + d) c d e - c d^2 e + a e^3 \right)^{\frac{3}{2}} a^3 e^6 \right)}{35c^4d^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex + d)^{\frac{7}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

## 14.268 Problem number 2058

$$\int \frac{(d+ex)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ex+d)^{\frac{3}{2}} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{5cd} \\ & + \frac{16(-ae^2+cd^2)^2 \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{15c^3 d^3 \sqrt{ex+d}} \\ & + \frac{8(-ae^2+cd^2) \sqrt{ex+d} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{15c^2 d^2} \end{aligned}$$

command

`integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(c^2 d^4 - 2acd^2 e^2 + a^2 e^4) \sqrt{(xe+d)cde - cd^2 e + ae^3} e^{(-1)}}{c^3 d^3} \\ & - \frac{16 \left( \sqrt{-cd^2 e + ae^3} c^2 d^4 - 2 \sqrt{-cd^2 e + ae^3} acd^2 e^2 + \sqrt{-cd^2 e + ae^3} a^2 e^4 \right) e^{(-1)}}{15 c^3 d^3} \\ & + \frac{2 \left( 10 \left( (xe+d)cde - cd^2 e + ae^3 \right)^{\frac{3}{2}} cd^2 e - 10 \left( (xe+d)cde - cd^2 e + ae^3 \right)^{\frac{3}{2}} ae^3 + 3 \left( (xe+d)cde - cd^2 e + ae^3 \right)^{\frac{5}{2}} \right) e^{(-1)}}{15 c^3 d^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cde x^2 + ade + (cd^2 + ae^2)x}} dx$$

## 14.269 Problem number 2059

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{4(-ae^2+cd^2) \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^2 d^2 \sqrt{ex+d}} + \frac{2\sqrt{ex+d} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3cd}$$

command

`integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{(xe+d)cde - cd^2e + ae^3} (cd^2 - ae^2) e^{(-1)}}{c^2 d^2} + \frac{2 ((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-2)}}{3 c^2 d^2} - \frac{4 \left( \sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2 \right) e^{(-1)}}{3 c^2 d^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

**14.270 Problem number 2060**

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{cd \sqrt{ex+d}}$$

command

`integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{(xe+d)cde - cd^2e + ae^3} e^{(-1)}}{cd} - \frac{2 \sqrt{-cd^2e + ae^3} e^{(-1)}}{cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$



## 14.271 Problem number 2061

$$\int \frac{1}{\sqrt{d+ex} \sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left( \frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}} \right)}{\sqrt{e} \sqrt{-ae^2 + cd^2}}$$

command

`integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan \left( \frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}} \right)}{\sqrt{cd^2e - ae^3}} - \frac{2 \arctan \left( \frac{\sqrt{-cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}} \right)}{\sqrt{cd^2e - ae^3}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d}} dx$$

## 14.272 Problem number 2062

$$\int \frac{1}{(d+ex)^{3/2} \sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{cd \arctan \left( \frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}} \right)}{(-ae^2 + cd^2)^{3/2} \sqrt{e}} + \frac{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{(-ae^2 + cd^2)(ex + d)^{3/2}}$$

command

`integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{c^2 d^2 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{\sqrt{cd^2e - ae^3} (cd^2 - ae^2)} + \frac{\sqrt{(xe+d)cde - cd^2e + ae^3} cd}{(cd^2 - ae^2)(xe+d)} \right) e^{(-1)}}{cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (ex + d)^{\frac{3}{2}}} dx$$

### 14.273 Problem number 2063

$$\int \frac{1}{(d + ex)^{5/2} \sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{3c^2 d^2 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{4(-ae^2 + cd^2)^{\frac{5}{2}} \sqrt{e}} + \frac{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{2(-ae^2 + cd^2)(ex + d)^{\frac{5}{2}}} + \frac{3cd \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{4(-ae^2 + cd^2)^2 (ex + d)^{\frac{3}{2}}}$$

command

`integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{3c^3 d^3 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^2 d^4 - 2acd^2 e^2 + a^2 e^4) \sqrt{cd^2e - ae^3}} + \frac{(5 \sqrt{(xe+d)cde - cd^2e + ae^3} c^4 d^5 e^2 - 5 \sqrt{(xe+d)cde - cd^2e + ae^3})}{(c^2 d^4 - 2acd^2 e^2 + a^2 e^4)(xe+d)^2 c^2 d} \right) e^{(-1)}}{4cd}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.274 Problem number 2064

$$\int \frac{1}{(d+ex)^{7/2} \sqrt{ade + (cd^2 + ae^2)x + cde x^2}} dx$$

Optimal antiderivative

$$\frac{5c^3 d^3 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{8(-ae^2 + cd^2)^{7/2} \sqrt{e}} + \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{3(-ae^2 + cd^2)(ex + d)^{7/2}} + \frac{5cd \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{12(-ae^2 + cd^2)^2 (ex + d)^{5/2}} + \frac{5c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{8(-ae^2 + cd^2)^3 (ex + d)^{3/2}}$$

command

```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{15c^4 d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^3 d^6 - 3ac^2 d^4 e^2 + 3a^2 cd^2 e^4 - a^3 e^6) \sqrt{cd^2e - ae^3}} + \frac{(33 \sqrt{(xe+d)cde - cd^2e + ae^3} c^6 d^8 e^3 - 66 \sqrt{(xe+d)cde - cd^2e + ae^3})}{(c^3 d^6 - 3ac^2 d^4 e^2 + 3a^2 cd^2 e^4 - a^3 e^6) \sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 14.275 Problem number 2065

$$\int \frac{(d+ex)^{7/2}}{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{8(-ae^2 + cd^2)(ex + d)^{3/2}}{3c^2 d^2 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} + \frac{2(ex + d)^{5/2}}{3cd \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} - \frac{16(-ae^2 + cd^2)^2 \sqrt{ex + d}}{3c^3 d^3 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}$$

command

```
integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2(c^2d^4e - 2acd^2e^3 + a^2e^5)}{\sqrt{(xe+d)cde - cd^2e + ae^3}c^3d^3} + \frac{16(c^2d^4e - 2acd^2e^3 + a^2e^5)}{3\sqrt{-cd^2e + ae^3}c^3d^3} + \frac{2\left(6\sqrt{(xe+d)cde - cd^2e + ae^3}c^7d^8e^3 - 6\sqrt{(xe+d)cde - cd^2e + ae^3}ac^6d^6e^5 + ((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^6\right)}{3c^9d^9}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.276 Problem number 2066

$$\int \frac{(d+ex)^{5/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}}{cd\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} - \frac{4(-ae^2 + cd^2)\sqrt{ex+d}}{c^2d^2\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

command

```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{cd}e - \frac{cd^2e^2 - ae^4}{\sqrt{(xe+d)cde - cd^2e + ae^3}cd}\right)e^{(-1)}}{cd} + \frac{4(cd^2e - ae^3)}{\sqrt{-cd^2e + ae^3}c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.277 Problem number 2067

$$\int \frac{(d+ex)^{3/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{ex+d}}{cd\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

command

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2e}{\sqrt{(xe+d)cde-cd^2e+ae^3}cd} + \frac{2e}{\sqrt{-cd^2e+ae^3}cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 14.278 Problem number 2068

$$\int \frac{\sqrt{d+ex}}{(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \arctan\left(\frac{\sqrt{e} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-ae^2+cd^2} \sqrt{ex+d}}\right) \sqrt{e}}{(-ae^2+cd^2)^{\frac{3}{2}}}$$

$$-\frac{2\sqrt{ex+d}}{(-ae^2+cd^2)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right) e}{\sqrt{cd^2e-ae^3}(cd^2-ae^2)}$$

$$+\frac{2\left(\sqrt{-cd^2e+ae^3} \arctan\left(\frac{\sqrt{-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right) e + \sqrt{cd^2e-ae^3} e\right)}{\sqrt{cd^2e-ae^3} \sqrt{-cd^2e+ae^3} cd^2 - \sqrt{cd^2e-ae^3} \sqrt{-cd^2e+ae^3} ae^2}$$

$$-\frac{2e}{\sqrt{(xe+d)cde-cd^2e+ae^3}(cd^2-ae^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.279 Problem number 2069

$$\int \frac{1}{\sqrt{d+ex} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3cd \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right) \sqrt{e}}{(-ae^2 + cd^2)^{\frac{5}{2}}} \\ & + \frac{1}{(-ae^2 + cd^2) \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{3cd\sqrt{ex + d}}{(-ae^2 + cd^2)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \end{aligned}$$

command

`integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3cd \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^2d^4 - 2acd^2e^2 + a^2e^4) \sqrt{cd^2e - ae^3}} \\ & - \frac{2c^2d^3e^2 - 2acde^4 + 3((xe+d)cde - cd^2e + ae^3)cde}{(c^2d^4 - 2acd^2e^2 + a^2e^4) \left(\sqrt{(xe+d)cde - cd^2e + ae^3} \sqrt{cd^2e - ae^3} + \sqrt{(xe+d)cde - cd^2e + ae^3} ae^3 + ((xe+d)cde - cd^2e + ae^3)\right)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.280 Problem number 2070

$$\int \frac{1}{(d+ex)^{3/2} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{15c^2d^2 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right) \sqrt{e}}{4(-ae^2 + cd^2)^{\frac{7}{2}}} \\ & + \frac{1}{2(-ae^2 + cd^2) (ex + d)^{\frac{3}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & + \frac{5cd}{4(-ae^2 + cd^2)^2 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{15c^2d^2 \sqrt{ex + d}}{4(-ae^2 + cd^2)^3 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 c^2 d^2 \arctan\left(\frac{\sqrt{(x e + d) c d e - c d^2 e + a e^3}}{\sqrt{c d^2 e - a e^3}}\right) e}{4 (c^3 d^6 - 3 a c^2 d^4 e^2 + 3 a^2 c d^2 e^4 - a^3 e^6) \sqrt{c d^2 e - a e^3}} - \frac{2 c^2 d^2 e}{(c^3 d^6 - 3 a c^2 d^4 e^2 + 3 a^2 c d^2 e^4 - a^3 e^6) \sqrt{(x e + d) c d e - c d^2 e + a e^3}} - \frac{(9 \sqrt{(x e + d) c d e - c d^2 e + a e^3} c^3 d^4 e^2 - 9 \sqrt{(x e + d) c d e - c d^2 e + a e^3} a c^2 d^2 e^4 + 7 ((x e + d) c d e - c d^2 e + a e^3)^{\frac{3}{2}} c)}{4 (c^3 d^6 - 3 a c^2 d^4 e^2 + 3 a^2 c d^2 e^4 - a^3 e^6) (x e + d)^2 c^2 d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.281 Problem number 2071

$$\int \frac{1}{(d + ex)^{5/2} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35c^3 d^3 \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right) \sqrt{e}}{8(-ae^2 + cd^2)^{\frac{9}{2}}} \\ & + \frac{1}{3(-ae^2 + cd^2)(ex + d)^{\frac{5}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & + \frac{7cd}{12(-ae^2 + cd^2)^2 (ex + d)^{\frac{3}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & + \frac{35c^2 d^2}{24(-ae^2 + cd^2)^3 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{35c^3 d^3 \sqrt{ex + d}}{8(-ae^2 + cd^2)^4 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35 c^3 d^3 \arctan \left( \frac{\sqrt{(x e + d) c d e - c d^2 e + a e^3}}{\sqrt{c d^2 e - a e^3}} \right) e}{8 (c^4 d^8 - 4 a c^3 d^6 e^2 + 6 a^2 c^2 d^4 e^4 - 4 a^3 c d^2 e^6 + a^4 e^8) \sqrt{c d^2 e - a e^3}}$$


---


$$\frac{2 c^3 d^3 e}{(c^4 d^8 - 4 a c^3 d^6 e^2 + 6 a^2 c^2 d^4 e^4 - 4 a^3 c d^2 e^6 + a^4 e^8) \sqrt{(x e + d) c d e - c d^2 e + a e^3}}$$


---


$$\frac{(87 \sqrt{(x e + d) c d e - c d^2 e + a e^3} c^5 d^7 e^3 - 174 \sqrt{(x e + d) c d e - c d^2 e + a e^3} a c^4 d^5 e^5 + 136 ((x e + d) c d e - c d^2 e + a e^3))}{24 (c^4 d^8 - 4 a c^3 d^6 e^2 + 6 a^2 c^2 d^4 e^4 - 4 a^3 c d^2 e^6 + a^4 e^8)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.282 Problem number 2072

$$\int \frac{1}{(d + ex)^{7/2} (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{315 c^4 d^4 \arctan \left( \frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}} \right) \sqrt{e}}{64 (-ae^2 + cd^2)^{\frac{11}{2}}}$$

$$+ \frac{1}{4 (-ae^2 + cd^2) (ex + d)^{\frac{7}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

$$+ \frac{3cd}{8 (-ae^2 + cd^2)^2 (ex + d)^{\frac{5}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

$$+ \frac{21c^2 d^2}{32 (-ae^2 + cd^2)^3 (ex + d)^{\frac{3}{2}} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

$$+ \frac{105c^3 d^3}{64 (-ae^2 + cd^2)^4 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

$$- \frac{315c^4 d^4 \sqrt{ex + d}}{64 (-ae^2 + cd^2)^5 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}$$

command

```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{315 c^4 d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{64 (c^5 d^{10} - 5 ac^4 d^8 e^2 + 10 a^2 c^3 d^6 e^4 - 10 a^3 c^2 d^4 e^6 + 5 a^4 c d^2 e^8 - a^5 e^{10}) \sqrt{cd^2e - ae^3}} - \frac{2 c^4 d^4 e}{(c^5 d^{10} - 5 ac^4 d^8 e^2 + 10 a^2 c^3 d^6 e^4 - 10 a^3 c^2 d^4 e^6 + 5 a^4 c d^2 e^8 - a^5 e^{10}) \sqrt{(xe+d)cde - cd^2e + ae^3}} - \frac{(325 \sqrt{(xe+d)cde - cd^2e + ae^3} c^7 d^{10} e^4 - 975 \sqrt{(xe+d)cde - cd^2e + ae^3} ac^6 d^8 e^6 + 765 ((xe+d)cde - cd^2e + ae^3)^{3/2})}{(c^5 d^{10} - 5 ac^4 d^8 e^2 + 10 a^2 c^3 d^6 e^4 - 10 a^3 c^2 d^4 e^6 + 5 a^4 c d^2 e^8 - a^5 e^{10}) \sqrt{(xe+d)cde - cd^2e + ae^3}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 14.283 Problem number 2073

$$\int \frac{(d+ex)^{7/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{4(-ae^2 + cd^2)(ex+d)^{3/2}}{3c^2d^2(ade + (ae^2 + cd^2)x + cdex^2)^{3/2}} - \frac{2(ex+d)^{5/2}}{cd(ade + (ae^2 + cd^2)x + cdex^2)^{3/2}}$$

command

`integrate((e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4e^2}{3\sqrt{-cd^2e + ae^3} c^2d^2} - \frac{2(cd^2e^3 - ae^5 + 3((xe+d)cde - cd^2e + ae^3)e^2)}{3((xe+d)cde - cd^2e + ae^3)^{3/2} c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 14.284 Problem number 2074

$$\int \frac{(d+ex)^{5/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex+d)^{3/2}}{3cd(ade + (ae^2 + cd^2)x + cdex^2)^{3/2}}$$

command

```
integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2e^2}{3\left(\sqrt{-cd^2e+ae^3}c^2d^3-\sqrt{-cd^2e+ae^3}acde^2\right)}-\frac{2e^3}{3\left((xe+d)cde-cd^2e+ae^3\right)^{\frac{3}{2}}cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 14.285 Problem number 2075

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cde x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-ae^2+cd^2} \sqrt{ex+d}}\right)}{(-ae^2+cd^2)^{\frac{5}{2}}}-\frac{2\sqrt{ex+d}}{3cd(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}}$$

$$-\frac{2e}{3cd(-ae^2+cd^2)\sqrt{ex+d}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

$$+\frac{2e\sqrt{ex+d}}{(-ae^2+cd^2)^2\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3}\left(\frac{3\arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)e}{(c^2d^4-2acd^2e^2+a^2e^4)\sqrt{cd^2e-ae^3}}-\frac{cd^2e^2-ae^4-3((xe+d)cde-cd^2e+ae^3)e}{(c^2d^4-2acd^2e^2+a^2e^4)((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}}\right)e$$

$$-\frac{2\left(3\sqrt{-cd^2e+ae^3}\arctan\left(\frac{\sqrt{-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right)e^2+4\sqrt{cd^2e-ae^3}e^2\right)}{3\left(\sqrt{cd^2e-ae^3}\sqrt{-cd^2e+ae^3}c^2d^4-2\sqrt{cd^2e-ae^3}\sqrt{-cd^2e+ae^3}acd^2e^2+\sqrt{cd^2e-ae^3}\sqrt{-cd^2e+ae^3}a^2\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.286 Problem number 2076

$$\int \frac{\sqrt{d+ex}}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5cd e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{(-ae^2 + cd^2)^{\frac{7}{2}} 2\sqrt{ex + d}} \\ & - \frac{3(-ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{5e} \\ & - \frac{3(-ae^2 + cd^2)^2 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{5cde \sqrt{ex + d}} \\ & + \frac{5cde \sqrt{ex + d}}{(-ae^2 + cd^2)^3 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \end{aligned}$$

command

```
integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} \left( \frac{15 cd \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)\sqrt{cd^2e - ae^3}} - \frac{2(c^2d^3e^2 - acde^4 - 6((xe+d)cde - cd^2e + ae^3))}{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)((xe+d)cde - cd^2e + ae^3)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.287 Problem number 2077

$$\int \frac{1}{\sqrt{d+ex} (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{35c^2d^2e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-ae^2+cd^2}\sqrt{ex+d}}\right)}{4(-ae^2+cd^2)^{\frac{9}{2}}} \\
& + \frac{1}{2(-ae^2+cd^2)(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}\sqrt{ex+d}} \\
& - \frac{7cd\sqrt{ex+d}}{6(-ae^2+cd^2)^2(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}} \\
& - \frac{35cde}{12(-ae^2+cd^2)^3\sqrt{ex+d}\sqrt{ade+(ae^2+cd^2)x+cde x^2}} \\
& + \frac{35c^2d^2e\sqrt{ex+d}}{4(-ae^2+cd^2)^4\sqrt{ade+(ae^2+cd^2)x+cde x^2}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{12} \left( \frac{105c^2d^2 \arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}}{\sqrt{cd^2e-ae^3}}\right) e}{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)\sqrt{cd^2e-ae^3}} - \frac{8(c^3d^4e^2-ac^2d^2e^4-9((xe+d)cde-cd^2e+ae^3))}{(c^4d^8-4ac^3d^6e^2+6a^2c^2d^4e^4-4a^3cd^2e^6+a^4e^8)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**14.288 Problem number 2078**

$$\int \frac{1}{(d+ex)^{3/2}(ade+(cd^2+ae^2)x+cde x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{1}{3(-ae^2 + cd^2)(ex + d)^{\frac{3}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{105c^3d^3e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2}\sqrt{ex + d}}\right)}{8(-ae^2 + cd^2)^{\frac{11}{2}}} \\
& + \frac{3cd}{4(-ae^2 + cd^2)^2(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}\sqrt{ex + d}} \\
& - \frac{7c^2d^2\sqrt{ex + d}}{4(-ae^2 + cd^2)^3(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& - \frac{35c^2d^2e}{8(-ae^2 + cd^2)^4\sqrt{ex + d}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \\
& + \frac{105c^3d^3e\sqrt{ex + d}}{8(-ae^2 + cd^2)^5\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{24} \left( \frac{315c^3d^3 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10})\sqrt{cd^2e - ae^3}} - \frac{16c^7d^{11}e^5 - 64ac^6d^9e^7}{(c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10})\sqrt{cd^2e - ae^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**14.289 Problem number 2079**

$$\int \frac{1}{(d + ex)^{5/2} (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{1}{4(-ae^2 + cd^2)(ex + d)^{\frac{5}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{11cd}{24(-ae^2 + cd^2)^2(ex + d)^{\frac{3}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{1155c^4d^4e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2}\sqrt{ex + d}}\right)}{64(-ae^2 + cd^2)^{\frac{13}{2}}} \\
& + \frac{33c^2d^2}{32(-ae^2 + cd^2)^3(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}\sqrt{ex + d}} \\
& - \frac{77c^3d^3\sqrt{ex + d}}{32(-ae^2 + cd^2)^4(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& - \frac{385c^3d^3e}{64(-ae^2 + cd^2)^5\sqrt{ex + d}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \\
& + \frac{1155c^4d^4e\sqrt{ex + d}}{64(-ae^2 + cd^2)^6\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{192} \left( \frac{3465c^4d^4 \arctan\left(\frac{\sqrt{(xe+d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^6d^{12} - 6ac^5d^{10}e^2 + 15a^2c^4d^8e^4 - 20a^3c^3d^6e^6 + 15a^4c^2d^4e^8 - 6a^5cd^2e^{10} + a^6e^{12})\sqrt{cd^2e - ae^3}} - \frac{1}{c^6d^{12}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**14.290 Problem number 2080**

$$\int \frac{1}{(d + ex)^{7/2} (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{1}{5(-ae^2 + cd^2)(ex + d)^{\frac{7}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{13cd}{40(-ae^2 + cd^2)^2(ex + d)^{\frac{5}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{143c^2d^2}{240(-ae^2 + cd^2)^3(ex + d)^{\frac{3}{2}}(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& + \frac{3003c^5d^5e^{\frac{3}{2}} \arctan\left(\frac{\sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{\sqrt{-ae^2 + cd^2} \sqrt{ex + d}}\right)}{128(-ae^2 + cd^2)^{\frac{15}{2}}} \\
& + \frac{429c^3d^3}{320(-ae^2 + cd^2)^4(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}} \sqrt{ex + d}} \\
& - \frac{1001c^4d^4 \sqrt{ex + d}}{320(-ae^2 + cd^2)^5(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}} \\
& - \frac{1001c^4d^4e}{128(-ae^2 + cd^2)^6 \sqrt{ex + d} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \\
& + \frac{3003c^5d^5e \sqrt{ex + d}}{128(-ae^2 + cd^2)^7 \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}
\end{aligned}$$

command

```
integrate(1/(e*x+d)^(7/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{1920} \left( \frac{45045 c^5 d^5 \arctan\left(\frac{\sqrt{(xe + d)cde - cd^2e + ae^3}}{\sqrt{cd^2e - ae^3}}\right) e}{(c^7 d^{14} - 7 a c^6 d^{12} e^2 + 21 a^2 c^5 d^{10} e^4 - 35 a^3 c^4 d^8 e^6 + 35 a^4 c^3 d^6 e^8 - 21 a^5 c^2 d^4 e^{10} + 7 a^6 c d^2 e^{12} - a^7 e^{14}) \sqrt{cd^2e}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.291 Problem number 2081

$$\int \frac{1}{\sqrt{d+ex} \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2} \sqrt{2}}{2\sqrt{d} \sqrt{ex + d}}\right) \sqrt{2}}{e\sqrt{d}}$$

command

`integrate(1/(e*x+d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-xe + d}}{2\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)}{\sqrt{-d}} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2} \sqrt{ex + d}} dx$$

## 14.292 Problem number 2082

$$\int \frac{1}{\sqrt{-d+ex} \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctan}\left(\frac{\sqrt{-e^2x^2 + d^2} \sqrt{2}}{2\sqrt{d} \sqrt{ex - d}}\right) \sqrt{2}}{e\sqrt{d}}$$

command

`integrate(1/(e*x-d)^(1/2)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-xe - d}}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{-d}}{\sqrt{d}}\right)}{\sqrt{d}} \right) e^{(-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2} \sqrt{ex - d}} dx$$



## 14.293 Problem number 2354

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-be + 2cd)(8c^2d^2 + 3b^2e^2 - 4ce(ae + 2bd))(bd - 2ae + (-be + 2cd)x)(cx^2 + bx + a)^{\frac{3}{2}}}{128(ae^2 - bde + cd^2)^4(ex + d)^4} \\ & - \frac{e(cx^2 + bx + a)^{\frac{5}{2}}}{7(ae^2 - bde + cd^2)(ex + d)^7} - \frac{3e(-be + 2cd)(cx^2 + bx + a)^{\frac{5}{2}}}{28(ae^2 - bde + cd^2)^2(ex + d)^6} \\ & - \frac{e(68c^2d^2 + 21b^2e^2 - 4ce(4ae + 17bd))(cx^2 + bx + a)^{\frac{5}{2}}}{280(ae^2 - bde + cd^2)^3(ex + d)^5} \\ & + \frac{3(-4ac + b^2)^2(-be + 2cd)(8c^2d^2 + 3b^2e^2 - 4ce(ae + 2bd)) \operatorname{arctanh}\left(\frac{bd - 2ae + (-be + 2cd)x}{2\sqrt{ae^2 - bde + cd^2}\sqrt{cx^2 + bx + a}}\right)}{2048(ae^2 - bde + cd^2)^{\frac{11}{2}}} \\ & - \frac{3(-4ac + b^2)(-be + 2cd)(8c^2d^2 + 3b^2e^2 - 4ce(ae + 2bd))(bd - 2ae + (-be + 2cd)x)\sqrt{cx^2 + bx + a}}{1024(ae^2 - bde + cd^2)^5(ex + d)^2} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.294 Problem number 2399

$$\int \frac{1}{(d + ex)^3 (a + bx + cx^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(bcd - b^2e + 2ace + c(-be + 2cd)x)}{3(-4ac + b^2)(ae^2 - bde + cd^2)(ex + d)^2(cx^2 + bx + a)^{\frac{3}{2}}} \\ & + \frac{5e^4(24c^2d^2 + 7b^2e^2 - 4ce(ae + 6bd)) \operatorname{arctanh}\left(\frac{bd - 2ae + (-be + 2cd)x}{2\sqrt{ae^2 - bde + cd^2}\sqrt{cx^2 + bx + a}}\right)}{8(ae^2 - bde + cd^2)^{\frac{9}{2}}} \\ & - \frac{2(8ace(-be + 2cd)^2 - (2ace - b^2e + bcd)(20ace^2 - 7b^2e^2 + 8c^2d^2) - c(-be + 2cd)(8c^2d^2 - 7b^2e^2 - 4ce(-9ae - 3(-4ac + b^2)^2(ae^2 - bde + cd^2)^2(ex + d)^2\sqrt{cx^2 + bx + a})))}{3(-4ac + b^2)^2(ae^2 - bde + cd^2)^2(ex + d)^2\sqrt{cx^2 + bx + a}} \\ & + \frac{e(64c^4d^4 - 35b^4e^4 - 128c^3d^2e(-3ae + bd) - 48ac^2e^3(5ae + 8bd) + 8b^2ce^3(27ae + 8bd))\sqrt{cx^2 + bx + a}}{6(-4ac + b^2)^2(ae^2 - bde + cd^2)^3(ex + d)^2} \\ & + \frac{e(-be + 2cd)(64c^4d^4 - 105b^4e^4 - 64c^3d^2e(-7ae + 2bd) + 40b^2ce^3(19ae + 2bd) - 16c^2e^2(81a^2e^2 + 28abde + b^2d^2))}{12(-4ac + b^2)^2(ae^2 - bde + cd^2)^4(ex + d)} \end{aligned}$$

command

```
integrate(1/(e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 14.295 Problem number 2407

$$\int \frac{1}{(d + ex)\sqrt{\frac{b^2}{4c} + bx + cx^2}} dx$$

Optimal antiderivative

$$\frac{2(2cx + b) \ln(2cx + b)}{(-be + 2cd)\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}} - \frac{2(2cx + b) \ln(ex + d)}{(-be + 2cd)\sqrt{\frac{b^2}{c} + 4bx + 4cx^2}}$$

command

```
integrate(2/(e*x+d)/(b^2/c+4*b*x+4*c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2c^{\frac{5}{2}} \log(|2cx + b|)}{2c^2d|c|\operatorname{sgn}(2cx + b) - bc|c|\operatorname{esgn}(2cx + b)} - \frac{2c^{\frac{3}{2}}e \log(|xe + d|)}{2cd|c|\operatorname{esgn}(2cx + b) - b|c|e^2\operatorname{sgn}(2cx + b)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 14.296 Problem number 2409

$$\int \frac{1}{(d+ex)\sqrt{\frac{-cd^2+bde}{e^2}+bx+cx^2}} dx$$

Optimal antiderivative

$$\frac{2e\sqrt{-\frac{d(-be+cd)}{e^2}+bx+cx^2}}{(-be+2cd)(ex+d)}$$

command

```
integrate(1/(e*x+d)/((b*d*e-c*d^2)/e^2+b*x+c*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\sqrt{c}xe + \sqrt{c}d - \sqrt{cx^2e^2 - cd^2 + bxe^2 + bde}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 14.297 Problem number 2421

$$\int \frac{1}{x\sqrt{-4+12x-9x^2}} dx$$

Optimal antiderivative

$$\frac{(2-3x)\operatorname{arctanh}(-1+3x)}{\sqrt{-(-2+3x)^2}}$$

command

```
integrate(1/x/(-(-2+3*x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i \log(|3x-2|)}{2 \operatorname{sgn}(-3x+2)} - \frac{i \log(|x|)}{2 \operatorname{sgn}(-3x+2)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 14.298 Problem number 2422

$$\int \frac{1}{x \sqrt{-4 - 12x - 9x^2}} dx$$

Optimal antiderivative

$$-\frac{(2 + 3x) \operatorname{arctanh}(1 + 3x)}{\sqrt{-(2 + 3x)^2}}$$

command

`integrate(1/x/(-(2+3*x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{i \log(|3x + 2|)}{2 \operatorname{sgn}(-3x - 2)} + \frac{i \log(|x|)}{2 \operatorname{sgn}(-3x - 2)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 14.299 Problem number 2425

$$\int \frac{1}{x \sqrt{-a^2 + 2abx - b^2x^2}} dx$$

Optimal antiderivative

$$\frac{(-bx + a) \ln(x)}{a \sqrt{-(bx - a)^2}} - \frac{(-bx + a) \ln(-bx + a)}{a \sqrt{-(bx - a)^2}}$$

command

`integrate(1/x/(-(b*x-a)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i \log(|bx - a|)}{\operatorname{asgn}(-bx + a)} - \frac{i \log(|x|)}{\operatorname{asgn}(-bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

### 14.300 Problem number 2426

$$\int \frac{1}{x \sqrt{-a^2 - 2abx - b^2x^2}} dx$$

Optimal antiderivative

$$\frac{(bx + a) \ln(x)}{a \sqrt{-(bx + a)^2}} - \frac{(bx + a) \ln(bx + a)}{a \sqrt{-(bx + a)^2}}$$

command

```
integrate(1/x/(-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{i \log(|bx + a|)}{\operatorname{asgn}(-bx - a)} + \frac{i \log(|x|)}{\operatorname{asgn}(-bx - a)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 15 Test file number 34

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.1_Quadratic/34_1.2.1.3-  
d+e_x^-m-f+g_x-a+b_x+c_x^2-^p
```

### 15.1 Problem number 714

$$\int \frac{x^4(A + Bx)}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(4Ab - 5aB)}{b^6 \sqrt{(bx + a)^2}} - \frac{a^4(Ab - aB)}{2b^6 (bx + a) \sqrt{(bx + a)^2}} - \frac{3a(Ab - 2aB)x(bx + a)}{b^5 \sqrt{(bx + a)^2}} \\ & + \frac{(Ab - 3aB)x^2(bx + a)}{2b^4 \sqrt{(bx + a)^2}} + \frac{Bx^3(bx + a)}{3b^3 \sqrt{(bx + a)^2}} + \frac{2a^2(3Ab - 5aB)(bx + a) \ln(bx + a)}{b^6 \sqrt{(bx + a)^2}} \end{aligned}$$

command

```
integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{2(5Ba^3 - 3Aa^2b)\log(|bx+a|)}{b^6\operatorname{sgn}(bx+a)} - \frac{9Ba^5 - 7Aa^4b + 2(5Ba^4b - 4Aa^3b^2)x}{2(bx+a)^2b^6\operatorname{sgn}(bx+a)} \\ & + \frac{2Bb^6x^3 - 9Bab^5x^2 + 3Ab^6x^2 + 36Ba^2b^4x - 18Aab^5x}{6b^9\operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.2 Problem number 715

$$\int \frac{x^3(A+Bx)}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^2(3Ab-4aB)}{b^5\sqrt{(bx+a)^2}} + \frac{a^3(Ab-aB)}{2b^5(bx+a)\sqrt{(bx+a)^2}} + \frac{(Ab-3aB)x(bx+a)}{b^4\sqrt{(bx+a)^2}} \\ & + \frac{Bx^2(bx+a)}{2b^3\sqrt{(bx+a)^2}} - \frac{3a(Ab-2aB)(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3(2Ba^2 - Aab)\log(|bx+a|)}{b^5\operatorname{sgn}(bx+a)} \\ & + \frac{Bb^3x^2\operatorname{sgn}(bx+a) - 6Bab^2x\operatorname{sgn}(bx+a) + 2Ab^3x\operatorname{sgn}(bx+a)}{2b^6} \\ & + \frac{7Ba^4 - 5Aa^3b + 2(4Ba^3b - 3Aa^2b^2)x}{2(bx+a)^2b^5\operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.3 Problem number 716

$$\int \frac{x^2(A + Bx)}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{a(2Ab - 3aB)}{b^4 \sqrt{(bx + a)^2}} - \frac{a^2(Ab - aB)}{2b^4 (bx + a) \sqrt{(bx + a)^2}} + \frac{Bx(bx + a)}{b^3 \sqrt{(bx + a)^2}} + \frac{(Ab - 3aB)(bx + a) \ln(bx + a)}{b^4 \sqrt{(bx + a)^2}}$$

command

`integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Bx}{b^3 \operatorname{sgn}(bx + a)} - \frac{(3Ba - Ab) \log(|bx + a|)}{b^4 \operatorname{sgn}(bx + a)} - \frac{5Ba^3 - 3Aa^2b + 2(3Ba^2b - 2Aab^2)x}{2(bx + a)^2 b^4 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.4 Problem number 717

$$\int \frac{x(A + Bx)}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{-Ab + 2aB}{b^3 \sqrt{(bx + a)^2}} + \frac{a(Ab - aB)}{2b^3 (bx + a) \sqrt{(bx + a)^2}} + \frac{B(bx + a) \ln(bx + a)}{b^3 \sqrt{(bx + a)^2}}$$

command

`integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{B \log(|bx + a|)}{b^3 \operatorname{sgn}(bx + a)} + \frac{2(2Ba - Ab)x + \frac{3Ba^2 - Aab}{b}}{2(bx + a)^2 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.5 Problem number 718

$$\int \frac{A + Bx}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{B}{b^2 \sqrt{(bx+a)^2}} + \frac{-Ab + aB}{2b^2 (bx+a) \sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2Bbx + Ba + Ab}{2(bx+a)^2 b^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.6 Problem number 719

$$\int \frac{A + Bx}{x(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{A}{a^2 \sqrt{(bx+a)^2}} + \frac{Ab - aB}{2ab(bx+a) \sqrt{(bx+a)^2}} + \frac{A(bx+a) \ln(x)}{a^3 \sqrt{(bx+a)^2}} - \frac{A(bx+a) \ln(bx+a)}{a^3 \sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{A \log(|bx+a|)}{a^3 \operatorname{sgn}(bx+a)} + \frac{A \log(|x|)}{a^3 \operatorname{sgn}(bx+a)} + \frac{2Aab^2x - Ba^3 + 3Aa^2b}{2(bx+a)^2 a^3 b \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 15.7 Problem number 720

$$\int \frac{A + Bx}{x^2 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{-2Ab + aB}{a^3 \sqrt{(bx + a)^2}} + \frac{-Ab + aB}{2a^2 (bx + a) \sqrt{(bx + a)^2}} - \frac{A(bx + a)}{a^3 x \sqrt{(bx + a)^2}} - \frac{(3Ab - aB)(bx + a) \ln(x)}{a^4 \sqrt{(bx + a)^2}} + \frac{(3Ab - aB)(bx + a) \ln(bx + a)}{a^4 \sqrt{(bx + a)^2}}$$

command

`integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Ba - 3Ab) \log(|x|)}{a^4 \operatorname{sgn}(bx + a)} - \frac{(Bab - 3Ab^2) \log(|bx + a|)}{a^4 b \operatorname{sgn}(bx + a)} - \frac{2Aa^3 - 2(Ba^2b - 3Aab^2)x^2 - 3(Ba^3 - 3Aa^2b)x}{2(bx + a)^2 a^4 x \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.8 Problem number 721

$$\int \frac{A + Bx}{x^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{b(3Ab - 2aB)}{a^4 \sqrt{(bx + a)^2}} + \frac{b(Ab - aB)}{2a^3 (bx + a) \sqrt{(bx + a)^2}} - \frac{A(bx + a)}{2a^3 x^2 \sqrt{(bx + a)^2}} + \frac{(3Ab - aB)(bx + a)}{a^4 x \sqrt{(bx + a)^2}} + \frac{3b(2Ab - aB)(bx + a) \ln(x)}{a^5 \sqrt{(bx + a)^2}} - \frac{3b(2Ab - aB)(bx + a) \ln(bx + a)}{a^5 \sqrt{(bx + a)^2}}$$

command

`integrate((B*x+A)/x^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3(Bab - 2Ab^2) \log(|x|)}{a^5 \operatorname{sgn}(bx + a)} + \frac{3(Bab^2 - 2Ab^3) \log(|bx + a|)}{a^5 b \operatorname{sgn}(bx + a)}$$

$$-\frac{6Bab^2x^3 - 12Ab^3x^3 + 9Ba^2bx^2 - 18Aab^2x^2 + 2Ba^3x - 4Aa^2bx + Aa^3}{2(bx^2 + ax)^2 a^4 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.9 Problem number 722

$$\int \frac{x^4(A + Bx)}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2a(2Ab - 5aB)}{b^6 \sqrt{(bx + a)^2}} - \frac{a^4(Ab - aB)}{4b^6 (bx + a)^3 \sqrt{(bx + a)^2}} + \frac{a^3(4Ab - 5aB)}{3b^6 (bx + a)^2 \sqrt{(bx + a)^2}}$$

$$- \frac{a^2(3Ab - 5aB)}{b^6 (bx + a) \sqrt{(bx + a)^2}} + \frac{Bx(bx + a)}{b^5 \sqrt{(bx + a)^2}} + \frac{(Ab - 5aB)(bx + a) \ln(bx + a)}{b^6 \sqrt{(bx + a)^2}}$$

command

`integrate(x^4*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Bx}{b^5 \operatorname{sgn}(bx + a)} - \frac{(5Ba - Ab) \log(|bx + a|)}{b^6 \operatorname{sgn}(bx + a)}$$

$$-\frac{77Ba^5 - 25Aa^4b + 24(5Ba^2b^3 - 2Aab^4)x^3 + 12(25Ba^3b^2 - 9Aa^2b^3)x^2 + 4(65Ba^4b - 22Aa^3b^2)x}{12(bx + a)^4 b^6 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.10 Problem number 723

$$\int \frac{x^3(A + Bx)}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{3aB}{b^5 \sqrt{(bx+a)^2}} + \frac{(Ab-aB)x^4}{4ab(bx+a)^3 \sqrt{(bx+a)^2}} + \frac{a^3B}{3b^5(bx+a)^2 \sqrt{(bx+a)^2}} - \frac{3a^2B}{2b^5(bx+a) \sqrt{(bx+a)^2}} + \frac{B(bx+a) \ln(bx+a)}{b^5 \sqrt{(bx+a)^2}}$$

command

`integrate(x^3*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{B \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)} + \frac{12(4Bab^2 - Ab^3)x^3 + 18(6Ba^2b - Aab^2)x^2 + 4(22Ba^3 - 3Aa^2b)x + \frac{25Ba^4 - 3Aa^3b}{b}}{12(bx+a)^4 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.11 Problem number 724

$$\int \frac{x^2(A+Bx)}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{Ax^3}{3a^2(b^2x^2+2abx+a^2)^{3/2}} - \frac{(Ab-aB)x^4}{4a^2(bx+a)^3 \sqrt{(bx+a)^2}}$$

command

`integrate(x^2*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12Bb^3x^3 + 18Bab^2x^2 + 6Ab^3x^2 + 12Ba^2bx + 4Aab^2x + 3Ba^3 + Aa^2b}{12(bx+a)^4 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.12 Problem number 725

$$\int \frac{x(A + Bx)}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{a(Ab - aB)}{4b^3 (bx + a)^3 \sqrt{(bx + a)^2}} + \frac{-Ab + 2aB}{3b^3 (bx + a)^2 \sqrt{(bx + a)^2}} - \frac{B}{2b^3 (bx + a) \sqrt{(bx + a)^2}}$$

command

`integrate(x*(B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{6Bb^2x^2 + 4Babx + 4Ab^2x + Ba^2 + Aab}{12(bx + a)^4 b^3 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.13 Problem number 726

$$\int \frac{A + Bx}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{B}{3b^2 (b^2x^2 + 2abx + a^2)^{3/2}} + \frac{-Ab + aB}{4b^2 (bx + a) (b^2x^2 + 2abx + a^2)^{3/2}}$$

command

`integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4Bbx + Ba + 3Ab}{12(bx + a)^4 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.14 Problem number 727

$$\int \frac{A + Bx}{x (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{A}{a^4 \sqrt{(bx+a)^2}} + \frac{Ab - aB}{4ab (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{A}{3a^2 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & + \frac{A}{2a^3 (bx+a) \sqrt{(bx+a)^2}} + \frac{A(bx+a) \ln(x)}{a^5 \sqrt{(bx+a)^2}} - \frac{A(bx+a) \ln(bx+a)}{a^5 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((B*x+A)/x/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{A \log(|bx+a|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{A \log(|x|)}{a^5 \operatorname{sgn}(bx+a)} + \frac{12 Aab^4x^3 + 42 Aa^2b^3x^2 + 52 Aa^3b^2x - 3Ba^5 + 25 Aa^4b}{12 (bx+a)^4 a^5 b \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

`sage0x`

## 15.15 Problem number 728

$$\int \frac{A + Bx}{x^2 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{-4Ab + aB}{a^5 \sqrt{(bx+a)^2}} + \frac{-Ab + aB}{4a^2 (bx+a)^3 \sqrt{(bx+a)^2}} + \frac{-2Ab + aB}{3a^3 (bx+a)^2 \sqrt{(bx+a)^2}} \\ & + \frac{-3Ab + aB}{2a^4 (bx+a) \sqrt{(bx+a)^2}} - \frac{A(bx+a)}{a^5 x \sqrt{(bx+a)^2}} \\ & - \frac{(5Ab - aB)(bx+a) \ln(x)}{a^6 \sqrt{(bx+a)^2}} + \frac{(5Ab - aB)(bx+a) \ln(bx+a)}{a^6 \sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((B*x+A)/x^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Ba - 5Ab) \log(|x|) - (Bab - 5Ab^2) \log(|bx + a|)}{a^6 \operatorname{sgn}(bx + a) - a^6 b \operatorname{sgn}(bx + a)} - \frac{12Aa^5 - 12(Ba^2b^3 - 5Aab^4)x^4 - 42(Ba^3b^2 - 5Aa^2b^3)x^3 - 52(Ba^4b - 5Aa^3b^2)x^2 - 25(Ba^5 - 5Aa^4b)x}{12(bx + a)^4 a^6 x \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.16 Problem number 1458

$$\int \frac{(A + Bx)(d + ex)^{7/2}}{(a - cx^2)^3} dx$$

Optimal antiderivative

$$\frac{(ex + d)^{\frac{5}{2}} (a(Ae + Bd) + (Acd + aBe)x)}{4ac(-cx^2 + a)^2} + \frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}\sqrt{ex + d}}{\sqrt{e\sqrt{a} + d\sqrt{c}}}\right) (e\sqrt{a} + d\sqrt{c})^{\frac{3}{2}} (7aBe(-3e\sqrt{a} + 2d\sqrt{c}) - A(12c^{\frac{3}{2}}d^2 - 18cde\sqrt{a} + 5ae^2\sqrt{c}))}{32a^{\frac{5}{2}}c^{\frac{11}{4}}} + \frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}\sqrt{ex + d}}{\sqrt{-e\sqrt{a} + d\sqrt{c}}}\right) (-e\sqrt{a} + d\sqrt{c})^{\frac{3}{2}} (7aBe(3e\sqrt{a} + 2d\sqrt{c}) - A(12c^{\frac{3}{2}}d^2 + 18cde\sqrt{a} + 5ae^2\sqrt{c}))}{32a^{\frac{5}{2}}c^{\frac{11}{4}}} + \frac{(ae(-5Aae^2 + 7Acd^2 - 14aBde) + (2Acd(-2ae^2 + 3cd^2) - 7aBe(ae^2 + cd^2))x)\sqrt{ex + d}}{16a^2c^2(-cx^2 + a)}$$

command

`integrate((B*x+A)*(e*x+d)^(7/2)/(-c*x^2+a)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(xe+d)^{\frac{7}{2}}Ac^3d^3e - 18(xe+d)^{\frac{5}{2}}Ac^3d^4e + 18(xe+d)^{\frac{3}{2}}Ac^3d^5e - 6\sqrt{xe+d}Ac^3d^6e - 7(xe+d)^{\frac{7}{2}}Bac^2d^2e^2 + 2$$

$$\frac{\left( (6ac^2d^2e - 12\sqrt{ac}c^2d^3 + 13\sqrt{ac}acde^2 - 5a^2ce^3) \sqrt{-c^2d - \sqrt{ac}ce} A|c| + 7(2\sqrt{ac}acd^2e - a^2cde^2 - 3\sqrt{ac}$$

---

 $32a^3c^5$ 

$$\frac{\left( (6ac^2d^2e + 12\sqrt{ac}c^2d^3 - 13\sqrt{ac}acde^2 - 5a^2ce^3) \sqrt{-c^2d + \sqrt{ac}ce} A|c| - 7(2\sqrt{ac}acd^2e + a^2cde^2 - 3\sqrt{ac}$$

---

 $32a^3c^5$ 

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

### 15.17 Problem number 1462

$$\int \frac{A+Bx}{\sqrt{d+ex}(a-cx^2)^3} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}\sqrt{ex+d}}{\sqrt{-e\sqrt{a}+d\sqrt{c}}}\right)\left(aBe(-5e\sqrt{a}+2d\sqrt{c})+3A\left(4c^{\frac{3}{2}}d^2-10cde\sqrt{a}+7ae^2\sqrt{c}\right)\right)}{32a^{\frac{5}{2}}c^{\frac{3}{4}}(-e\sqrt{a}+d\sqrt{c})^{\frac{5}{2}}}$$

$$+ \frac{\operatorname{arctanh}\left(\frac{c^{\frac{1}{4}}\sqrt{ex+d}}{\sqrt{e\sqrt{a}+d\sqrt{c}}}\right)\left(aBe(5e\sqrt{a}+2d\sqrt{c})+3A\left(4c^{\frac{3}{2}}d^2+10cde\sqrt{a}+7ae^2\sqrt{c}\right)\right)}{32a^{\frac{5}{2}}c^{\frac{3}{4}}(e\sqrt{a}+d\sqrt{c})^{\frac{5}{2}}}$$

$$+ \frac{(a(-Ae+Bd)+(Acd-aBe)x)\sqrt{ex+d}}{4a(-ae^2+cd^2)(-cx^2+a)^2}$$

$$- \frac{(ae(-7Aae^2+Ac d^2+6aBde)-(6Acd(-2ae^2+cd^2)+aBe(5ae^2+cd^2))x)\sqrt{ex+d}}{16a^2(-ae^2+cd^2)^2(-cx^2+a)}$$

command

`integrate((B*x+A)/(e*x+d)^(1/2)/(-c*x^2+a)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 15.18 Problem number 1767

$$\int \frac{(A+Bx)(d+ex)^4}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ae+bd)^3(4Abe-5aBe+Bbd)}{b^6\sqrt{(bx+a)^2}} - \frac{(Ab-aB)(-ae+bd)^4}{2b^6(bx+a)\sqrt{(bx+a)^2}} \\ & + \frac{e^2(6a^2B e^2-3abe(Ae+4Bd)+2b^2d(2Ae+3Bd))x(bx+a)}{b^5\sqrt{(bx+a)^2}} \\ & + \frac{e^3(Abe-3aBe+4Bbd)x^2(bx+a)}{2b^4\sqrt{(bx+a)^2}} + \frac{B e^4x^3(bx+a)}{3b^3\sqrt{(bx+a)^2}} \\ & + \frac{2e(-ae+bd)^2(3Abe-5aBe+2Bbd)(bx+a)\ln(bx+a)}{b^6\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(2Bb^3d^3e-9Bab^2d^2e^2+3Ab^3d^2e^2+12Ba^2bde^3-6Aab^2de^3-5Ba^3e^4+3Aa^2be^4)\log(|bx+a|)}{b^6\operatorname{sgn}(bx+a)} \\ & - \frac{Bab^4d^4+Ab^5d^4-12Ba^2b^3d^3e+4Aab^4d^3e+30Ba^3b^2d^2e^2-18Aa^2b^3d^2e^2-28Ba^4bde^3+20Aa^3b^2de^3+9Ba^4e^4}{6b^9\operatorname{sgn}(bx+a)} \\ & + \frac{2Bb^6x^3e^4+12Bb^6dx^2e^3+36Bb^6d^2xe^2-9Bab^5x^2e^4+3Ab^6x^2e^4-72Bab^5dxe^3+24Ab^6dxe^3+36Ba^2b^4xe^4}{6b^9\operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 15.19 Problem number 1768

$$\int \frac{(A+Bx)(d+ex)^3}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ae+bd)^2(3Abe-4aBe+Bbd)}{b^5\sqrt{(bx+a)^2}} - \frac{(Ab-aB)(-ae+bd)^3}{2b^5(bx+a)\sqrt{(bx+a)^2}} \\ & + \frac{e^2(Abe-3aBe+3Bbd)x(bx+a)}{b^4\sqrt{(bx+a)^2}} + \frac{Be^3x^2(bx+a)}{2b^3\sqrt{(bx+a)^2}} \\ & + \frac{3e(-ae+bd)(Abe-2aBe+Bbd)(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((B*x+A)*(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3(Bb^2d^2e-3Babde^2+Ab^2de^2+2Ba^2e^3-Aabe^3)\log(|bx+a|)}{b^5\operatorname{sgn}(bx+a)} \\ & + \frac{Bb^3x^2e^3\operatorname{sgn}(bx+a)+6Bb^3dx^2e^2\operatorname{sgn}(bx+a)-6Bab^2xe^3\operatorname{sgn}(bx+a)+2Ab^3xe^3\operatorname{sgn}(bx+a)}{2b^6} \\ & - \frac{Bab^3d^3+Ab^4d^3-9Ba^2b^2d^2e+3Aab^3d^2e+15Ba^3bde^2-9Aa^2b^2de^2-7Ba^4e^3+5Aa^3be^3+2(Bb^4d^3-6Ba^2b^2d^2e+3Aab^3d^2e+15Ba^3bde^2-9Aa^2b^2de^2-7Ba^4e^3+5Aa^3be^3)}{2(bx+a)^2b^5\operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

`sage_0x`

## 15.20 Problem number 1769

$$\int \frac{(A+Bx)(d+ex)^2}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ae+bd)(2Abe-3aBe+Bbd)}{b^4\sqrt{(bx+a)^2}} - \frac{(Ab-aB)(-ae+bd)^2}{2b^4(bx+a)\sqrt{(bx+a)^2}} \\ & + \frac{Be^2x(bx+a)}{b^3\sqrt{(bx+a)^2}} + \frac{e(Abe-3aBe+2Bbd)(bx+a)\ln(bx+a)}{b^4\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Bxe^2}{b^3 \operatorname{sgn}(bx+a)} + \frac{(2Bbde - 3Bae^2 + Abe^2) \log(|bx+a|)}{b^4 \operatorname{sgn}(bx+a)}$$


---


$$\frac{Bab^2d^2 + Ab^3d^2 - 6Ba^2bde + 2Aab^2de + 5Ba^3e^2 - 3Aa^2be^2 + 2(Bb^3d^2 - 4Bab^2de + 2Ab^3de + 3Ba^2be^2 - 2Aa^2e^2)}{2(bx+a)^2 b^4 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.21 Problem number 1770

$$\int \frac{(A+Bx)(d+ex)}{(a^2+2abx+b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{-Abe + 2aBe - Bbd}{b^3 \sqrt{(bx+a)^2}} - \frac{(Ab - aB)(-ae + bd)}{2b^3(bx+a) \sqrt{(bx+a)^2}} + \frac{Be(bx+a) \ln(bx+a)}{b^3 \sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)*(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Be \log(|bx+a|)}{b^3 \operatorname{sgn}(bx+a)} - \frac{2(Bbd - 2Bae + Abe)x + \frac{Babd + Ab^2d - 3Ba^2e + Aabe}{b}}{2(bx+a)^2 b^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.22 Problem number 1771

$$\int \frac{A + Bx}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{B}{b^2 \sqrt{(bx+a)^2}} + \frac{-Ab + aB}{2b^2 (bx+a) \sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2Bbx + Ba + Ab}{2(bx+a)^2 b^2 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.23 Problem number 1772

$$\int \frac{A + Bx}{(d + ex)(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{Ae - Bd}{(-ae + bd)^2 \sqrt{(bx+a)^2}} + \frac{-Ab + aB}{2b(-ae + bd)(bx+a) \sqrt{(bx+a)^2}} \\ & - \frac{e(-Ae + Bd)(bx+a) \ln(bx+a)}{(-ae + bd)^3 \sqrt{(bx+a)^2}} + \frac{e(-Ae + Bd)(bx+a) \ln(ex+d)}{(-ae + bd)^3 \sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{(Bbde - Abe^2) \log(|bx+a|)}{b^4 d^3 \operatorname{sgn}(bx+a) - 3ab^3 d^2 e \operatorname{sgn}(bx+a) + 3a^2 b^2 d e^2 \operatorname{sgn}(bx+a) - a^3 b e^3 \operatorname{sgn}(bx+a)} \\ & + \frac{(Bde^2 - Ae^3) \log(|xe+d|)}{b^3 d^3 e \operatorname{sgn}(bx+a) - 3ab^2 d^2 e^2 \operatorname{sgn}(bx+a) + 3a^2 b d e^3 \operatorname{sgn}(bx+a) - a^3 e^4 \operatorname{sgn}(bx+a)} \\ & - \frac{Bab^2 d^2 + Ab^3 d^2 - 4Aab^2 de - Ba^3 e^2 + 3Aa^2 be^2 + 2(Bb^3 d^2 - Bab^2 de - Ab^3 de + Aab^2 e^2)x}{2(bd - ae)^3 (bx+a)^2 b \operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.24 Problem number 1774

$$\int \frac{A + Bx}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b(-3Abe + 2aBe + Bbd)}{(-ae + bd)^4 \sqrt{(bx + a)^2}} - \frac{b(Ab - aB)}{2(-ae + bd)^3 (bx + a) \sqrt{(bx + a)^2}} \\ & - \frac{e(-Ae + Bd)(bx + a)}{2(-ae + bd)^3 (ex + d)^2 \sqrt{(bx + a)^2}} - \frac{e(-3Abe + aBe + 2Bbd)(bx + a)}{(-ae + bd)^4 (ex + d) \sqrt{(bx + a)^2}} \\ & - \frac{3be(-2Abe + aBe + Bbd)(bx + a) \ln(bx + a)}{(-ae + bd)^5 \sqrt{(bx + a)^2}} \\ & + \frac{3be(-2Abe + aBe + Bbd)(bx + a) \ln(ex + d)}{(-ae + bd)^5 \sqrt{(bx + a)^2}} \end{aligned}$$

command

`integrate((B*x+A)/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3(Bb^3de + Bab^2e^2 - 2Ab^3e^2) \log(|bx + a|)}{b^6d^5\operatorname{sgn}(bx + a) - 5ab^5d^4e\operatorname{sgn}(bx + a) + 10a^2b^4d^3e^2\operatorname{sgn}(bx + a) - 10a^3b^3d^2e^3\operatorname{sgn}(bx + a) + 5a^4b^2de^4\operatorname{sgn}(bx + a)} \\ & + \frac{3(Bb^2de^2 + Babe^3 - 2Ab^2e^3) \log(|xe + d|)}{b^5d^5e\operatorname{sgn}(bx + a) - 5ab^4d^4e^2\operatorname{sgn}(bx + a) + 10a^2b^3d^3e^3\operatorname{sgn}(bx + a) - 10a^3b^2d^2e^4\operatorname{sgn}(bx + a) + 5a^4bde^5\operatorname{sgn}(bx + a)} \\ & - \frac{6Bb^3dx^3e^2 + 9Bb^3d^2x^2e + 2Bb^3d^3x + 6Bab^2x^3e^3 - 12Ab^3x^3e^3 + 18Bab^2dx^2e^2 - 18Ab^3dx^2e^2 + 16Bab^2d^2xe^2}{2(b^4d^4\operatorname{sgn}(bx + a) - 4ab^3d^3e\operatorname{sgn}(bx + a))} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

`sage_0x`

## 15.25 Problem number 1775

$$\int \frac{(A + Bx)(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{10e^2(-ae + bd)^2 (Abe - 2aBe + Bbd)}{b^7 \sqrt{(bx + a)^2}} - \frac{(Ab - aB)(-ae + bd)^5}{4b^7 (bx + a)^3 \sqrt{(bx + a)^2}} \\ & - \frac{(-ae + bd)^4 (5Abe - 6aBe + Bbd)}{3b^7 (bx + a)^2 \sqrt{(bx + a)^2}} - \frac{5e(-ae + bd)^3 (2Abe - 3aBe + Bbd)}{2b^7 (bx + a) \sqrt{(bx + a)^2}} \\ & + \frac{e^4 (Abe - 5aBe + 5Bbd) x (bx + a)}{b^6 \sqrt{(bx + a)^2}} + \frac{B e^5 x^2 (bx + a)}{2b^5 \sqrt{(bx + a)^2}} \\ & + \frac{5e^3(-ae + bd) (Abe - 3aBe + 2Bbd) (bx + a) \ln (bx + a)}{b^7 \sqrt{(bx + a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5 (2 B b^2 d^2 e^3 - 5 B a b d e^4 + A b^2 d e^4 + 3 B a^2 e^5 - A a b e^5) \log (|b x + a|)}{b^7 \operatorname{sgn}(b x + a)} \\ & + \frac{B b^5 x^2 e^5 \operatorname{sgn}(b x + a) + 10 B b^5 d x e^4 \operatorname{sgn}(b x + a) - 10 B a b^4 x e^5 \operatorname{sgn}(b x + a) + 2 A b^5 x e^5 \operatorname{sgn}(b x + a)}{2 b^{10}} \\ & - \frac{B a b^5 d^5 + 3 A b^6 d^5 + 5 B a^2 b^4 d^4 e + 5 A a b^5 d^4 e + 30 B a^3 b^3 d^3 e^2 + 10 A a^2 b^4 d^3 e^2 - 250 B a^4 b^2 d^2 e^3 + 30 A a^3 b^3 d^2 e^3 + \dots}{\dots} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.26 Problem number 1776

$$\int \frac{(A + Bx)(d + ex)^4}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2e^2(-ae + bd) (2Abe - 5aBe + 3Bbd)}{b^6 \sqrt{(bx + a)^2}} - \frac{(Ab - aB)(-ae + bd)^4}{4b^6 (bx + a)^3 \sqrt{(bx + a)^2}} \\ & - \frac{(-ae + bd)^3 (4Abe - 5aBe + Bbd)}{3b^6 (bx + a)^2 \sqrt{(bx + a)^2}} - \frac{e(-ae + bd)^2 (3Abe - 5aBe + 2Bbd)}{b^6 (bx + a) \sqrt{(bx + a)^2}} \\ & + \frac{B e^4 x (bx + a)}{b^5 \sqrt{(bx + a)^2}} + \frac{e^3 (Abe - 5aBe + 4Bbd) (bx + a) \ln (bx + a)}{b^6 \sqrt{(bx + a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Bxe^4}{b^5 \operatorname{sgn}(bx+a)} + \frac{(4Bbde^3 - 5Bae^4 + Abe^4) \log(|bx+a|)}{b^6 \operatorname{sgn}(bx+a)}$$


---


$$Bab^4d^4 + 3Ab^5d^4 + 4Ba^2b^3d^3e + 4Aab^4d^3e + 18Ba^3b^2d^2e^2 + 6Aa^2b^3d^2e^2 - 100Ba^4bde^3 + 12Aa^3b^2de^3 + 77A$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.27 Problem number 1777

$$\int \frac{(A+Bx)(d+ex)^3}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3Be^2(-ae+bd)}{b^5\sqrt{(bx+a)^2}} - \frac{B(-ae+bd)^3}{3b^5(bx+a)^2\sqrt{(bx+a)^2}} - \frac{3Be(-ae+bd)^2}{2b^5(bx+a)\sqrt{(bx+a)^2}} \\ & - \frac{(Ab-aB)(ex+d)^4}{4b(-ae+bd)(bx+a)^3\sqrt{(bx+a)^2}} + \frac{Be^3(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((B*x+A)*(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{Be^3 \log(|bx+a|)}{b^5 \operatorname{sgn}(bx+a)}$$


---


$$12(3Bb^3de^2 - 4Bab^2e^3 + Ab^3e^3)x^3 + 18(Bb^3d^2e + 3Bab^2de^2 + Ab^3de^2 - 6Ba^2be^3 + Aab^2e^3)x^2 + 4(Bb^3d^3 +$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.28 Problem number 1778

$$\int \frac{(A+Bx)(d+ex)^2}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(-Ae+Bd)(ex+d)^3}{3(-ae+bd)^2(b^2x^2+2abx+a^2)^{3/2}} - \frac{(Ab-aB)(ex+d)^4}{4(-ae+bd)^2(bx+a)^3\sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)*(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12Bb^3x^3e^2 + 12Bb^3dx^2e + 4Bb^3d^2x + 18Bab^2x^2e^2 + 6Ab^3x^2e^2 + 8Bab^2dxe + 8Ab^3dxe + Bab^2d^2 + 3Ab^3d^2}{12(bx+a)^4b^4\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.29 Problem number 1779

$$\int \frac{(A+Bx)(d+ex)}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(Ab-aB)(-ae+bd)}{4b^3(bx+a)^3\sqrt{(bx+a)^2}} + \frac{-Abe+2aBe-Bbd}{3b^3(bx+a)^2\sqrt{(bx+a)^2}} - \frac{Be}{2b^3(bx+a)\sqrt{(bx+a)^2}}$$

command

```
integrate((B*x+A)*(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6Bb^2x^2e + 4Bb^2dx + 4Babxe + 4Ab^2xe + Babd + 3Ab^2d + Ba^2e + Aabe}{12(bx+a)^4b^3\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.30 Problem number 1780

$$\int \frac{A + Bx}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{B}{3b^2 (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}} + \frac{-Ab + aB}{4b^2 (bx + a) (b^2x^2 + 2abx + a^2)^{\frac{3}{2}}}$$

command

`integrate((B*x+A)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4Bbx + Ba + 3Ab}{12(bx + a)^4 b^2 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.31 Problem number 1781

$$\int \frac{A + Bx}{(d + ex)(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e^2(-Ae + Bd)}{(-ae + bd)^4 \sqrt{(bx + a)^2}} + \frac{-Ab + aB}{4b(-ae + bd)(bx + a)^3 \sqrt{(bx + a)^2}} \\ & + \frac{Ae - Bd}{3(-ae + bd)^2 (bx + a)^2 \sqrt{(bx + a)^2}} + \frac{e(-Ae + Bd)}{2(-ae + bd)^3 (bx + a) \sqrt{(bx + a)^2}} \\ & - \frac{e^3(-Ae + Bd)(bx + a) \ln(bx + a)}{(-ae + bd)^5 \sqrt{(bx + a)^2}} + \frac{e^3(-Ae + Bd)(bx + a) \ln(ex + d)}{(-ae + bd)^5 \sqrt{(bx + a)^2}} \end{aligned}$$

command

`integrate((B*x+A)/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{(Bbde^3 - Abe^4) \log(|bx + a|)}{b^6 d^5 \operatorname{sgn}(bx + a) - 5 ab^5 d^4 e \operatorname{sgn}(bx + a) + 10 a^2 b^4 d^3 e^2 \operatorname{sgn}(bx + a) - 10 a^3 b^3 d^2 e^3 \operatorname{sgn}(bx + a) + 5 a^4 b^2 d e^4 \operatorname{sgn}(bx + a) - 5 a^5 d e^5 \operatorname{sgn}(bx + a)} + \frac{(Bde^4 - Ae^5) \log(|xe + d|)}{b^5 d^5 \operatorname{sgn}(bx + a) - 5 ab^4 d^4 e^2 \operatorname{sgn}(bx + a) + 10 a^2 b^3 d^3 e^3 \operatorname{sgn}(bx + a) - 10 a^3 b^2 d^2 e^4 \operatorname{sgn}(bx + a) + 5 a^4 b d e^5 \operatorname{sgn}(bx + a) - 5 a^5 d e^5 \operatorname{sgn}(bx + a)} + \frac{Bab^4 d^4 + 3 Ab^5 d^4 - 6 Ba^2 b^3 d^3 e - 16 Aab^4 d^3 e + 18 Ba^3 b^2 d^2 e^2 + 36 Aa^2 b^3 d^2 e^2 - 10 Ba^4 b d e^3 - 48 Aa^3 b^2 d e^3 - 3 B a^5 d e^5}{b^6 d^5 \operatorname{sgn}(bx + a) - 5 ab^5 d^4 e \operatorname{sgn}(bx + a) + 10 a^2 b^4 d^3 e^2 \operatorname{sgn}(bx + a) - 10 a^3 b^3 d^2 e^3 \operatorname{sgn}(bx + a) + 5 a^4 b^2 d e^4 \operatorname{sgn}(bx + a) - 5 a^5 d e^5 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.32 Problem number 1783

$$\int \frac{A + Bx}{(d + ex)^3 (a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2b e^2(-5Abe + 2aBe + 3Bbd)}{(-ae + bd)^6 \sqrt{(bx + a)^2}} - \frac{b(Ab - aB)}{4(-ae + bd)^3 (bx + a)^3 \sqrt{(bx + a)^2}} \\ & - \frac{b(-3Abe + 2aBe + Bbd)}{3(-ae + bd)^4 (bx + a)^2 \sqrt{(bx + a)^2}} + \frac{3be(-2Abe + aBe + Bbd)}{2(-ae + bd)^5 (bx + a) \sqrt{(bx + a)^2}} \\ & - \frac{e^3(-Ae + Bd)(bx + a)}{2(-ae + bd)^5 (ex + d)^2 \sqrt{(bx + a)^2}} - \frac{e^3(-5Abe + aBe + 4Bbd)(bx + a)}{(-ae + bd)^6 (ex + d) \sqrt{(bx + a)^2}} \\ & - \frac{5b e^3(-3Abe + aBe + 2Bbd)(bx + a) \ln(bx + a)}{(-ae + bd)^7 \sqrt{(bx + a)^2}} \\ & + \frac{5b e^3(-3Abe + aBe + 2Bbd)(bx + a) \ln(ex + d)}{(-ae + bd)^7 \sqrt{(bx + a)^2}} \end{aligned}$$

command

`integrate((B*x+A)/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.33 Problem number 2029

$$\int \frac{a + bx}{(a^2 + 2abx + b^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{b\sqrt{(bx+a)^2}}$$

command

```
integrate((b*x+a)/(b^2*x^2+2*a*b*x+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{(bx+a)b\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 15.34 Problem number 2033

$$\int \frac{(a + bx)(d + ex)^5}{(a^2 + 2abx + b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ex+d)^5}{3b(b^2x^2+2abx+a^2)^{\frac{3}{2}}} - \frac{20e^2(-ae+bd)^3}{3b^6\sqrt{(bx+a)^2}} - \frac{5e(-ae+bd)^4}{6b^6(bx+a)\sqrt{(bx+a)^2}} \\ & + \frac{5e^4(-3ae+4bd)x(bx+a)}{3b^5\sqrt{(bx+a)^2}} + \frac{5e^5x^2(bx+a)}{6b^4\sqrt{(bx+a)^2}} + \frac{10e^3(-ae+bd)^2(bx+a)\ln(bx+a)}{b^6\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((b*x+a)*(e*x+d)^5/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{10(b^2d^2e^3 - 2abde^4 + a^2e^5)\log(|bx+a|)}{b^6\operatorname{sgn}(bx+a)} \\ & + \frac{b^4x^2e^5\operatorname{sgn}(bx+a) + 10b^4dxe^4\operatorname{sgn}(bx+a) - 8ab^3xe^5\operatorname{sgn}(bx+a)}{2b^8} \\ & - \frac{2b^5d^5 + 5ab^4d^4e + 20a^2b^3d^3e^2 - 110a^3b^2d^2e^3 + 130a^4bde^4 - 47a^5e^5 + 60(b^5d^3e^2 - 3ab^4d^2e^3 + 3a^2b^3de^4 - a^3e^5)}{6(bx+a)^3b^6\operatorname{sgn}(bx+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx+a)(ex+d)^5}{(b^2x^2+2abx+a^2)^{\frac{5}{2}}} dx$$

### 15.35 Problem number 2034

$$\int \frac{(a+bx)(d+ex)^4}{(a^2+2abx+b^2x^2)^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ex+d)^4}{3b(b^2x^2+2abx+a^2)^{\frac{3}{2}}} - \frac{4e^2(-ae+bd)^2}{b^5\sqrt{(bx+a)^2}} - \frac{2e(-ae+bd)^3}{3b^5(bx+a)\sqrt{(bx+a)^2}} \\ & + \frac{4e^4x(bx+a)}{3b^4\sqrt{(bx+a)^2}} + \frac{4e^3(-ae+bd)(bx+a)\ln(bx+a)}{b^5\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((b*x+a)*(e*x+d)^4/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^4}{b^4\operatorname{sgn}(bx+a)} + \frac{4(bde^3 - ae^4)\log(|bx+a|)}{b^5\operatorname{sgn}(bx+a)}$$


---


$$\frac{b^4d^4 + 2ab^3d^3e + 6a^2b^2d^2e^2 - 22a^3bde^3 + 13a^4e^4 + 18(b^4d^2e^2 - 2ab^3de^3 + a^2b^2e^4)x^2 + 6(b^4d^3e + 3ab^3d^2e^2 - 3(bx+a)^3b^5\operatorname{sgn}(bx+a))}{3(bx+a)^3b^5\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx+a)(ex+d)^4}{(b^2x^2+2abx+a^2)^{\frac{5}{2}}} dx$$

### 15.36 Problem number 2035

$$\int \frac{(a+bx)(d+ex)^3}{(a^2+2abx+b^2x^2)^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ex+d)^3}{3b(b^2x^2+2abx+a^2)^{\frac{3}{2}}} - \frac{2e^2(-ae+bd)}{b^4\sqrt{(bx+a)^2}} - \frac{e(-ae+bd)^2}{2b^4(bx+a)\sqrt{(bx+a)^2}} + \frac{e^3(bx+a)\ln(bx+a)}{b^4\sqrt{(bx+a)^2}} \end{aligned}$$

command

```
integrate((b*x+a)*(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^3 \log(|bx + a|)}{b^4 \operatorname{sgn}(bx + a)} - \frac{18(b^2 de^2 - abe^3)x^2 + 9(b^2 d^2 e + 2abde^2 - 3a^2 e^3)x + \frac{2b^3 d^3 + 3ab^2 d^2 e + 6a^2 bde^2 - 11a^3 e^3}{b}}{6(bx + a)^3 b^3 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)(ex + d)^3}{(b^2 x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

### 15.37 Problem number 2036

$$\int \frac{(a + bx)(d + ex)^2}{(a^2 + 2abx + b^2 x^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(ex + d)^3}{3(-ae + bd)(b^2 x^2 + 2abx + a^2)^{\frac{3}{2}}}$$

command

```
integrate((b*x+a)*(e*x+d)^2/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3b^2 x^2 e^2 + 3b^2 dx e + b^2 d^2 + 3abx e^2 + abde + a^2 e^2}{3(bx + a)^3 b^3 \operatorname{sgn}(bx + a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx + a)(ex + d)^2}{(b^2 x^2 + 2abx + a^2)^{\frac{5}{2}}} dx$$

### 15.38 Problem number 2037

$$\int \frac{(a+bx)(d+ex)}{(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{-ex-d}{3b(b^2x^2+2abx+a^2)^{3/2}} - \frac{e}{6b^2(bx+a)\sqrt{(bx+a)^2}}$$

command

`integrate((b*x+a)*(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3bxe+2bd+ae}{6(bx+a)^3b^2\operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(bx+a)(ex+d)}{(b^2x^2+2abx+a^2)^{5/2}} dx$$

### 15.39 Problem number 2039

$$\int \frac{a+bx}{(d+ex)(a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e^2}{(-ae+bd)^3\sqrt{(bx+a)^2}} - \frac{1}{3(-ae+bd)(bx+a)^2\sqrt{(bx+a)^2}} \\ & + \frac{e}{2(-ae+bd)^2(bx+a)\sqrt{(bx+a)^2}} - \frac{e^3(bx+a)\ln(bx+a)}{(-ae+bd)^4\sqrt{(bx+a)^2}} + \frac{e^3(bx+a)\ln(ex+d)}{(-ae+bd)^4\sqrt{(bx+a)^2}} \end{aligned}$$

command

`integrate((b*x+a)/(e*x+d)/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{be^3 \log(|bx+a|)}{b^5 d^4 \operatorname{sgn}(bx+a) - 4ab^4 d^3 e \operatorname{sgn}(bx+a) + 6a^2 b^3 d^2 e^2 \operatorname{sgn}(bx+a) - 4a^3 b^2 d e^3 \operatorname{sgn}(bx+a) + a^4 b e^4 \operatorname{sgn}(bx+a)}$$

$$+ \frac{e^4 \log(|xe+d|)}{b^4 d^4 \operatorname{sgn}(bx+a) - 4ab^3 d^3 e^2 \operatorname{sgn}(bx+a) + 6a^2 b^2 d^2 e^3 \operatorname{sgn}(bx+a) - 4a^3 b d e^4 \operatorname{sgn}(bx+a) + a^4 e^5 \operatorname{sgn}(bx+a)}$$

$$\frac{2b^3 d^3 - 9ab^2 d^2 e + 18a^2 b d e^2 - 11a^3 e^3 + 6(b^3 d e^2 - ab^2 e^3)x^2 - 3(b^3 d^2 e - 6ab^2 d e^2 + 5a^2 b e^3)x}{6(bd-ae)^4 (bx+a)^3 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx+a}{(b^2x^2+2abx+a^2)^{\frac{5}{2}}(ex+d)} dx$$

### 15.40 Problem number 2041

$$\int \frac{a+bx}{(d+ex)^3 (a^2+2abx+b^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{6b^2e^2}{(-ae+bd)^5 \sqrt{(bx+a)^2}} - \frac{b^2}{3(-ae+bd)^3 (bx+a)^2 \sqrt{(bx+a)^2}}$$

$$+ \frac{3b^2e}{2(-ae+bd)^4 (bx+a) \sqrt{(bx+a)^2}}$$

$$- \frac{e^3(bx+a)}{2(-ae+bd)^4 (ex+d)^2 \sqrt{(bx+a)^2}} - \frac{4be^3(bx+a)}{(-ae+bd)^5 (ex+d) \sqrt{(bx+a)^2}}$$

$$- \frac{10b^2e^3(bx+a) \ln(bx+a)}{(-ae+bd)^6 \sqrt{(bx+a)^2}} + \frac{10b^2e^3(bx+a) \ln(ex+d)}{(-ae+bd)^6 \sqrt{(bx+a)^2}}$$

command

`integrate((b*x+a)/(e*x+d)^3/(b^2*x^2+2*a*b*x+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10b^3e^3 \log(|bx+a|)}{b^7 d^6 \operatorname{sgn}(bx+a) - 6ab^6 d^5 e \operatorname{sgn}(bx+a) + 15a^2 b^5 d^4 e^2 \operatorname{sgn}(bx+a) - 20a^3 b^4 d^3 e^3 \operatorname{sgn}(bx+a) + 15a^4 b^3 d^2 e^4 \operatorname{sgn}(bx+a)}$$

$$+ \frac{10b^2e^4 \log(|xe+d|)}{b^6 d^6 \operatorname{sgn}(bx+a) - 6ab^5 d^5 e^2 \operatorname{sgn}(bx+a) + 15a^2 b^4 d^4 e^3 \operatorname{sgn}(bx+a) - 20a^3 b^3 d^3 e^4 \operatorname{sgn}(bx+a) + 15a^4 b^2 d^2 e^5 \operatorname{sgn}(bx+a)}$$

$$\frac{2b^5 d^5 - 15ab^4 d^4 e + 60a^2 b^3 d^3 e^2 - 20a^3 b^2 d^2 e^3 - 30a^4 b d e^4 + 3a^5 e^5 + 60(b^5 d e^4 - ab^4 e^5)x^4 + 30(3b^5 d^2 e^3 + 2ab^4 d e^4 - 5a^2 b^3 d e^5)x^3}{6(bd-ae)^4 (bx+a)^3 \operatorname{sgn}(bx+a)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{bx+a}{(b^2x^2+2abx+a^2)^{\frac{5}{2}}(ex+d)^3} dx$$

## 15.41 Problem number 2175

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{2ce^2(ex + d)} \\ & + \frac{(-be + 2cd)(-beg - 2cdg + 4cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{8c^{\frac{3}{2}}e^2} \\ & + \frac{(-beg - 2cdg + 4cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4ce^2} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{4} \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( 2gxe^{(-1)} - \frac{(4cdge - 4cfe^2 - bge^2)e^{(-3)}}{c} \right) \\ & - \frac{(4c^2d^2g - 8c^2dfe + 4bcfe^2 - b^2ge^2) \sqrt{-c} e^{(-2)} \log\left(\left| -b\sqrt{-c}e - 2\left(\sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}\right) \right|\right)}{8c^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.42 Problem number 2176

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{e^2(-be + 2cd)(ex + d)^2} \\ & - \frac{(beg - 4cdg + 2cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{2e^2\sqrt{c}} \\ & - \frac{(beg - 4cdg + 2cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2(-be + 2cd)} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(4cdg\operatorname{sgn}\left(\frac{1}{xe+d}\right)-2cf\operatorname{esgn}\left(\frac{1}{xe+d}\right)-bg\operatorname{esgn}\left(\frac{1}{xe+d}\right)\right)\arctan\left(\frac{\sqrt{-c+\frac{2cd}{xe+d}-\frac{be}{xe+d}}}{\sqrt{c}}\right)e^{(-3)}}{\sqrt{c}}-2\left(\sqrt{-c}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.43 Problem number 2177

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^3} dx$$

Optimal antiderivative

$$\frac{2(-dg+ef)(d(-be+cd)-be^2x-ce^2x^2)^{\frac{3}{2}}}{3e^2(-be+2cd)(ex+d)^3} - \frac{g\arctan\left(\frac{e(2cx+b)}{2\sqrt{c}\sqrt{d(-be+cd)-be^2x-ce^2x^2}}\right)\sqrt{c}}{e^2} - \frac{2g\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{e^2(ex+d)}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{cge^{(-2)}\log\left(\left|be-2\left(\sqrt{-ce^2}x-\sqrt{-cx^2e^2+cd^2-bxe^2-bde}\right)\sqrt{-c}\right|\right)}{\sqrt{-c}} - \frac{2\left(7c^2d^3g-c^2d^2fe-4bcd^2ge-12\left(\sqrt{-ce^2}x-\sqrt{-cx^2e^2+cd^2-bxe^2-bde}\right)\sqrt{-c}cd^2g+3\left(\sqrt{-ce^2}x-\sqrt{-cx^2e^2+cd^2-bxe^2-bde}\right)\sqrt{-c}\right)}{e^2(ex+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 15.44 Problem number 2178

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{5e^2 (-be + 2cd) (ex + d)^4} \\ & - \frac{2(-5beg + 8cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{15e^2 (-be + 2cd)^2 (ex + d)^3} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 4c^3d^4g + c^3d^3fe - 6bc^2d^3ge - 5 \left( \sqrt{-ce^2} x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \sqrt{-c} c^2d^3g - 5 \left( \sqrt{-ce^2} x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 15.45 Problem number 2179

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{7e^2 (-be + 2cd) (ex + d)^5} \\ & - \frac{2(-7beg + 10cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{35e^2 (-be + 2cd)^2 (ex + d)^4} \\ & - \frac{4c(-7beg + 10cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{105e^2 (-be + 2cd)^3 (ex + d)^3} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 15.46 Problem number 2180

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{9e^2 (-be + 2cd) (ex + d)^6} \\ & - \frac{2(-3beg + 4cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{21e^2 (-be + 2cd)^2 (ex + d)^5} \\ & - \frac{8c(-3beg + 4cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{105e^2 (-be + 2cd)^3 (ex + d)^4} \\ & - \frac{16c^2(-3beg + 4cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{315e^2 (-be + 2cd)^4 (ex + d)^3} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.47 Problem number 2181

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{11e^2 (-be + 2cd) (ex + d)^7} \\ & - \frac{2(-11beg + 14cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{99e^2 (-be + 2cd)^2 (ex + d)^6} \\ & - \frac{4c(-11beg + 14cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{231e^2 (-be + 2cd)^3 (ex + d)^5} \\ & - \frac{16c^2(-11beg + 14cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{1155e^2 (-be + 2cd)^4 (ex + d)^4} \\ & - \frac{32c^3(-11beg + 14cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3465e^2 (-be + 2cd)^5 (ex + d)^3} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.48 Problem number 2182

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{13e^2(-be + 2cd)(ex + d)^8} \\
& - \frac{2(-13beg + 16cdg + 10cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{143e^2(-be + 2cd)^2(ex + d)^7} \\
& + \frac{16c(13beg - 2c(8dg + 5ef))(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{1287e^2(-be + 2cd)^3(ex + d)^6} \\
& - \frac{32c^2(-13beg + 16cdg + 10cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3003e^2(-be + 2cd)^4(ex + d)^5} \\
& + \frac{128c^3(13beg - 2c(8dg + 5ef))(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{15015e^2(-be + 2cd)^5(ex + d)^4} \\
& - \frac{256c^4(-13beg + 16cdg + 10cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{45045e^2(-be + 2cd)^6(ex + d)^3}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.49 Problem number 2186**

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(-3beg - 2cdg + 8cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{24ce^2} - \frac{g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{4ce^2(ex + d)} \\
& + \frac{(-be + 2cd)^3(-3beg - 2cdg + 8cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c}\sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{128c^{\frac{5}{2}}e^2} \\
& + \frac{(-be + 2cd)(-3beg - 2cdg + 8cef)(2cx + b)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{64c^2e}
\end{aligned}$$

command



## 15.51 Problem number 2188

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(beg - 6cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{3/2}}{2e^2 (-be + 2cd) (ex + d)} \\ & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{e^2 (-be + 2cd) (ex + d)^3} \\ & - \frac{3(-be + 2cd) (beg - 6cdg + 4cef) \arctan \left( \frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \right)}{8e^2 \sqrt{c}} \\ & - \frac{3(beg - 6cdg + 4cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{3(12c^2d^2g - 8c^2dfe - 8bcdge + 4bcfe^2 + b^2ge^2)e^{(-2)} \log \left( \left| -be + 2 \left( \sqrt{-ce^2} x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \right| \right)}{8\sqrt{-c}} \\ & - \frac{1}{4} \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( 2cgxe^{(-1)} - \frac{(12c^2dge^2 - 4c^2fe^3 - 5bcge^3)e^{(-4)}}{c} \right) \\ & - \frac{2(4c^2d^3g - 4c^2d^2fe - 4bcd^2ge + 4bcdfe^2 + b^2dge^2 - b^2fe^3)e^{(-2)}}{\sqrt{-c}d + \sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 15.52 Problem number 2189

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(3beg - 8cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2 (-be + 2cd) (ex + d)^2} \\ & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3e^2 (-be + 2cd) (ex + d)^4} \\ & + \frac{(3beg - 8cdg + 2cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right) \sqrt{c}}{2e^2} \\ & + \frac{c(3beg - 8cdg + 2cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 (-be + 2cd)} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} cge^{(-2)} \\ & + \frac{(8c^2dg - 2c^2fe - 3bcge)e^{(-2)} \log\left(\left| -be + 2\left(\sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}\right)\sqrt{-c} \right|\right)}{2\sqrt{-c}} \\ & - \frac{2\left(20c^3d^4g - 8c^3d^3fe - 18bc^2d^3ge - 36\left(\sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}\right)\sqrt{-c}c^2d^3g + 12\left(\sqrt{-ce^2}\right)\right)}{\dots} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 15.53 Problem number 2191

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{7e^2 (-be + 2cd) (ex + d)^6} \\ & + \frac{2(7beg - 2c(6dg + ef)) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{35e^2 (-be + 2cd)^2 (ex + d)^5} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.54 Problem number 2192

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{9e^2 (-be + 2cd) (ex + d)^7} \\ & - \frac{2(-9beg + 14cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{63e^2 (-be + 2cd)^2 (ex + d)^6} \\ & - \frac{4c(-9beg + 14cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{315e^2 (-be + 2cd)^3 (ex + d)^5} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.55 Problem number 2193

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^8} dx$$



Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{11e^2(-be + 2cd)(ex + d)^8} \\ & - \frac{2(-11beg + 16cdg + 6cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{99e^2(-be + 2cd)^2(ex + d)^7} \\ & - \frac{8c(-11beg + 16cdg + 6cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{693e^2(-be + 2cd)^3(ex + d)^6} \\ & - \frac{16c^2(-11beg + 16cdg + 6cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3465e^2(-be + 2cd)^4(ex + d)^5} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.56 Problem number 2194

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^9} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{13e^2(-be + 2cd)(ex + d)^9} \\ & - \frac{2(-13beg + 18cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{143e^2(-be + 2cd)^2(ex + d)^8} \\ & - \frac{4c(-13beg + 18cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{429e^2(-be + 2cd)^3(ex + d)^7} \\ & - \frac{16c^2(-13beg + 18cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3003e^2(-be + 2cd)^4(ex + d)^6} \\ & - \frac{32c^3(-13beg + 18cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{15015e^2(-be + 2cd)^5(ex + d)^5} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^9,x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.57 Problem number 2198

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(-be + 2cd) (5beg - 2c(-dg + 6ef)) (2cx + b) (d(-be + cd) - be^2x - ce^2x^2)^{3/2}}{192c^2e} \\ & + \frac{(-5beg - 2cdg + 12cef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{60ce^2} \\ & - \frac{g(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{6ce^2(ex + d)} \\ & - \frac{(-be + 2cd)^5 (5beg - 2c(-dg + 6ef)) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{1024c^{7/2}e^2} \\ & - \frac{(-be + 2cd)^3 (5beg - 2c(-dg + 6ef)) (2cx + b) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{512c^3e} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d),x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{7680} \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10c^2gxe^3 - \frac{(12c^7dge^{10} - 12c^7fe^{11} - 25bc^6ge^{11})e^{(-8)}}{c^5} \right) x - \frac{(1}{\right. \right. \right. \\ & \left. \left. \left. (64c^6d^6g - 384c^6d^5fe + 960bc^5d^4fe^2 - 240b^2c^4d^4ge^2 - 960b^2c^4d^3fe^3 + 320b^3c^3d^3ge^3 + 480b^3c^3d^2fe^4 - 180b \right. \right. \right. \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.58 Problem number 2199

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-3beg - 4cdg + 10cef) (2cx + b) (d(-be + cd) - be^2x - ce^2x^2)^{3/2}}{48ce} \\ & + \frac{(-3beg - 4cdg + 10cef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{15e^2(-be + 2cd)} \\ & + \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{3e^2(-be + 2cd)(ex + d)^2} \\ & + \frac{(-be + 2cd)^4 (-3beg - 4cdg + 10cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{256c^2e^2} \\ & + \frac{(-be + 2cd)^2 (-3beg - 4cdg + 10cef) (2cx + b) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{128c^2e} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.59 Problem number 2200

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{5(-beg - 6cdg + 8cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{24e^2} \\
& + \frac{(-beg - 6cdg + 8cef) (-cex - be + cd) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{4e^2 (-be + 2cd)} \\
& + \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{e^2 (-be + 2cd) (ex + d)^3} \\
& + \frac{5(-be + 2cd)^3 (-beg - 6cdg + 8cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{128c^{\frac{3}{2}}e^2} \\
& + \frac{5(-be + 2cd) (-beg - 6cdg + 8cef) (2cx + b) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{64ce}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{192} \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \left( 2 \left( 4 \left( 6c^2gxe - \frac{(24c^5dge^4 - 8c^5fe^5 - 17bc^4ge^5)e^{(-4)}}{c^3} \right) x + \frac{(180c^5d^2ge^3 - 1}{c^3} \right) \right)$$


---


$$\frac{5(48c^4d^4g - 64c^4d^3fe - 64bc^3d^3ge + 96bc^3d^2fe^2 + 24b^2c^2d^2ge^2 - 48b^2c^2dfe^3 + 8b^3cfe^4 - b^4ge^4) \sqrt{-c} e^{(-2)}}{128c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**15.60 Problem number 2201**

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^4} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{5c(3beg - 10cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{6e^2 (-be + 2cd) (ex + d)} \\
& + \frac{2(3beg - 10cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3e^2 (-be + 2cd) (ex + d)^3} \\
& - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{3e^2 (-be + 2cd) (ex + d)^5} \\
& + \frac{5(-be + 2cd) (3beg - 10cdg + 4cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right) \sqrt{c}}{8e^2} \\
& + \frac{5c(3beg - 10cdg + 4cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.62 Problem number 2203**

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2c(5beg - 12cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2 (-be + 2cd) (ex + d)^2} \\
& + \frac{2(5beg - 12cdg + 2cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{15e^2 (-be + 2cd) (ex + d)^4} \\
& - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{5e^2 (-be + 2cd) (ex + d)^6} \\
& - \frac{c^{\frac{3}{2}}(5beg - 12cdg + 2cef) \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{2e^2} \\
& - \frac{c^2(5beg - 12cdg + 2cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 (-be + 2cd)}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.63 Problem number 2204

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^7} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2cg(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2(ex + d)^3} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{5e^2(ex + d)^5} \\ & - \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{7e^2(-be + 2cd)(ex + d)^7} \\ & - \frac{c^{\frac{5}{2}}g \arctan\left(\frac{e(2cx+b)}{2\sqrt{c}\sqrt{d(-be + cd) - be^2x - ce^2x^2}}\right)}{e^2} \\ & - \frac{2c^2g\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2(ex + d)} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.64 Problem number 2205

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{9e^2 (-be + 2cd) (ex + d)^8} \\ & + \frac{2(9beg - 2c(8dg + ef)) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{63e^2 (-be + 2cd)^2 (ex + d)^7} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.65 Problem number 2206

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^9} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{11e^2 (-be + 2cd) (ex + d)^9} \\ & - \frac{2(-11beg + 18cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{99e^2 (-be + 2cd)^2 (ex + d)^8} \\ & - \frac{4c(-11beg + 18cdg + 4cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{693e^2 (-be + 2cd)^3 (ex + d)^7} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^9,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 15.66 Problem number 2207

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{10}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{13e^2(-be + 2cd)(ex + d)^{10}} \\ & - \frac{2(-13beg + 20cdg + 6cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{143e^2(-be + 2cd)^2(ex + d)^9} \\ & - \frac{8c(-13beg + 20cdg + 6cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{1287e^2(-be + 2cd)^3(ex + d)^8} \\ & - \frac{16c^2(-13beg + 20cdg + 6cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{9009e^2(-be + 2cd)^4(ex + d)^7} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^10,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.67 Problem number 2208

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{11}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{15e^2(-be + 2cd)(ex + d)^{11}} \\ & - \frac{2(-15beg + 22cdg + 8cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{195e^2(-be + 2cd)^2(ex + d)^{10}} \\ & - \frac{4c(-15beg + 22cdg + 8cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{715e^2(-be + 2cd)^3(ex + d)^9} \\ & - \frac{16c^2(-15beg + 22cdg + 8cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{6435e^2(-be + 2cd)^4(ex + d)^8} \\ & - \frac{32c^3(-15beg + 22cdg + 8cef)(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{45045e^2(-be + 2cd)^5(ex + d)^7} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^11,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.68 Problem number 2212

$$\int \frac{f + gx}{(d + ex) \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{g \arctan\left(\frac{e(2cx+b)}{2\sqrt{c} \sqrt{d(-be+cd) - be^2x - ce^2x^2}}\right)}{e^2\sqrt{c}} - \frac{2(-dg+ef) \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{e^2(-be+2cd)(ex+d)}$$

command

```
integrate((g*x+f)/(e*x+d)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{ge^{(-2)} \log\left(\left|be - 2\left(\sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}\right)\sqrt{-c}\right|\right)}{\sqrt{-c}} - \frac{2(dg - fe)e^{(-2)}}{\sqrt{-c}d + \sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 15.69 Problem number 2213

$$\int \frac{f + gx}{(d + ex)^2 \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{2(-dg+ef) \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{3e^2(-be+2cd)(ex+d)^2} - \frac{2(-3beg+4cdg+2cef) \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{3e^2(-be+2cd)^2(ex+d)}$$

command

```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{(4\sqrt{-c}cdg + 2\sqrt{-c}cfe - 3b\sqrt{-c}ge)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{4c^2d^2e - 4bcde^2 + b^2e^3} + \frac{\left(3c\sqrt{-c + \frac{2cd}{xe+d} - \frac{be}{xe+d}} + \left(-c + \frac{2cd}{xe+d} - \frac{be}{xe+d}\right)^{\frac{3}{2}}\right)dge}{2cde - be^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.70 Problem number 2214

$$\int \frac{f + gx}{(d + ex)^3 \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{5e^2(-be + 2cd)(ex + d)^3} \\ & - \frac{2(-5beg + 6cdg + 4cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{15e^2(-be + 2cd)^2(ex + d)^2} \\ & - \frac{4c(-5beg + 6cdg + 4cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{15e^2(-be + 2cd)^3(ex + d)} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^3/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 3c^2d^3g + 2c^2d^2fe + 2bcd^2ge - 15 \left( \sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \sqrt{-c}cd^2g - 10 \left( \sqrt{-ce^2}x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 15.71 Problem number 2215

$$\int \frac{f + gx}{(d + ex)^4 \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{7e^2(-be + 2cd)(ex + d)^4} \\ & - \frac{2(-7beg + 8cdg + 6cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{35e^2(-be + 2cd)^2(ex + d)^3} \\ & - \frac{8c(-7beg + 8cdg + 6cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{105e^2(-be + 2cd)^3(ex + d)^2} \\ & - \frac{16c^2(-7beg + 8cdg + 6cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{105e^2(-be + 2cd)^4(ex + d)} \end{aligned}$$

command

`integrate((g*x+f)/(e*x+d)^4/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 8c^3d^4g + 6c^3d^3fe + 9bc^2d^3ge - 56 \left( \sqrt{-ce^2} x - \sqrt{-cx^2e^2 + cd^2 - bxe^2 - bde} \right) \sqrt{-c} c^2d^3g - 42 \left( \sqrt{-ce^2} x \right. \right.}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 15.72 Problem number 2216

$$\int \frac{f + gx}{(d + ex)^5 \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{9e^2(-be + 2cd)(ex + d)^5} \\ & - \frac{2(-9beg + 10cdg + 8cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{63e^2(-be + 2cd)^2(ex + d)^4} \\ & - \frac{4c(-9beg + 10cdg + 8cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{105e^2(-be + 2cd)^3(ex + d)^3} \\ & - \frac{16c^2(-9beg + 10cdg + 8cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{315e^2(-be + 2cd)^4(ex + d)^2} \\ & - \frac{32c^3(-9beg + 10cdg + 8cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{315e^2(-be + 2cd)^5(ex + d)} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^5/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{315} \left( \frac{16(10\sqrt{-c}c^4dg + 8\sqrt{-c}c^4fe - 9b\sqrt{-c}c^3ge)\operatorname{sgn}\left(\frac{1}{xe+d}\right)}{32c^5d^5e - 80bc^4d^4e^2 + 80b^2c^3d^3e^3 - 40b^3c^2d^2e^4 + 10b^4cde^5 - b^5e^6} + \frac{\left(35\left(c - \frac{2cd}{xe+d} + \frac{be}{xe+d}\right)^4 \sqrt{-c + \frac{2cd}{xe+d}}\right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.73 Problem number 2221

$$\int \frac{f + gx}{(d + ex)^2 (cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8c(-5beg + 4cdg + 6cef)(2cx + b)}{15e(-be + 2cd)^4 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & - \frac{2(-dg + ef)}{5e^2(-be + 2cd)(ex + d)^2 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & - \frac{2(-5beg + 4cdg + 6cef)}{15e^2(-be + 2cd)^2 (ex + d) \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.74 Problem number 2229

$$\int \frac{f + gx}{(d + ex)^2 (cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16c(-7beg + 4cdg + 10cef)(2cx + b)}{105e(-be + 2cd)^4 (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} \\ & - \frac{2(-dg + ef)}{7e^2(-be + 2cd)(ex + d)^2 (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} \\ & - \frac{35e^2(-be + 2cd)^2 (ex + d)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{2(-7beg + 4cdg + 10cef)} \\ & + \frac{128c^2(-7beg + 4cdg + 10cef)(2cx + b)}{105e(-be + 2cd)^6 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^2/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 15.75 Problem number 2231

$$\int (d + ex)^{5/2} (f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{32(-be + 2cd)^3 (-8beg + 5cdg + 11cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3465c^5e^2(ex + d)^{\frac{3}{2}}} \\ & - \frac{2(-8beg + 5cdg + 11cef)(ex + d)^{\frac{3}{2}}(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{99c^2e^2} \\ & - \frac{2g(ex + d)^{\frac{5}{2}}(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{11ce^2} \\ & - \frac{16(-be + 2cd)^2(-8beg + 5cdg + 11cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{1155c^4e^2\sqrt{ex + d}} \\ & - \frac{4(-be + 2cd)(-8beg + 5cdg + 11cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}\sqrt{ex + d}}{231c^3e^2} \end{aligned}$$

command

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (ex + d)^{\frac{5}{2}}(gx + f) dx$$

### 15.76 Problem number 2232

$$\int (d + ex)^{3/2}(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16(-be + 2cd)^2 (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{315c^4e^2 (ex + d)^{\frac{3}{2}}} \\ & - \frac{2g(ex + d)^{\frac{3}{2}} (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{9ce^2} \\ & - \frac{8(-be + 2cd) (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{105c^3e^2 \sqrt{ex + d}} \\ & - \frac{2(-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}} \sqrt{ex + d}}{21c^2e^2} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (ex + d)^{\frac{3}{2}}(gx + f) dx$$

## 15.77 Problem number 2233

$$\int \sqrt{d+ex} (f+gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(-be + 2cd)(-4beg + cdg + 7cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{105c^3e^2(ex + d)^{\frac{3}{2}}} \\ & - \frac{2(-4beg + cdg + 7cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{35c^2e^2\sqrt{ex + d}} \\ & - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}\sqrt{ex + d}}{7ce^2} \end{aligned}$$

command

`integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{105} \left( 7dg \left( \frac{2\sqrt{2cd-be}c^2d^2 + 3\sqrt{2cd-be}bcde - 2\sqrt{2cd-be}b^2e^2}{c^2} + \frac{5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd - 5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd - 5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd}{c^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.78 Problem number 2234

$$\int \frac{(f+gx)\sqrt{cd^2 - bde - be^2x - ce^2x^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$-\frac{2(-2beg - cdg + 5cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{15c^2e^2(ex + d)^{\frac{3}{2}}} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{5ce^2\sqrt{ex + d}}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( g \left( \frac{2\sqrt{2cd-be}c^2d^2 + 3\sqrt{2cd-be}bcde - 2\sqrt{2cd-be}b^2e^2}{c^2} + \frac{5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd - 5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd - 5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd}{c^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 - be^2x + cd^2 - bde}(gx + f)}{\sqrt{ex + d}} dx$$



## 15.79 Problem number 2235

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3ce^2(ex + d)^{\frac{3}{2}}} \\ & - \frac{2(-dg + ef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right) \sqrt{-be + 2cd}}{e^2} \\ & + \frac{2(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 \sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{3} \left( \frac{3(2cd^2g - 2cdf e - bdge + bfe^2) \operatorname{arctan}\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{\sqrt{-2cd + be}} + 3\sqrt{-(xe+d)c + 2cd - be} c^3 dg - \right. \\ & \left. + \frac{2\left(6c^2d^2g \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) - 6c^2df \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) e - 3bcdg \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) e + 3bcf\right)}{\sqrt{-2cd + be}} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (gx + f)}{(ex + d)^{\frac{3}{2}}} dx$$

## 15.80 Problem number 2236

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{e^2 (-be + 2cd) (ex + d)^{\frac{5}{2}}} \\ & + \frac{(2beg - 5cdg + cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right)}{e^2 \sqrt{-be + 2cd}} \\ & - \frac{(2beg - 5cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 (-be + 2cd) \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 2 \sqrt{-(xe+d)c+2cd-be} \, cg + \frac{(5c^2dg-c^2fe-2bcge) \arctan\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{\sqrt{-2cd+be}} + \frac{\sqrt{-(xe+d)c+2cd-be}}{c} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: AttributeError

### 15.81 Problem number 2237

$$\int \frac{(f+gx)\sqrt{cd^2-bde-be^2x-ce^2x^2}}{(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-dg+ef)(d(-be+cd)-be^2x-ce^2x^2)^{\frac{3}{2}}}{2e^2(-be+2cd)(ex+d)^{\frac{7}{2}}} \\ & + \frac{c(-4beg+7cdg+cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{\sqrt{-be+2cd}\sqrt{ex+d}}\right)}{4e^2(-be+2cd)^{\frac{3}{2}}} \\ & - \frac{(-4beg+7cdg+cef)\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{4e^2(-be+2cd)(ex+d)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(7c^3dg+c^3fe-4bc^2ge) \arctan\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{(2cd-be)\sqrt{-2cd+be}} + \frac{14\sqrt{-(xe+d)c+2cd-be} \, c^4d^2g+2\sqrt{-(xe+d)c+2cd-be}}{c^4d^2g+2\sqrt{-(xe+d)c+2cd-be}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2-be^2x+cd^2-bde}(gx+f)}{(ex+d)^{\frac{7}{2}}} dx$$

## 15.82 Problem number 2238

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2 (-be + 2cd) (ex + d)^{\frac{9}{2}}} \\ & + \frac{c^2(-2beg + 3cdg + cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{8e^2 (-be + 2cd)^{\frac{5}{2}}} \\ & - \frac{(-2beg + 3cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2 (-be + 2cd) (ex + d)^{\frac{5}{2}}} \\ & + \frac{c(-2beg + 3cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{8e^2 (-be + 2cd)^2 (ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3(3c^4dg + c^4fe - 2bc^3ge) \operatorname{arctan} \left( \frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{(4c^2d^2 - 4bcde + b^2e^2) \sqrt{-2cd + be}} + 36 \sqrt{-(xe + d)c + 2cd - be} c^6 d^3 g + 12 \sqrt{-(xe + d)c + 2cd - be} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (gx + f)}{(ex + d)^{\frac{9}{2}}} dx$$

## 15.83 Problem number 2239

$$\int \frac{(f + gx) \sqrt{cd^2 - bde - be^2x - ce^2x^2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{4e^2 (-be + 2cd) (ex + d)^{\frac{11}{2}}} \\
& + \frac{c^3(-8beg + 11cdg + 5cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{64e^2 (-be + 2cd)^{\frac{7}{2}}} \\
& - \frac{(-8beg + 11cdg + 5cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{24e^2 (-be + 2cd) (ex + d)^{\frac{7}{2}}} \\
& + \frac{c(-8beg + 11cdg + 5cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{96e^2 (-be + 2cd)^2 (ex + d)^{\frac{5}{2}}} \\
& + \frac{c^2(-8beg + 11cdg + 5cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{64e^2 (-be + 2cd)^3 (ex + d)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3(11c^5dg + 5c^5fe - 8bc^4ge) \operatorname{arctan} \left( \frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{(8c^3d^3 - 12bc^2d^2e + 6b^2cde^2 - b^3e^3) \sqrt{-2cd + be}} \right) + \frac{264 \sqrt{-(xe+d)c + 2cd - be} c^8d^4g + 120 \sqrt{-(xe+d)}}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (gx + f)}{(ex + d)^{\frac{11}{2}}} dx$$

**15.84 Problem number 2240**

$$\int (d + ex)^{5/2} (f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{256(-be + 2cd)^4 (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{45045c^6e^2 (ex + d)^{\frac{5}{2}}} \\
& - \frac{128(-be + 2cd)^3 (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{9009c^5e^2 (ex + d)^{\frac{3}{2}}} \\
& - \frac{2(-2beg + cdg + 3cef) (ex + d)^{\frac{3}{2}} (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{39c^2e^2} \\
& - \frac{2g(ex + d)^{\frac{5}{2}} (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{15c^2e^2} \\
& - \frac{32(-be + 2cd)^2 (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{1287c^4e^2 \sqrt{ex + d}} \\
& - \frac{16(-be + 2cd) (-2beg + cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}} \sqrt{ex + d}}{429c^3e^2}
\end{aligned}$$

command

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.85 Problem number 2241**

$$\int (d + ex)^{3/2} (f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{32(-be + 2cd)^3 (-8beg + 3cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{15015c^5e^2 (ex + d)^{\frac{5}{2}}} \\
& - \frac{16(-be + 2cd)^2 (-8beg + 3cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3003c^4e^2 (ex + d)^{\frac{3}{2}}} \\
& - \frac{2g(ex + d)^{\frac{3}{2}} (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{13c^2e^2} \\
& - \frac{4(-be + 2cd) (-8beg + 3cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{429c^3e^2 \sqrt{ex + d}} \\
& - \frac{2(-8beg + 3cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}} \sqrt{ex + d}}{143c^2e^2}
\end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{3}{2}}(ex + d)^{\frac{3}{2}}(gx + f) dx$$

### 15.86 Problem number 2242

$$\int \sqrt{d+ex} (f+gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16(-be + 2cd)^2 (-6beg + cdg + 11cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3465c^4e^2 (ex + d)^{\frac{5}{2}}} \\ & - \frac{8(-be + 2cd) (-6beg + cdg + 11cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{693c^3e^2 (ex + d)^{\frac{3}{2}}} \\ & - \frac{2(-6beg + cdg + 11cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{99c^2e^2\sqrt{ex + d}} \\ & - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}\sqrt{ex + d}}{11ce^2} \end{aligned}$$

command

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{3}{2}}\sqrt{ex + d} (gx + f) dx$$

## 15.87 Problem number 2243

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(-be + 2cd)(-4beg - cdg + 9cef)(d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{315c^3e^2(ex + d)^{5/2}} \\ & - \frac{2(-4beg - cdg + 9cef)(d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{63c^2e^2(ex + d)^{3/2}} \\ & - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{9ce^2\sqrt{ex + d}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 - be^2x + cd^2 - bde)^{3/2}(gx + f)}{\sqrt{ex + d}} dx$$

## 15.88 Problem number 2244

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(-2beg - 3cdg + 7cef)(d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{35c^2e^2(ex + d)^{5/2}} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{7ce^2(ex + d)^{3/2}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{105} \left( 7cdg \left( \frac{2\sqrt{2cd-be}c^2d^2 + 3\sqrt{2cd-be}bcde - 2\sqrt{2cd-be}b^2e^2}{c^2} + \frac{5(-(xe+d)c + 2cd - be)^{\frac{3}{2}}cd - 5(-}{c^2} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{3}{2}}(gx + f)}{(ex + d)^{\frac{3}{2}}} dx$$

### 15.89 Problem number 2245

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-dg + ef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2(ex + d)^{\frac{3}{2}}} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{5ce^2(ex + d)^{\frac{5}{2}}} \\ & - \frac{2(-be + 2cd)^{\frac{3}{2}}(-dg + ef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd}\sqrt{ex + d}}\right)}{e^2} \\ & + \frac{2(-be + 2cd)(-dg + ef)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2\sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{2}{15} \left( \frac{15(4c^2d^3g - 4c^2d^2fe - 4bcd^2ge + 4bcdf e^2 + b^2dge^2 - b^2fe^3) \operatorname{arctan}\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{\sqrt{-2cd + be}} + 30 \sqrt{-2cd + be} \right) \\ & + \frac{2 \left( 60c^3d^3g \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) - 60c^3d^2f \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) e - 60bc^2d^2g \operatorname{arctan}\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) e \right)}{\sqrt{-2cd + be}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 15.90 Problem number 2246

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(2beg - 7cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{3e^2 (-be + 2cd) (ex + d)^{\frac{3}{2}}} \\ & - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{e^2 (-be + 2cd) (ex + d)^{\frac{7}{2}}} \\ & + \frac{(2beg - 7cdg + 3cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right) \sqrt{-be + 2cd}}{e^2} \\ & - \frac{(2beg - 7cdg + 3cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 \sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 18 \sqrt{-(xe + d)c + 2cd - be} c^2 dg - 6 \sqrt{-(xe + d)c + 2cd - be} c^2 fe - 6 \sqrt{-(xe + d)c + 2cd - be} bcge + 2(-a \right.$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.91 Problem number 2247

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4beg - 9cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{4e^2 (-be + 2cd) (ex + d)^{\frac{5}{2}}} \\ & - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{2e^2 (-be + 2cd) (ex + d)^{\frac{9}{2}}} \\ & - \frac{3c(4beg - 9cdg + cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{4e^2 \sqrt{-be + 2cd}} \\ & + \frac{3c(4beg - 9cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2 (-be + 2cd) \sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 8 \sqrt{-(x+d)c + 2cd - be} c^2 g + \frac{3(9c^3 dg - c^3 fe - 4bc^2 ge) \operatorname{arctan} \left( \frac{\sqrt{-(x+d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{\sqrt{-2cd + be}} + \frac{22 \sqrt{-(x+d)c + be}}{\sqrt{-2cd + be}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

## 15.92 Problem number 2248

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{3/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-6beg + 11cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{12e^2 (-be + 2cd) (ex + d)^{\frac{7}{2}}} \\ & - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{3e^2 (-be + 2cd) (ex + d)^{\frac{11}{2}}} \\ & - \frac{c^2(-6beg + 11cdg + cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{8e^2 (-be + 2cd)^{\frac{3}{2}}} \\ & + \frac{c(-6beg + 11cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{8e^2 (-be + 2cd) (ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3(11c^4dg+c^4fe-6bc^3ge) \arctan\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{(2cd-be)\sqrt{-2cd+be}} + \frac{132\sqrt{-(xe+d)c+2cd-be}c^6d^3g+12\sqrt{-(xe+d)c+}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.93 Problem number 2249

$$\int \frac{(f+gx)(cd^2-bde-be^2x-ce^2x^2)^{3/2}}{(d+ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-8beg+13cdg+3cef)(d(-be+cd)-be^2x-ce^2x^2)^{\frac{3}{2}}}{24e^2(-be+2cd)(ex+d)^{\frac{9}{2}}} \\ & - \frac{(-dg+ef)(d(-be+cd)-be^2x-ce^2x^2)^{\frac{5}{2}}}{4e^2(-be+2cd)(ex+d)^{\frac{13}{2}}} \\ & - \frac{c^3(-8beg+13cdg+3cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{\sqrt{-be+2cd}\sqrt{ex+d}}\right)}{64e^2(-be+2cd)^{\frac{5}{2}}} \\ & + \frac{c(-8beg+13cdg+3cef)\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{32e^2(-be+2cd)(ex+d)^{\frac{5}{2}}} \\ & - \frac{c^2(-8beg+13cdg+3cef)\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{64e^2(-be+2cd)^2(ex+d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{3(13c^5dg+3c^5fe-8bc^4ge) \arctan\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{(4c^2d^2-4bcde+b^2e^2)\sqrt{-2cd+be}} + \frac{312\sqrt{-(xe+d)c+2cd-be}c^8d^4g+72\sqrt{-(xe+d)c-}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.94 Problem number 2250

$$\int (d + ex)^{5/2} (f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{512(-be + 2cd)^5 (-12beg + 5cdg + 19cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{2909907c^7e^2 (ex + d)^{7/2}} \\ & - \frac{256(-be + 2cd)^4 (-12beg + 5cdg + 19cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{415701c^6e^2 (ex + d)^{5/2}} \\ & - \frac{64(-be + 2cd)^3 (-12beg + 5cdg + 19cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{46189c^5e^2 (ex + d)^{3/2}} \\ & - \frac{2(-12beg + 5cdg + 19cef) (ex + d)^{3/2} (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{323c^2e^2} \\ & - \frac{2g(ex + d)^{5/2} (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{19ce^2} \\ & - \frac{32(-be + 2cd)^2 (-12beg + 5cdg + 19cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{12597c^4e^2\sqrt{ex + d}} \\ & - \frac{4(-be + 2cd) (-12beg + 5cdg + 19cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2} \sqrt{ex + d}}{969c^3e^2} \end{aligned}$$

command

```
integrate((e*x+d)^(5/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 - be^2x + cd^2 - bde)^{5/2} (ex + d)^{5/2} (gx + f) dx$$

### 15.95 Problem number 2251

$$\int (d + ex)^{3/2} (f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{256(-be + 2cd)^4 (-10beg + 3cdg + 17cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{765765c^6e^2 (ex + d)^{7/2}} \\ & - \frac{128(-be + 2cd)^3 (-10beg + 3cdg + 17cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{109395c^5e^2 (ex + d)^{5/2}} \\ & - \frac{32(-be + 2cd)^2 (-10beg + 3cdg + 17cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{12155c^4e^2 (ex + d)^{3/2}} \\ & - \frac{2g(ex + d)^{3/2} (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{17ce^2} \\ & - \frac{16(-be + 2cd) (-10beg + 3cdg + 17cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{3315c^3e^2\sqrt{ex + d}} \\ & - \frac{2(-10beg + 3cdg + 17cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2} \sqrt{ex + d}}{255c^2e^2} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 - be^2x + cd^2 - bde)^{5/2} (ex + d)^{3/2} (gx + f) dx$$

### 15.96 Problem number 2252

$$\int \sqrt{d + ex} (f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{32(-be + 2cd)^3 (-8beg + cdg + 15cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{45045c^5e^2 (ex + d)^{\frac{7}{2}}} \\
& - \frac{16(-be + 2cd)^2 (-8beg + cdg + 15cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{6435c^4e^2 (ex + d)^{\frac{5}{2}}} \\
& - \frac{4(-be + 2cd) (-8beg + cdg + 15cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{715c^3e^2 (ex + d)^{\frac{3}{2}}} \\
& - \frac{2(-8beg + cdg + 15cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{195c^2e^2\sqrt{ex + d}} \\
& - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}} \sqrt{ex + d}}{15c^2e^2}
\end{aligned}$$

command

```
integrate((e*x+d)^(1/2)*(g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{5}{2}} \sqrt{ex + d} (gx + f) dx$$

**15.97 Problem number 2253**

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{16(-be + 2cd)^2 (-6beg - cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{9009c^4e^2 (ex + d)^{\frac{7}{2}}} \\
& - \frac{8(-be + 2cd) (-6beg - cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{1287c^3e^2 (ex + d)^{\frac{5}{2}}} \\
& - \frac{2(-6beg - cdg + 13cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{143c^2e^2 (ex + d)^{\frac{3}{2}}} \\
& - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{13c^2e^2\sqrt{ex + d}}
\end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{5}{2}}(gx + f)}{\sqrt{ex + d}} dx$$

### 15.98 Problem number 2254

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(-be + 2cd)(-4beg - 3cdg + 11cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{693c^3e^2(ex + d)^{\frac{7}{2}}} \\ & - \frac{2(-4beg - 3cdg + 11cef)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{99c^2e^2(ex + d)^{\frac{5}{2}}} \\ & - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{11ce^2(ex + d)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(-ce^2x^2 - be^2x + cd^2 - bde)^{\frac{5}{2}}(gx + f)}{(ex + d)^{\frac{3}{2}}} dx$$

### 15.99 Problem number 2255

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-2beg - 5cdg + 9cef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{63c^2e^2 (ex + d)^{7/2}} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{9ce^2 (ex + d)^{5/2}}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 15.100 Problem number 2256

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-be + 2cd) (-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{3/2}}{3e^2 (ex + d)^{3/2}} \\ & + \frac{2(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{5e^2 (ex + d)^{5/2}} - \frac{2g(d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{7ce^2 (ex + d)^{7/2}} \\ & - \frac{2(-be + 2cd)^{5/2} (-dg + ef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right)}{e^2} \\ & + \frac{2(-be + 2cd)^2 (-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 15.101 Problem number 2257

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(2beg - 9cdg + 5cef) (d(-be + cd) - be^2x - ce^2x^2)^{3/2}}{3e^2 (ex + d)^{3/2}} \\ & - \frac{(2beg - 9cdg + 5cef) (d(-be + cd) - be^2x - ce^2x^2)^{5/2}}{5e^2 (-be + 2cd) (ex + d)^{5/2}} \\ & - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{7/2}}{e^2 (-be + 2cd) (ex + d)^{7/2}} \\ & + \frac{(-be + 2cd)^{3/2} (2beg - 9cdg + 5cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{e^2} \\ & - \frac{(-be + 2cd) (2beg - 9cdg + 5cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 \sqrt{ex + d}} \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 240 \sqrt{-(xe + d)c + 2cd - be} c^3 d^2 g - 120 \sqrt{-(xe + d)c + 2cd - be} c^3 d f e - 180 \sqrt{-(xe + d)c + 2cd - be} b c^2 d g \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.102 Problem number 2258

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{5c(4beg - 11cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{12e^2(-be + 2cd)(ex + d)^{\frac{3}{2}}} \\
& + \frac{(4beg - 11cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{4e^2(-be + 2cd)(ex + d)^{\frac{7}{2}}} \\
& - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{2e^2(-be + 2cd)(ex + d)^{\frac{11}{2}}} \\
& - \frac{5c(4beg - 11cdg + 3cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd}\sqrt{ex + d}}\right) \sqrt{-be + 2cd}}{4e^2} \\
& + \frac{5c(4beg - 11cdg + 3cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2\sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 120 \sqrt{-(xe + d)c + 2cd - be} c^3 dg - 24 \sqrt{-(xe + d)c + 2cd - be} c^3 fe - 48 \sqrt{-(xe + d)c + 2cd - be} bc^2 ge + \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.103 Problem number 2259**

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{13/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{5c(6beg - 13cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{24e^2(-be + 2cd)(ex + d)^{\frac{5}{2}}} \\
& + \frac{(6beg - 13cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{12e^2(-be + 2cd)(ex + d)^{\frac{9}{2}}} \\
& - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{3e^2(-be + 2cd)(ex + d)^{\frac{13}{2}}} \\
& + \frac{5c^2(6beg - 13cdg + cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right)}{8e^2\sqrt{-be + 2cd}} \\
& - \frac{5c^2(6beg - 13cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{8e^2(-be + 2cd) \sqrt{ex + d}}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 48 \sqrt{-(xe + d)c + 2cd - be} c^3 g + \frac{15 (13c^4 dg - c^4 fe - 6bc^3 ge) \operatorname{arctan}\left(\frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{\sqrt{-2cd + be}} + \frac{396 \sqrt{-(xe + d)c}}{\sqrt{-2cd + be}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.104 Problem number 2260**

$$\int \frac{(f + gx) (cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{15/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{5c(-8beg + 15cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{96e^2(-be + 2cd)(ex + d)^{\frac{7}{2}}} \\
& - \frac{(-8beg + 15cdg + cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{24e^2(-be + 2cd)(ex + d)^{\frac{11}{2}}} \\
& - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{4e^2(-be + 2cd)(ex + d)^{\frac{15}{2}}} \\
& + \frac{5c^3(-8beg + 15cdg + cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd}\sqrt{ex + d}}\right)}{64e^2(-be + 2cd)^{\frac{3}{2}}} \\
& - \frac{5c^2(-8beg + 15cdg + cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{64e^2(-be + 2cd)(ex + d)^{\frac{3}{2}}}
\end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(15/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{15(15c^5dg + c^5fe - 8bc^4ge) \operatorname{arctan}\left(\frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{(2cd - be)\sqrt{-2cd + be}} + \frac{1800\sqrt{-(xe + d)c + 2cd - be} c^8d^4g + 120\sqrt{-(xe + d)c + 2cd - be}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**15.105 Problem number 2261**

$$\int \frac{(f + gx)(cd^2 - bde - be^2x - ce^2x^2)^{5/2}}{(d + ex)^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{c(-10beg + 17cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{48e^2 (-be + 2cd) (ex + d)^{\frac{9}{2}}} \\
 & - \frac{(-10beg + 17cdg + 3cef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{5}{2}}}{40e^2 (-be + 2cd) (ex + d)^{\frac{13}{2}}} \\
 & - \frac{(-dg + ef) (d(-be + cd) - be^2x - ce^2x^2)^{\frac{7}{2}}}{5e^2 (-be + 2cd) (ex + d)^{\frac{17}{2}}} \\
 & + \frac{c^4(-10beg + 17cdg + 3cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{128e^2 (-be + 2cd)^{\frac{5}{2}}} \\
 & - \frac{c^2(-10beg + 17cdg + 3cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{64e^2 (-be + 2cd) (ex + d)^{\frac{5}{2}}} \\
 & + \frac{c^3(-10beg + 17cdg + 3cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{128e^2 (-be + 2cd)^2 (ex + d)^{\frac{3}{2}}}
 \end{aligned}$$

command

```
integrate((g*x+f)*(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2)/(e*x+d)^(17/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.106 Problem number 2262

$$\int \frac{(d + ex)^{5/2} (f + gx)}{\sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{2(-6beg + 5cdg + 7cef) (ex + d)^{\frac{3}{2}} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{35c^2e^2} \\
 & - \frac{2g(ex + d)^{\frac{5}{2}} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{7ce^2} \\
 & - \frac{16(-be + 2cd)^2 (-6beg + 5cdg + 7cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{105c^4e^2 \sqrt{ex + d}} \\
 & - \frac{8(-be + 2cd) (-6beg + 5cdg + 7cef) \sqrt{ex + d} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{105c^3e^2}
 \end{aligned}$$

command

`integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{105} \left( \frac{105 (4c^3d^3g + 4c^3d^2fe - 8bc^2d^2ge - 4bc^2dfe^2 + 5b^2cdge^2 + b^2cfe^3 - b^3ge^3) \sqrt{-(xe+d)c + 2cd - be}}{c^4} e^{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.107 Problem number 2263

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2g(ex+d)^{\frac{3}{2}} \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{5ce^2} \\ & -\frac{4(-be+2cd)(-4beg+3cdg+5cef) \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{15c^3e^2\sqrt{ex+d}} \\ & -\frac{2(-4beg+3cdg+5cef) \sqrt{ex+d} \sqrt{d(-be+cd) - be^2x - ce^2x^2}}{15c^2e^2} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( \frac{15 (2c^2d^2g + 2c^2dfe - 3bcdge - bcf e^2 + b^2ge^2) \sqrt{-(xe+d)c + 2cd - be}}{c^3} e^{(-1)} - \frac{2 (6\sqrt{2cd - be} c^2d^2g + \dots)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.108 Problem number 2264

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{cd^2-bde-be^2x-ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{2(-2beg+cdg+3cef)\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{3c^2e^2\sqrt{ex+d}} - \frac{2g\sqrt{ex+d}\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{3ce^2}$$

command

`integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{(-(xe+d)c+2cd-be)^{\frac{3}{2}}ge^{(-1)}}{c^2} - \frac{3(cdg+cfe-bge)\sqrt{-(xe+d)c+2cd-be}e^{(-1)}}{c^2} + \frac{(\sqrt{2cd-be}cdg+3}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.109 Problem number 2265

$$\int \frac{f+gx}{\sqrt{d+ex}\sqrt{cd^2-bde-be^2x-ce^2x^2}} dx$$

Optimal antiderivative

$$-\frac{2(-dg+ef)\operatorname{arctanh}\left(\frac{\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{\sqrt{-be+2cd}\sqrt{ex+d}}\right)}{e^2\sqrt{-be+2cd}} - \frac{2g\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{ce^2\sqrt{ex+d}}$$

command

`integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-2 \left( \left( \frac{(dg-fe)\operatorname{arctan}\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{\sqrt{-2cd+be}} + \frac{\sqrt{-(xe+d)c+2cd-be}g}{c} \right) e^{(-1)} - \frac{(cdg\operatorname{arctan}\left(\frac{\sqrt{2cd-be}}{\sqrt{-2cd+be}}\right)+3}{c^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{gx+f}{\sqrt{-ce^2x^2-be^2x+cd^2-bde}\sqrt{ex+d}} dx$$

## 15.110 Problem number 2266

$$\int \frac{f + gx}{(d + ex)^{3/2} \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{(-2beg + 3cdg + cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{e^2 (-be + 2cd)^{\frac{3}{2}}} - \frac{(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{e^2 (-be + 2cd) (ex + d)^{\frac{3}{2}}}$$

command

`integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(3c^2dg + c^2fe - 2bcge) \operatorname{arctan} \left( \frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{(2cd - be) \sqrt{-2cd + be}} + \frac{\sqrt{-(xe + d)c + 2cd - be} c^2dg - \sqrt{-(xe + d)c + 2cd - be}}{(2cd - be)(xe + d)c} \right) c$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{gx + f}{\sqrt{-ce^2x^2 - be^2x + cd^2 - bde} (ex + d)^{\frac{3}{2}}} dx$$

## 15.111 Problem number 2267

$$\int \frac{f + gx}{(d + ex)^{5/2} \sqrt{cd^2 - bde - be^2x - ce^2x^2}} dx$$

Optimal antiderivative

$$\frac{c(-4beg + 5cdg + 3cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{4e^2 (-be + 2cd)^{\frac{5}{2}}} - \frac{(-dg + ef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{2e^2 (-be + 2cd) (ex + d)^{\frac{5}{2}}} - \frac{(-4beg + 5cdg + 3cef) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{4e^2 (-be + 2cd)^2 (ex + d)^{\frac{3}{2}}}$$



command

```
integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(5c^3dg+3c^3fe-4bc^2ge) \arctan\left(\frac{\sqrt{-(xe+d)c+2cd-be}}{\sqrt{-2cd+be}}\right)}{(4c^2d^2-4bcde+b^2e^2)\sqrt{-2cd+be}} - \frac{6\sqrt{-(xe+d)c+2cd-be}c^4d^2g+10\sqrt{-(xe+d)c+2cd-be}c^4d^2f}{(4c^2d^2-4bcde+b^2e^2)\sqrt{-2cd+be}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 15.112 Problem number 2268

$$\int \frac{(d+ex)^{9/2}(f+gx)}{(cd^2-bde-be^2x-ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-beg+cdg+cef)(ex+d)^{\frac{9}{2}}}{ce^2(-be+2cd)\sqrt{d(-be+cd)-be^2x-ce^2x^2}} \\ & + \frac{12(-8beg+9cdg+7cef)(ex+d)^{\frac{3}{2}}\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{35c^3e^2} \\ & + \frac{2(-8beg+9cdg+7cef)(ex+d)^{\frac{5}{2}}\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{7c^2e^2(-be+2cd)} \\ & + \frac{32(-be+2cd)^2(-8beg+9cdg+7cef)\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{35c^5e^2\sqrt{ex+d}} \\ & + \frac{16(-be+2cd)(-8beg+9cdg+7cef)\sqrt{ex+d}\sqrt{d(-be+cd)-be^2x-ce^2x^2}}{35c^4e^2} \end{aligned}$$

command

```
integrate((e*x+d)^(9/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(8c^4d^4g+8c^4d^3fe-20bc^3d^3ge-12bc^3d^2fe^2+18b^2c^2d^2ge^2+6b^2c^2dfe^3-7b^3cdge^3-b^3cfe^4+b^4ge^4)e^{(-2)}}{\sqrt{-(xe+d)c+2cd-be}c^5} \\ & - \frac{32(72c^4d^4g+56c^4d^3fe-172bc^3d^3ge-84bc^3d^2fe^2+150b^2c^2d^2ge^2+42b^2c^2dfe^3-57b^3cdge^3-7b^3cfe^4+8b^4ge^4)e^{(-2)}}{35\sqrt{2cd-be}c^5} \\ & + \frac{2\left(700\sqrt{-(xe+d)c+2cd-be}c^{33}d^3ge^{12}+420\sqrt{-(xe+d)c+2cd-be}c^{33}d^2fe^{13}-1260\sqrt{-(xe+d)c+2cd-be}c^{33}dfe^{14}\right)}{35\sqrt{2cd-be}c^5} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.113 Problem number 2269

$$\int \frac{(d+ex)^{7/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-beg + cdg + cef)(ex + d)^{7/2}}{ce^2(-be + 2cd)\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{2(-6beg + 7cdg + 5cef)(ex + d)^{5/2}\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{5c^2e^2(-be + 2cd)} \\ & + \frac{16(-be + 2cd)(-6beg + 7cdg + 5cef)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{15c^4e^2\sqrt{ex + d}} \\ & + \frac{8(-6beg + 7cdg + 5cef)\sqrt{ex + d}\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{15c^3e^2} \end{aligned}$$

command

`integrate((e*x+d)^(7/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(4c^3d^3g + 4c^3d^2fe - 8bc^2d^2ge - 4bc^2dfe^2 + 5b^2cdge^2 + b^2cfe^3 - b^3ge^3)e^{(-2)}}{\sqrt{-(xe+d)c + 2cd - be}c^4} \\ & - \frac{16(28c^3d^3g + 20c^3d^2fe - 52bc^2d^2ge - 20bc^2dfe^2 + 31b^2cdge^2 + 5b^2cfe^3 - 6b^3ge^3)e^{(-2)}}{15\sqrt{2cd - be}c^4} \\ & + \frac{2(120\sqrt{-(xe+d)c + 2cd - be}c^{18}d^2ge^8 + 60\sqrt{-(xe+d)c + 2cd - be}c^{18}dfe^9 - 150\sqrt{-(xe+d)c + 2cd - be}}{\dots} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.114 Problem number 2270

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-beg + cdg + cef)(ex + d)^{5/2}}{ce^2(-be + 2cd)\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{4(-4beg + 5cdg + 3cef)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{3c^3e^2\sqrt{ex + d}} \\ & + \frac{2(-4beg + 5cdg + 3cef)\sqrt{ex + d}\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{3c^2e^2(-be + 2cd)} \end{aligned}$$

command

```
integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(2c^2d^2g + 2c^2dfe - 3bcdge - bcfe^2 + b^2ge^2)e^{(-2)}}{\sqrt{-(xe+d)c + 2cd - be} c^3} - \frac{4(10c^2d^2g + 6c^2dfe - 13bcdge - 3bcfe^2 + 4b^2ge^2)e^{(-2)}}{3\sqrt{2cd - be} c^3} + \frac{2\left(9\sqrt{-(xe+d)c + 2cd - be} c^7dge^4 + 3\sqrt{-(xe+d)c + 2cd - be} c^7fe^5 - 6\sqrt{-(xe+d)c + 2cd - be} bc^6ge^5 - \dots\right)}{3c^9}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.115 Problem number 2271

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-beg + cdg + cef)(ex + d)^{\frac{3}{2}}}{ce^2(-be + 2cd)\sqrt{d(-be + cd) - be^2x - ce^2x^2}} + \frac{2(-2beg + 3cdg + cef)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{c^2e^2(-be + 2cd)\sqrt{ex + d}}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{-(xe+d)c + 2cd - be} ge^{(-2)}}{c^2} + \frac{2(cdg + cfe - bge)e^{(-2)}}{\sqrt{-(xe+d)c + 2cd - be} c^2} - \frac{2(3cdg + cfe - 2bge)e^{(-2)}}{\sqrt{2cd - be} c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.116 Problem number 2272

$$\int \frac{\sqrt{d+ex} (f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(-dg + ef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{e^2 (-be + 2cd)^{\frac{3}{2}}} + \frac{2(-beg + cdg + cef) \sqrt{ex + d}}{ce^2 (-be + 2cd) \sqrt{d(-be + cd) - be^2x - ce^2x^2}}$$

command

`integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(dg - fe) \arctan \left( \frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{(2cde^2 - be^3) \sqrt{-2cd + be}} + \frac{2 \left( \sqrt{2cd - be} cdg \arctan \left( \frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}} \right) - \sqrt{2cd - be} cf \arctan \left( \frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}} \right) e - \sqrt{-2cd + be} cdg - \sqrt{-2cd + be} bce^3 \right)}{2 \sqrt{2cd - be} \sqrt{-2cd + be} c^2de^2 - \sqrt{2cd - be} \sqrt{-2cd + be} bce^3} + \frac{2(cdg + cfe - bge)}{(2c^2de^2 - bce^3) \sqrt{-(xe + d)c + 2cd - be}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.117 Problem number 2273

$$\int \frac{f+gx}{\sqrt{d+ex} (cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-2beg + cdg + 3cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{e^2 (-be + 2cd)^{\frac{5}{2}}} + \frac{dg - ef}{e^2 (-be + 2cd) \sqrt{ex + d} \sqrt{d(-be + cd) - be^2x - ce^2x^2}} + \frac{(-2beg + cdg + 3cef) \sqrt{ex + d}}{e^2 (-be + 2cd)^2 \sqrt{d(-be + cd) - be^2x - ce^2x^2}}$$

command

```
integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(cdg + 3cfe - 2bge) \arctan\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{(4c^2d^2e^2 - 4bcde^3 + b^2e^4)\sqrt{-2cd + be}} + \frac{4c^2d^2g + 4c^2dfe - 6bcdge + ((xe+d)c - 2cd + be)cdg - 2bcfe^2 + 2b^2ge^2 + 3((xe+d)c - 2cd + be)cfe - 2((xe+d)c - 2cd + be)cdg}{(4c^2d^2e^2 - 4bcde^3 + b^2e^4)\left(2\sqrt{-(xe+d)c + 2cd - be}cd - \sqrt{-(xe+d)c + 2cd - be}be - (-(xe+d)c + 2cd + be)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 15.118 Problem number 2274

$$\int \frac{f + gx}{(d + ex)^{3/2} (cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3c(-4beg + 3cdg + 5cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right)}{4e^2(-be + 2cd)^{\frac{7}{2}}} \\ & + \frac{dg - ef}{2e^2(-be + 2cd)(ex + d)^{\frac{3}{2}} \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{4beg - 3cdg - 5cef}{4e^2(-be + 2cd)^2 \sqrt{ex + d} \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{3c(-4beg + 3cdg + 5cef) \sqrt{ex + d}}{4e^2(-be + 2cd)^3 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3(3c^2dg + 5c^2fe - 4bcge) \arctan\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{4(8c^3d^3e^2 - 12bc^2d^2e^3 + 6b^2cde^4 - b^3e^5)\sqrt{-2cd + be}} \\ & + \frac{2(c^2dg + c^2fe - bcge)}{(8c^3d^3e^2 - 12bc^2d^2e^3 + 6b^2cde^4 - b^3e^5)\sqrt{-(xe+d)c + 2cd - be}} \\ & + \frac{2\sqrt{-(xe+d)c + 2cd - be}c^3d^2g - 18\sqrt{-(xe+d)c + 2cd - be}c^3dfe + 7\sqrt{-(xe+d)c + 2cd - be}bc^2dge + (-}{ \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 15.119 Problem number 2275

$$\int \frac{f + gx}{(d + ex)^{5/2} (cd^2 - bde - be^2x - ce^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5c^2(-6beg + 5cdg + 7cef) \operatorname{arctanh} \left( \frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}} \right)}{8e^2(-be + 2cd)^{\frac{9}{2}} (dg - ef)} \\ & + \frac{3e^2(-be + 2cd)(ex + d)^{\frac{5}{2}} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{6beg - 5cdg - 7cef} \\ & + \frac{12e^2(-be + 2cd)^2(ex + d)^{\frac{3}{2}} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{5c(-6beg + 5cdg + 7cef)} \\ & - \frac{24e^2(-be + 2cd)^3 \sqrt{ex + d} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{5c^2(-6beg + 5cdg + 7cef) \sqrt{ex + d}} \\ & + \frac{5c^2(-6beg + 5cdg + 7cef) \sqrt{ex + d}}{8e^2(-be + 2cd)^4 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

`integrate((g*x+f)/(e*x+d)^(5/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5(5c^3dg + 7c^3fe - 6bc^2ge) \operatorname{arctan} \left( \frac{\sqrt{-(x+d)c + 2cd - be}}{\sqrt{-2cd + be}} \right)}{8(16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6) \sqrt{-2cd + be}} \\ & + \frac{2(c^3dg + c^3fe - bc^2ge)}{(16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6) \sqrt{-(x+d)c + 2cd - be}} \\ & - \frac{84 \sqrt{-(x+d)c + 2cd - be} c^5d^3g + 348 \sqrt{-(x+d)c + 2cd - be} c^5d^2fe - 300 \sqrt{-(x+d)c + 2cd - be} bc^4d^2}{(16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6) \sqrt{-(x+d)c + 2cd - be}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 15.120 Problem number 2276

$$\int \frac{(d+ex)^{13/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-beg + cdg + cef)(ex + d)^{\frac{13}{2}}}{3ce^2(-be + 2cd)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} \\ & + \frac{128(-be + 2cd)^2(-10beg + 13cdg + 7cef)(ex + d)^{\frac{3}{2}}}{105c^5e^2\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{32(-be + 2cd)(-10beg + 13cdg + 7cef)(ex + d)^{\frac{5}{2}}}{105c^4e^2\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{16(-10beg + 13cdg + 7cef)(ex + d)^{\frac{7}{2}}}{105c^3e^2\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{2(-10beg + 13cdg + 7cef)(ex + d)^{\frac{9}{2}}}{21c^2e^2(-be + 2cd)\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & - \frac{256(-be + 2cd)^3(-10beg + 13cdg + 7cef)\sqrt{ex + d}}{105c^6e^2\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

```
integrate((e*x+d)^(13/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.121 Problem number 2277

$$\int \frac{(d+ex)^{11/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$







## 15.124 Problem number 2280

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-beg + cdg + cef)(ex + d)^{\frac{5}{2}}}{3ce^2(-be + 2cd)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} + \frac{2(2beg - 5cdg + cef)\sqrt{ex + d}}{3c^2e^2(-be + 2cd)\sqrt{d(-be + cd) - be^2x - ce^2x^2}}$$

command

`integrate((e*x+d)^(5/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(5cdg - cfe - 2bge)}{3\left(2\sqrt{2cd - be}c^3de^2 - \sqrt{2cd - be}bc^2e^3\right)} - \frac{2(cdg + cfe - bge + 3((xe + d)c - 2cd + be)g)e^{(-2)}}{3((xe + d)c - 2cd + be)\sqrt{-(xe + d)c + 2cd - be}c^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.125 Problem number 2281

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(-beg + cdg + cef)(ex + d)^{\frac{3}{2}}}{3ce^2(-be + 2cd)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} - \frac{2(-dg + ef)\operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd}\sqrt{ex + d}}\right)}{e^2(-be + 2cd)^{\frac{5}{2}}} + \frac{2(-dg + ef)\sqrt{ex + d}}{e^2(-be + 2cd)^2\sqrt{d(-be + cd) - be^2x - ce^2x^2}}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(dg - fe) \arctan\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{(4c^2d^2e^2 - 4bcde^3 + b^2e^4)\sqrt{-2cd + be}} + \frac{2\left(3\sqrt{2cd - be} cdg \arctan\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) - 3\sqrt{2cd - be} cf \arctan\left(\frac{\sqrt{2cd - be}}{\sqrt{-2cd + be}}\right) e + 2\sqrt{-2cd + be} cdg - \sqrt{-2cd + be} cfe\right)}{3\left(4\sqrt{2cd - be} \sqrt{-2cd + be} c^3d^2e^2 - 4\sqrt{2cd - be} \sqrt{-2cd + be} bc^2de^3 + \sqrt{2cd - be} \sqrt{-2cd + be} c^2d^2g + 2c^2dfe - 3bcdge + 3((xe+d)c - 2cd + be)cdg - bcfe^2 + b^2ge^2 - 3((xe+d)c - 2cd + be)cfe\right)}{3(4c^3d^2e^2 - 4bc^2de^3 + b^2ce^4)((xe+d)c - 2cd + be)\sqrt{-(xe+d)c + 2cd - be}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.126 Problem number 2282

$$\int \frac{\sqrt{d+ex}(f+gx)}{(cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{(-2beg - cdg + 5cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be+cd) - be^2x - ce^2x^2}}{\sqrt{-be+2cd} \sqrt{ex+d}}\right)}{e^2(-be+2cd)^{7/2}} + \frac{2(-beg + cdg + cef) \sqrt{ex+d}}{3ce^2(-be+2cd)(d(-be+cd) - be^2x - ce^2x^2)^{3/2}} + \frac{2beg + cdg - 5cef}{3ce^2(-be+2cd)^2 \sqrt{ex+d} \sqrt{d(-be+cd) - be^2x - ce^2x^2}} + \frac{(-2beg - cdg + 5cef) \sqrt{ex+d}}{e^2(-be+2cd)^3 \sqrt{d(-be+cd) - be^2x - ce^2x^2}}$$

command

`integrate((e*x+d)^(1/2)*(g*x+f)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(cdg - 5cfe + 2bge) \arctan\left(\frac{\sqrt{-(xe+d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{(8c^3d^3e^2 - 12bc^2d^2e^3 + 6b^2cde^4 - b^3e^5)\sqrt{-2cd + be}} + \frac{2(2c^2d^2g + 2c^2dfe - 3bcdge - bcfe^2 + b^2ge^2 - 6((xe+d)c - 2cd + be)cfe + 3((xe+d)c - 2cd + be)bge)}{3(8c^3d^3e^2 - 12bc^2d^2e^3 + 6b^2cde^4 - b^3e^5)((xe+d)c - 2cd + be)\sqrt{-(xe+d)c + 2cd - be}} + \frac{\sqrt{-(xe+d)c + 2cd - be} cdg - \sqrt{-(xe+d)c + 2cd - be} cfe}{(8c^3d^3e^2 - 12bc^2d^2e^3 + 6b^2cde^4 - b^3e^5)(xe+d)c}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 15.127 Problem number 2283

$$\int \frac{f + gx}{\sqrt{d + ex} (cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5c(-4beg + cdg + 7cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd} \sqrt{ex + d}}\right)}{4e^2(-be + 2cd)^{\frac{9}{2}} dg - ef} \\ & + \frac{2e^2(-be + 2cd)(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}} \sqrt{ex + d}}{(-4beg + cdg + 7cef) \sqrt{ex + d}} \\ & + \frac{6e^2(-be + 2cd)^2(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}}{5(-4beg + cdg + 7cef)} \\ & - \frac{12e^2(-be + 2cd)^3 \sqrt{ex + d} \sqrt{d(-be + cd) - be^2x - ce^2x^2}}{5c(-4beg + cdg + 7cef) \sqrt{ex + d}} \\ & + \frac{5c(-4beg + cdg + 7cef) \sqrt{ex + d}}{4e^2(-be + 2cd)^4 \sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

`integrate((g*x+f)/(e*x+d)^(1/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5(c^2dg + 7c^2fe - 4bcge) \arctan\left(\frac{\sqrt{-(xe + d)c + 2cd - be}}{\sqrt{-2cd + be}}\right)}{4(16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6)\sqrt{-2cd + be}} \\ & - \frac{2(2c^3d^2g + 2c^3dfe - 3bc^2dge - 3((xe + d)c - 2cd + be)c^2dg - bc^2fe^2 + b^2cge^2 - 9((xe + d)c - 2cd + be)c^2fe)}{3(16c^4d^4e^2 - 32bc^3d^3e^3 + 24b^2c^2d^2e^4 - 8b^3cde^5 + b^4e^6)((xe + d)c - 2cd + be)\sqrt{-(xe + d)}} \\ & + \frac{10\sqrt{-(xe + d)c + 2cd - be}c^3d^2g - 26\sqrt{-(xe + d)c + 2cd - be}c^3dfe + 3\sqrt{-(xe + d)c + 2cd - be}bc^2dge - 3}{4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.128 Problem number 2284

$$\int \frac{f + gx}{(d + ex)^{3/2} (cd^2 - bde - be^2x - ce^2x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{dg - ef}{3e^2(-be + 2cd)(ex + d)^{\frac{3}{2}}(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} \\ & - \frac{35c^2(-2beg + cdg + 3cef) \operatorname{arctanh}\left(\frac{\sqrt{d(-be + cd) - be^2x - ce^2x^2}}{\sqrt{-be + 2cd}\sqrt{ex + d}}\right)}{8e^2(-be + 2cd)^{\frac{11}{2}}} \\ & + \frac{2beg - cdg - 3cef}{4e^2(-be + 2cd)^2(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}\sqrt{ex + d}} \\ & + \frac{7c(-2beg + cdg + 3cef)\sqrt{ex + d}}{12e^2(-be + 2cd)^3(d(-be + cd) - be^2x - ce^2x^2)^{\frac{3}{2}}} \\ & - \frac{35c(-2beg + cdg + 3cef)}{24e^2(-be + 2cd)^4\sqrt{ex + d}\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \\ & + \frac{35c^2(-2beg + cdg + 3cef)\sqrt{ex + d}}{8e^2(-be + 2cd)^5\sqrt{d(-be + cd) - be^2x - ce^2x^2}} \end{aligned}$$

command

```
integrate((g*x+f)/(e*x+d)^(3/2)/(-c*e^2*x^2-b*e^2*x-b*d*e+c*d^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 15.129 Problem number 2494

$$\int \frac{A + Bx}{(d + ex)(a + bx + cx^2)^{7/2}} dx$$

Optimal antiderivative

$$\frac{2aB(-be+2cd)}{5} - \frac{2A(2ace-b^2e+bcd)}{5} + \frac{2c(Abe-2Acd-2aBe+Bbd)x}{5}$$

$$\frac{(-4ac + b^2)(ae^2 - bde + cd^2)(cx^2 + bx + a)^{\frac{5}{2}}}{15} + \frac{16ace(-be+2cd)(Abe-2Acd-2aBe+Bbd)}{15} + \frac{2(2ace-b^2e+bcd)(5b^2e(-Ae+Bd)-8bcd(Ae+Bd)+4c(5Aae^2+4Acd^2-aBde))}{15} + \frac{2c(8ce(-2ae-(-4ac + b^2)(ae^2 - bde + cd^2)^2)(cx^2 + bx + a)^{\frac{5}{2}}}{15}$$

$$- \frac{e^5(-Ae + Bd) \operatorname{arctanh}\left(\frac{bd-2ae+(-be+2cd)x}{2\sqrt{ae^2 - bde + cd^2}\sqrt{cx^2 + bx + a}}\right)}{(ae^2 - bde + cd^2)^{\frac{7}{2}}}$$

$$+ \frac{8ace(-be+2cd)(8ce(-2ae+bd)(Abe-2Acd-2aBe+Bbd)+(-be+2cd)(5b^2e(-Ae+Bd)-8bcd(Ae+Bd)+4c(5Aae^2+4Acd^2-aBde)))}{15} - \frac{2(2ace-b^2e+bcd)(cx^2 + bx + a)^{\frac{5}{2}}}{15}$$

command

```
integrate((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 16 Test file number 35

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/35\_1.2.1.4-d+e\_x-^m-f+g\_x-^n-a+b\_x+c\_x^2-^p

### 16.1 Problem number 99

$$\int \frac{\sqrt{d^2 - e^2x^2}}{x^3(d + ex)} dx$$

Optimal antiderivative

$$-\frac{e^2 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{2d^2} - \frac{\sqrt{-e^2x^2 + d^2}}{2dx^2} + \frac{e\sqrt{-e^2x^2 + d^2}}{d^2x}$$

command

```
integrate((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2 \left( \frac{4 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)}{x} - e^2 \right) e^4 - e^2 \log \left( \frac{\left| -2de - 2 \sqrt{-x^2 e^2 + d^2} e \right| e^{(-2)}}{2|x|} \right)}{8 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^2} - \frac{2 d^2}{2 d^2}$$

$$- \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^2 e^{(-2)}}{x^2} - \frac{4 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^2}{x}}{8 d^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.2 Problem number 100

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x^4(d + ex)} dx$$

Optimal antiderivative

$$\frac{e^3 \operatorname{arctanh} \left( \frac{\sqrt{-e^2 x^2 + d^2}}{d} \right)}{2d^3} - \frac{\sqrt{-e^2 x^2 + d^2}}{3d x^3} + \frac{e \sqrt{-e^2 x^2 + d^2}}{2d^2 x^2} - \frac{2e^2 \sqrt{-e^2 x^2 + d^2}}{3d^3 x}$$

command

`integrate((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3 \left( \frac{3 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e}{x} - \frac{9 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-1)}}{x^2} - e^3 \right) e^6}{24 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^3}$$

$$+ \frac{e^3 \log \left( \frac{\left| -2de - 2 \sqrt{-x^2 e^2 + d^2} e \right| e^{(-2)}}{2|x|} \right)}{2 d^3}$$

$$- \frac{9 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^6 e}{x} - \frac{3 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^6 e^{(-1)}}{x^2} + \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^6 e^{(-3)}}{x^3}}{24 d^9}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.3 Problem number 101

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x^5(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3e^4 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{8d^4} - \frac{\sqrt{-e^2 x^2 + d^2}}{4d x^4} \\ & + \frac{e \sqrt{-e^2 x^2 + d^2}}{3d^2 x^3} - \frac{3e^2 \sqrt{-e^2 x^2 + d^2}}{8d^3 x^2} + \frac{2e^3 \sqrt{-e^2 x^2 + d^2}}{3d^4 x} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & x^4 \left( \frac{8 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^2}{x} + \frac{72 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-2)}}{x^3} - \frac{24 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2}{x^2} - 3e^4 \right) e^8 \\ & - \frac{192 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 d^4}{8d^4} \\ & - \frac{3e^4 \log\left(\frac{\left| -2de - 2\sqrt{-x^2 e^2 + d^2} e \right| e^{(-2)}}{2|x|}\right)}{8d^4} \\ & + \frac{72 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^{12} e^2}{x} + \frac{8 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^{12} e^{(-2)}}{x^3} - \frac{3 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 d^{12} e^{(-4)}}{x^4} - \frac{24 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^5 d^{12} e^{(-6)}}{x^5} \\ & \frac{\hspace{10em}}{192d^{16}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.4 Problem number 102

$$\int \frac{x^2 (d^2 - e^2 x^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{d(-3ex + 4d) (-e^2 x^2 + d^2)^{\frac{3}{2}}}{12e^3} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}}}{5e^3} + \frac{d^5 \operatorname{arctan}\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{8e^3} + \frac{d^3 x \sqrt{-e^2 x^2 + d^2}}{8e^2}$$





## 16.6 Problem number 104

$$\int \frac{x^3(d^2 - e^2x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d^4x(-e^2x^2 + d^2)^{\frac{3}{2}}}{64e^3} - \frac{dx^2(-e^2x^2 + d^2)^{\frac{5}{2}}}{7e^2} + \frac{x^3(-e^2x^2 + d^2)^{\frac{5}{2}}}{8e} \\ & - \frac{d^2(-35ex + 32d)(-e^2x^2 + d^2)^{\frac{5}{2}}}{560e^4} - \frac{3d^8 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{128e^4} - \frac{3d^6x\sqrt{-e^2x^2 + d^2}}{128e^3} \end{aligned}$$

command

```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{3}{128}d^8 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sgn}(d) \\ & -\frac{1}{4480} \left( 256d^7e^{(-4)} - \left( 105d^6e^{(-3)} - 2 \left( 64d^5e^{(-2)} - \left( 35d^4e^{(-1)} + 4(128d^3 - 5(21d^2e - 2(7xe^3 - 8de^2)x)x \right) \right) \right) \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.7 Problem number 105

$$\int \frac{x^2(d^2 - e^2x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3x(-e^2x^2 + d^2)^{\frac{3}{2}}}{24e^2} + \frac{d(-5ex + 6d)(-e^2x^2 + d^2)^{\frac{5}{2}}}{30e^3} - \frac{(-e^2x^2 + d^2)^{\frac{7}{2}}}{7e^3} \\ & + \frac{d^7 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{16e^3} + \frac{d^5x\sqrt{-e^2x^2 + d^2}}{16e^2} \end{aligned}$$

command

```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{16} d^7 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sgn}(d) + \frac{1}{1680} \left(96 d^6 e^{(-3)} - \left(105 d^5 e^{(-2)} - 2 \left(24 d^4 e^{(-1)} + (245 d^3 - 4 (48 d^2 e - 5 (6 x e^3 - 7 d e^2) x) x) x\right) x\right) \sqrt{-x^2 e^2 + d^2}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.8 Problem number 106

$$\int \frac{x(d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$-\frac{d^2 x (-e^2 x^2 + d^2)^{3/2}}{24e} - \frac{(-5ex + 6d) (-e^2 x^2 + d^2)^{5/2}}{30e^2} - \frac{d^6 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{16e^2} - \frac{d^4 x \sqrt{-e^2 x^2 + d^2}}{16e}$$

command

`integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \operatorname{sgn}(d) - \frac{1}{240} \left(48 d^5 e^{(-2)} - \left(15 d^4 e^{(-1)} + 2 (48 d^3 - (35 d^2 e - 4 (5 x e^3 - 6 d e^2) x) x) x\right) \sqrt{-x^2 e^2 + d^2}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.9 Problem number 107

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{dx(-e^2 x^2 + d^2)^{3/2}}{4} + \frac{(-e^2 x^2 + d^2)^{5/2}}{5e} + \frac{3d^5 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{8e} + \frac{3d^3 x \sqrt{-e^2 x^2 + d^2}}{8}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3}{8}d^5 \arcsin\left(\frac{xe}{d}\right) e^{(-1)\operatorname{sgn}(d)} + \frac{1}{40}\left(8d^4e^{(-1)} + (25d^3 - 2(8d^2e - (4xe^3 - 5de^2)x)x)\right)\sqrt{-x^2e^2 + d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.10 Problem number 108

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-3ex + 4d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{12} - \frac{3d^4 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{8} - d^4 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right) + \frac{d^2(-3ex + 8d)\sqrt{-e^2x^2 + d^2}}{8}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3}{8}d^4 \arcsin\left(\frac{xe}{d}\right) \operatorname{sgn}(d) - d^4 \log\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right) + \frac{1}{24}(32d^3 - (15d^2e - 2(3xe^3 - 4de^2)x)x)\sqrt{-x^2e^2 + d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.11 Problem number 109

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ex + 3d)(-e^2 x^2 + d^2)^{\frac{3}{2}}}{3x} - \frac{3d^3 e \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{2} \\ & + d^3 e \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right) - \frac{de(3ex + 2d)\sqrt{-e^2 x^2 + d^2}}{2} \end{aligned}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{3}{2} d^3 \arcsin\left(\frac{xe}{d}\right) \operatorname{esgn}(d) + d^3 e \log\left(\frac{|-2de - 2\sqrt{-x^2 e^2 + d^2} e| e^{(-2)}}{2|x|}\right) \\ & + \frac{d^3 x e^3}{2(de + \sqrt{-x^2 e^2 + d^2} e)} - \frac{(de + \sqrt{-x^2 e^2 + d^2} e) d^3 e^{(-1)}}{2x} \\ & - \frac{1}{6} \sqrt{-x^2 e^2 + d^2} (8d^2 e - (2xe^3 - 3de^2)x) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.12 Problem number 110

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^3(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(ex + d)(-e^2 x^2 + d^2)^{\frac{3}{2}}}{2x^2} + \frac{3d^2 e^2 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{2} \\ & + \frac{3d^2 e^2 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{2} + \frac{3de(-ex + d)\sqrt{-e^2 x^2 + d^2}}{2x} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^2 \operatorname{sgn}(d) + \frac{3}{2} d^2 e^2 \log\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right) \\ & - \frac{(de + \sqrt{-x^2e^2 + d^2}e)^2 d^2 e^{(-2)}}{8x^2} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)d^2}{2x} \\ & + \frac{\left(d^2e^2 - \frac{4(de + \sqrt{-x^2e^2 + d^2}e)d^2}{x}\right)x^2e^4}{8(de + \sqrt{-x^2e^2 + d^2}e)^2} + \frac{1}{2}\sqrt{-x^2e^2 + d^2}(xe^3 - 2de^2) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.13 Problem number 111

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^4(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-3ex + 2d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{6x^3} + de^3 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right) \\ & - \frac{3de^3 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{2} + \frac{e^2(3ex + 2d)\sqrt{-e^2x^2 + d^2}}{2x} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& d \arcsin\left(\frac{xe}{d}\right) e^3 \operatorname{sgn}(d) - \frac{3}{2} de^3 \log\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right) \\
& + \frac{\left(de^3 - \frac{3\left(de + \sqrt{-x^2e^2 + d^2}e\right)de}{x} - \frac{15\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2 de^{(-1)}}{x^2}\right) x^3 e^6}{24\left(de + \sqrt{-x^2e^2 + d^2}e\right)^3} \\
& + \frac{5\left(de + \sqrt{-x^2e^2 + d^2}e\right)de}{8x} + \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2 de^{(-1)}}{8x^2} \\
& - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^3 de^{(-3)}}{24x^3} + \sqrt{-x^2e^2 + d^2}e^3
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.14 Problem number 112

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{x^5(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{(-4ex + 3d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{12x^4} - e^4 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right) \\
& - \frac{3e^4 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{8} + \frac{e^2(-8ex + 3d)\sqrt{-e^2x^2 + d^2}}{8x^2}
\end{aligned}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -\arcsin\left(\frac{xe}{d}\right) e^4 \operatorname{sgn}(d) \\
& x^4 \left( \frac{8\left(de + \sqrt{-x^2e^2 + d^2}e\right)e^2}{x} - \frac{120\left(de + \sqrt{-x^2e^2 + d^2}e\right)^3 e^{(-2)}}{x^3} + \frac{24\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2}{x^2} - 3e^4 \right) e^8 \\
& - \frac{192\left(de + \sqrt{-x^2e^2 + d^2}e\right)^4}{192\left(de + \sqrt{-x^2e^2 + d^2}e\right)^4} \\
& - \frac{3}{8} e^4 \log\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right) - \frac{5\left(de + \sqrt{-x^2e^2 + d^2}e\right)e^2}{8x} \\
& + \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^3 e^{(-2)}}{24x^3} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^4 e^{(-4)}}{64x^4} + \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2}{8x^2}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.15 Problem number 113

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6 (d + ex)} dx$$

Optimal antiderivative

$$\frac{e(-e^2 x^2 + d^2)^{\frac{3}{2}}}{4x^4} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}}}{5d x^5} + \frac{3e^5 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{8d} - \frac{3e^3 \sqrt{-e^2 x^2 + d^2}}{8x^2}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^5 \left( \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^3}{x} + \frac{10 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e}{x^2} - \frac{40 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-1)}}{x^3} - \frac{20 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-2)}}{x^4} \right)}{320 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^5 d} + \frac{3 e^5 \log \left( \frac{\left| \frac{-2 de - 2 \sqrt{-x^2 e^2 + d^2} e}{2|x|} \right| e^{(-2)}}{2|x|} \right)}{8d} - \frac{20 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^4 e^3}{x} + \frac{40 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^4 e}{x^2} - \frac{10 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^4 e^{(-1)}}{x^3} - \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 d^4 e^{(-2)}}{x^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 16.16 Problem number 114

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^7(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{e^2(-e^2x^2 + d^2)^{\frac{3}{2}}}{24dx^4} - \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{6dx^6} + \frac{e(-e^2x^2 + d^2)^{\frac{5}{2}}}{5d^2x^5} \\ & - \frac{e^6 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{16d^2} + \frac{e^4 \sqrt{-e^2x^2 + d^2}}{16dx^2} \end{aligned}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & x^6 \left( \frac{12 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e^4}{x} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^2}{x^2} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^4 e^{(-2)}}{x^4} + \frac{120 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^6}{x^5} \right) \\ & - \frac{1920 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^6 d^2}{e^6 \log\left(\frac{\left| \frac{-2de - 2\sqrt{-x^2e^2 + d^2} e}{2|x|} \right| e^{(-2)}}{16d^2}\right)} \\ & + \frac{120 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^{10} e^4}{x} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 d^{10} e^2}{x^2} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^4 d^{10} e^{(-2)}}{x^4} + \frac{12 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^6 d^{12}}{1920 d^{12}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.17 Problem number 115

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^8(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{e^3(-e^2x^2 + d^2)^{\frac{3}{2}}}{24d^2x^4} - \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{7dx^7} + \frac{e(-e^2x^2 + d^2)^{\frac{5}{2}}}{6d^2x^6} \\ & - \frac{2e^2(-e^2x^2 + d^2)^{\frac{5}{2}}}{35d^3x^5} + \frac{e^7 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{16d^3} - \frac{e^5 \sqrt{-e^2x^2 + d^2}}{16d^2x^2} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & x^7 \left( \frac{35 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^5}{x} + \frac{21 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^3}{x^2} - \frac{105 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e}{x^3} + \frac{105 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4}{x^4} \right. \\
 & \left. - \frac{13440 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^5}{16 d^3} \right) \\
 & + \frac{e^7 \log \left( \frac{-2 de - 2 \sqrt{-x^2 e^2 + d^2} e |e^{(-2)}|}{2|x|} \right)}{16 d^3} \\
 & - \frac{315 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^{18} e^5}{x} + \frac{105 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 d^{18} e^3}{x^2} - \frac{105 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^{18} e}{x^3} + \frac{105 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 d^{18}}{x^4}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.18 Problem number 116

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^9 (d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{e^4 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{64 d^3 x^4} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}}}{8 d x^8} + \frac{e (-e^2 x^2 + d^2)^{\frac{5}{2}}}{7 d^2 x^7} - \frac{e^2 (-e^2 x^2 + d^2)^{\frac{5}{2}}}{16 d^3 x^6} \\
 & + \frac{2 e^3 (-e^2 x^2 + d^2)^{\frac{5}{2}}}{35 d^4 x^5} - \frac{3 e^8 \operatorname{arctanh} \left( \frac{\sqrt{-e^2 x^2 + d^2}}{d} \right)}{128 d^4} + \frac{3 e^6 \sqrt{-e^2 x^2 + d^2}}{128 d^3 x^2}
 \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& x^8 \left( \frac{80 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^6}{x} - \frac{112 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^2}{x^3} - \frac{560 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^5 e^{(-2)}}{x^5} + \frac{1680 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^7}{x^7} \right) \\
& \frac{71680 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^8 d^4}{128 d^4} \\
& 3 e^8 \log \left( \frac{\left| \frac{-2 de - 2 \sqrt{-x^2 e^2 + d^2} e}{2|x|} e^{(-2)} \right|}{128 d^4} \right) \\
& + \frac{1680 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^{28} e^6}{x} - \frac{560 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 d^{28} e^2}{x^3} - \frac{112 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^5 d^{28} e^{(-2)}}{x^5} + \frac{80 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^7}{x^7} \\
& \frac{71680 d^{32}}{71680 d^{32}}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.19 Problem number 119

$$\int \frac{x^4}{(d + ex) \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{3d^3 \arctan \left( \frac{ex}{\sqrt{-e^2 x^2 + d^2}} \right)}{2e^5} + \frac{x^3(-ex + d)}{e^2 \sqrt{-e^2 x^2 + d^2}} \\
& - \frac{4x^2 \sqrt{-e^2 x^2 + d^2}}{3e^3} - \frac{d(-9ex + 16d) \sqrt{-e^2 x^2 + d^2}}{6e^5}
\end{aligned}$$

command

`integrate(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -\frac{3}{2} d^3 \arcsin \left( \frac{xe}{d} \right) e^{(-5)} \operatorname{sgn}(d) + \frac{2 d^3 e^{(-5)}}{\left( \frac{de + \sqrt{-x^2 e^2 + d^2} e}{x} \right) e^{(-2)} + 1} \\
& - \frac{1}{6} \sqrt{-x^2 e^2 + d^2} \left( 10 d^2 e^{(-5)} + \left( 2 x e^{(-3)} - 3 d e^{(-4)} \right) x \right)
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.20 Problem number 120

$$\int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$\frac{3d^2 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{2e^4} + \frac{x^2(-ex+d)}{e^2\sqrt{-e^2x^2+d^2}} + \frac{(-3ex+4d)\sqrt{-e^2x^2+d^2}}{2e^4}$$

command

```
integrate(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sgn}(d) - \frac{2d^2 e^{(-4)}}{\left(\frac{de + \sqrt{-x^2 e^2 + d^2}}{e}\right) e^{(-2)} + 1} - \frac{1}{2} \sqrt{-x^2 e^2 + d^2} \left(xe^{(-3)} - 2de^{(-4)}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.21 Problem number 121

$$\int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{d \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^3} - \frac{\sqrt{-e^2x^2+d^2}}{e^3} - \frac{d\sqrt{-e^2x^2+d^2}}{e^3(ex+d)}$$

command

```
integrate(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-d \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sgn}(d) - \sqrt{-x^2 e^2 + d^2} e^{(-3)} + \frac{2de^{(-3)}}{\left(\frac{de + \sqrt{-x^2 e^2 + d^2}}{e}\right) e^{(-2)} + 1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.22 Problem number 122

$$\int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^2} + \frac{\sqrt{-e^2x^2+d^2}}{e^2(ex+d)}$$

command

```
integrate(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\arcsin\left(\frac{xe}{d}\right)e^{(-2)}\operatorname{sgn}(d) - \frac{2e^{(-2)}}{\left(\frac{de+\sqrt{-x^2e^2+d^2}}{e}\right)e^{(-2)} + 1} + 1$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.23 Problem number 123

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2x^2+d^2}}{de(ex+d)}$$

command

```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$d\left(\frac{2e^{(-1)}}{\left(\frac{de+\sqrt{-x^2e^2+d^2}}{e}\right)e^{(-2)} + 1}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.24 Problem number 124

$$\int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^2} + \frac{\sqrt{-e^2x^2+d^2}}{d^2(ex+d)}$$

command

```
integrate(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log\left(\frac{-2de^{-2}\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|}\right)}{d^2} - \frac{2}{d^2\left(\frac{(de+\sqrt{-x^2e^2+d^2}e)e^{(-2)}}{x}+1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.25 Problem number 125

$$\int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$\frac{e \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^3} - \frac{2\sqrt{-e^2x^2+d^2}}{d^3x} + \frac{\sqrt{-e^2x^2+d^2}}{d^2x(ex+d)}$$

command

```
integrate(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e \log \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2} e| e^{(-2)}}{2|x|} \right)}{d^3} + \frac{x \left( \frac{5 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-1)}}{x} + e \right) e^2}{2 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^3 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)} - \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-1)}}{2d^3x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.26 Problem number 126

$$\int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}} dx$$

Optimal antiderivative

$$-\frac{3e^2 \operatorname{arctanh} \left( \frac{\sqrt{-e^2x^2 + d^2}}{d} \right)}{2d^4} - \frac{3\sqrt{-e^2x^2 + d^2}}{2d^3x^2} + \frac{2e\sqrt{-e^2x^2 + d^2}}{d^4x} + \frac{\sqrt{-e^2x^2 + d^2}}{d^2x^2(ex+d)}$$

command

`integrate(1/x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3e^2 \log \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2} e| e^{(-2)}}{2|x|} \right)}{2d^4} - \frac{x^2 \left( \frac{20 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^{(-2)}}{x^2} + \frac{3 \left( de + \sqrt{-x^2e^2 + d^2} e \right)}{x} - e^2 \right) e^4}{8 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 d^4 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)} - \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 d^4 e^{(-2)}}{x^2} - \frac{4 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^4}{x}}{8d^8}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.27 Problem number 157

$$\int \frac{x^5 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4d^4 x^2 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{21e^4} + \frac{5d^3 x^3 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{24e^3} - \frac{5d^2 x^4 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{21e^2} \\ & + \frac{d x^5 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{4e} - \frac{x^6 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{9} - \frac{d^5 (-315ex + 256d) (-e^2 x^2 + d^2)^{\frac{3}{2}}}{2016e^6} \\ & - \frac{5d^9 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{64e^6} - \frac{5d^7 x \sqrt{-e^2 x^2 + d^2}}{64e^5} \end{aligned}$$

command

`integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 161280 d^{10} \arctan\left(\sqrt{\frac{2d}{xe+d}-1}\right) e^{10} \operatorname{sgn}\left(\frac{1}{xe+d}\right) + \frac{\left(315 d^{10} \left(\frac{2d}{xe+d}-1\right)^{\frac{17}{2}} e^{10} \operatorname{sgn}\left(\frac{1}{xe+d}\right) - 18774 d^{10} \left(\frac{2d}{xe+d}-1\right)^{\frac{15}{2}} e^{10} \operatorname{sgn}\left(\frac{1}{xe+d}\right)\right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.28 Problem number 158

$$\int \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8d^3 x^2 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{35e^3} - \frac{13d^2 x^3 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{48e^2} + \frac{2d x^4 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{7e} \\ & - \frac{x^5 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{8} + \frac{d^4 (-1365ex + 1024d) (-e^2 x^2 + d^2)^{\frac{3}{2}}}{6720e^5} \\ & + \frac{13d^8 \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right)}{128e^5} + \frac{13d^6 x \sqrt{-e^2 x^2 + d^2}}{128e^4} \end{aligned}$$



command

```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 349440 d^9 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^9 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \frac{\left( 1365 d^9 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{15}{2}} e^9 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 61215 d^9 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{13}{2}} e^9 \operatorname{sgn} \left( \frac{1}{xe+d} \right) \right)}{1} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 16.29 Problem number 159

$$\int \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{11d^2 x^2 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{35e^2} + \frac{dx^3 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{3e} - \frac{x^4 (-e^2 x^2 + d^2)^{\frac{3}{2}}}{7} \\ & - \frac{d^3 (-105ex + 88d) (-e^2 x^2 + d^2)^{\frac{3}{2}}}{420e^4} - \frac{d^7 \arctan \left( \frac{ex}{\sqrt{-e^2 x^2 + d^2}} \right)}{8e^4} - \frac{d^5 x \sqrt{-e^2 x^2 + d^2}}{8e^3} \end{aligned}$$

command

```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 13440 d^8 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^8 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \frac{\left( 105 d^8 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{13}{2}} e^8 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 3780 d^8 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{11}{2}} e^8 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + 1890 d^8 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{9}{2}} e^8 \operatorname{sgn} \left( \frac{1}{xe+d} \right) \right)}{1} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.30 Problem number 160

$$\int \frac{x^2(d^2 - e^2x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\frac{2dx^2(-e^2x^2 + d^2)^{\frac{3}{2}}}{5e} - \frac{x^3(-e^2x^2 + d^2)^{\frac{3}{2}}}{6} + \frac{d^2(-45ex + 32d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{120e^3}$$

$$+ \frac{3d^6 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{16e^3} + \frac{3d^4x\sqrt{-e^2x^2 + d^2}}{16e^2}$$

command

```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 2880 d^7 \arctan\left(\sqrt{\frac{2d}{xe+d} - 1}\right) e^7 \operatorname{sgn}\left(\frac{1}{xe+d}\right) + \frac{\left(45 d^7 \left(\frac{2d}{xe+d} - 1\right)^{\frac{11}{2}} e^7 \operatorname{sgn}\left(\frac{1}{xe+d}\right) - 1025 d^7 \left(\frac{2d}{xe+d} - 1\right)^{\frac{9}{2}} e^7 \operatorname{sgn}\left(\frac{1}{xe+d}\right) - 174 d^7\right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.31 Problem number 161

$$\int \frac{x(d^2 - e^2x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$-\frac{dx(-e^2x^2 + d^2)^{\frac{3}{2}}}{6e} - \frac{2(-e^2x^2 + d^2)^{\frac{5}{2}}}{15e^2} - \frac{(-e^2x^2 + d^2)^{\frac{7}{2}}}{3e^2(ex + d)^2}$$

$$- \frac{d^5 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{4e^2} - \frac{d^3x\sqrt{-e^2x^2 + d^2}}{4e}$$

command

```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 480 d^6 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \frac{\left( 15 d^6 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{9}{2}} e^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 250 d^6 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{7}{2}} e^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 128 d^6 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} e^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right) \right)}{960 d}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.32 Problem number 162

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\frac{5d(-e^2x^2 + d^2)^{\frac{3}{2}}}{12e} + \frac{(-ex + d)(-e^2x^2 + d^2)^{\frac{3}{2}}}{4e} + \frac{5d^4 \arctan \left( \frac{ex}{\sqrt{-e^2x^2 + d^2}} \right)}{8e} + \frac{5d^2x \sqrt{-e^2x^2 + d^2}}{8}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 240 d^5 \arctan \left( \sqrt{\frac{2d}{xe+d} - 1} \right) e^5 \operatorname{sgn} \left( \frac{1}{xe+d} \right) + \frac{\left( 15 d^5 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{7}{2}} e^5 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 73 d^5 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} e^5 \operatorname{sgn} \left( \frac{1}{xe+d} \right) - 55 d^5 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{3}{2}} e^5 \operatorname{sgn} \left( \frac{1}{xe+d} \right) \right)}{d^4}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.33 Problem number 167

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^5 (d + ex)^2} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{\frac{3}{2}}}{4x^4} + \frac{2e(-e^2 x^2 + d^2)^{\frac{3}{2}}}{3d x^3} + \frac{5e^4 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{8d} - \frac{5e^2 \sqrt{-e^2 x^2 + d^2}}{8x^2}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{192} \left( \frac{120 e^3 \log\left(\sqrt{\frac{2d}{xe+d}} - 1 + 1\right) \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d} - \frac{120 e^3 \log\left(\left|\sqrt{\frac{2d}{xe+d}} - 1 - 1\right|\right) \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d} + \frac{4(15 e^3 \log(2))}{d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.34 Problem number 168

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6 (d + ex)^2} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{\frac{3}{2}}}{5x^5} + \frac{e(-e^2 x^2 + d^2)^{\frac{3}{2}}}{2d x^4} - \frac{7e^2(-e^2 x^2 + d^2)^{\frac{3}{2}}}{15d^2 x^3} - \frac{e^5 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{4d^2} + \frac{e^3 \sqrt{-e^2 x^2 + d^2}}{4d x^2}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{960} \left( \frac{240 e^4 \log \left( \sqrt{\frac{2d}{xe+d}} - 1 + 1 \right) \operatorname{sgn} \left( \frac{1}{xe+d} \right)}{d^2} - \frac{240 e^4 \log \left( \left| \sqrt{\frac{2d}{xe+d}} - 1 - 1 \right| \right) \operatorname{sgn} \left( \frac{1}{xe+d} \right)}{d^2} + \frac{8 (15 e^4 \log \dots)}{d^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.35 Problem number 169

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^7 (d + ex)^2} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{3/2}}{6x^6} + \frac{2e(-e^2 x^2 + d^2)^{3/2}}{5d x^5} - \frac{3e^2(-e^2 x^2 + d^2)^{3/2}}{8d^2 x^4} + \frac{4e^3(-e^2 x^2 + d^2)^{3/2}}{15d^3 x^3} + \frac{3e^6 \operatorname{arctanh} \left( \frac{\sqrt{-e^2 x^2 + d^2}}{d} \right)}{16d^3} - \frac{3e^4 \sqrt{-e^2 x^2 + d^2}}{16d^2 x^2}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^7/(e*x+d)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{7680} \left( \frac{1440 e^5 \log \left( \sqrt{\frac{2d}{xe+d}} - 1 + 1 \right) \operatorname{sgn} \left( \frac{1}{xe+d} \right)}{d^3} - \frac{1440 e^5 \log \left( \left| \sqrt{\frac{2d}{xe+d}} - 1 - 1 \right| \right) \operatorname{sgn} \left( \frac{1}{xe+d} \right)}{d^3} + \frac{16 (45 e^5 \log \dots)}{d^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.36 Problem number 170

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^8 (d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-e^2 x^2 + d^2)^{\frac{3}{2}}}{7x^7} + \frac{e(-e^2 x^2 + d^2)^{\frac{3}{2}}}{3d x^6} - \frac{11e^2(-e^2 x^2 + d^2)^{\frac{3}{2}}}{35d^2 x^5} + \frac{e^3(-e^2 x^2 + d^2)^{\frac{3}{2}}}{4d^3 x^4} \\ & - \frac{22e^4(-e^2 x^2 + d^2)^{\frac{3}{2}}}{105d^4 x^3} - \frac{e^7 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{8d^4} + \frac{e^5 \sqrt{-e^2 x^2 + d^2}}{8d^3 x^2} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{53760} \left( \frac{6720 e^6 \log\left(\sqrt{\frac{2d}{xe+d}} - 1 + 1\right) \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^4} - \frac{6720 e^6 \log\left(\left|\sqrt{\frac{2d}{xe+d}} - 1 - 1\right|\right) \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^4} + \frac{32(10}{d^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.37 Problem number 172

$$\int \frac{x^3}{(d + ex)^2 (d^2 - e^2 x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{d^2(-ex + d)^2}{5e^4(-e^2 x^2 + d^2)^{\frac{5}{2}}} - \frac{4d(-ex + d)}{5e^4(-e^2 x^2 + d^2)^{\frac{3}{2}}} + \frac{-2ex + 5d}{5de^4 \sqrt{-e^2 x^2 + d^2}}$$

command

```
integrate(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{40} \left( \frac{16i e^{(-3)} \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d} - \frac{5 e^{(-3)}}{d \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{\left(d^4 \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} e^{12} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 - 5 d^4 \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} e^{12}\right)}{d^5 \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.38 Problem number 173

$$\int \frac{x^2}{(d+ex)^2 (d^2 - e^2 x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{15d^2 e^2 \sqrt{-e^2 x^2 + d^2}} - \frac{d}{5e^3 (ex+d)^2 \sqrt{-e^2 x^2 + d^2}} + \frac{7}{15e^3 (ex+d) \sqrt{-e^2 x^2 + d^2}}$$

command

`integrate(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{120} \left( -\frac{8i e^{(-2)} \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^2} - \frac{15 e^{(-2)}}{d^2 \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} + \frac{\left(3 d^8 \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} e^8 \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 - 5 d^8 \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} e^8\right)}{d^{10} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.39 Problem number 174

$$\int \frac{x}{(d+ex)^2 (d^2 - e^2 x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{4x}{15d^3 e \sqrt{-e^2 x^2 + d^2}} + \frac{1}{5e^2 (ex+d)^2 \sqrt{-e^2 x^2 + d^2}} - \frac{2}{15d e^2 (ex+d) \sqrt{-e^2 x^2 + d^2}}$$

command

```
integrate(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{120} \left( -\frac{32i \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^3} - \frac{15}{d^3 \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{3d^{12} \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 + 5d^{12} \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{d^{15} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 16.40 Problem number 175

$$\int \frac{1}{(d+ex)^2 (d^2 - e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x}{5d^4 \sqrt{-e^2x^2 + d^2}} - \frac{1}{5de (ex + d)^2 \sqrt{-e^2x^2 + d^2}} - \frac{1}{5d^2e (ex + d) \sqrt{-e^2x^2 + d^2}}$$

command

```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{40} \left( \left( \frac{5e^{(-3)}}{d^4 \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{\left( d^{16} \left(\frac{2d}{xe+d} - 1\right)^{\frac{5}{2}} e^{12} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 + 5d^{16} \left(\frac{2d}{xe+d} - 1\right)^{\frac{3}{2}} e^{12} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 + 15d^{16} \left(\frac{2d}{xe+d} - 1\right)^{\frac{1}{2}} e^{12} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^4 \right)}{d^{20} \operatorname{sgn}\left(\frac{1}{xe+d}\right)^5} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 16.41 Problem number 176

$$\int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\frac{-\frac{2ex}{5} + \frac{2d}{5}}{d(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{-8ex + 5d}{15d^3(-e^2x^2 + d^2)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2 + d^2}}{d}\right)}{d^5} + \frac{-16ex + 15d}{15d^5\sqrt{-e^2x^2 + d^2}}$$

command

`integrate(1/x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{120} \left( \frac{120 e^{(-4)} \log\left(\sqrt{\frac{2d}{xe+d}} - 1 + 1\right)}{d^5 \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{120 e^{(-4)} \log\left(\left|\sqrt{\frac{2d}{xe+d}} - 1 - 1\right|\right)}{d^5 \operatorname{sgn}\left(\frac{1}{xe+d}\right)} - \frac{15 e^{(-4)}}{d^5 \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn}\left(\frac{1}{xe+d}\right)} \right) - \frac{(15 \log(2) - 30 \log(i+1) + 32i) \operatorname{sgn}\left(\frac{1}{xe+d}\right)}{30 d^5}$$

Giac 1.7.0 via sagemath 9.3 output

`sage_0x`

## 16.42 Problem number 177

$$\int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2e(-ex+d)}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{e(-13ex+10d)}{15d^4(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{2e \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{d^6} - \frac{e(-41ex+30d)}{15d^6\sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{d^6x}$$

command

`integrate(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{120} \left( \left( \frac{240 e^{(-5)} \log \left( \sqrt{\frac{2d}{xe+d}} - 1 + 1 \right)}{d^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right)} - \frac{240 e^{(-5)} \log \left( \left| \sqrt{\frac{2d}{xe+d}} - 1 - 1 \right| \right)}{d^6 \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right) + \frac{30 \left( \frac{17d}{xe+d} - 9 \right)}{\left( \left( \frac{2d}{xe+d} - 1 \right)^{\frac{3}{2}} - \sqrt{\frac{2d}{xe+d}} \right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 16.43 Problem number 178

$$\int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2e^2(-ex+d)}{5d^3(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{e^2(-6ex+5d)}{5d^5(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{9e^2 \operatorname{arctanh} \left( \frac{\sqrt{-e^2x^2+d^2}}{d} \right)}{2d^7} \\ & + \frac{2e^2(-11ex+10d)}{5d^7 \sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{2d^6 x^2} + \frac{2e \sqrt{-e^2x^2+d^2}}{d^7 x} \end{aligned}$$

command

`integrate(1/x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{40} \left( \left( \frac{180 e^{(-6)} \log \left( \sqrt{\frac{2d}{xe+d}} - 1 + 1 \right)}{d^7 \operatorname{sgn} \left( \frac{1}{xe+d} \right)} - \frac{180 e^{(-6)} \log \left( \left| \sqrt{\frac{2d}{xe+d}} - 1 - 1 \right| \right)}{d^7 \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right) - \frac{5 e^{(-6)}}{d^7 \sqrt{\frac{2d}{xe+d} - 1} \operatorname{sgn} \left( \frac{1}{xe+d} \right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 16.44 Problem number 179

$$\int \frac{x^5}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^4(-ex+d)^3}{5e^6(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{23d^3(-ex+d)^2}{15e^6(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{13d^2 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{2e^6} \\ & + \frac{127d^2(-ex+d)}{15e^6\sqrt{-e^2x^2+d^2}} + \frac{3d\sqrt{-e^2x^2+d^2}}{e^6} - \frac{x\sqrt{-e^2x^2+d^2}}{2e^5} \end{aligned}$$

command

```
integrate(x^5/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{13}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \operatorname{sgn}(d) - \frac{1}{2} \sqrt{-x^2 e^2 + d^2} \left(xe^{(-5)} - 6de^{(-6)}\right) \\ & 2 \left( \frac{445 \left(de + \sqrt{-x^2 e^2 + d^2} e\right) d^2 e^{(-2)}}{x} + \frac{665 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^2 d^2 e^{(-4)}}{x^2} + \frac{405 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^3 d^2 e^{(-6)}}{x^3} + \frac{90 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^4 d^2 e^{(-8)}}{x^4} \right) \\ & \frac{15 \left( \frac{\left(de + \sqrt{-x^2 e^2 + d^2} e\right) e^{(-2)}}{x} + 1 \right)^5}{15} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.45 Problem number 180

$$\int \frac{x^4}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d^3(-ex+d)^3}{5e^5(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{6d^2(-ex+d)^2}{5e^5(-e^2x^2+d^2)^{\frac{3}{2}}} \\ & -\frac{3d \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^5} - \frac{24d(-ex+d)}{5e^5\sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{e^5} \end{aligned}$$

command

```
integrate(x^4/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-3d \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sgn}(d) - \sqrt{-x^2e^2 + d^2} e^{(-5)}$$

$$+ \frac{2 \left( \frac{80 \left( de + \sqrt{-x^2e^2 + d^2} e \right) de^{(-2)}}{x} + \frac{120 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 de^{(-4)}}{x^2} + \frac{70 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 de^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^4 de^{(-8)}}{x^4} \right)}{5 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.46 Problem number 181

$$\int \frac{x^3}{(d+ex)^3 \sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$\frac{d^2(-ex+d)^3}{5e^4(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{13d(-ex+d)^2}{15e^4(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^4} + \frac{-\frac{32ex}{15} + \frac{32d}{15}}{e^4 \sqrt{-e^2x^2+d^2}}$$

command

```
integrate(x^3/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sgn}(d)$$

$$+ \frac{2 \left( \frac{95 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{145 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{75 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right)}{5 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.47 Problem number 182

$$\int \frac{x^2}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$-\frac{d\sqrt{-e^2x^2+d^2}}{5e^3(ex+d)^3} + \frac{8\sqrt{-e^2x^2+d^2}}{15e^3(ex+d)^2} - \frac{7\sqrt{-e^2x^2+d^2}}{15de^3(ex+d)}$$

command

```
integrate(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( \frac{5 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{10 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + 1 \right) e^{(-3)}}{15 d \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.48 Problem number 183

$$\int \frac{x}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-e^2x^2+d^2}}{5e^2(ex+d)^3} - \frac{\sqrt{-e^2x^2+d^2}}{5de^2(ex+d)^2} - \frac{\sqrt{-e^2x^2+d^2}}{5d^2e^2(ex+d)}$$

command

```
integrate(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + 1 \right) e^{(-2)}$$


---


$$5 d^2 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.49 Problem number 184

$$\int \frac{1}{(d + ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{-e^2 x^2 + d^2}}{5de (ex + d)^3} - \frac{2\sqrt{-e^2 x^2 + d^2}}{15d^2 e (ex + d)^2} - \frac{2\sqrt{-e^2 x^2 + d^2}}{15d^3 e (ex + d)}$$

command

`integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{20 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{40 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{30 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} + 1 \right) e^{(-2)}$$


---


$$15 d^3 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.50 Problem number 185

$$\int \frac{1}{x(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\frac{-\frac{4ex}{5} + \frac{4d}{5}}{(-e^2 x^2 + d^2)^{\frac{5}{2}}} + \frac{-11ex + 5d}{15d^2 (-e^2 x^2 + d^2)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{d^4} + \frac{-22ex + 15d}{15d^4 \sqrt{-e^2 x^2 + d^2}}$$

command

`integrate(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\frac{|-2de-2\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|}\right)}{d^4} - 2\left(\frac{115\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x} + \frac{185\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{(-4)}}{x^2} + \frac{135\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{(-6)}}{x^3} + \frac{45\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{(-8)}}{x^4}\right) - 15d^4\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x} + 1\right)^5$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.51 Problem number 188

$$\int \frac{x^5 \sqrt{d^2 - e^2 x^2}}{(d+ex)^4} dx$$

Optimal antiderivative

$$\frac{d^4(-ex+d)^4}{5e^6(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{8d^3(-ex+d)^3}{5e^6(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{18d^3 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{e^6} + \frac{10d^2(-ex+d)^2}{e^6\sqrt{-e^2x^2+d^2}} + \frac{59d^2\sqrt{-e^2x^2+d^2}}{3e^6} - \frac{2dx\sqrt{-e^2x^2+d^2}}{e^5} + \frac{x^2\sqrt{-e^2x^2+d^2}}{3e^4}$$

command

```
integrate(x^5*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$18 d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \operatorname{sgn}(d) + \frac{1}{3} \sqrt{-x^2 e^2 + d^2} \left(29 d^2 e^{(-6)} + \left(xe^{(-4)} - 6 de^{(-5)}\right)x\right) \\ 2 \left( \frac{385 \left(de + \sqrt{-x^2 e^2 + d^2} e\right) d^3 e^{(-2)}}{x} + \frac{575 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^2 d^3 e^{(-4)}}{x^2} + \frac{355 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^3 d^3 e^{(-6)}}{x^3} + \frac{80 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^4 d^3 e^{(-8)}}{x^4} \right) \\ \frac{5 \left( \frac{\left(de + \sqrt{-x^2 e^2 + d^2} e\right) e^{(-2)}}{x} + 1 \right)^5}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.52 Problem number 189

$$\int \frac{x^4 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{d^3(-ex+d)^4}{5e^5(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{19d^2(-ex+d)^3}{15e^5(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{19d^2 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{2e^5} \\ - \frac{6d(-ex+d)^2}{e^5 \sqrt{-e^2x^2+d^2}} - \frac{(-ex+20d) \sqrt{-e^2x^2+d^2}}{2e^5}$$

command

```
integrate(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{19}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sgn}(d) + \frac{1}{2} \sqrt{-x^2 e^2 + d^2} \left(xe^{(-4)} - 8 de^{(-5)}\right) \\ 2 \left( \frac{685 \left(de + \sqrt{-x^2 e^2 + d^2} e\right) d^2 e^{(-2)}}{x} + \frac{1025 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^2 d^2 e^{(-4)}}{x^2} + \frac{615 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^3 d^2 e^{(-6)}}{x^3} + \frac{135 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)^4 d^2 e^{(-8)}}{x^4} \right) \\ \frac{15 \left( \frac{\left(de + \sqrt{-x^2 e^2 + d^2} e\right) e^{(-2)}}{x} + 1 \right)^5}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 16.53 Problem number 190

$$\int \frac{x^3 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\frac{d^2(-e^2x^2 + d^2)^{\frac{3}{2}}}{5e^4(ex + d)^4} - \frac{14d(-e^2x^2 + d^2)^{\frac{3}{2}}}{15e^4(ex + d)^3} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}}}{e^4(ex + d)^2} + \frac{4d \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{e^4} + \frac{8d\sqrt{-e^2x^2 + d^2}}{e^4(ex + d)}$$

command

`integrate(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4d \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sgn}(d) + \sqrt{-x^2e^2 + d^2} e^{(-4)} - \frac{2 \left( \frac{335 \left( de + \sqrt{-x^2e^2 + d^2} e \right) de^{(-2)}}{x} + \frac{505 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 de^{(-4)}}{x^2} + \frac{285 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 de^{(-6)}}{x^3} + \frac{60 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^4 de^{(-8)}}{x^4} \right)}{15 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.54 Problem number 191

$$\int \frac{x^2 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{d(-e^2x^2 + d^2)^{\frac{3}{2}}}{5e^3(ex + d)^4} + \frac{3(-e^2x^2 + d^2)^{\frac{3}{2}}}{5e^3(ex + d)^3} - \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{e^3} - \frac{2\sqrt{-e^2x^2 + d^2}}{e^3(ex + d)}$$

command

`integrate(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & -\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\operatorname{sgn}(d) \\
 & 2\left(\frac{35\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x} + \frac{55\left(de+\sqrt{-x^2e^2+d^2}\right)^2e^{(-4)}}{x^2} + \frac{25\left(de+\sqrt{-x^2e^2+d^2}\right)^3e^{(-6)}}{x^3} + \frac{5\left(de+\sqrt{-x^2e^2+d^2}\right)^4e^{(-8)}}{x^4}\right) \\
 & + \frac{\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x} + 1\right)^5}{5}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.55 Problem number 192

$$\int \frac{x\sqrt{d^2 - e^2x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\frac{(-e^2x^2 + d^2)^{\frac{3}{2}}}{5e^2(ex + d)^4} - \frac{4(-e^2x^2 + d^2)^{\frac{3}{2}}}{15de^2(ex + d)^3}$$

command

`integrate(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & 2\left(\frac{5\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x} - \frac{5\left(de+\sqrt{-x^2e^2+d^2}\right)^2e^{(-4)}}{x^2} + \frac{15\left(de+\sqrt{-x^2e^2+d^2}\right)^3e^{(-6)}}{x^3} + 1\right)e^{(-2)} \\
 & + \frac{\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}\right)e^{(-2)}}{x} + 1\right)^5}{15d}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.56 Problem number 193

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{(-e^2 x^2 + d^2)^{\frac{3}{2}}}{5de (ex + d)^4} - \frac{(-e^2 x^2 + d^2)^{\frac{3}{2}}}{15d^2 e (ex + d)^3}$$

command

```
integrate((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{5 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + \frac{25 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 e^{(-4)}}{x^2} + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 e^{(-6)}}{x^3} + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^4 e^{(-8)}}{x^4} \right) - \frac{15 d^2 \left( \frac{\left( de + \sqrt{-x^2 e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)^5}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.57 Problem number 194

$$\int \frac{\sqrt{d^2 - e^2 x^2}}{x(d + ex)^4} dx$$

Optimal antiderivative

$$\frac{8d(-ex + d)}{5(-e^2 x^2 + d^2)^{\frac{5}{2}}} - \frac{4ex}{5d(-e^2 x^2 + d^2)^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{d^3} + \frac{-8ex + 5d}{5d^3 \sqrt{-e^2 x^2 + d^2}}$$

command

```
integrate((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\frac{-2de-2\sqrt{-x^2e^2+d^2}e^{(-2)}}{2|x|}\right)}{d^3} - 2\left(\frac{45\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x} + \frac{75\left(de+\sqrt{-x^2e^2+d^2}e\right)^2e^{(-4)}}{x^2} + \frac{55\left(de+\sqrt{-x^2e^2+d^2}e\right)^3e^{(-6)}}{x^3} + \frac{20\left(de+\sqrt{-x^2e^2+d^2}e\right)^4e^{(-8)}}{x^4}\right) - \frac{5d^3\left(\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x} + 1\right)^5}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.58 Problem number 198

$$\int \frac{x^5(d^2 - e^2x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{65d^7 \arctan\left(\frac{ex}{\sqrt{-e^2x^2+d^2}}\right)}{4e^6} + \frac{d^4(-ex+d)^4}{e^6\sqrt{-e^2x^2+d^2}} + \frac{515d^6\sqrt{-e^2x^2+d^2}}{21e^6} \\ & - \frac{49d^5x\sqrt{-e^2x^2+d^2}}{4e^5} + \frac{121d^4x^2\sqrt{-e^2x^2+d^2}}{21e^4} - \frac{17d^3x^3\sqrt{-e^2x^2+d^2}}{6e^3} \\ & + \frac{11d^2x^4\sqrt{-e^2x^2+d^2}}{7e^2} - \frac{2dx^5\sqrt{-e^2x^2+d^2}}{3e} + \frac{x^6\sqrt{-e^2x^2+d^2}}{7} \end{aligned}$$

command

`integrate(x^5*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{65}{4}d^7 \arcsin\left(\frac{xe}{d}\right)e^{(-6)}\operatorname{sgn}(d) - \frac{16d^7e^{(-6)}}{\frac{\left(de+\sqrt{-x^2e^2+d^2}e\right)e^{(-2)}}{x} + 1} \\ & + \frac{1}{84}\left(1472d^6e^{(-6)} - \left(693d^5e^{(-5)} - 2\left(200d^4e^{(-4)} - \left(119d^3e^{(-3)} - 2\left(33d^2e^{(-2)} - \left(14de^{(-1)} - 3x\right)x\right)x\right)x\right)x\right)x\right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.59 Problem number 199

$$\int \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{239d^6 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{16e^5} - \frac{d^3(-ex + d)^4}{e^5 \sqrt{-e^2x^2 + d^2}} \\ & - \frac{337d^5 \sqrt{-e^2x^2 + d^2}}{15e^5} + \frac{175d^4 x \sqrt{-e^2x^2 + d^2}}{16e^4} - \frac{71d^3 x^2 \sqrt{-e^2x^2 + d^2}}{15e^3} \\ & + \frac{47d^2 x^3 \sqrt{-e^2x^2 + d^2}}{24e^2} - \frac{4d x^4 \sqrt{-e^2x^2 + d^2}}{5e} + \frac{x^5 \sqrt{-e^2x^2 + d^2}}{6} \end{aligned}$$

command

```
integrate(x^4*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{239}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sgn}(d) + \frac{16 d^6 e^{(-5)}}{\left(\frac{de + \sqrt{-x^2 e^2 + d^2}}{e}\right) e^{(-2)} + 1} \\ & -\frac{1}{240} \left(3712 d^5 e^{(-5)} - \left(1665 d^4 e^{(-4)} - 2 \left(448 d^3 e^{(-3)} - \left(235 d^2 e^{(-2)} - 4 \left(24 d e^{(-1)} - 5 x\right) x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.60 Problem number 200

$$\int \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{27d^5 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2e^4} + \frac{d^2(-ex + d)^4}{e^4 \sqrt{-e^2x^2 + d^2}} + \frac{101d^4 \sqrt{-e^2x^2 + d^2}}{5e^4} \\ & - \frac{19d^3 x \sqrt{-e^2x^2 + d^2}}{2e^3} + \frac{18d^2 x^2 \sqrt{-e^2x^2 + d^2}}{5e^2} - \frac{d x^3 \sqrt{-e^2x^2 + d^2}}{e} + \frac{x^4 \sqrt{-e^2x^2 + d^2}}{5} \end{aligned}$$

command

```
integrate(x^3*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{27}{2} d^5 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sgn}(d) - \frac{16 d^5 e^{(-4)}}{\left(\frac{de + \sqrt{-x^2 e^2 + d^2} e}{x}\right) e^{(-2)} + 1} + \frac{1}{10} \left(132 d^4 e^{(-4)} - \left(55 d^3 e^{(-3)} - 2 \left(13 d^2 e^{(-2)} - \left(5 d e^{(-1)} - x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.61 Problem number 201

$$\int \frac{x^2 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{95d^4 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{8e^3} - \frac{d(-ex + d)^4}{e^3 \sqrt{-e^2x^2 + d^2}} - \frac{95d^3 \sqrt{-e^2x^2 + d^2}}{8e^3} \\ & - \frac{95d^2(-ex + d) \sqrt{-e^2x^2 + d^2}}{24e^3} - \frac{19d(-ex + d)^2 \sqrt{-e^2x^2 + d^2}}{12e^3} - \frac{(-ex + d)^3 \sqrt{-e^2x^2 + d^2}}{4e^3} \end{aligned}$$

command

```
integrate(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{95}{8} d^4 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sgn}(d) + \frac{16 d^4 e^{(-3)}}{\left(\frac{de + \sqrt{-x^2 e^2 + d^2} e}{x}\right) e^{(-2)} + 1} \\ & - \frac{1}{24} \left(256 d^3 e^{(-3)} - \left(93 d^2 e^{(-2)} - 2 \left(16 d e^{(-1)} - 3 x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.62 Problem number 202

$$\int \frac{x(d^2 - e^2x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{20(-e^2x^2 + d^2)^{\frac{3}{2}}}{3e^2} + \frac{8(-e^2x^2 + d^2)^{\frac{5}{2}}}{e^2(ex + d)^2} + \frac{(-e^2x^2 + d^2)^{\frac{7}{2}}}{e^2(ex + d)^4} \\ & + \frac{10d^3 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{e^2} + \frac{10dx\sqrt{-e^2x^2 + d^2}}{e} \end{aligned}$$

command

```
integrate(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & 10d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-2)\operatorname{sgn}(d)} - \frac{16d^3e^{(-2)}}{\frac{(de + \sqrt{-x^2e^2 + d^2}e)^{e^{(-2)}}}{x} + 1} \\ & + \frac{1}{3} \sqrt{-x^2e^2 + d^2} \left(23d^2e^{(-2)} - (6de^{(-1)} - x)x\right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.63 Problem number 203

$$\int \frac{(d^2 - e^2x^2)^{5/2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$-\frac{5(-e^2x^2 + d^2)^{\frac{3}{2}}}{2e(ex + d)} - \frac{2(-e^2x^2 + d^2)^{\frac{5}{2}}}{e(ex + d)^3} - \frac{15d^2 \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2e} - \frac{15d\sqrt{-e^2x^2 + d^2}}{2e}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-1)} \operatorname{sgn}(d) + \frac{16 d^2 e^{(-1)}}{\frac{(de + \sqrt{-x^2 e^2 + d^2}) e^{(-2)}}{x} + 1} - \frac{1}{2} \sqrt{-x^2 e^2 + d^2} (8 d e^{(-1)} - x)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.64 Problem number 204

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x(d + ex)^4} dx$$

Optimal antiderivative

$$4d \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right) - d \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right) + \frac{8d(-ex + d)}{\sqrt{-e^2 x^2 + d^2}} + \sqrt{-e^2 x^2 + d^2}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4d \arcsin\left(\frac{xe}{d}\right) \operatorname{sgn}(d) - d \log\left(\frac{|-2de - 2\sqrt{-x^2 e^2 + d^2} e| e^{(-2)}}{2|x|}\right) + \sqrt{-x^2 e^2 + d^2} - \frac{16d}{\frac{(de + \sqrt{-x^2 e^2 + d^2}) e^{(-2)}}{x} + 1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.65 Problem number 205

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2(d + ex)^4} dx$$

Optimal antiderivative

$$-e \arctan\left(\frac{ex}{\sqrt{-e^2 x^2 + d^2}}\right) + 4e \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right) - \frac{8e(-ex + d)}{\sqrt{-e^2 x^2 + d^2}} - \frac{\sqrt{-e^2 x^2 + d^2}}{x}$$





Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.67 Problem number 207

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^4 (d + ex)^4} dx$$

Optimal antiderivative

$$\frac{10e^3 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{d^2} - \frac{8e^3(-ex + d)}{d^2 \sqrt{-e^2 x^2 + d^2}} - \frac{\sqrt{-e^2 x^2 + d^2}}{3x^3} + \frac{2e \sqrt{-e^2 x^2 + d^2}}{dx^2} - \frac{23e^2 \sqrt{-e^2 x^2 + d^2}}{3d^2 x}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10e^3 \log\left(\frac{|-2de-2\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|}\right)}{d^2} - \frac{x^3 \left( \frac{11 \left( de + \sqrt{-x^2e^2 + d^2} e \right) e}{x} - \frac{81 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 e^{(-1)}}{x^2} - \frac{477 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 e^{(-3)}}{x^3} - e^3 \right) e^6}{24 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 d^2 \left( \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right) e^{(-2)}}{x} + 1 \right)} - \frac{93 \left( de + \sqrt{-x^2e^2 + d^2} e \right) d^4 e}{x} - \frac{12 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 d^4 e^{(-1)}}{x^2} + \frac{\left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 d^4 e^{(-3)}}{x^3}}{24d^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.68 Problem number 208

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^5 (d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{95e^4 \operatorname{arctanh}\left(\frac{\sqrt{-e^2 x^2 + d^2}}{d}\right)}{8d^3} + \frac{8e^4(-ex + d)}{d^3 \sqrt{-e^2 x^2 + d^2}} - \frac{\sqrt{-e^2 x^2 + d^2}}{4x^4} \\ & + \frac{4e \sqrt{-e^2 x^2 + d^2}}{3d x^3} - \frac{31e^2 \sqrt{-e^2 x^2 + d^2}}{8d^2 x^2} + \frac{32e^3 \sqrt{-e^2 x^2 + d^2}}{3d^3 x} \end{aligned}$$

command

```
integrate((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{95 e^4 \log\left(\frac{|-2de-2\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|}\right)}{8d^3} \\ & x^4 \left( \frac{29 \left( de + \sqrt{-x^2e^2+d^2}e \right) e^2}{x} + \frac{864 \left( de + \sqrt{-x^2e^2+d^2}e \right)^3 e^{(-2)}}{x^3} + \frac{4128 \left( de + \sqrt{-x^2e^2+d^2}e \right)^4 e^{(-4)}}{x^4} - \frac{160 \left( de + \sqrt{-x^2e^2+d^2}e \right)^5 e^{(-6)}}{x^5} \right) \\ & + \frac{1056 \left( de + \sqrt{-x^2e^2+d^2}e \right) d^9 e^2}{x} + \frac{32 \left( de + \sqrt{-x^2e^2+d^2}e \right)^3 d^9 e^{(-2)}}{x^3} - \frac{3 \left( de + \sqrt{-x^2e^2+d^2}e \right)^4 d^9 e^{(-4)}}{x^4} - \frac{192 \left( de + \sqrt{-x^2e^2+d^2}e \right)^5 d^9 e^{(-6)}}{x^5} \\ & \frac{192 \left( de + \sqrt{-x^2e^2+d^2}e \right)^4 d^3 \left( \frac{\left( de + \sqrt{-x^2e^2+d^2}e \right) e^{(-2)}}{x} + 1 \right)}{192 d^{12}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.69 Problem number 209

$$\int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6 (d + ex)^4} dx$$

Optimal antiderivative

$$\frac{27e^5 \operatorname{arctanh}\left(\frac{\sqrt{-e^2x^2+d^2}}{d}\right)}{2d^4} - \frac{8e^5(-ex+d)}{d^4\sqrt{-e^2x^2+d^2}} - \frac{\sqrt{-e^2x^2+d^2}}{5x^5} + \frac{e\sqrt{-e^2x^2+d^2}}{dx^4} - \frac{13e^2\sqrt{-e^2x^2+d^2}}{5d^2x^3} + \frac{11e^3\sqrt{-e^2x^2+d^2}}{2d^3x^2} - \frac{66e^4\sqrt{-e^2x^2+d^2}}{5d^4x}$$

command

`integrate((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{27e^5 \log\left(\frac{\left| \frac{-2de-2\sqrt{-x^2e^2+d^2}}{2|x|} e^{(-2)} \right|}{2d^4}\right)}{2d^4} - \frac{x^5 \left( \frac{9 \left( de + \sqrt{-x^2e^2+d^2} \right) e^3}{x} - \frac{45 \left( de + \sqrt{-x^2e^2+d^2} \right)^2 e}{x^2} + \frac{185 \left( de + \sqrt{-x^2e^2+d^2} \right)^3 e^{(-1)}}{x^3} - \frac{870 \left( de + \sqrt{-x^2e^2+d^2} \right)^4 e^{(-2)}}{x^4} + \frac{160 \left( de + \sqrt{-x^2e^2+d^2} \right)^5 d^4 \left( \frac{\left( de + \sqrt{-x^2e^2+d^2} \right) e^{(-2)}}{x} + \frac{1110 \left( de + \sqrt{-x^2e^2+d^2} \right) d^{16} e^3}{x} - \frac{240 \left( de + \sqrt{-x^2e^2+d^2} \right)^2 d^{16} e}{x^2} + \frac{55 \left( de + \sqrt{-x^2e^2+d^2} \right)^3 d^{16} e^{(-1)}}{x^3} - \frac{10 \left( de + \sqrt{-x^2e^2+d^2} \right)^4 d^{16} e^{(-2)}}{x^4} \right)}{160d^{20}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.70 Problem number 437

$$\int \frac{x^3 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)(5a^3e^6 + 9a^2cd^2e^4 + 15a^2c^2d^4e^2 + 35c^3d^6) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{128c^{\frac{7}{2}}d^{\frac{7}{2}}e^{\frac{9}{2}}} + \frac{\left(\frac{a}{cd} - \frac{7d}{e^2}\right)x^2\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{24} + \frac{x^3\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{4e} - \frac{(105c^3d^6 - 25a^2c^2d^4e^2 - 17a^2cd^2e^4 - 15a^3e^6 - 2cde(-5a^2e^4 - 6acd^2e^2 + 35c^2d^4)x)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{192c^3d^3e^4}$$

command

```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{192} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 6xe^{(-1)} - \frac{(7c^3d^4e^2 - ac^2d^2e^4)e^{(-4)}}{c^3d^3} \right) x + \frac{(35c^3d^5e - 6ac^2d^3e^3 - 5a^2e^5)}{c^3d^3} \right) \right. \\ \left. + \frac{(35c^4d^8 - 20ac^3d^6e^2 - 6a^2c^2d^4e^4 - 4a^3cd^2e^6 - 5a^4e^8)e^{(-\frac{9}{2})} \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right)}{128 \sqrt{cd} c^3d^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.71 Problem number 438

$$\int \frac{x^2 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \operatorname{arctanh} \left( \frac{2cdex + ae^2 + cd^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \right)}{16c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \\ + \frac{x^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3e} \\ + \frac{((-3ae^2 + 5cd^2)(ae^2 + 3cd^2) - 2cde(-ae^2 + 5cd^2)x) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{24c^2d^2e^3}$$

command

```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{24} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4xe^{(-1)} - \frac{(5c^2d^3e - acde^3)e^{(-3)}}{c^2d^2} \right) x + \frac{(15c^2d^4 - 4acd^2e^2 - 3a^2e^4)e^{(-3)}}{c^2d^2} \right) \\ + \frac{(5c^3d^6 - 3ac^2d^4e^2 - a^2cd^2e^4 - a^3e^6)e^{(-\frac{7}{2})} \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right) \sqrt{cd} e^{\frac{1}{2}}}{16 \sqrt{cd} c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.72 Problem number 439

$$\int \frac{x \sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{d + ex} dx$$

Optimal antiderivative

$$\frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{2cde(ex + d)} + \frac{(-ae^2 + cd^2)(ae^2 + 3cd^2) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{8c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{5}{2}}} - \frac{\left(\frac{a}{cd} + \frac{3d}{e^2}\right)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4}$$

command

`integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2xe^{(-1)} - \frac{(3cd^2 - ae^2)e^{(-2)}}{cd} \right) - \frac{(3c^2d^4 - 2acd^2e^2 - a^2e^4)e^{(-\frac{5}{2})} \log\left(\left| -cd^2 - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)\sqrt{cd}e^{\frac{1}{2}} - ae^2 \right|\right)}{8\sqrt{cd}cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.73 Problem number 440

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{d + ex} dx$$

Optimal antiderivative

$$-\frac{(-ae^2 + cd^2) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{2e^{\frac{3}{2}}\sqrt{c}\sqrt{d}} + \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{e}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{cdx^2e + cd^2x + axe^2 + ade} e^{(-1)} (cd^2 - ae^2) \sqrt{cd} e^{(-\frac{3}{2})} \log \left( \left| -\sqrt{cd} cd^2e^{\frac{1}{2}} - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) cde - \sqrt{cd} ae^{\frac{5}{2}} \right| \right)}{2cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.74 Problem number 442

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{x^2(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2) \operatorname{arctanh} \left( \frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cde x^2}} \right)}{2d^{\frac{3}{2}} \sqrt{a} \sqrt{e}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{dx}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(cd^2 - ae^2) \arctan \left( -\frac{\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}} \right)}{\sqrt{-ade} d} - \frac{\left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) cd^2 + 2 \sqrt{cd} ade^{\frac{3}{2}} + \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) a}{\left( ade - \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right)^2 \right) d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.75 Problem number 443

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdx^2}}{x^3(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)(3ae^2 + cd^2) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}\right)}{8a^{\frac{3}{2}}d^{\frac{5}{2}}e^{\frac{3}{2}}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{2dx^2} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right)\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{4x}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^3/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(c^2d^4 + 2acd^2e^2 - 3a^2e^4) \operatorname{arctan}\left(\frac{-\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right) e^{(-1)}}{4\sqrt{-ade}ad^2} + \frac{\left(\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)ac^2d^5e + \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3c^2d^4 + 8\right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.76 Problem number 444

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdx^2}}{x^4(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)(5a^2e^4 + 2acd^2e^2 + c^2d^4) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}\right)}{16a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}} - \frac{\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{3dx^3} - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right)\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{12x^2} + \frac{(-5ae^2 + 3cd^2)(3ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdx^2}}{24a^2d^3e^2x}$$



command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(c^3 d^6 + a c^2 d^4 e^2 + 3 a^2 c d^2 e^4 - 5 a^3 e^6) \arctan\left(-\frac{\sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2 e + cd^2 x + axe^2 + ade}}{\sqrt{-ade}}\right) e^{(-2)}}{8 \sqrt{-ade} a^2 d^3} \\ - \frac{\left(3 \left(\sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2 e + cd^2 x + axe^2 + ade}\right) a^2 c^3 d^8 e^2 + 8 \left(\sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2 e + cd^2 x + axe^2 + ade}\right)^3 a c^3 d\right)}{192 a^3 d^4 e^3 x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.77 Problem number 445

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{x^5(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-ae^2 + cd^2)(35a^3e^6 + 15a^2cd^2e^4 + 9ac^2d^4e^2 + 5c^3d^6) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{128a^{\frac{7}{2}}d^{\frac{9}{2}}e^{\frac{7}{2}}} \\ - \frac{\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4dx^4} - \frac{\left(\frac{c}{ae} - \frac{7e}{d^2}\right) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{24x^3} \\ + \frac{(-35a^2e^4 + 6acd^2e^2 + 5c^2d^4) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{96a^2d^3e^2x^2} \\ - \frac{(-105a^3e^6 + 25a^2cd^2e^4 + 17ac^2d^4e^2 + 15c^3d^6) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{192a^3d^4e^3x}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.78 Problem number 446

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{a}{cd} - \frac{3d}{e^2}\right) x^2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{20} + \frac{x^3(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{6e} \\ & - \frac{(105c^3d^6 - 21a^2c^2d^4e^2 - 33a^2cd^2e^4 - 35a^3e^6 - 6cde(-7a^2e^4 - 6acd^2e^2 + 21c^2d^4)x)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{960c^3d^3e^4} \\ & - \frac{(-ae^2 + cd^2)^3(7a^3e^6 + 15a^2cd^2e^4 + 21a^2c^2d^4e^2 + 21c^3d^6) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{1024c^{\frac{9}{2}}d^{\frac{9}{2}}e^{\frac{11}{2}}} \\ & + \frac{(-7a^4e^8 - 8a^3cd^2e^6 - 6a^2c^2d^4e^4 + 21c^4d^8)(2cde x + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{512c^4d^4e^5} \end{aligned}$$

command

```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{7680} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10cdx + \frac{(c^6d^7e^4 + 13ac^5d^5e^6)e^{(-5)}}{c^5d^5} \right) x - \frac{(9c^6d^8e^3 - 14ac^5d^6)}{c^5} \right) \right) \right) \right) \\ & + \frac{(21c^6d^{12} - 42ac^5d^{10}e^2 + 15a^2c^4d^8e^4 + 4a^3c^3d^6e^6 + 3a^4c^2d^4e^8 + 6a^5cd^2e^{10} - 7a^6e^{12})e^{(-\frac{11}{2})} \log\left(\left| -cd^2 - 2\left(\sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) \right|\right)}{1024\sqrt{cd}c^4d^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.79 Problem number 447

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{5e} \\ & + \frac{(35c^2d^4 - 12acd^2e^2 - 15a^2e^4 - 6cde(-3ae^2 + 7cd^2)x)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{240c^2d^2e^3} \\ & + \frac{(-ae^2 + cd^2)^3(3a^2e^4 + 6acd^2e^2 + 7c^2d^4) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{256c^{\frac{7}{2}}d^{\frac{7}{2}}e^{\frac{9}{2}}} \\ & - \frac{(-ae^2 + cd^2)(3a^2e^4 + 6acd^2e^2 + 7c^2d^4)(2cde x + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{128c^3d^3e^4} \end{aligned}$$

command

```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{1920} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 6 \left( 8cdx + \frac{(c^5d^6e^3 + 11ac^4d^4e^5)e^{(-4)}}{c^4d^4} \right) x - \frac{(7c^5d^7e^2 - 12ac^4d^5e^4 - (7c^5d^{10} - 15ac^4d^8e^2 + 6a^2c^3d^6e^4 + 2a^3c^2d^4e^6 + 3a^4cd^2e^8 - 3a^5e^{10})e^{(-\frac{9}{2})}}{c^4d^4} \right) \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right) \right) \right) \right) \frac{1}{256 \sqrt{cd} c^3 d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.80 Problem number 448

$$\int \frac{x(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\frac{3a}{cd} + \frac{5d}{e^2}\right) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{24} + \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{4cde(ex + d)} \\ & - \frac{(-ae^2 + cd^2)^3 (3ae^2 + 5cd^2) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{128c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \\ & + \frac{(-ae^2 + cd^2)(3ae^2 + 5cd^2)(2cde x + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{64c^2d^2e^3} \end{aligned}$$

command

```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{192} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 6cdx + \frac{(c^4d^5e^2 + 9ac^3d^3e^4)e^{(-3)}}{c^3d^3} \right) x - \frac{(5c^4d^6e - 10ac^3d^4e^3 - 3a^2c^2d^2e^5 - (5c^4d^8 - 12ac^3d^6e^2 + 6a^2c^2d^4e^4 + 4a^3cd^2e^6 - 3a^4e^8)e^{(-\frac{7}{2})}}{c^3d^3} \right) \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right) \right) \right) \frac{1}{128 \sqrt{cd} c^2 d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.81 Problem number 449

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3e} \\ & + \frac{(-ae^2 + cd^2)^3 \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{16c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{5}{2}}} \\ & + \frac{\left(\frac{a}{cd} - \frac{d}{e^2}\right)(2cdex + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{8} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{24} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4cdx + \frac{(c^3d^4e + 7ac^2d^2e^3)e^{(-2)}}{c^2d^2} \right) x - \frac{(3c^3d^5 - 8ac^2d^3e^2 - 3a^2cde^4)e^{(-2)}}{c^2d^2} \right. \\ & \left. - \frac{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6)e^{(-\frac{5}{2})} \log\left(\left| -cd^2 - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)\sqrt{cd}e^{\frac{1}{2}} \right.\right)}{16\sqrt{cd}cd} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.82 Problem number 451

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3ae^2 + cd^2) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right) \sqrt{c}\sqrt{d}}{2\sqrt{e}} \\ & - \frac{(ae^2 + 3cd^2) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right) \sqrt{a}\sqrt{e}}{2\sqrt{d}} \\ & - \frac{(-cdx + ae)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{x} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{cdx^2e + cd^2x + axe^2 + ade} cd}{(3acd^2e + a^2e^3) \arctan\left(-\frac{\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)} + \frac{\sqrt{-ade}}{\left(\sqrt{cd} c^2 d^3 e^{\frac{1}{2}} + 3\sqrt{cd} acde^{\frac{5}{2}}\right) e^{(-1)} \log\left(\left|-\sqrt{cd} cd^2 e^{\frac{1}{2}} - 2\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) cde - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right|\right)} - \frac{2cd}{\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) acd^2e + 2\sqrt{cd} a^2 de^{\frac{5}{2}} + \left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2} - \frac{ade - \left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2}{ade - \left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.83 Problem number 452

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d + ex)} dx$$

Optimal antiderivative

$$\frac{(-a^2e^4 + 6acd^2e^2 + 3c^2d^4) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{8d^{\frac{3}{2}}\sqrt{a}\sqrt{e}} + \frac{c^{\frac{3}{2}}d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right) \sqrt{e}}{(2ade + (ae^2 + 5cd^2)x) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} - \frac{2ade + (ae^2 + 5cd^2)x}{4dx^2}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\sqrt{cd} cde^{\frac{1}{2}} \log\left(\left|-\sqrt{cd} cd^2 e^{\frac{1}{2}} - 2\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) cde - \sqrt{cd} ae^{\frac{5}{2}}\right|\right) + \frac{(3c^2d^4 + 6acd^2e^2 - a^2e^4) \arctan\left(-\frac{\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}d} - \frac{3\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right) ac^2d^5e - 5\left(\sqrt{cd} xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3 c^2d^4 + 8}{4\sqrt{-ade}d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.84 Problem number 453

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{x^4(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{3dx^3} \\ & + \frac{(-ae^2 + cd^2)^3 \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}} \\ & - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{8x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^4/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(c^3d^6 - 3ac^2d^4e^2 + 3a^2cd^2e^4 - a^3e^6) \operatorname{arctan}\left(\frac{\sqrt{cd}xe^{1/2} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right) e^{(-1)}}{8\sqrt{-ade}ad^2} \\ & + \frac{\left(3\left(\sqrt{cd}xe^{1/2} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^2c^3d^8e^2 - 8\left(\sqrt{cd}xe^{1/2} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3ac^3d^7\right)}{8\sqrt{-ade}ad^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.85 Problem number 454

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{x^5(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{4dx^4} - \frac{(\frac{3c}{ae} - \frac{5e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{24x^3} \\ & - \frac{(-ae^2 + cd^2)^3(5ae^2 + 3cd^2) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{128a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}} \\ & + \frac{(-ae^2 + cd^2)(5ae^2 + 3cd^2)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{64a^2d^3e^2x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.86 Problem number 455

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{x^6(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{5dx^5} - \frac{(\frac{3c}{ae} - \frac{7e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{40x^4} \\ & + \frac{(-35a^2e^4 + 12acd^2e^2 + 15c^2d^4)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{240a^2d^3e^2x^3} \\ & + \frac{(-ae^2 + cd^2)^3(7a^2e^4 + 6acd^2e^2 + 3c^2d^4) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{256a^{\frac{7}{2}}d^{\frac{9}{2}}e^{\frac{7}{2}}} \\ & - \frac{(-ae^2 + cd^2)(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{128a^3d^4e^3x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.87 Problem number 456

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{x^7(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{6dx^6} - \frac{(\frac{c}{ae} - \frac{3e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{20x^5} \\ & + \frac{(-21a^2e^4 + 6acd^2e^2 + 7c^2d^4)(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{160a^2d^3e^2x^4} \\ & - \frac{(-105a^3e^6 + 21a^2cd^2e^4 + 33a^2c^2d^4e^2 + 35c^3d^6)(ade + (ae^2 + cd^2)x + cde x^2)^{3/2}}{960a^3d^4e^3x^3} \\ & - \frac{(-ae^2 + cd^2)^3(21a^3e^6 + 21a^2cd^2e^4 + 15a^2c^2d^4e^2 + 7c^3d^6) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{1024a^{\frac{9}{2}}d^{\frac{11}{2}}e^{\frac{9}{2}}} \\ & + \frac{(-21a^4e^8 + 6a^2c^2d^4e^4 + 8a^3c^3d^6e^2 + 7c^4d^8)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{512a^4d^5e^4x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 16.88 Problem number 457

$$\int \frac{x^3(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ae^2 + cd^2)(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(2cdex + ae^2 + cd^2)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{2048c^4d^4e^5} \\ & + \frac{(\frac{5a}{cd} - \frac{11d}{e^2})x^2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{112} + \frac{x^3(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{8e} \\ & - \frac{(231c^3d^6 - 15ac^2d^4e^2 - 95a^2cd^2e^4 - 105a^3e^6 - 10cde(-15a^2e^4 - 10acd^2e^2 + 33c^2d^4)x)(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{4480c^3d^3e^4} \\ & + \frac{3(-ae^2 + cd^2)^5(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}\right)}{32768c^{\frac{11}{2}}d^{\frac{11}{2}}e^{\frac{13}{2}}} \\ & - \frac{3(-ae^2 + cd^2)^3(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(2cdex + ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{16384c^5d^5e^6} \end{aligned}$$

command

```
integrate(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{573440} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10 \left( 4 \left( 14c^2d^2xe + \frac{(17c^9d^{10}e^7 + 33ac^8d^8e^9)e^{(-7)}}{c^7d^7} \right) x + \frac{(c^9d^{10}e^7 + 33ac^8d^8e^9)e^{(-7)}}{c^7d^7} \right) \right) \right) \right) \right) \right) \\ & - \frac{3(33c^8d^{16} - 120ac^7d^{14}e^2 + 140a^2c^6d^{12}e^4 - 40a^3c^5d^{10}e^6 - 10a^4c^4d^8e^8 - 8a^5c^3d^6e^{10} - 20a^6c^2d^4e^{12} + 40a^7cd^2e^{14} - 33c^3d^6)}{32768\sqrt{cd}c^5d^5} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.89 Problem number 458

$$\int \frac{x^2(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(-a e^2 + c d^2) (5a^2 e^4 + 10ac d^2 e^2 + 9c^2 d^4) (2cdex + a e^2 + c d^2) (ade + (a e^2 + c d^2) x + cde x^2)^{\frac{3}{2}}}{384c^3 d^3 e^4} \\ & + \frac{x^2 (ade + (a e^2 + c d^2) x + cde x^2)^{\frac{5}{2}}}{7e} \\ & + \frac{(63c^2 d^4 - 20ac d^2 e^2 - 35a^2 e^4 - 10cde(-5a e^2 + 9c d^2) x) (ade + (a e^2 + c d^2) x + cde x^2)^{\frac{5}{2}}}{840c^2 d^2 e^3} \\ & - \frac{(-a e^2 + c d^2)^5 (5a^2 e^4 + 10ac d^2 e^2 + 9c^2 d^4) \operatorname{arctanh} \left( \frac{2cdex + a e^2 + c d^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (a e^2 + c d^2) x + cde x^2}} \right)}{2048c^{\frac{9}{2}} d^{\frac{9}{2}} e^{\frac{11}{2}}} \\ & + \frac{(-a e^2 + c d^2)^3 (5a^2 e^4 + 10ac d^2 e^2 + 9c^2 d^4) (2cdex + a e^2 + c d^2) \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{1024c^4 d^4 e^5} \end{aligned}$$

command

```
integrate(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{107520} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10 \left( 12c^2d^2xe + \frac{(15c^8d^9e^6 + 29ac^7d^7e^8)e^{(-6)}}{c^6d^6} \right) x + \frac{(3c^8d^{10}}{c^6d^6} \right) \right) \right) \right) \right) \\ & + \frac{(9c^7d^{14} - 35ac^6d^{12}e^2 + 45a^2c^5d^{10}e^4 - 15a^3c^4d^8e^6 - 5a^4c^3d^6e^8 - 9a^5c^2d^4e^{10} + 15a^6cd^2e^{12} - 5a^7e^{14})e^{(-\frac{11}{2})} \log}{2048 \sqrt{cd} c^4 d^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.90 Problem number 459

$$\int \frac{x(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-a e^2 + c d^2) (5a e^2 + 7c d^2) (2cdex + a e^2 + c d^2) (ade + (a e^2 + c d^2) x + cde x^2)^{\frac{3}{2}}}{192c^2 d^2 e^3} \\ & - \frac{\left(\frac{5a}{cd} + \frac{7d}{e^2}\right) (ade + (a e^2 + c d^2) x + cde x^2)^{\frac{5}{2}}}{60} + \frac{(ade + (a e^2 + c d^2) x + cde x^2)^{\frac{7}{2}}}{6cde (ex + d)} \\ & + \frac{(-a e^2 + c d^2)^5 (5a e^2 + 7c d^2) \operatorname{arctanh} \left( \frac{2cdex + a e^2 + c d^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (a e^2 + c d^2) x + cde x^2}} \right)}{1024c^{\frac{7}{2}} d^{\frac{7}{2}} e^{\frac{9}{2}}} \\ & - \frac{(-a e^2 + c d^2)^3 (5a e^2 + 7c d^2) (2cdex + a e^2 + c d^2) \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{512c^3 d^3 e^4} \end{aligned}$$

command

```
integrate(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{7680} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8 \left( 10c^2d^2xe + \frac{(13c^7d^8e^5 + 25ac^6d^6e^7)e^{(-5)}}{c^5d^5} \right) x + \frac{(3c^7d^9e^4 + 27c^6d^8e^3 + 18c^5d^7e^2 + 12c^4d^6e) e^{(-4)}}{c^4d^4} \right) x + \frac{(c^6d^8e^3 + 64ac^5d^6e^5 + 12c^4d^5e^4 + 12c^3d^4e^3 + 12c^2d^3e^2 + 12cd^2e) e^{(-3)}}{c^4d^4} \right) x + \frac{(7c^6d^{12} - 30ac^5d^{10}e^2 + 45a^2c^4d^8e^4 - 20a^3c^3d^6e^6 - 15a^4c^2d^4e^8 + 18a^5cd^2e^{10} - 5a^6e^{12}) e^{(-\frac{9}{2})} \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right)}{1024 \sqrt{cd} c^3d^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.91 Problem number 460

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{a}{cd} - \frac{d}{e^2}\right) (2cdex + ae^2 + cd^2) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{16} \\ & + \frac{(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{5e} \\ & - \frac{3(-ae^2 + cd^2)^5 \operatorname{arctanh} \left( \frac{2cdex + ae^2 + cd^2}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \right)}{256c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \\ & + \frac{3(-ae^2 + cd^2)^3 (2cdex + ae^2 + cd^2) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{128c^2d^2e^3} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{640} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4 \left( 2 \left( 8c^2d^2xe + \frac{(11c^6d^7e^4 + 21ac^5d^5e^6)e^{(-4)}}{c^4d^4} \right) x + \frac{(c^6d^8e^3 + 64ac^5d^6e^5 + 12c^4d^5e^4 + 12c^3d^4e^3 + 12c^2d^3e^2 + 12cd^2e) e^{(-3)}}{c^4d^4} \right) x + \frac{(c^5d^{10} - 5ac^4d^8e^2 + 10a^2c^3d^6e^4 - 10a^3c^2d^4e^6 + 5a^4cd^2e^8 - a^5e^{10}) e^{(-\frac{7}{2})} \log \left( \left| -cd^2 - 2 \left( \sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade} \right) \right| \right)}{256 \sqrt{cd} c^2d^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.92 Problem number 462

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^2(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-cdx + 3ae)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{3x} \\ & - \frac{(-5a^3e^6 - 45a^2cd^2e^4 - 15a^2c^2d^4e^2 + c^3d^6) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{16e^{\frac{3}{2}}\sqrt{c}\sqrt{d}} \\ & - \frac{a^{\frac{3}{2}}e^{\frac{3}{2}}(3ae^2 + 5cd^2) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right) \sqrt{d}}{2} \\ & + \frac{(c^2d^4 + 28acd^2e^2 + 19a^2e^4 + 2cde(7ae^2 + cd^2)x) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{8e} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{24} \sqrt{cdx^2e + cd^2x + axe^2 + ade} \left( 2 \left( 4c^2d^2xe + \frac{(7c^4d^5e^2 + 13ac^3d^3e^4)e^{(-2)}}{c^2d^2} \right) x + \frac{(3c^4d^6e + 68ac^3d^4e^3 + 33a^2c^2d^2e^2 + 3cd^2e^2)}{c^2d^2} \right) \\ & + \frac{(5a^2cd^3e^2 + 3a^3de^4) \arctan\left(-\frac{\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{\sqrt{-ade}} \\ & + \frac{\left(\sqrt{cd}c^3d^6e^{\frac{1}{2}} - 15\sqrt{cd}ac^2d^4e^{\frac{5}{2}} - 45\sqrt{cd}a^2cd^2e^{\frac{9}{2}} - 5\sqrt{cd}a^3e^{\frac{13}{2}}\right)e^{(-2)} \log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)\right|\right)}{\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2} \\ & - \frac{16cd}{ade - \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.93 Problem number 463

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^3(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-cdx + ae)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{2x^2} \\ & + \frac{3(5a^2e^4 + 10acd^2e^2 + c^2d^4) \operatorname{arctanh}\left(\frac{2cdex + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right) \sqrt{c}\sqrt{d}}{8\sqrt{e}} \\ & - \frac{3(a^2e^4 + 10acd^2e^2 + 5c^2d^4) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right) \sqrt{a}\sqrt{e}}{8\sqrt{d}} \\ & - \frac{3(ae(ae^2 + 3cd^2) - cd(3ae^2 + cd^2)x) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{4x} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{4} \left( 2c^2d^2xe + \frac{(5c^3d^4e + 9ac^2d^2e^3)e^{(-1)}}{cd} \right) \sqrt{cdx^2e + cd^2x + axe^2 + ade} \\ & + \frac{3(5ac^2d^4e + 10a^2cd^2e^3 + a^3e^5) \operatorname{arctan}\left(-\frac{\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}}{\sqrt{-ade}}\right)}{4\sqrt{-ade}} \\ & - \frac{3\left(\sqrt{cd}c^3d^5e^{\frac{1}{2}} + 10\sqrt{cd}ac^2d^3e^{\frac{5}{2}} + 5\sqrt{cd}a^2cde^{\frac{9}{2}}\right)e^{(-1)} \log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}} - 2\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)\right|\right)}{8cd} \\ & - \frac{7\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)a^2c^2d^5e^2 - 9\left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e + cd^2x + axe^2 + ade}\right)^3ac^2d^4e}{8cd} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.94 Problem number 464

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^4(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(4ade + 3(ae^2 + 3cd^2)x)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{12dx^3} \\ & - \frac{(-a^3e^6 + 15a^2cd^2e^4 + 45a^2c^2d^4e^2 + 5c^3d^6) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{16d^{\frac{3}{2}}\sqrt{a}\sqrt{e}} \\ & + \frac{c^{\frac{3}{2}}d^{\frac{3}{2}}(5ae^2 + 3cd^2) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)\sqrt{e}}{2} \\ & - \frac{(5c^2d^4 + 12acd^2e^2 - a^2e^4 - 2cde(ae^2 + 7cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{8dx} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^4/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.95 Problem number 465

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^5(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(6ade + (3ae^2 + 11cd^2)x)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{24dx^4} \\ & + \frac{(-3a^4e^8 + 20a^3cd^2e^6 - 90a^2c^2d^4e^4 - 60a^3c^3d^6e^2 + 5c^4d^8) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{128a^{\frac{3}{2}}d^{\frac{5}{2}}e^{\frac{3}{2}}} \\ & + c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right) \\ & - \frac{(2ade(-ae^2 + 5cd^2)(3ae^2 + cd^2) + (-3a^3e^6 + 11a^2cd^2e^4 + 83a^2c^2d^4e^2 + 5c^3d^6)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{64ad^2ex^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^5/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.96 Problem number 466

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^6(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right) (2ade + (ae^2 + cd^2)x) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{16x^4} \\ & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{5dx^5} \\ & - \frac{3(-ae^2 + cd^2)^5 \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{256a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}} \\ & + \frac{3(-ae^2 + cd^2)^3 (2ade + (ae^2 + cd^2)x) \sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{128a^2d^3e^2x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.97 Problem number 467

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^7(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ae^2 + cd^2)(7ae^2 + 5cd^2)(2ade + (ae^2 + cd^2)x)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{192a^2d^3e^2x^4} \\ & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{6dx^6} - \frac{(\frac{5c}{ae} - \frac{7e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{60x^5} \\ & + \frac{(-ae^2 + cd^2)^5(7ae^2 + 5cd^2) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{1024a^{\frac{7}{2}}d^{\frac{9}{2}}e^{\frac{7}{2}}} \\ & - \frac{(-ae^2 + cd^2)^3(7ae^2 + 5cd^2)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{512a^3d^4e^3x^2} \end{aligned}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 16.98 Problem number 468

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^8(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ae^2 + cd^2)(9a^2e^4 + 10acd^2e^2 + 5c^2d^4)(2ade + (ae^2 + cd^2)x)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{384a^3d^4e^3x^4} \\ & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{7dx^7} - \frac{(\frac{5c}{ae} - \frac{9e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{84x^6} \\ & + \frac{(-63a^2e^4 + 20acd^2e^2 + 35c^2d^4)(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{840a^2d^3e^2x^5} \\ & - \frac{(-ae^2 + cd^2)^5(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{2048a^{\frac{9}{2}}d^{\frac{11}{2}}e^{\frac{9}{2}}} \\ & + \frac{(-ae^2 + cd^2)^3(9a^2e^4 + 10acd^2e^2 + 5c^2d^4)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{1024a^4d^5e^4x^2} \end{aligned}$$



command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 16.99 Problem number 469

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{x^9(d + ex)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-ae^2 + cd^2)(33a^3e^6 + 45a^2cd^2e^4 + 35a^2c^2d^4e^2 + 15c^3d^6)(2ade + (ae^2 + cd^2)x)(ade + (ae^2 + cd^2)x + cde x^2)}{2048a^4d^5e^4x^4} \\ & - \frac{(ade + (ae^2 + cd^2)x + cde x^2)^{5/2}}{8dx^8} - \frac{(\frac{5c}{ae} - \frac{11e}{d^2})(ade + (ae^2 + cd^2)x + cde x^2)^{5/2}}{112x^7} \\ & + \frac{(-33a^2e^4 + 10acd^2e^2 + 15c^2d^4)(ade + (ae^2 + cd^2)x + cde x^2)^{5/2}}{448a^2d^3e^2x^6} \\ & - \frac{(-231a^3e^6 + 15a^2cd^2e^4 + 95a^2c^2d^4e^2 + 105c^3d^6)(ade + (ae^2 + cd^2)x + cde x^2)^{5/2}}{4480a^3d^4e^3x^5} \\ & + \frac{3(-ae^2 + cd^2)^5(33a^3e^6 + 45a^2cd^2e^4 + 35a^2c^2d^4e^2 + 15c^3d^6) \operatorname{arctanh}\left(\frac{2ade + (ae^2 + cd^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}\right)}{32768a^{\frac{11}{2}}d^{\frac{13}{2}}e^{\frac{11}{2}}} \\ & - \frac{3(-ae^2 + cd^2)^3(33a^3e^6 + 45a^2cd^2e^4 + 35a^2c^2d^4e^2 + 15c^3d^6)(2ade + (ae^2 + cd^2)x)\sqrt{ade + (ae^2 + cd^2)x + cde x^2}}{16384a^5d^6e^5x^2} \end{aligned}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 16.100 Problem number 470

$$\int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{3(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}\right)}{8c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} - \frac{3(ae^2 + 3cd^2)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{4c^2d^2e^3} - \frac{2d^3\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{e^3(-ae^2+cd^2)(ex+d)} + \frac{(ex+d)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{2cde^3}$$

command

```
integrate(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2d^3e^{(-3)}}{\sqrt{cd}de^{\frac{1}{2}} + \left(\sqrt{cd}xe^{\frac{1}{2}} - \sqrt{cdx^2e+cd^2x+axe^2+ade}\right)e} + \frac{1}{4}\sqrt{cdx^2e+cd^2x+axe^2+ade}\left(\frac{2xe^{(-2)}}{cd} - \frac{(7cd^2e^5+3ae^7)e^{(-8)}}{c^2d^2}\right) - \frac{3\left(5\sqrt{cd}c^2d^4e^{\frac{1}{2}}+2\sqrt{cd}acd^2e^{\frac{5}{2}}+\sqrt{cd}a^2e^{\frac{9}{2}}\right)e^{(-4)}\log\left(\left|-\sqrt{cd}cd^2e^{\frac{1}{2}}-2\left(\sqrt{cd}xe^{\frac{1}{2}}-\sqrt{cdx^2e+cd^2x+axe^2}\right)\right|\right)}{8c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.101 Problem number 471

$$\int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{(ae^2 + 3cd^2) \operatorname{arctanh}\left(\frac{2cde x + ae^2 + cd^2}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}\right)}{2c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{5}{2}}} + \frac{\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{cde^2} + \frac{2d^2\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{e^2(-ae^2+cd^2)(ex+d)}$$

command

```
integrate(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 d^2 e^{(-2)}}{\sqrt{cd} d e^{\frac{1}{2}} + \left( \sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cd x^2 e + cd^2 x + a x e^2 + a d e} \right) e} + \frac{\sqrt{cd x^2 e + cd^2 x + a x e^2 + a d e} e^{(-2)}}{cd}$$

$$+ \frac{\left( 3 \sqrt{cd} cd^2 e^{\frac{1}{2}} + \sqrt{cd} a e^{\frac{5}{2}} \right) e^{(-3)} \log \left( \left| -\sqrt{cd} cd^2 e^{\frac{1}{2}} - 2 \left( \sqrt{cd} x e^{\frac{1}{2}} - \sqrt{cd x^2 e + cd^2 x + a x e^2 + a d e} \right) c d e - \sqrt{cd} a \right. \right.}{2 c^2 d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.102 Problem number 553

$$\int \frac{(f + gx)^2}{(d + ex)(d^2 - e^2 x^2)} dx$$

Optimal antiderivative

$$-\frac{(-dg + ef)^2}{2d e^3 (ex + d)} - \frac{(dg + ef)^2 \ln(-ex + d)}{4d^2 e^3} + \frac{(-dg + ef)(3dg + ef) \ln(ex + d)}{4d^2 e^3}$$

command

```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(3 d^2 g^2 - 2 d f g e - f^2 e^2) e^{(-3)} \log(|x e + d|)}{4 d^2}$$

$$-\frac{(d^2 g^2 + 2 d f g e + f^2 e^2) e^{(-3)} \log(|x e - d|)}{4 d^2} - \frac{(d^3 g^2 - 2 d^2 f g e + d f^2 e^2) e^{(-3)}}{2 (x e + d) d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.103 Problem number 554

$$\int \frac{(f + gx)^2}{(d + ex)^2 (d^2 - e^2 x^2)} dx$$

Optimal antiderivative

$$-\frac{(-dg + ef)^2}{4d e^3 (ex + d)^2} - \frac{(-dg + ef)(3dg + ef)}{4d^2 e^3 (ex + d)} + \frac{(dg + ef)^2 \operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^3 e^3}$$

command

`integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{3d^2 g^2 e^3}{xe+d} - \frac{d^3 g^2 e^3}{(xe+d)^2} - \frac{2dfge^4}{xe+d} + \frac{2d^2 fge^4}{(xe+d)^2} - \frac{f^2 e^5}{xe+d} - \frac{df^2 e^5}{(xe+d)^2}\right) e^{(-6)}}{4d^2} - \frac{(d^2 g^2 + 2dfge + f^2 e^2) e^{(-3)} \log\left(\left|-\frac{2d}{xe+d} + 1\right|\right)}{8d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.104 Problem number 555

$$\int \frac{(f + gx)^2}{(d + ex)^3 (d^2 - e^2 x^2)} dx$$

Optimal antiderivative

$$-\frac{(-dg + ef)^2}{6d e^3 (ex + d)^3} - \frac{(-dg + ef)(3dg + ef)}{8d^2 e^3 (ex + d)^2} - \frac{(dg + ef)^2}{8d^3 e^3 (ex + d)} + \frac{(dg + ef)^2 \operatorname{arctanh}\left(\frac{ex}{d}\right)}{8d^4 e^3}$$

command

`integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d^2 g^2 + 2dfge + f^2 e^2) e^{(-3)} \log(|xe + d|)}{16d^4} - \frac{(d^2 g^2 + 2dfge + f^2 e^2) e^{(-3)} \log(|xe - d|)}{16d^4} + \frac{(2d^5 g^2 - 4d^4 fge - 10d^3 f^2 e^2 - 3(d^3 g^2 e^2 + 2d^2 fge^3 + df^2 e^4) x^2 + 3(d^4 g^2 e - 6d^3 fge^2 - 3d^2 f^2 e^3) x) e^{(-3)}}{24(ex + d)^3 d^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 16.107 Problem number 566

$$\int \frac{(f + gx)^2}{(d + ex)^2 (d^2 - e^2 x^2)^2} dx$$

Optimal antiderivative

$$\frac{(dg + ef)^2}{16d^4 e^3 (-ex + d)} - \frac{(-dg + ef)^2}{12d^2 e^3 (ex + d)^3} + \frac{d^2 g^2 - e^2 f^2}{8d^3 e^3 (ex + d)^2} - \frac{(-dg + 3ef)(dg + ef)}{16d^4 e^3 (ex + d)} + \frac{f(dg + ef) \operatorname{arctanh}\left(\frac{ex}{d}\right)}{4d^5 e^2}$$

command

`integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(dfg + f^2e)e^{(-2) \log\left(\left|-\frac{2d}{xe+d} + 1\right|\right)}}{8d^5} + \frac{(d^2g^2 + 2dfge + f^2e^2)e^{(-3)}}{32d^5\left(\frac{2d}{xe+d} - 1\right)} + \frac{\left(\frac{3d^4g^2e^3}{xe+d} + \frac{6d^5g^2e^3}{(xe+d)^2} - \frac{4d^6g^2e^3}{(xe+d)^3} - \frac{6d^3fge^4}{xe+d} + \frac{8d^5fge^4}{(xe+d)^3} - \frac{9d^2f^2e^5}{xe+d} - \frac{6d^3f^2e^5}{(xe+d)^2} - \frac{4d^4f^2e^5}{(xe+d)^3}\right)e^{(-6)}}{48d^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 16.108 Problem number 567

$$\int \frac{(f + gx)^2}{(d + ex)^3 (d^2 - e^2 x^2)^2} dx$$

Optimal antiderivative

$$\frac{(dg + ef)^2}{32d^5 e^3 (-ex + d)} - \frac{(-dg + ef)^2}{16d^2 e^3 (ex + d)^4} + \frac{d^2 g^2 - e^2 f^2}{12d^3 e^3 (ex + d)^3} - \frac{(-dg + 3ef)(dg + ef)}{32d^4 e^3 (ex + d)^2} - \frac{f(dg + ef)}{8d^5 e^2 (ex + d)} + \frac{(dg + ef)(dg + 5ef) \operatorname{arctanh}\left(\frac{ex}{d}\right)}{32d^6 e^3}$$

command

`integrate((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d^2g^2 + 6dfge + 5f^2e^2)e^{(-3)\log(|xe+d|)}}{64d^6} - \frac{(d^2g^2 + 6dfge + 5f^2e^2)e^{(-3)\log(|xe-d|)}}{64d^6} - \frac{(8d^7g^2 - 32d^5f^2e^2 + 3(d^3g^2e^4 + 6d^2fge^5 + 5df^2e^6)x^4 + 9(d^4g^2e^3 + 6d^3fge^4 + 5d^2f^2e^5)x^3 + 7(d^5g^2e^2 + 6d^4fge^3 + 5d^3f^2e^4)x^2 + 2(d^6g^2e + 6d^5fge^2 + 5d^4f^2e^3)x + 7d^6f^2e^2)}{96(xe+d)^4(xe-d)d^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.109 Problem number 568

$$\int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2} dx$$

Optimal antiderivative

$$\frac{(dg+ef)^2}{64d^6e^3(-ex+d)} - \frac{(-dg+ef)^2}{20d^2e^3(ex+d)^5} + \frac{d^2g^2-e^2f^2}{16d^3e^3(ex+d)^4} - \frac{(-dg+3ef)(dg+ef)}{48d^4e^3(ex+d)^3} - \frac{f(dg+ef)}{16d^5e^2(ex+d)^2} - \frac{(dg+ef)(dg+5ef)}{64d^6e^3(ex+d)} + \frac{(dg+ef)(dg+3ef)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{32d^7e^3}$$

command

`integrate((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d^2g^2 + 4dfge + 3f^2e^2)e^{(-3)\log(|xe+d|)}}{64d^7} - \frac{(d^2g^2 + 4dfge + 3f^2e^2)e^{(-3)\log(|xe-d|)}}{64d^7} - \frac{(16d^8g^2 - 32d^7fge - 144d^6f^2e^2 + 15(d^3g^2e^5 + 4d^2fge^6 + 3df^2e^7)x^5 + 60(d^4g^2e^4 + 4d^3fge^5 + 3d^2f^2e^6)x^4 + 2(d^5g^2e^3 + 6d^4fge^4 + 5d^3f^2e^5)x^3 + 7(d^6g^2e^2 + 6d^5fge^3 + 5d^4f^2e^4)x^2 + 7d^6f^2e^2)}{480(xe+d)^4(xe-d)d^7}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.110 Problem number 577

$$\int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3} dx$$

Optimal antiderivative

$$\frac{(dg+ef)^2}{32d^4e^3(-ex+d)^2} + \frac{f(dg+ef)}{8d^5e^2(-ex+d)} - \frac{(-dg+ef)^2}{24d^3e^3(ex+d)^3} - \frac{(-dg+ef)(dg+3ef)}{32d^4e^3(ex+d)^2} + \frac{d^2g^2-3e^2f^2}{16d^5e^3(ex+d)} + \frac{(-d^2g^2+2defg+5e^2f^2)\operatorname{arctanh}\left(\frac{ex}{d}\right)}{16d^6e^3}$$

command

```
integrate((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d^2 g^2 - 2 d f g e - 5 f^2 e^2) e^{(-3)} \log(|x e + d|)}{32 d^6} + \frac{(d^2 g^2 - 2 d f g e - 5 f^2 e^2) e^{(-3)} \log(|x e - d|)}{32 d^6} + \frac{(4 d^7 g^2 + 16 d^6 f g e - 8 d^5 f^2 e^2 + 3 (d^3 g^2 e^4 - 2 d^2 f g e^5 - 5 d f^2 e^6) x^4 + 3 (d^4 g^2 e^3 - 2 d^3 f g e^4 - 5 d^2 f^2 e^5) x^3 - 5 (d^5 g^2 e^2 - 2 d^4 f g e^3 - 5 d^3 f^2 e^4) x^2 + 5 (d^6 g^2 e - 2 d^5 f g e^2 - 5 d^4 f^2 e^3) x - 5 (d^7 g^2 - 2 d^6 f g e - 5 d^5 f^2 e^2)}{48 (x e + d)^3 (x e - d)^2 d^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 16.111 Problem number 578

$$\int \frac{(f + g x)^2}{(d + e x)^2 (d^2 - e^2 x^2)^3} dx$$

Optimal antiderivative

$$\frac{(d g + e f)^2}{64 d^5 e^3 (-e x + d)^2} + \frac{(d g + e f) (d g + 5 e f)}{64 d^6 e^3 (-e x + d)} - \frac{(-d g + e f)^2}{32 d^3 e^3 (e x + d)^4} - \frac{(-d g + e f) (d g + 3 e f)}{48 d^4 e^3 (e x + d)^3} + \frac{d^2 g^2 - 3 e^2 f^2}{32 d^5 e^3 (e x + d)^2} + \frac{d^2 g^2 - 2 d e f g - 5 e^2 f^2}{32 d^6 e^3 (e x + d)} + \frac{(-d^2 g^2 + 10 d e f g + 15 e^2 f^2) \operatorname{arctanh}\left(\frac{e x}{d}\right)}{64 d^7 e^3}$$

command

```
integrate((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d^2 g^2 - 10 d f g e - 15 f^2 e^2) e^{(-3)} \log\left(\left|-\frac{2 d}{x e + d} + 1\right|\right)}{128 d^7} - \frac{\left(3 d^2 g^2 + 14 d f g e + 11 f^2 e^2 - \frac{8 (d^3 g^2 e + 4 d^2 f g e^2 + 3 d f^2 e^3) e^{(-1)}}{x e + d}\right) e^{(-3)}}{256 d^7 \left(\frac{2 d}{x e + d} - 1\right)^2} + \frac{\left(\frac{3 d^8 g^2 e^9}{x e + d} + \frac{3 d^9 g^2 e^9}{(x e + d)^2} + \frac{2 d^{10} g^2 e^9}{(x e + d)^3} - \frac{3 d^{11} g^2 e^9}{(x e + d)^4} - \frac{6 d^7 f g e^{10}}{x e + d} + \frac{4 d^9 f g e^{10}}{(x e + d)^3} + \frac{6 d^{10} f g e^{10}}{(x e + d)^4} - \frac{15 d^6 f^2 e^{11}}{x e + d} - \frac{9 d^7 f^2 e^{11}}{(x e + d)^2} - \frac{6 d^8 f^2 e^{11}}{(x e + d)^3} - \frac{3 d^9 f^2 e^{11}}{(x e + d)^4}\right)}{96 d^{12}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 16.112 Problem number 618

$$\int \frac{\sqrt{x}}{\sqrt{1+x}(1+x^2)} dx$$

Optimal antiderivative

$$-\frac{(1-i)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{1+x}}\right)}{2} - \frac{(1+i)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{1+x}}\right)}{2}$$

command

`integrate(x^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{4} \left( \sqrt{2\sqrt{2}+2} + \sqrt{2\sqrt{2}-2} \right) \operatorname{arctan} \left( \frac{2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} + 2 \sqrt{-\frac{1}{x+1}+1} \right)}{\sqrt{-\sqrt{2}+2}} \right) \\ & + \frac{1}{4} \left( \sqrt{2\sqrt{2}+2} + \sqrt{2\sqrt{2}-2} \right) \operatorname{arctan} \left( -\frac{2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} - 2 \sqrt{-\frac{1}{x+1}+1} \right)}{\sqrt{-\sqrt{2}+2}} \right) \\ & - \frac{1}{8} \left( \sqrt{2\sqrt{2}+2} - \sqrt{2\sqrt{2}-2} \right) \log \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{-\frac{1}{x+1}+1} + \sqrt{\frac{1}{2} - \frac{1}{x+1} + 1} \right) \\ & + \frac{1}{8} \left( \sqrt{2\sqrt{2}+2} - \sqrt{2\sqrt{2}-2} \right) \log \left( -\left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{-\frac{1}{x+1}+1} + \sqrt{\frac{1}{2} - \frac{1}{x+1} + 1} \right) \\ & - \frac{1}{4} \sqrt{2\sqrt{2}+2} \operatorname{arctan} \left( \frac{2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} + 2 \right)}{\sqrt{-\sqrt{2}+2}} \right) \\ & - \frac{1}{4} \sqrt{2\sqrt{2}+2} \operatorname{arctan} \left( -\frac{2 \left(\frac{1}{2}\right)^{\frac{3}{4}} \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} - 2 \right)}{\sqrt{-\sqrt{2}+2}} \right) \\ & - \frac{1}{8} \sqrt{2\sqrt{2}-2} \log \left( \left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} + \sqrt{\frac{1}{2} + 1} \right) \\ & + \frac{1}{8} \sqrt{2\sqrt{2}-2} \log \left( -\left(\frac{1}{2}\right)^{\frac{1}{4}} \sqrt{\sqrt{2}+2} + \sqrt{\frac{1}{2} + 1} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 16.113 Problem number 657

$$\int \frac{\sqrt{d+ex} (f+gx)^3}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16(-aeg + cdf)^2 (2ae^2g - cd(-dg + 3ef)) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^4d^4e\sqrt{ex+d}} \\ & + \frac{12(-aeg + cdf)(gx+f)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^2d^2\sqrt{ex+d}} \\ & + \frac{2(gx+f)^3 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{7cd\sqrt{ex+d}} \\ & + \frac{16g(-aeg + cdf)^2 \sqrt{ex+d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{35c^3d^3e} \end{aligned}$$

command

`integrate((g*x+f)^3*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(c^3d^3f^3 - 3ac^2d^2f^2ge + 3a^2cdfg^2e^2 - a^3g^3e^3) \sqrt{(xe+d)cde - cd^2e + ae^3} e^{(-1)}}{c^4d^4} \\ & + \frac{2(5\sqrt{-cd^2e + ae^3} c^3d^6g^3 - 21\sqrt{-cd^2e + ae^3} c^3d^5fg^2e + 35\sqrt{-cd^2e + ae^3} c^3d^4f^2ge^2 + 6\sqrt{-cd^2e + ae^3} ac^2d^2e^2)}{c^4d^4} \\ & + \frac{2(35((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^2f^2ge^4 - 70((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}acdfg^2e^5 + 21((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^2f^2ge^4)}{c^4d^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d} (gx+f)^3}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

## 16.114 Problem number 658

$$\int \frac{\sqrt{d+ex} (f+gx)^2}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(-aeg + cdf)(2ae^2g - cd(-dg + 3ef)) \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{15c^3d^3e\sqrt{ex+d}} \\ & + \frac{2(gx+f)^2 \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{5cd\sqrt{ex+d}} \\ & + \frac{8g(-aeg + cdf) \sqrt{ex+d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{15c^2d^2e} \end{aligned}$$

command

`integrate((g*x+f)^2*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^2d^2f^2 - 2acdfge + a^2g^2e^2)\sqrt{(xe+d)cde - cd^2e + ae^3}e^{(-1)}}{c^3d^3} - \frac{2\left(3\sqrt{-cd^2e + ae^3}c^2d^4g^2 - 10\sqrt{-cd^2e + ae^3}c^2d^3fge + 15\sqrt{-cd^2e + ae^3}c^2d^2f^2e^2 + 4\sqrt{-cd^2e + ae^3}acd^2g^2\right)}{15c^3d^3} + \frac{2\left(10((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}cdfge^2 - 10((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}ag^2e^3 + 3((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}\right)}{15c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}(gx+f)^2}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

### 16.115 Problem number 659

$$\int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$-\frac{2(2ae^2g - cd(-dg + 3ef))\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3c^2d^2e\sqrt{ex+d}} + \frac{2g\sqrt{ex+d}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{3cde}$$

command

`integrate((g*x+f)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}ge^{(-3)}}{3c^2d^2} + \frac{2\sqrt{(xe+d)cde - cd^2e + ae^3}(cdf - age)e^{(-1)}}{c^2d^2} + \frac{2\left(\sqrt{-cd^2e + ae^3}cd^2g - 3\sqrt{-cd^2e + ae^3}cdf e + 2\sqrt{-cd^2e + ae^3}age^2\right)e^{(-2)}}{3c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}(gx+f)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

## 16.116 Problem number 660

$$\int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{2\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{cd\sqrt{ex+d}}$$

command

`integrate((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{(xe+d)cde-cd^2e+ae^3}e^{(-1)}}{cd} - \frac{2\sqrt{-cd^2e+ae^3}e^{(-1)}}{cd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

## 16.117 Problem number 661

$$\int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{\sqrt{g}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-aeg+cdf}\sqrt{ex+d}}\right)}{\sqrt{g}\sqrt{-aeg+cdf}}$$

command

`integrate((e*x+d)^(1/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3}ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)}{\sqrt{cdfg-ag^2e}} - \frac{2 \arctan\left(\frac{\sqrt{-cd^2e+ae^3}ge^{(-1)}}{\sqrt{cdfg-ag^2e}}\right)}{\sqrt{cdfg-ag^2e}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)} dx$$

## 16.118 Problem number 665

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(gx+f)^3 \sqrt{ex+d}}{cd \sqrt{ade+(ae^2+cd^2)x+cde x^2}} \\ & - \frac{16g(-aeg+cdf)(2ae^2g-cd(-dg+3ef)) \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{5c^4 d^4 e \sqrt{ex+d}} \\ & + \frac{12g(gx+f)^2 \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{5c^2 d^2 \sqrt{ex+d}} \\ & + \frac{16g^2(-aeg+cdf) \sqrt{ex+d} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{5c^3 d^3 e} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2(c^3 d^6 g^3 - 5c^3 d^5 f g^2 e + 15c^3 d^4 f^2 g e^2 + 2ac^2 d^4 g^3 e^2 + 5c^3 d^3 f^3 e^3 - 20ac^2 d^3 f g^2 e^3 - 30ac^2 d^2 f^2 g e^4 + 8a^2 c d^2 g^3 e^4}{5 \sqrt{-cd^2 e + ae^3} c^4 d^4} \\ & - \frac{2(c^3 d^3 f^3 e - 3ac^2 d^2 f^2 g e^2 + 3a^2 c d f g^2 e^3 - a^3 g^3 e^4)}{\sqrt{(xe+d)cde - cd^2 e + ae^3} c^4 d^4} \\ & + \frac{2(15 \sqrt{(xe+d)cde - cd^2 e + ae^3} c^{18} d^{18} f^2 g e^{24} - 30 \sqrt{(xe+d)cde - cd^2 e + ae^3} ac^{17} d^{17} f g^2 e^{25} + 5((xe+d)cde$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.119 Problem number 666

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(gx+f)^2 \sqrt{ex+d}}{cd \sqrt{ade+(ae^2+cd^2)x+cde x^2}} \\ & - \frac{8g(2ae^2g-cd(-dg+3ef)) \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^3 d^3 e \sqrt{ex+d}} \\ & + \frac{8g^2 \sqrt{ex+d} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^2 d^2 e} \end{aligned}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^2d^4g^2 - 6c^2d^3fge - 3c^2d^2f^2e^2 + 4acd^2g^2e^2 + 12acdfge^3 - 8a^2g^2e^4)e^{(-1)}}{3\sqrt{-cd^2e + ae^3}c^3d^3} - \frac{2(c^2d^2f^2e - 2acdfge^2 + a^2g^2e^3)}{\sqrt{(xe+d)cde - cd^2e + ae^3}c^3d^3} + \frac{2\left(6\sqrt{(xe+d)cde - cd^2e + ae^3}c^7d^7fge^8 - 6\sqrt{(xe+d)cde - cd^2e + ae^3}ac^6d^6g^2e^9 + ((xe+d)cde - cd^2e + ae^3)c^9d^9\right)}{3c^9d^9}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.120 Problem number 667

$$\int \frac{(d+ex)^{3/2}(f+gx)}{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2(-aeg + cdf)(ex+d)^{\frac{3}{2}}}{cd(-ae^2 + cd^2)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} - \frac{2(2ae^2g - cd(dg + ef))\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{c^2d^2(-ae^2 + cd^2)\sqrt{ex+d}}$$

command

```
integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{(xe+d)cde - cd^2e + ae^3}ge^{(-1)}}{c^2d^2} + \frac{2(cd^2g + cdfe - 2age^2)}{\sqrt{-cd^2e + ae^3}c^2d^2} - \frac{2(cdfe - age^2)}{\sqrt{(xe+d)cde - cd^2e + ae^3}c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.121 Problem number 668

$$\int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2\sqrt{ex+d}}{cd\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

```
integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2e}{\sqrt{(xe+d)cde-cd^2e+ae^3}cd} + \frac{2e}{\sqrt{-cd^2e+ae^3}cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.122 Problem number 669

$$\int \frac{(d+ex)^{3/2}}{(f+gx)(ade+(cd^2+ae^2)x+cde x^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \arctan\left(\frac{\sqrt{g}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-aeg+cdf}\sqrt{ex+d}}\right)\sqrt{g}}{(-aeg+cdf)^{\frac{3}{2}}\sqrt{ex+d}} - \frac{2\sqrt{ex+d}}{(-aeg+cdf)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

```
integrate((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -2 \left( \frac{g \arctan \left( \frac{\sqrt{(xe+d)cde - cd^2e + ae^3} ge^{(-1)}}{\sqrt{cdfg - ag^2e}} \right) e^{(-1)}}{\sqrt{cdfg - ag^2e} (cdf - age)} + \frac{1}{\sqrt{(xe+d)cde - cd^2e + ae^3} (cdf - age)} \right) e \\
& + \frac{2 \left( \sqrt{-cd^2e + ae^3} g \arctan \left( \frac{\sqrt{-cd^2e + ae^3} ge^{(-1)}}{\sqrt{cdfg - ag^2e}} \right) + \sqrt{cdfg - ag^2e} e \right)}{\sqrt{cdfg - ag^2e} \sqrt{-cd^2e + ae^3} cdf - \sqrt{cdfg - ag^2e} \sqrt{-cd^2e + ae^3} age}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.123 Problem number 672

$$\int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cde x^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(ex+d)^{\frac{3}{2}}(gx+f)^3}{3cd(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}} - \frac{4g(gx+f)^2 \sqrt{ex+d}}{c^2 d^2 \sqrt{ade+(ae^2+cd^2)x+cde x^2}} \\
& - \frac{16g^2(2ae^2g - cd(-dg+3ef)) \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^4 d^4 e \sqrt{ex+d}} \\
& + \frac{16g^3 \sqrt{ex+d} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^3 d^3 e}
\end{aligned}$$

command

`integrate((e*x+d)^(5/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{2(c^3 d^6 g^3 - 9c^3 d^5 f g^2 e - 9c^3 d^4 f^2 g e^2 + 6ac^2 d^4 g^3 e^2 + c^3 d^3 f^3 e^3 + 36ac^2 d^3 f g^2 e^3 + 6ac^2 d^2 f^2 g e^4 - 24a^2 c d^2 g^3 e^4 - 3(\sqrt{-cd^2e + ae^3} c^5 d^6 e - \sqrt{-cd^2e + ae^3} ac^4 d^4 e^3)}{3((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}} c^4 d^4} \\
& \frac{2(c^3 d^3 f^3 e^3 - 3ac^2 d^2 f^2 g e^4 + 9((xe+d)cde - cd^2e + ae^3)c^2 d^2 f^2 g e + 3a^2 cdf g^2 e^5 - 18((xe+d)cde - cd^2e + ae^3)c^9 d^9 f g^2 e^8 - 9\sqrt{(xe+d)cde - cd^2e + ae^3} ac^8 d^8 g^3 e^9 + ((xe+d)cde - cd^2e + ae^3)c^{12} d^{12})}{3c^{12} d^{12}}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 16.124 Problem number 673

$$\int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cde x^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ex+d)^{\frac{3}{2}}(gx+f)^2}{3cd(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}} - \frac{8g(-aeg+cdf)(ex+d)^{\frac{3}{2}}}{3c^2d^2(-ae^2+cd^2)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

$$- \frac{8g(2ae^2g-cd(dg+ef))\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^3d^3(-ae^2+cd^2)\sqrt{ex+d}}$$

command

`integrate((e*x+d)^(5/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(3c^2d^4g^2+6c^2d^3fge-c^2d^2f^2e^2-12acd^2g^2e^2-4acdfge^3+8a^2g^2e^4)}{3(\sqrt{-cd^2e+ae^3}c^4d^5-\sqrt{-cd^2e+ae^3}ac^3d^3e^2)}$$

$$+ \frac{2\sqrt{(xe+d)cde-cd^2e+ae^3}g^2e^{(-1)}}{c^3d^3}$$

$$- \frac{2(c^2d^2f^2e^3-2acdfge^4+6((xe+d)cde-cd^2e+ae^3)cdfge+a^2g^2e^5-6((xe+d)cde-cd^2e+ae^3)ag^2e^2)}{3((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.125 Problem number 674

$$\int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)x+cde x^2)^{5/2}} dx$$

Optimal antiderivative

$$- \frac{2(-aeg+cdf)(ex+d)^{\frac{5}{2}}}{3cd(-ae^2+cd^2)(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}}$$

$$+ \frac{2(2ae^2g+cd(-3dg+ef))\sqrt{ex+d}}{3c^2d^2(-ae^2+cd^2)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}$$

command

`integrate((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(3cd^2ge - cdf e^2 - 2age^3)}{3\left(\sqrt{-cd^2e + ae^3}c^3d^4 - \sqrt{-cd^2e + ae^3}ac^2d^2e^2\right)} - \frac{2(cdf e^3 - age^4 + 3((xe+d)cde - cd^2e + ae^3)ge)}{3((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.126 Problem number 675

$$\int \frac{(d+ex)^{5/2}}{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2(ex+d)^{\frac{3}{2}}}{3cd(ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}$$

command

`integrate((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2e^2}{3\left(\sqrt{-cd^2e + ae^3}c^2d^3 - \sqrt{-cd^2e + ae^3}acde^2\right)} - \frac{2e^3}{3((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}cd}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.127 Problem number 676

$$\int \frac{(d+ex)^{5/2}}{(f+gx)(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2(ex+d)^{\frac{3}{2}}}{3(-aeg+cdf)(ade+(ae^2+cd^2)x+cde x^2)^{\frac{3}{2}}} \\
& + \frac{2g^{\frac{3}{2}} \arctan\left(\frac{\sqrt{g} \sqrt{ade+(ae^2+cd^2)x+cde x^2}}{\sqrt{-aeg+cdf} \sqrt{ex+d}}\right)}{(-aeg+cdf)^{\frac{5}{2}}} \\
& + \frac{2g\sqrt{ex+d}}{(-aeg+cdf)^2 \sqrt{ade+(ae^2+cd^2)x+cde x^2}}
\end{aligned}$$

command

```
integrate((e*x+d)^(5/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{2}{3} \left( \frac{3g^2 \arctan\left(\frac{\sqrt{(xe+d)cde-cd^2e+ae^3} g e^{(-1)}}{\sqrt{cdfg-ag^2e}}\right) e^{(-1)}}{(c^2d^2f^2e-2acdfge^2+a^2g^2e^3)\sqrt{cdfg-ag^2e}} - \frac{cdfe^2-age^3-3((xe+d)cde-cd^2e+ae^3)g}{(c^2d^2f^2e-2acdfge^2+a^2g^2e^3)((xe+d)cde-cd^2e+ae^3)} \right) \\
& - \frac{2 \left( 3\sqrt{-cd^2e+ae^3} cd^2g^2 \arctan\left(\frac{\sqrt{-cd^2e+ae^3} g e^{(-1)}}{\sqrt{cdfg-ag^2e}}\right) - 3\sqrt{-cd^2e+ae^3} \right)}{3 \left( \sqrt{cdfg-ag^2e} \sqrt{-cd^2e+ae^3} c^3d^4f^2 - 2\sqrt{cdfg-ag^2e} \sqrt{-cd^2e+ae^3} ac^2d^3fge - \sqrt{cdfg-ag^2e} \sqrt{-cd^2e+ae^3} \right)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**16.128 Problem number 679**

$$\int \frac{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cde x^2}}{\sqrt{d+ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{128(-aeg + cdf)^3 (2ae^2g - cd(-3dg + 5ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{3465c^5d^5e (ex + d)^{\frac{3}{2}}} \\ & + \frac{32(-aeg + cdf)^2 (gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{231c^3d^3 (ex + d)^{\frac{3}{2}}} \\ & + \frac{16(-aeg + cdf) (gx + f)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{99c^2d^2 (ex + d)^{\frac{3}{2}}} \\ & + \frac{2(gx + f)^4 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{11cd (ex + d)^{\frac{3}{2}}} \\ & + \frac{128g(-aeg + cdf)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{1155c^4d^4e\sqrt{ex + d}} \end{aligned}$$

command

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (gx + f)^4}{\sqrt{ex + d}} dx$$

**16.129 Problem number 680**

$$\int \frac{(f + gx)^3 \sqrt{ade + (cd^2 + ae^2)x + cde x^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16(-aeg + cdf)^2 (2ae^2g - cd(-3dg + 5ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{315c^4d^4e (ex + d)^{\frac{3}{2}}} \\ & + \frac{4(-aeg + cdf) (gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{21c^2d^2 (ex + d)^{\frac{3}{2}}} \\ & + \frac{2(gx + f)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{9cd (ex + d)^{\frac{3}{2}}} \\ & + \frac{16g(-aeg + cdf)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{3}{2}}}{105c^3d^3e\sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{315} \left( 105 f^3 \left( \frac{((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} + 9 f g^2 \left( \frac{(15 \sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2)}{cd} \right) e^{(-1)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (gx + f)^3}{\sqrt{ex + d}} dx$$

### 16.130 Problem number 681

$$\int \frac{(f + gx)^2 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8(-aeg + cdf) (2ae^2g - cd(-3dg + 5ef)) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{105c^3d^3e (ex + d)^{\frac{3}{2}}} \\ & + \frac{2(gx + f)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{7cd (ex + d)^{\frac{3}{2}}} \\ & + \frac{8g(-aeg + cdf) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{35c^2d^2e\sqrt{ex + d}} \end{aligned}$$

command

`integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( 35 f^2 \left( \frac{((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} + g^2 \left( \frac{(15 \sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2)}{cd} \right) e^{(-1)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (gx + f)^2}{\sqrt{ex + d}} dx$$

## 16.131 Problem number 682

$$\int \frac{(f + gx) \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(2ae^2g - cd(-3dg + 5ef)) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{15c^2d^2e(ex + d)^{\frac{3}{2}}} + \frac{2g(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{5cde\sqrt{ex + d}}$$

command

`integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{15} \left( 5f \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-1)} - g \left( \frac{5((xe + d)}{\right. \right.$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (gx + f)}{\sqrt{ex + d}} dx$$

## 16.132 Problem number 683

$$\int \frac{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{\sqrt{d + ex}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}}}{3cd(ex + d)^{\frac{3}{2}}}$$

command

`integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{3} \left( \frac{((xe + d)cde - cd^2e + ae^3)^{\frac{3}{2}} e^{(-1)}}{cd} + \frac{\sqrt{-cd^2e + ae^3} cd^2 - \sqrt{-cd^2e + ae^3} ae^2}{cd} \right) e^{(-2)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}} dx$$

## 16.133 Problem number 689

$$\int \frac{(f + gx)^4 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{128(-aeg + cdf)^3 (2ae^2g - cd(-5dg + 7ef)) (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{15015c^5d^5e (ex + d)^{\frac{5}{2}}} \\ & + \frac{128g(-aeg + cdf)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{3003c^4d^4e (ex + d)^{\frac{3}{2}}} \\ & + \frac{32(-aeg + cdf)^2 (gx + f)^2 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{429c^3d^3 (ex + d)^{\frac{5}{2}}} \\ & + \frac{16(-aeg + cdf) (gx + f)^3 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{143c^2d^2 (ex + d)^{\frac{5}{2}}} \\ & + \frac{2(gx + f)^4 (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{13cd (ex + d)^{\frac{5}{2}}} \end{aligned}$$

command

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac"
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^4}{(ex + d)^{\frac{3}{2}}} dx$$

## 16.134 Problem number 690

$$\int \frac{(f + gx)^3 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{16(-aeg + cdf)^2 (2ae^2g - cd(-5dg + 7ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{1155c^4d^4e (ex + d)^{\frac{5}{2}}} \\
& + \frac{16g(-aeg + cdf)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{231c^3d^3e (ex + d)^{\frac{3}{2}}} \\
& + \frac{4(-aeg + cdf)(gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{33c^2d^2 (ex + d)^{\frac{5}{2}}} \\
& + \frac{2(gx + f)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{11cd (ex + d)^{\frac{5}{2}}}
\end{aligned}$$

command

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac"
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^3}{(ex + d)^{\frac{3}{2}}} dx$$

**16.135 Problem number 691**

$$\int \frac{(f + gx)^2 (ade + (cd^2 + ae^2)x + cde x^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{8(-aeg + cdf) (2ae^2g - cd(-5dg + 7ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{315c^3d^3e (ex + d)^{\frac{5}{2}}} \\
& + \frac{8g(-aeg + cdf) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{63c^2d^2e (ex + d)^{\frac{3}{2}}} \\
& + \frac{2(gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{5}{2}}}{9cd (ex + d)^{\frac{5}{2}}}
\end{aligned}$$

command

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac"
```



Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^2}{(ex + d)^{\frac{3}{2}}} dx$$

### 16.136 Problem number 692

$$\int \frac{(f + gx)(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(2ae^2g - cd(-5dg + 7ef))(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{35c^2d^2e(ex + d)^{\frac{5}{2}}} + \frac{2g(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{7cde(ex + d)^{\frac{3}{2}}}$$

command

`integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( cdg \left( \frac{(15 \sqrt{-cd^2e + ae^3} c^3 d^6 - 3 \sqrt{-cd^2e + ae^3} ac^2 d^4 e^2 - 4 \sqrt{-cd^2e + ae^3} a^2 cd^2 e^4 - 8 \sqrt{-cd^2e + ae^3} a^3)}{c^3 d^3} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 16.137 Problem number 693

$$\int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d + ex)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}}}{5cd(ex + d)^{\frac{5}{2}}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{15} \left( cd \left( \frac{\left( 5 \left( (xe + d)cde - cd^2e + ae^3 \right)^{\frac{3}{2}} ae^3 - 3 \left( (xe + d)cde - cd^2e + ae^3 \right)^{\frac{5}{2}} \right) e^{(-2)}}{c^2 d^2} + \frac{3 \sqrt{-cd^2e + ae^3} c^2 d^4 - \dots}{\dots} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(cde x^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}} dx$$

### 16.138 Problem number 700

$$\int \frac{(f + gx)^4 (ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{128(-aeg + cdf)^3 (2ae^2g - cd(-7dg + 9ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{7}{2}}}{45045c^5d^5e (ex + d)^{\frac{7}{2}}} \\ & + \frac{128g(-aeg + cdf)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{7}{2}}}{6435c^4d^4e (ex + d)^{\frac{5}{2}}} \\ & + \frac{32(-aeg + cdf)^2 (gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{7}{2}}}{715c^3d^3 (ex + d)^{\frac{7}{2}}} \\ & + \frac{16(-aeg + cdf) (gx + f)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{7}{2}}}{195c^2d^2 (ex + d)^{\frac{7}{2}}} \\ & + \frac{2(gx + f)^4 (ade + (ae^2 + cd^2)x + cde x^2)^{\frac{7}{2}}}{15cd (ex + d)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.139 Problem number 701

$$\int \frac{(f + gx)^3 (ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{16(-aeg + cdf)^2 (2ae^2g - cd(-7dg + 9ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{3003c^4d^4e (ex + d)^{7/2}} \\ & + \frac{16g(-aeg + cdf)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{429c^3d^3e (ex + d)^{5/2}} \\ & + \frac{12(-aeg + cdf)(gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{143c^2d^2 (ex + d)^{7/2}} \\ & + \frac{2(gx + f)^3 (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{13cd (ex + d)^{7/2}} \end{aligned}$$

command

```
integrate((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac"
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 16.140 Problem number 702

$$\int \frac{(f + gx)^2 (ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8(-aeg + cdf) (2ae^2g - cd(-7dg + 9ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{693c^3d^3e (ex + d)^{7/2}} \\ & + \frac{8g(-aeg + cdf) (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{99c^2d^2e (ex + d)^{5/2}} \\ & + \frac{2(gx + f)^2 (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{11cd (ex + d)^{7/2}} \end{aligned}$$

command

```
integrate((g*x+f)^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.141 Problem number 703

$$\int \frac{(f + gx) (ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(2ae^2g - cd(-7dg + 9ef)) (ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{63c^2d^2e (ex + d)^{7/2}} + \frac{2g(ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{9cde (ex + d)^{5/2}}$$

command

```
integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.142 Problem number 704

$$\int \frac{(ade + (cd^2 + ae^2)x + cde x^2)^{5/2}}{(d + ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{2(ade + (ae^2 + cd^2)x + cde x^2)^{7/2}}{7cd (ex + d)^{7/2}}$$

command

```
integrate((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{105} \left( c^2 d^2 \left( \frac{(15 \sqrt{-cd^2e + ae^3} c^3 d^6 - 3 \sqrt{-cd^2e + ae^3} ac^2 d^4 e^2 - 4 \sqrt{-cd^2e + ae^3} a^2 cd^2 e^4 - 8 \sqrt{-cd^2e + ae^3} a^3)}{c^3 d^3} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 16.143 Problem number 783

$$\int \frac{(d + ex)^{3/2} (f + gx)^4}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{128(-aeg + cdf)^3 (10a e^2 g + cd(-11dg + ef)) (2a e^2 g - cd(-dg + 3ef)) \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{3465c^6 d^6 eg \sqrt{ex + d}} \\ & - \frac{32(-aeg + cdf)^2 (10a e^2 g + cd(-11dg + ef)) (gx + f)^2 \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{1155c^4 d^4 g \sqrt{ex + d}} \\ & - \frac{16(-aeg + cdf) (10a e^2 g + cd(-11dg + ef)) (gx + f)^3 \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{693c^3 d^3 g \sqrt{ex + d}} \\ & - \frac{2(10a e^2 g + cd(-11dg + ef)) (gx + f)^4 \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{99c^2 d^2 g \sqrt{ex + d}} \\ & + \frac{2e(gx + f)^5 \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{11cdg \sqrt{ex + d}} \\ & - \frac{128(-aeg + cdf)^3 (10a e^2 g + cd(-11dg + ef)) \sqrt{ex + d} \sqrt{ade + (a e^2 + c d^2) x + cde x^2}}{3465c^5 d^5 e} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex + d)^{\frac{3}{2}} (gx + f)^4}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

## 16.144 Problem number 784

$$\int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16(-aeg+cdf)^2(8ae^2g+cd(-9dg+ef))(2ae^2g-cd(-dg+3ef))\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{315c^5d^5eg\sqrt{ex+d}} \\ & - \frac{4(-aeg+cdf)(8ae^2g+cd(-9dg+ef))(gx+f)^2\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{105c^3d^3g\sqrt{ex+d}} \\ & - \frac{2(8ae^2g+cd(-9dg+ef))(gx+f)^3\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{63c^2d^2g\sqrt{ex+d}} \\ & + \frac{2e(gx+f)^4\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{9cdg\sqrt{ex+d}} \\ & - \frac{16(-aeg+cdf)^2(8ae^2g+cd(-9dg+ef))\sqrt{ex+d}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{315c^4d^4e} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{3/2}(gx+f)^3}{\sqrt{cde x^2+ade+(cd^2+ae^2)x}} dx$$

## 16.145 Problem number 785

$$\int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(-aeg+cdf)(6ae^2g+cd(-7dg+ef))(2ae^2g-cd(-dg+3ef))\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{105c^4d^4eg\sqrt{ex+d}} \\ & - \frac{2(6ae^2g+cd(-7dg+ef))(gx+f)^2\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{35c^2d^2g\sqrt{ex+d}} \\ & + \frac{2e(gx+f)^3\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{7cdg\sqrt{ex+d}} \\ & - \frac{8(-aeg+cdf)(6ae^2g+cd(-7dg+ef))\sqrt{ex+d}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{105c^3d^3e} \end{aligned}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^3d^4f^2 - 2ac^2d^3fge - ac^2d^2f^2e^2 + a^2cd^2g^2e^2 + 2a^2cdfge^3 - a^3g^2e^4)\sqrt{(xe+d)cde - cd^2e + ae^3}e^{(-1)}}{c^4d^4} + \frac{4\left(3\sqrt{-cd^2e + ae^3}c^3d^6g^2 - 14\sqrt{-cd^2e + ae^3}c^3d^5fge + 35\sqrt{-cd^2e + ae^3}c^3d^4f^2e^2 + 5\sqrt{-cd^2e + ae^3}ac^2d^4g^2\right)}{15c^3d^3} + \frac{2\left(70((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^3fge^3 + 35((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^2f^2e^4 - 70((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}c^2d^2f^2e^4\right)}{15c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)^2}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

### 16.146 Problem number 786

$$\int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade + (cd^2 + ae^2)x + cdex^2}} dx$$

Optimal antiderivative

$$\frac{2g(ex+d)^{\frac{3}{2}}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{5cde} - \frac{4(-ae^2 + cd^2)(4ae^2g - cd(-dg + 5ef))\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{15c^3d^3e\sqrt{ex+d}} - \frac{2(4ae^2g - cd(-dg + 5ef))\sqrt{ex+d}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}}{15c^2d^2e}$$

command

`integrate((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(c^2d^3f - acd^2ge - acdf e^2 + a^2ge^3)\sqrt{(xe+d)cde - cd^2e + ae^3}e^{(-1)}}{c^3d^3} + \frac{4\left(\sqrt{-cd^2e + ae^3}c^2d^4g - 5\sqrt{-cd^2e + ae^3}c^2d^3fe + 3\sqrt{-cd^2e + ae^3}acd^2ge^2 + 5\sqrt{-cd^2e + ae^3}acdf e^3 - 4\sqrt{-cd^2e + ae^3}a^2ge^3\right)}{15c^3d^3} + \frac{2\left(5((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}cd^2ge + 5((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}cdf e^2 - 10((xe+d)cde - cd^2e + ae^3)^{\frac{3}{2}}a^2ge^3\right)}{15c^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{3}{2}}(gx+f)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

## 16.147 Problem number 787

$$\int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cde x^2}} dx$$

Optimal antiderivative

$$\frac{4(-ae^2+cd^2)\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3c^2d^2\sqrt{ex+d}} + \frac{2\sqrt{ex+d}\sqrt{ade+(ae^2+cd^2)x+cde x^2}}{3cd}$$

command

`integrate((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{(xe+d)cde-cd^2e+ae^3}(cd^2-ae^2)e^{(-1)}}{c^2d^2} + \frac{2((xe+d)cde-cd^2e+ae^3)^{\frac{3}{2}}e^{(-2)}}{3c^2d^2} - \frac{4\left(\sqrt{-cd^2e+ae^3}cd^2-\sqrt{-cd^2e+ae^3}ae^2\right)e^{(-1)}}{3c^2d^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cde x^2+ade+(cd^2+ae^2)x}} dx$$

## 16.148 Problem number 847

$$\int \frac{15d^2+20dex+8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}} dx$$

Optimal antiderivative

$$\frac{16 \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{bx+a}}{\sqrt{b}\sqrt{ex+d}}\right)}{\sqrt{b}\sqrt{e}} + \frac{2d^2\sqrt{bx+a}}{(-ae+bd)(ex+d)^{\frac{3}{2}}} + \frac{4d(-2ae+3bd)\sqrt{bx+a}}{(-ae+bd)^2\sqrt{ex+d}}$$

command

`integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{16\sqrt{b}e^{(-\frac{1}{2})}\log\left(\left|-\sqrt{bx+a}\sqrt{b}e^{\frac{1}{2}}+\sqrt{b^2d+(bx+a)be-abe}\right|\right)}{|b|} + \frac{2\sqrt{bx+a}\left(\frac{2(3b^6d^2e^2-2ab^5de^3)(bx+a)}{b^4d^2|b|e-2ab^3d|b|e^2+a^2b^2|b|e^3}+\frac{7b^7d^3e-11ab^6d^2e^2+4a^2b^5de^3}{b^4d^2|b|e-2ab^3d|b|e^2+a^2b^2|b|e^3}\right)}{(b^2d+(bx+a)be-abe)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 16.149 Problem number 848

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}} dx$$

Optimal antiderivative

$$\frac{6d^2\sqrt{bx+a}}{5(-ae+bd)(ex+d)^{\frac{5}{2}}} + \frac{8d(-5ae+8bd)\sqrt{bx+a}}{15(-ae+bd)^2(ex+d)^{\frac{3}{2}}} + \frac{16(15a^2e^2-35abde+23b^2d^2)\sqrt{bx+a}}{15(-ae+bd)^3\sqrt{ex+d}}$$

command

`integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(4(bx+a)\left(\frac{2(23b^8d^2e^4-35ab^7de^5+15a^2b^6e^6)(bx+a)}{b^5d^3|b|e^2-3ab^4d^2|b|e^3+3a^2b^3d|b|e^4-a^3b^2|b|e^5}+\frac{5(20b^9d^3e^3-49ab^8d^2e^4+41a^2b^7de^5-12a^3b^6e^6)}{b^5d^3|b|e^2-3ab^4d^2|b|e^3+3a^2b^3d|b|e^4-a^3b^2|b|e^5}\right)+\frac{15(15b^{10}d^4e^2-50ab^9d^3e^3-35ab^8d^2e^4+41a^2b^7de^5-12a^3b^6e^6)}{b^5d^3|b|e^2-3ab^4d^2|b|e^3+3a^2b^3d|b|e^4-a^3b^2|b|e^5}\right)}{15(b^2d+(bx+a)be-abe)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 16.150 Problem number 849

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}} dx$$

Optimal antiderivative

$$\frac{6d^2\sqrt{bx+a}}{7(-ae+bd)(ex+d)^{\frac{7}{2}}} + \frac{4d(-14ae+23bd)\sqrt{bx+a}}{35(-ae+bd)^2(ex+d)^{\frac{5}{2}}} + \frac{16(35a^2e^2-84abde+58b^2d^2)\sqrt{bx+a}}{105(-ae+bd)^3(ex+d)^{\frac{3}{2}}} + \frac{32b(35a^2e^2-84abde+58b^2d^2)\sqrt{bx+a}}{105(-ae+bd)^4\sqrt{ex+d}}$$

command

```
integrate((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( 2 \left( 4 (bx + a) \left( \frac{2 (58 b^{10} d^2 e^6 - 84 ab^9 de^7 + 35 a^2 b^8 e^8)(bx+a)}{b^6 d^4 |b| e^3 - 4 ab^5 d^3 |b| e^4 + 6 a^2 b^4 d^2 |b| e^5 - 4 a^3 b^3 d |b| e^6 + a^4 b^2 |b| e^7} + \frac{7 (58 b^{11} d^3 e^5 - 142 ab^{10} d^2 e^6 + 119 a^2 b^9 de^7 - 35 a^3 b^8 e^8)}{b^6 d^4 |b| e^3 - 4 ab^5 d^3 |b| e^4 + 6 a^2 b^4 d^2 |b| e^5 - 4 a^3 b^3 d |b| e^6 + a^4 b^2 |b| e^7} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 16.151 Problem number 861

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)(f + gx)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{g^2 (cx^2 + bx + a)^{\frac{3}{2}}}{3(-dg + ef)(ag^2 - bfg + cf^2)(gx + f)^3} \\
& + \frac{(-4ac + b^2)g(-bg + 2cf) \operatorname{arctanh}\left(\frac{bf - 2ag + (-bg + 2cf)x}{2\sqrt{ag^2 - bfg + cf^2}\sqrt{cx^2 + bx + a}}\right)}{16(-dg + ef)(ag^2 - bfg + cf^2)^{\frac{5}{2}}} \\
& + \frac{(-4ac + b^2)eg \operatorname{arctanh}\left(\frac{bf - 2ag + (-bg + 2cf)x}{2\sqrt{ag^2 - bfg + cf^2}\sqrt{cx^2 + bx + a}}\right)}{8(-dg + ef)^2(ag^2 - bfg + cf^2)^{\frac{3}{2}}} \\
& - \frac{e^2(-be + 2cd) \operatorname{arctanh}\left(\frac{2cx + b}{2\sqrt{c}\sqrt{cx^2 + bx + a}}\right)}{2(-dg + ef)^4\sqrt{c}} \\
& + \frac{e^3(-bg + 2cf) \operatorname{arctanh}\left(\frac{2cx + b}{2\sqrt{c}\sqrt{cx^2 + bx + a}}\right)}{2g(-dg + ef)^4\sqrt{c}} - \frac{e^2 \operatorname{arctanh}\left(\frac{2cx + b}{2\sqrt{c}\sqrt{cx^2 + bx + a}}\right)\sqrt{c}}{g(-dg + ef)^3} \\
& + \frac{e^2 \operatorname{arctanh}\left(\frac{bd - 2ae + (-be + 2cd)x}{2\sqrt{ae^2 - bde + cd^2}\sqrt{cx^2 + bx + a}}\right)\sqrt{ae^2 - bde + cd^2}}{(-dg + ef)^4} \\
& + \frac{e^2(-bg + 2cf) \operatorname{arctanh}\left(\frac{bf - 2ag + (-bg + 2cf)x}{2\sqrt{ag^2 - bfg + cf^2}\sqrt{cx^2 + bx + a}}\right)}{2g(-dg + ef)^3\sqrt{ag^2 - bfg + cf^2}} \\
& - \frac{e^3 \operatorname{arctanh}\left(\frac{bf - 2ag + (-bg + 2cf)x}{2\sqrt{ag^2 - bfg + cf^2}\sqrt{cx^2 + bx + a}}\right)\sqrt{ag^2 - bfg + cf^2}}{g(-dg + ef)^4} \\
& + \frac{e^2\sqrt{cx^2 + bx + a}}{(-dg + ef)^3(gx + f)} - \frac{g(-bg + 2cf)(bf - 2ag + (-bg + 2cf)x)\sqrt{cx^2 + bx + a}}{8(-dg + ef)(ag^2 - bfg + cf^2)^2(gx + f)^2} \\
& - \frac{eg(bf - 2ag + (-bg + 2cf)x)\sqrt{cx^2 + bx + a}}{4(-dg + ef)^2(ag^2 - bfg + cf^2)(gx + f)^2}
\end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 17 Test file number 36

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/36\_1.2.1.5-a+b\_x+c\_x^2-^p-d+e\_x+f\_x^2-^q

### 17.1 Problem number 6

$$\int \frac{1}{\sqrt{a+bx+cx^2} (d+bx+cx^2)^4} dx$$

Optimal antiderivative

$$\frac{(b^2 + 4c(a - 2d)) (5b^4 - 8b^2c(a + 4d) + 16c^2(5a^2 - 8ad + 8d^2)) \operatorname{arctanh}\left(\frac{(2cx+b)\sqrt{a-d}}{\sqrt{b^2-4cd}\sqrt{cx^2+bx+a}}\right) - \frac{8(a-d)^{\frac{7}{2}}(b^2-4cd)^{\frac{7}{2}}}{3(a-d)(b^2-4cd)(cx^2+bx+d)^3} + \frac{5(b^2+4c(a-2d))(2cx+b)\sqrt{cx^2+bx+a}}{12(a-d)^2(b^2-4cd)^2(cx^2+bx+d)^2} - \frac{(15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))(2cx+b)\sqrt{cx^2+bx+a}}{24(a-d)^3(b^2-4cd)^3(cx^2+bx+d)}}{1}$$

command

```
integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 18 Test file number 38

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1\_Quadratic/38\_1.2.1.9\_P-x-d+e\_x-^m-a+b\_x+c\_x^2-^p

### 18.1 Problem number 4

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{d + ex} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{C(-e^2x^2 + d^2)^{\frac{3}{2}}}{3e^3} + \frac{(-Be + Cd)(-e^2x^2 + d^2)^{\frac{3}{2}}}{2e^3(ex + d)} \\ & + \frac{d(Cd^2 - e(-2Ae + Bd)) \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2e^3} \\ & + \frac{(Cd^2 - e(-2Ae + Bd)) \sqrt{-e^2x^2 + d^2}}{2e^3} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} (Cd^3 - Bd^2e + 2Ade^2) \arcsin\left(\frac{xe}{d}\right) e^{(-3)\operatorname{sgn}(d)} \\ & + \frac{1}{6} \sqrt{-x^2e^2 + d^2} \left( (2Cxe^{(-1)} - 3(Cde^3 - Be^4)e^{(-5)})x + 2(2Cd^2e^2 - 3Bde^3 + 3Ae^4)e^{(-5)} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 18.2 Problem number 5

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ae^2 - Bde + Cd^2)(-e^2x^2 + d^2)^{\frac{3}{2}}}{de^3(ex + d)^2} - \frac{C(-e^2x^2 + d^2)^{\frac{3}{2}}}{2e^3(ex + d)} \\ & - \frac{(5Cd^2 - 2e(-Ae + 2Bd)) \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{2e^3} \\ & - \frac{(5Cd^2 - 2e(-Ae + 2Bd)) \sqrt{-e^2x^2 + d^2}}{2de^3} \end{aligned}$$



### 18.4 Problem number 7

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ae^2 - Bde + Cd^2)(-e^2x^2 + d^2)^{\frac{3}{2}}}{5de^3(ex + d)^4} + \frac{(-Be + 2Cd)(-e^2x^2 + d^2)^{\frac{3}{2}}}{3de^3(ex + d)^3} \\ & -\frac{(Ae^2 - Bde + Cd^2)(-e^2x^2 + d^2)^{\frac{3}{2}}}{15d^2e^3(ex + d)^3} - \frac{C \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{e^3} - \frac{2C\sqrt{-e^2x^2 + d^2}}{e^3(ex + d)} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -C \arcsin\left(\frac{xe}{d}\right) e^{(-3)\operatorname{sgn}(d)} \\ & 2 \left( \frac{105 \left( de + \sqrt{-x^2e^2 + d^2} e \right) Cd^2e^{(-2)}}{x} + \frac{165 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^2 Cd^2e^{(-4)}}{x^2} + \frac{75 \left( de + \sqrt{-x^2e^2 + d^2} e \right)^3 Cd^2e^{(-6)}}{x^3} + \dots \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 18.5 Problem number 8

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ae^2 - Bde + Cd^2)(-e^2x^2 + d^2)^{\frac{3}{2}}}{7de^3(ex + d)^5} + \frac{C(-e^2x^2 + d^2)^{\frac{3}{2}}}{e^3(ex + d)^4} \\ & -\frac{(23Cd^2 + e(2Ae + 5Bd))(-e^2x^2 + d^2)^{\frac{3}{2}}}{35d^2e^3(ex + d)^4} - \frac{(23Cd^2 + e(2Ae + 5Bd))(-e^2x^2 + d^2)^{\frac{3}{2}}}{105d^3e^3(ex + d)^3} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{420} \left( \left( 3 \left( 5 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{7}{2}} + 21 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} + 35 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{3}{2}} + 35 \sqrt{\frac{2d}{xe+d} - 1} \right) C \operatorname{sgn}\left(\frac{1}{xe+d}\right) - 35 \left( 3 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{7}{2}} + 21 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{5}{2}} + 35 \left( \frac{2d}{xe+d} - 1 \right)^{\frac{3}{2}} + 35 \sqrt{\frac{2d}{xe+d} - 1} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 18.6 Problem number 9

$$\int \frac{(A + Bx + Cx^2) \sqrt{d^2 - e^2x^2}}{(d + ex)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ae^2 - Bde + Cd^2)(-e^2x^2 + d^2)^{\frac{3}{2}}}{9de^3(ex + d)^6} + \frac{C(-e^2x^2 + d^2)^{\frac{3}{2}}}{2e^3(ex + d)^5} \\ & -\frac{(11Cd^2 + 2e(Ae + 2Bd))(-e^2x^2 + d^2)^{\frac{3}{2}}}{42d^2e^3(ex + d)^5} - \frac{(11Cd^2 + 2e(Ae + 2Bd))(-e^2x^2 + d^2)^{\frac{3}{2}}}{105d^3e^3(ex + d)^4} \\ & -\frac{(11Cd^2 + 2e(Ae + 2Bd))(-e^2x^2 + d^2)^{\frac{3}{2}}}{315d^4e^3(ex + d)^3} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{36 \left( de + \sqrt{-x^2e^2 + d^2} \right) e C d^2 e^{(-2)}}{x} + \frac{144 \left( de + \sqrt{-x^2e^2 + d^2} \right)^2 C d^2 e^{(-4)}}{x^2} - \frac{84 \left( de + \sqrt{-x^2e^2 + d^2} \right)^3 C d^2 e^{(-6)}}{x^3} + \frac{504 \left( de + \sqrt{-x^2e^2 + d^2} \right)^4 C d^2 e^{(-8)}}{x^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 18.7 Problem number 14

$$\int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$\frac{(-Be + Cd) \arctan\left(\frac{ex}{\sqrt{-e^2x^2 + d^2}}\right)}{e^3} - \frac{C\sqrt{-e^2x^2 + d^2}}{e^3} - \frac{(Ae^2 - Bde + Cd^2)\sqrt{-e^2x^2 + d^2}}{de^3(ex + d)}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-(Cd - Be) \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sgn}(d) - \sqrt{-x^2e^2 + d^2} Ce^{(-3)} + \frac{2(Cd^2 - Bde + Ae^2)e^{(-3)}}{d\left(\frac{(de + \sqrt{-x^2e^2 + d^2})e^{(-2)}}{x} + 1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 18.8 Problem number 16

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3\sqrt{d^2 - e^2x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(Ae^2 - Bde + Cd^2)\sqrt{-e^2x^2 + d^2}}{5de^3(ex + d)^3} + \frac{C\sqrt{-e^2x^2 + d^2}}{e^3(ex + d)^2} \\ & -\frac{(7Cd^2 + e(2Ae + 3Bd))\sqrt{-e^2x^2 + d^2}}{15d^2e^3(ex + d)^2} - \frac{(7Cd^2 + e(2Ae + 3Bd))\sqrt{-e^2x^2 + d^2}}{15d^3e^3(ex + d)} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{10 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) C d^2 e^{(-2)}}{x} + \frac{20 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 C d^2 e^{(-4)}}{x^2} + 2 C d^2 + 3 B d e + \frac{15 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)}{x} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 18.9 Problem number 17

$$\int \frac{A + Bx + Cx^2}{(d + ex)^4 \sqrt{d^2 - e^2 x^2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A e^2 - B d e + C d^2) \sqrt{-e^2 x^2 + d^2}}{7 d e^3 (e x + d)^4} + \frac{C \sqrt{-e^2 x^2 + d^2}}{2 e^3 (e x + d)^3} \\ & - \frac{(6 A e^2 + 8 B d e + 13 C d^2) \sqrt{-e^2 x^2 + d^2}}{70 d^2 e^3 (e x + d)^3} - \frac{(6 A e^2 + 8 B d e + 13 C d^2) \sqrt{-e^2 x^2 + d^2}}{105 d^3 e^3 (e x + d)^2} \\ & - \frac{(6 A e^2 + 8 B d e + 13 C d^2) \sqrt{-e^2 x^2 + d^2}}{105 d^4 e^3 (e x + d)} \end{aligned}$$

command

```
integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{56 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) C d^2 e^{(-2)}}{x} + \frac{168 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2 C d^2 e^{(-4)}}{x^2} + \frac{140 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^3 C d^2 e^{(-6)}}{x^3} + \frac{140}{x} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 18.10 Problem number 195

$$\int \frac{\sqrt{a+bx+cx^2} (d+ex+fx^2)}{(g+hx)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(fg^2 - h(-dh + eg)) (cx^2 + bx + a)^{\frac{3}{2}}}{5h (ah^2 - bgh + cg^2) (hx + g)^5} \\ & + \frac{(2cg(3fg^2 + h(-7dh + 2eg)) + h(10ah(-eh + 2fg) - b(-7dh^2 - 3egh + 13fg^2))) (cx^2 + bx + a)^{\frac{3}{2}}}{40h (ah^2 - bgh + cg^2)^2 (hx + g)^4} \\ & + \frac{(4c^2g^2(3fg^2 + h(-27dh + 2eg)) - 5h^2(16a^2fh^2 - 2abh(5eh + 6fg) + b^2(7dh^2 + 3egh + 3fg^2)) - 2ch(bg(-54 \\ & \quad (-4ac + b^2) (32c^3dg^3 - 8c^2g(2bg(3dh + eg) + a(3dh^2 - 6egh + fg^2)) - bh(16a^2fh^2 - 2abh(5eh + 6fg) + b^2(7dh^2 + 3egh \\ & \quad (32c^3dg^3 - 8c^2g(2bg(3dh + eg) + a(3dh^2 - 6egh + fg^2)) - bh(16a^2fh^2 - 2abh(5eh + 6fg) + b^2(7dh^2 + 3egh \\ & \quad 12 \end{aligned}$$

command

```
integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 18.11 Problem number 206

$$\int \frac{(a+bx+cx^2)^{3/2} (d+ex+fx^2)}{(g+hx)^7} dx$$

Optimal antiderivative

$$\frac{(24c^2d g^2 + 24a^2f h^2 - 12abh(eh + 2fg) + b^2(7dh^2 + 5egh + 7fg^2) - 4c(3bg(2dh + eg) + a(dh^2 - 7egh + fg^2)))}{192(a h^2 - bgh + cg^2)^3 (hx + g)^4}$$

$$- \frac{(fg^2 - h(-dh + eg))(cx^2 + bx + a)^{\frac{5}{2}}}{6h(a h^2 - bgh + cg^2)(hx + g)^6}$$

$$+ \frac{(2cg(5fg^2 + h(-7dh + eg)) + h(12ah(-eh + 2fg) - b(-7dh^2 - 5egh + 17fg^2))) (cx^2 + bx + a)^{\frac{5}{2}}}{60h(a h^2 - bgh + cg^2)^2 (hx + g)^5}$$

$$+ \frac{(-4ac + b^2)^2 (24c^2d g^2 + 24a^2f h^2 - 12abh(eh + 2fg) + b^2(7dh^2 + 5egh + 7fg^2) - 4c(3bg(2dh + eg) + a(dh^2 - 7egh + fg^2)))}{1024(a h^2 - bgh + cg^2)^{\frac{9}{2}}}$$

$$- \frac{(-4ac + b^2) (24c^2d g^2 + 24a^2f h^2 - 12abh(eh + 2fg) + b^2(7dh^2 + 5egh + 7fg^2) - 4c(3bg(2dh + eg) + a(dh^2 - 7egh + fg^2)))}{512(a h^2 - bgh + cg^2)^4 (hx + g)^2}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**18.12 Problem number 207**

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^8} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(48c^3d g^3 - 8c^2g(3bg(3dh + eg) + a(3dh^2 - 8egh + fg^2)) - bh(24a^2f h^2 - 2abh(7eh + 10fg) + b^2(9dh^2 + 5egh \\ & - \frac{(fg^2 - h(-dh + eg))(cx^2 + bx + a)^{\frac{5}{2}}}{7h(ah^2 - bgh + cg^2)(hx + g)^7} \\ & + \frac{(2cg(5fg^2 + h(-9dh + 2eg)) + h(14ah(-eh + 2fg) - b(-9dh^2 - 5egh + 19fg^2))) (cx^2 + bx + a)^{\frac{5}{2}}}{84h(ah^2 - bgh + cg^2)^2 (hx + g)^6} \\ & + \frac{(4c^2g^2(5fg^2 + h(-51dh + 2eg)) - 7h^2(24a^2f h^2 - 2abh(7eh + 10fg) + b^2(9dh^2 + 5egh + 5fg^2)) - 2ch(3bg(- \\ & (-4ac + b^2)^2 (48c^3d g^3 - 8c^2g(3bg(3dh + eg) + a(3dh^2 - 8egh + fg^2)) - bh(24a^2f h^2 - 2abh(7eh + 10fg) + b^2 \\ & + \frac{(-4ac + b^2) (48c^3d g^3 - 8c^2g(3bg(3dh + eg) + a(3dh^2 - 8egh + fg^2)) - bh(24a^2f h^2 - 2abh(7eh + 10fg) + b^2}{840h(ah^2 - bgh + cg^2)^3 (hx + g)^5} \end{aligned}$$

command

```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 19 Test file number 39

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.2_Quartic/39_1.2.2.2-d_x-
~m-a+b_x^2+c_x^4-~p
```

### 19.1 Problem number 268

$$\int \frac{x^4}{\sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$-\frac{2b\sqrt{cx^4 + bx^2}}{3c^2x} + \frac{x\sqrt{cx^4 + bx^2}}{3c}$$

command

```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2b^{\frac{3}{2}}\operatorname{sgn}(x)}{3c^2} + \frac{(cx^2+b)^{\frac{3}{2}}}{3c^2\operatorname{sgn}(x)} - \frac{\sqrt{cx^2+b}b}{c^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}} dx$$

## 19.2 Problem number 271

$$\int \frac{1}{x^2\sqrt{bx^2+cx^4}} dx$$

Optimal antiderivative

$$\frac{c \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4+bx^2}}\right)}{2b^{\frac{3}{2}}} - \frac{\sqrt{cx^4+bx^2}}{2bx^3}$$

command

```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 \operatorname{arctan}\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{cx^2+b}c}{bx^2} - \frac{\sqrt{cx^2+b}c}{2c\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 19.3 Problem number 272

$$\int \frac{1}{x^4 \sqrt{bx^2 + cx^4}} dx$$

Optimal antiderivative

$$-\frac{3c^2 \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{8b^{\frac{5}{2}}} - \frac{\sqrt{cx^4 + bx^2}}{4bx^5} + \frac{3c\sqrt{cx^4 + bx^2}}{8b^2x^3}$$

command

```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3c^3 \operatorname{arctan}\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2} + \frac{3(cx^2 + b)^{\frac{3}{2}} c^3 - 5\sqrt{cx^2 + b} bc^3}{b^2 c^2 x^4}$$

8 csgn(x)

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 19.4 Problem number 275

$$\int \frac{x^5}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{x^2\sqrt{c}}{\sqrt{cx^4 + bx^2}}\right)}{c^{\frac{3}{2}}} - \frac{x^2}{c\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|b| \operatorname{sgn}(x))}{2c^{\frac{3}{2}}} - \frac{x}{\sqrt{cx^2 + b} \operatorname{csgn}(x)} - \frac{\log\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right)}{c^{\frac{3}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 19.5 Problem number 278

$$\int \frac{1}{x (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{bx^2 \sqrt{cx^4 + bx^2}} - \frac{4\sqrt{cx^4 + bx^2}}{3b^2x^4} + \frac{8c\sqrt{cx^4 + bx^2}}{3b^3x^2}$$

command

`integrate(1/x/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2x}{\sqrt{cx^2 + b} b^3 \operatorname{sgn}(x)} - \frac{2 \left( 3 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^4 c^{\frac{3}{2}} - 12 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 bc^{\frac{3}{2}} + 5 b^2 c^{\frac{3}{2}} \right)}{3 \left( \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 - b \right)^3 b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x} dx$$

## 19.6 Problem number 279

$$\int \frac{1}{x^3 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{bx^4 \sqrt{cx^4 + bx^2}} - \frac{6\sqrt{cx^4 + bx^2}}{5b^2x^6} + \frac{8c\sqrt{cx^4 + bx^2}}{5b^3x^4} - \frac{16c^2\sqrt{cx^4 + bx^2}}{5b^4x^2}$$

command

`integrate(1/x^3/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^3x}{\sqrt{cx^2 + b} b^4 \operatorname{sgn}(x)} + \frac{2 \left( 5 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^8 c^{\frac{5}{2}} - 30 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^6 bc^{\frac{5}{2}} + 80 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^4 b^2 c^{\frac{5}{2}} - 50 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 b^3 \operatorname{sgn}(x) \right)}{5 \left( \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 - b \right)^5 b^3 \operatorname{sgn}(x)}$$



Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x^3} dx$$

### 19.7 Problem number 280

$$\int \frac{1}{x^5 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{bx^6 \sqrt{cx^4 + bx^2}} - \frac{8\sqrt{cx^4 + bx^2}}{7b^2x^8} + \frac{48c\sqrt{cx^4 + bx^2}}{35b^3x^6} - \frac{64c^2\sqrt{cx^4 + bx^2}}{35b^4x^4} + \frac{128c^3\sqrt{cx^4 + bx^2}}{35b^5x^2}$$

command

```
integrate(1/x^5/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^4 x}{\sqrt{cx^2 + b} b^5 \operatorname{sgn}(x)} - \frac{2 \left( 35 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{12} c^{\frac{7}{2}} - 280 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^{10} bc^{\frac{7}{2}} + 1015 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^8 b^2 c^{\frac{7}{2}} - 2240 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^6 b^3 c^{\frac{7}{2}} + 1120 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^4 b^4 c^{\frac{7}{2}} - 224 \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 b^5 c^{\frac{7}{2}} + 35 b^6 c^{\frac{7}{2}} \right)}{35 \left( \left( \sqrt{c} x - \sqrt{cx^2 + b} \right)^2 + b \right)^{\frac{7}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x^5} dx$$

### 19.8 Problem number 283

$$\int \frac{x^2}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{b^{\frac{3}{2}}} + \frac{x}{b\sqrt{cx^4 + bx^2}}$$

command

```
integrate(x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(\sqrt{b} \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\right) \operatorname{sgn}(x)}{\sqrt{-b} b^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} b \operatorname{sgn}(x)} + \frac{1}{\sqrt{cx^2+b} b \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 19.9 Problem number 284

$$\int \frac{1}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3c \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{2b^{\frac{5}{2}}} + \frac{1}{bx \sqrt{cx^4 + bx^2}} - \frac{3\sqrt{cx^4 + bx^2}}{2b^2 x^3}$$

command

```
integrate(1/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3c \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{2\sqrt{-b} b^2 \operatorname{sgn}(x)} - \frac{3(cx^2+b)c - 2bc}{2\left((cx^2+b)^{\frac{3}{2}} - \sqrt{cx^2+b} b\right) b^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.10 Problem number 285

$$\int \frac{1}{x^2 (bx^2 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{15c^2 \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{8b^{\frac{7}{2}}} + \frac{1}{bx^3 \sqrt{cx^4 + bx^2}} - \frac{5\sqrt{cx^4 + bx^2}}{4b^2x^5} + \frac{15c\sqrt{cx^4 + bx^2}}{8b^3x^3}$$

command

`integrate(1/x^2/(c*x^4+b*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15c^2 \arctan\left(\frac{\sqrt{cx^2 + b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^3\operatorname{sgn}(x)} + \frac{c^2}{\sqrt{cx^2 + b}b^3\operatorname{sgn}(x)} + \frac{7(cx^2 + b)^{\frac{3}{2}}c^2 - 9\sqrt{cx^2 + b}bc^2}{8b^3c^2x^4\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}}x^2} dx$$

## 19.11 Problem number 640

$$\int \frac{x^4}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{3x}{8b^2\sqrt{(bx^2 + a)^2}} - \frac{x^3}{4b(bx^2 + a)\sqrt{(bx^2 + a)^2}} + \frac{3(bx^2 + a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}\sqrt{a}\sqrt{(bx^2 + a)^2}}$$

command

`integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}b^2\operatorname{sgn}(bx^2 + a)} - \frac{5bx^3 + 3ax}{8(bx^2 + a)^2b^2\operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.12 Problem number 641

$$\int \frac{x^2}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{8ab\sqrt{(bx^2+a)^2}} - \frac{x}{4b(bx^2+a)\sqrt{(bx^2+a)^2}} + \frac{(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}b^{\frac{3}{2}}\sqrt{(bx^2+a)^2}}$$

command

```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}ab\operatorname{sgn}(bx^2+a)} + \frac{bx^3 - ax}{8(bx^2+a)^2ab\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.13 Problem number 642

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(bx^2+a)}{4a(b^2x^4+2abx^2+a^2)^{\frac{3}{2}}} + \frac{3x(bx^2+a)^2}{8a^2(b^2x^4+2abx^2+a^2)^{\frac{3}{2}}} + \frac{3(bx^2+a)^3\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}(b^2x^4+2abx^2+a^2)^{\frac{3}{2}}\sqrt{b}}$$

command

```
integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2\operatorname{sgn}(bx^2+a)} + \frac{3bx^3 + 5ax}{8(bx^2+a)^2a^2\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.14 Problem number 643

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5}{8a^2x\sqrt{(bx^2+a)^2}} + \frac{1}{4ax(bx^2+a)\sqrt{(bx^2+a)^2}} \\ & - \frac{15(bx^2+a)}{8a^3x\sqrt{(bx^2+a)^2}} - \frac{15(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)\sqrt{b}}{8a^{\frac{7}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

```
integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{15b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^3\operatorname{sgn}(bx^2+a)} - \frac{7b^2x^3+9abx}{8(bx^2+a)^2a^3\operatorname{sgn}(bx^2+a)} - \frac{1}{a^3x\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.15 Problem number 644

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7}{8a^2x^3\sqrt{(bx^2+a)^2}} + \frac{1}{4ax^3(bx^2+a)\sqrt{(bx^2+a)^2}} - \frac{35(bx^2+a)}{24a^3x^3\sqrt{(bx^2+a)^2}} \\ & + \frac{35b(bx^2+a)}{8a^4x\sqrt{(bx^2+a)^2}} + \frac{35b^{\frac{3}{2}}(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{9}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^4 \operatorname{sgn}(bx^2 + a)} + \frac{11 b^3 x^3 + 13 ab^2 x}{8 (bx^2 + a)^2 a^4 \operatorname{sgn}(bx^2 + a)} + \frac{9 bx^2 - a}{3 a^4 x^3 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 19.16 Problem number 653

$$\int \frac{x^6}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5x}{128ab^3 \sqrt{(bx^2 + a)^2}} - \frac{x^5}{8b(bx^2 + a)^3 \sqrt{(bx^2 + a)^2}} - \frac{5x^3}{48b^2(bx^2 + a)^2 \sqrt{(bx^2 + a)^2}} \\ & - \frac{5x}{64b^3(bx^2 + a) \sqrt{(bx^2 + a)^2}} + \frac{5(bx^2 + a) \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}b^{\frac{7}{2}} \sqrt{(bx^2 + a)^2}} \end{aligned}$$

command

`integrate(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab} ab^3 \operatorname{sgn}(bx^2 + a)} + \frac{15 b^3 x^7 - 73 ab^2 x^5 - 55 a^2 bx^3 - 15 a^3 x}{384 (bx^2 + a)^4 ab^3 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.17 Problem number 654

$$\int \frac{x^4}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3x}{128a^2b^2\sqrt{(bx^2+a)^2}} - \frac{x^3}{8b(bx^2+a)^3\sqrt{(bx^2+a)^2}} - \frac{x}{16b^2(bx^2+a)^2\sqrt{(bx^2+a)^2}} \\ & + \frac{x}{64ab^2(bx^2+a)\sqrt{(bx^2+a)^2}} + \frac{3(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{128a^{\frac{5}{2}}b^{\frac{5}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

```
integrate(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}a^2b^2\operatorname{sgn}(bx^2+a)} + \frac{3b^3x^7 + 11ab^2x^5 - 11a^2bx^3 - 3a^3x}{128(bx^2+a)^4a^2b^2\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.18 Problem number 655

$$\int \frac{x^2}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5x}{128a^3b\sqrt{(bx^2+a)^2}} - \frac{x}{8b(bx^2+a)^3\sqrt{(bx^2+a)^2}} + \frac{x}{48ab(bx^2+a)^2\sqrt{(bx^2+a)^2}} \\ & + \frac{5x}{192a^2b(bx^2+a)\sqrt{(bx^2+a)^2}} + \frac{5(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{128a^{\frac{7}{2}}b^{\frac{3}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

```
integrate(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab} a^3 \operatorname{sgn}(bx^2 + a)} + \frac{15 b^3 x^7 + 55 ab^2 x^5 + 73 a^2 b x^3 - 15 a^3 x}{384 (bx^2 + a)^4 a^3 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 19.19 Problem number 656

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(bx^2 + a)}{8a(b^2x^4 + 2abx^2 + a^2)^{5/2}} + \frac{7x(bx^2 + a)^2}{48a^2(b^2x^4 + 2abx^2 + a^2)^{5/2}} + \frac{35x(bx^2 + a)^3}{192a^3(b^2x^4 + 2abx^2 + a^2)^{5/2}} \\ & + \frac{35x(bx^2 + a)^4}{128a^4(b^2x^4 + 2abx^2 + a^2)^{5/2}} + \frac{35(bx^2 + a)^5 \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{128a^{9/2}(b^2x^4 + 2abx^2 + a^2)^{5/2} \sqrt{b}} \end{aligned}$$

command

`integrate(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab} a^4 \operatorname{sgn}(bx^2 + a)} + \frac{105 b^3 x^7 + 385 ab^2 x^5 + 511 a^2 b x^3 + 279 a^3 x}{384 (bx^2 + a)^4 a^4 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*



## 19.20 Problem number 657

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105}{128a^4x\sqrt{(bx^2+a)^2}} + \frac{1}{8ax(bx^2+a)^3\sqrt{(bx^2+a)^2}} + \frac{3}{16a^2x(bx^2+a)^2\sqrt{(bx^2+a)^2}} \\ & + \frac{21}{64a^3x(bx^2+a)\sqrt{(bx^2+a)^2}} - \frac{315(bx^2+a)}{128a^5x\sqrt{(bx^2+a)^2}} - \frac{315(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)\sqrt{b}}{128a^{\frac{11}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

`integrate(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{315b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}a^5\operatorname{sgn}(bx^2+a)} - \frac{1}{a^5x\operatorname{sgn}(bx^2+a)} - \frac{187b^4x^7 + 643ab^3x^5 + 765a^2b^2x^3 + 325a^3bx}{128(bx^2+a)^4a^5\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.21 Problem number 658

$$\int \frac{1}{x^4 (a^2 + 2abx^2 + b^2x^4)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{231}{128a^4x^3\sqrt{(bx^2+a)^2}} + \frac{1}{8ax^3(bx^2+a)^3\sqrt{(bx^2+a)^2}} \\ & + \frac{11}{48a^2x^3(bx^2+a)^2\sqrt{(bx^2+a)^2}} + \frac{33}{64a^3x^3(bx^2+a)\sqrt{(bx^2+a)^2}} \\ & - \frac{385(bx^2+a)}{128a^5x^3\sqrt{(bx^2+a)^2}} + \frac{1155b(bx^2+a)}{128a^6x\sqrt{(bx^2+a)^2}} + \frac{1155b^{\frac{3}{2}}(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{128a^{\frac{13}{2}}\sqrt{(bx^2+a)^2}} \end{aligned}$$

command

```
integrate(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1155 b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{128 \sqrt{ab} a^6 \operatorname{sgn}(bx^2 + a)} + \frac{15 bx^2 - a}{3 a^6 x^3 \operatorname{sgn}(bx^2 + a)} + \frac{1545 b^5 x^7 + 5153 ab^4 x^5 + 5855 a^2 b^3 x^3 + 2295 a^3 b^2 x}{384 (bx^2 + a)^4 a^6 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 19.22 Problem number 994

$$\int \frac{x^4}{\sqrt{2 + 2a - 2(1 + a) + bx^2 + cx^4}} dx$$

Optimal antiderivative

$$-\frac{2b\sqrt{cx^4 + bx^2}}{3c^2x} + \frac{x\sqrt{cx^4 + bx^2}}{3c}$$

command

```
integrate(x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 b^{\frac{3}{2}} \operatorname{sgn}(x)}{3 c^2} + \frac{(cx^2 + b)^{\frac{3}{2}}}{3 c^2 \operatorname{sgn}(x)} - \frac{\sqrt{cx^2 + b} b}{c^2 \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2}} dx$$

## 19.23 Problem number 1000

$$\int \frac{1}{x^2 \sqrt{2 + 2a - 2(1 + a) + bx^2 + cx^4}} dx$$

Optimal antiderivative

$$\frac{c \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4 + bx^2}}\right)}{2b^{\frac{3}{2}}} - \frac{\sqrt{cx^4 + bx^2}}{2bx^3}$$

command

```
integrate(1/x^2/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right) + \frac{\sqrt{cx^2+b} c}{bx^2}}{\sqrt{-b} b} - \frac{\sqrt{cx^2+b} c}{bx^2}$$

$$- \frac{\sqrt{cx^2+b} c}{bx^2}$$

$$2 \operatorname{csgn}(x)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 19.24 Problem number 1002

$$\int \frac{1}{x^4 \sqrt{2+2a-2(1+a)+bx^2+cx^4}} dx$$

Optimal antiderivative

$$-\frac{3c^2 \operatorname{arctanh}\left(\frac{x\sqrt{b}}{\sqrt{cx^4+bx^2}}\right)}{8b^{\frac{5}{2}}} - \frac{\sqrt{cx^4+bx^2}}{4bx^5} + \frac{3c\sqrt{cx^4+bx^2}}{8b^2x^3}$$

command

```
integrate(1/x^4/(c*x^4+b*x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3c^3 \arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2} + \frac{3(cx^2+b)^{\frac{3}{2}} c^3 - 5\sqrt{cx^2+b} bc^3}{b^2 c^2 x^4}$$

$$8 \operatorname{csgn}(x)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 20 Test file number 40

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/40\_1.2.2.3-d+e\_x^2-  
^m-a+b\_x^2+c\_x^4-^p

### 20.1 Problem number 193

$$\int \frac{1}{(d+ex^2)(d^2-e^2x^4)} dx$$

Optimal antiderivative

$$\frac{x}{4d^2(e x^2 + d)} + \frac{\arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{2d^{\frac{5}{2}}\sqrt{e}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{4d^{\frac{5}{2}}\sqrt{e}}$$

command

`integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{(-\frac{1}{2})}}{2 d^{\frac{5}{2}}} - \frac{\arctan\left(\frac{x e}{\sqrt{-d e}}\right)}{4 \sqrt{-d e} d^2} + \frac{x}{4 (x^2 e + d) d^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 20.2 Problem number 194

$$\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)} dx$$

Optimal antiderivative

$$\frac{x}{8d^2(e x^2 + d)^2} + \frac{5x}{16d^3(e x^2 + d)} + \frac{7 \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{16d^{\frac{7}{2}}\sqrt{e}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{8d^{\frac{7}{2}}\sqrt{e}}$$

command

`integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{7 \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{(-\frac{1}{2})}}{16 d^{\frac{7}{2}}} - \frac{\arctan\left(\frac{x e}{\sqrt{-d e}}\right)}{8 \sqrt{-d e} d^3} + \frac{5 x^3 e + 7 d x}{16 (x^2 e + d)^2 d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 20.3 Problem number 198

$$\int \frac{1}{(d + ex^2)^{3/2} (d^2 - e^2 x^4)} dx$$

Optimal antiderivative

$$\frac{x}{6d^2 (ex^2 + d)^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}\sqrt{e}}{\sqrt{ex^2 + d}}\right) \sqrt{2}}{8d^3 \sqrt{e}} + \frac{7x}{12d^3 \sqrt{ex^2 + d}}$$

command

`integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x\left(\frac{7x^2e}{d^3} + \frac{9}{d^2}\right)}{12(x^2e + d)^{\frac{3}{2}}} + \frac{\sqrt{2} e^{(-\frac{1}{2})} \log\left(\frac{2\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 - 4\sqrt{2}|d| - 6d}{2\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 + 4\sqrt{2}|d| - 6d}\right)}{16d^2|d|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 20.4 Problem number 218

$$\int \frac{1}{(d + ex^2)(-cd^2 + bde + be^2x^2 + ce^2x^4)} dx$$

Optimal antiderivative

$$-\frac{x}{2d(-be+2cd)(ex^2+d)} - \frac{(-be+4cd)\arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}(-be+2cd)^2\sqrt{e}} - \frac{c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{c}\sqrt{e}}{\sqrt{-be+cd}}\right)}{(-be+2cd)^2\sqrt{e}\sqrt{-be+cd}}$$

command

`integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2\arctan\left(\frac{cx e}{\sqrt{-c^2de+bce^2}}\right)}{(4c^2d^2-4bcde+b^2e^2)\sqrt{-c^2de+bce^2}} - \frac{(4cd-be)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{(-\frac{1}{2})}}{2(4c^2d^3-4bcd^2e+b^2de^2)\sqrt{d}} - \frac{x}{2(2cd^2-bde)(x^2e+d)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 20.5 Problem number 219

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal antiderivative

$$-\frac{x}{4d(-be+2cd)(ex^2+d)^2} - \frac{(-3be+10cd)x}{8d^2(-be+2cd)^2(ex^2+d)} \\ - \frac{(3b^2e^2-16bcde+28c^2d^2)\arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{8d^{\frac{5}{2}}(-be+2cd)^3\sqrt{e}} - \frac{c^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{x\sqrt{c}\sqrt{e}}{\sqrt{-be+cd}}\right)}{(-be+2cd)^3\sqrt{e}\sqrt{-be+cd}}$$

command

`integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^3\arctan\left(\frac{cx e}{\sqrt{-c^2de+bce^2}}\right)}{(8c^3d^3-12bc^2d^2e+6b^2cde^2-b^3e^3)\sqrt{-c^2de+bce^2}} \\ - \frac{(28c^2d^2-16bcde+3b^2e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{(-\frac{1}{2})}}{8(8c^3d^5-12bc^2d^4e+6b^2cd^3e^2-b^3d^2e^3)\sqrt{d}} - \frac{10cdx^3e-3bx^3e^2+14cd^2x-5bdxe}{8(4c^2d^4-4bcd^3e+b^2d^2e^2)(x^2e+d)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 20.6 Problem number 222

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{e}\sqrt{-be+2cd}}{\sqrt{-be+cd}\sqrt{ex^2+d}}\right)}{\sqrt{e}\sqrt{-be+cd}\sqrt{-be+2cd}}$$

command

`integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\operatorname{arctan}\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3cd+2be}{2\sqrt{-2c^2d^2+3bcde-b^2e^2}}\right)e^{(-\frac{1}{2})}}{\sqrt{-2c^2d^2+3bcde-b^2e^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 20.7 Problem number 223

$$\int \frac{1}{\sqrt{d+ex^2}(-cd^2+bde+be^2x^2+ce^2x^4)} dx$$

Optimal antiderivative

$$\frac{c \operatorname{arctanh}\left(\frac{x\sqrt{e}\sqrt{-be+2cd}}{\sqrt{-be+cd}\sqrt{ex^2+d}}\right)}{(-be+2cd)^{\frac{3}{2}}\sqrt{e}\sqrt{-be+cd}} - \frac{x}{d(-be+2cd)\sqrt{ex^2+d}}$$

command

`integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c \operatorname{arctan}\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3cd+2be}{2\sqrt{-2c^2d^2+3bcde-b^2e^2}}\right)e^{\frac{1}{2}}}{\sqrt{-2c^2d^2+3bcde-b^2e^2}(2cde-be^2)} - \frac{x}{(2cd^2-bde)\sqrt{x^2e+d}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 20.8 Problem number 224

$$\int \frac{1}{(d + ex^2)^{3/2} (-cd^2 + bde + be^2x^2 + ce^2x^4)} dx$$

Optimal antiderivative

$$\frac{x}{3d(-be + 2cd)(ex^2 + d)^{\frac{3}{2}}} - \frac{c^2 \operatorname{arctanh}\left(\frac{x\sqrt{e}\sqrt{-be + 2cd}}{\sqrt{-be + cd}\sqrt{ex^2 + d}}\right)}{(-be + 2cd)^{\frac{5}{2}}\sqrt{e}\sqrt{-be + cd}} - \frac{(-2be + 7cd)x}{3d^2(-be + 2cd)^2\sqrt{ex^2 + d}}$$

command

`integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 \arctan\left(\frac{\left(xe^{\frac{1}{2}} - \sqrt{x^2e + d}\right)^2 c - 3cd + 2be}{2\sqrt{-2c^2d^2 + 3bcde - b^2e^2}}\right) e^{\frac{1}{2}}}{(4c^2d^2e - 4bcde^2 + b^2e^3)\sqrt{-2c^2d^2 + 3bcde - b^2e^2}} - \frac{\left(\frac{(28c^3d^3e^2 - 36bc^2d^2e^3 + 15b^2cde^4 - 2b^3e^5)x^2}{16c^4d^6e - 32bc^3d^5e^2 + 24b^2c^2d^4e^3 - 8b^3cd^3e^4 + b^4d^2e^5} + \frac{3(12c^3d^4e - 16bc^2d^3e^2 + 7b^2cd^2e^3 - b^3de^4)}{16c^4d^6e - 32bc^3d^5e^2 + 24b^2c^2d^4e^3 - 8b^3cd^3e^4 + b^4d^2e^5}\right)x}{3(x^2e + d)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 20.9 Problem number 274

$$\int \frac{1}{(d + ex^2)(a + bx^2 + cx^4)^2} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{x(b^2cd - 2ac^2d - b^3e + 3abce + c(2ace - b^2e + bcd)x^2)}{2a(-4ac + b^2)(ae^2 - bde + cd^2)(cx^4 + bx^2 + a)} + \frac{e^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{(ae^2 - bde + cd^2)^2 \sqrt{d}} \\
& - \frac{e^2 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{be - 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)^2 \sqrt{b - \sqrt{-4ac + b^2}}} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(bcd - b^2e + 2ace + \frac{8abce - 12ac^2d - b^3e + b^2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{4a(-4ac + b^2)(ae^2 - bde + cd^2) \sqrt{b - \sqrt{-4ac + b^2}}} \\
& - \frac{e^2 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(e + \frac{-be + 2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{2(ae^2 - bde + cd^2)^2 \sqrt{b + \sqrt{-4ac + b^2}}} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(bcd - b^2e + 2ace + \frac{-8abce + 12ac^2d + b^3e - b^2cd}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{4a(-4ac + b^2)(ae^2 - bde + cd^2) \sqrt{b + \sqrt{-4ac + b^2}}}
\end{aligned}$$

command

```
integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**20.10 Problem number 275**

$$\int \frac{1}{(d + ex^2)^2 (a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{e^4 x}{2d(ae^2 - bde + cd^2)^2 (ex^2 + d)} \\
& + \frac{x(abce(-be + 2cd) + (-2ac + b^2)(c^2d^2 + b^2e^2 - ce(ae + 2bd)) - c(2b^2cde - 4ac^2de - b^3e^2 - bc(-3ae^2 + cd^2)))}{2a(-4ac + b^2)(ae^2 - bde + cd^2)^2 (cx^4 + bx^2 + a)} \\
& + \frac{e^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{2d^{\frac{3}{2}}(ae^2 - bde + cd^2)^2} + \frac{2e^{\frac{7}{2}}(-be + 2cd) \arctan\left(\frac{x\sqrt{e}}{\sqrt{d}}\right)}{(ae^2 - bde + cd^2)^3 \sqrt{d}} \\
& + \frac{e^2 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{2}\sqrt{c} (3c^2d^2 + be^2(b + \sqrt{-4ac + b^2})) - ce(3bd + ae + 2d\sqrt{-4ac + b^2})}{(ae^2 - bde + cd^2)^3 \sqrt{-4ac + b^2} \sqrt{b - \sqrt{-4ac + b^2}}} \\
& + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^4e^2 - b^3e(2cd - e\sqrt{-4ac + b^2})) - 4ac^2(3cd^2 - e(3ae + d\sqrt{-4ac + b^2}))}{4a(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)} \\
& - \frac{e^2 \arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{2}\sqrt{c} (3c^2d^2 + be^2(b - \sqrt{-4ac + b^2})) - ce(3bd + ae - 2d\sqrt{-4ac + b^2})}{(ae^2 - bde + cd^2)^3 \sqrt{-4ac + b^2} \sqrt{b + \sqrt{-4ac + b^2}}} \\
& - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} (b^4e^2 - b^3e(2cd + e\sqrt{-4ac + b^2})) + bc(3ae^2\sqrt{-4ac + b^2} - cd(-16ae + cd^2))}{4a(-4ac + b^2)^{\frac{3}{2}}(ae^2 - bde + cd^2)}
\end{aligned}$$

command

```
integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 21 Test file number 41

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/41\_1.2.2.4-f\_x-  
 $\hat{m}-d+e_x^2-\hat{q}-a+b_x^2+c_x^4-\hat{p}$

### 21.1 Problem number 81

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-5ae+bd)x}{8ab^2\sqrt{(bx^2+a)^2}} - \frac{(-ae+bd)x}{4b^2(bx^2+a)\sqrt{(bx^2+a)^2}} + \frac{(3ae+bd)(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}b^{\frac{5}{2}}\sqrt{(bx^2+a)^2}}$$

command

`integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(bd+3ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}ab^2\operatorname{sgn}(bx^2+a)} + \frac{b^2dx^3-5abx^3e-abdx-3a^2xe}{8(bx^2+a)^2ab^2\operatorname{sgn}(bx^2+a)}$$

Giac 1.7.0 via sagemath 9.3 output

`sage_0x`

### 21.2 Problem number 83

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(ae+3bd)x}{8a^2b\sqrt{(bx^2+a)^2}} + \frac{(-ae+bd)x}{4ab(bx^2+a)\sqrt{(bx^2+a)^2}} + \frac{(ae+3bd)(bx^2+a)\arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{3}{2}}\sqrt{(bx^2+a)^2}}$$

command

`integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(3bd + ae) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab} a^2 \operatorname{sgn}(bx^2 + a)} + \frac{3b^2 dx^3 + abx^3 e + 5abdx - a^2 x e}{8(bx^2 + a)^2 a^2 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 21.3 Problem number 85

$$\int \frac{d + ex^2}{x^2 (a^2 + 2abx^2 + b^2x^4)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-3ae + 7bd)x}{8a^3 \sqrt{(bx^2 + a)^2}} - \frac{(-ae + bd)x}{4a^2 (bx^2 + a) \sqrt{(bx^2 + a)^2}} \\ & - \frac{d(bx^2 + a)}{a^3 x \sqrt{(bx^2 + a)^2}} - \frac{3(-ae + 5bd)(bx^2 + a) \arctan\left(\frac{x\sqrt{b}}{\sqrt{a}}\right)}{8a^{7/2} \sqrt{b} \sqrt{(bx^2 + a)^2}} \end{aligned}$$

command

`integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3(5bd - ae) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab} a^3 \operatorname{sgn}(bx^2 + a)} - \frac{d}{a^3 x \operatorname{sgn}(bx^2 + a)} - \frac{7b^2 dx^3 - 3abx^3 e + 9abdx - 5a^2 x e}{8(bx^2 + a)^2 a^3 \operatorname{sgn}(bx^2 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 22 Test file number 42

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.2\_Quartic/42\_1.2.2.5\_P-x-a+b\_x^2+c\_x^4-~p

### 22.1 Problem number 40

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x(b^2d - abf - 2a(-ah + cd) + (abh - 2acf + bcd)x^2)}{2a(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{2acg - b(ai + ce) - (-2aci + b^2i - bcg + 2c^2e)x^2}{2c(-4ac + b^2)(cx^4 + bx^2 + a)} + \frac{(2ai - bg + 2ce) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{\frac{3}{2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(bcd - 2acf + abh + \frac{4abcf + b^2(-ah + cd) - 4ac(ah + 3cd)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{4a(-4ac + b^2)\sqrt{c}\sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(bcd - 2acf + abh + \frac{-4abcf - b^2(-ah + cd) + 4ac(ah + 3cd)}{\sqrt{-4ac + b^2}}\right) \sqrt{2}}{4a(-4ac + b^2)\sqrt{c}\sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 22.2 Problem number 41

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{mx}{c^2} + \frac{-bc(aj + ce) + ab^2l + 2ac(-al + cg) - (2c^3e - c^2(2aj + bg) - b^3l + bc(3al + bj))x^2}{2c^2(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & - \frac{x(abc(ak + cf) - b^2(a^2m + c^2d) + 2ac(a^2m - ach + c^2d) + (ab^2ck + 2ac^2(-ak + cf) - ab^3m - bc(-3a^2m + c^2d)))}{2ac^2(-4ac + b^2)(cx^4 + bx^2 + a)} \\ & + \frac{(4c^3e - c^2(-4aj + 2bg) + b^3l - 6abcl) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right) + l \ln(cx^4 + bx^2 + a)}{2c^2(-4ac + b^2)^{\frac{3}{2}}} + \frac{l \ln(cx^4 + bx^2 + a)}{4c^2} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(a b^2ck - 2ac^2(3ak + cf) - 3ab^3m + bc(13a^2m + ach + c^2d) + \frac{-ab^3ck + 4abc^2(2ak + c^2d)}{4ac^2(-4ac + b^2)}\right)}{4ac^{\frac{5}{2}}(-4ac + b^2)\sqrt{b - \sqrt{-4ac + b^2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(a b^2ck - 2ac^2(3ak + cf) - 3ab^3m + bc(13a^2m + ach + c^2d) + \frac{ab^3ck - 4abc^2(2ak + c^2d)}{4ac^2(-4ac + b^2)}\right)}{4ac^{\frac{5}{2}}(-4ac + b^2)\sqrt{b + \sqrt{-4ac + b^2}}} \end{aligned}$$

command

```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 22.3 Problem number 56

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x(b^2d - abf - 2a(-ah + cd) + (abh - 2acf + bcd)x^2)}{4a(-4ac + b^2)(cx^4 + bx^2 + a)^2} \\
& + \frac{2acg - b(ai + ce) - (-2aci + b^2i - bcg + 2c^2e)x^2}{4c(-4ac + b^2)(cx^4 + bx^2 + a)^2} + \frac{\left(6ce - 3bg + 2ai + \frac{b^2i}{c}\right)(2cx^2 + b)}{4(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\
& + \frac{x(3b^4d + ab^3f + 8a^2bcf + 4a^2c(ah + 7cd) - ab^2(7ah + 25cd) + c(3b^3d + ab^2f + 20a^2cf - 12ab(ah + 2cd))x^2)}{8a^2(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\
& - \frac{(2aci + b^2i - 3bcg + 6c^2e) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{\frac{5}{2}}} \\
& + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(3b^3d + ab^2f + 20a^2cf - 12ab(ah + 2cd) + \frac{3b^4d + ab^3f - 52a^2bcf - 6ab^2(-3ah + 5ca)}{\sqrt{-4ac + b^2}}\right)}{16a^2(-4ac + b^2)^2 \sqrt{b - \sqrt{-4ac + b^2}}} \\
& + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \sqrt{c} \left(3b^3d + ab^2f + 20a^2cf - 12ab(ah + 2cd) + \frac{-3b^4d - ab^3f + 52a^2bcf + 6ab^2(-3ah + 5ca)}{\sqrt{-4ac + b^2}}\right)}{16a^2(-4ac + b^2)^2 \sqrt{b + \sqrt{-4ac + b^2}}}
\end{aligned}$$

command

```
integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**22.4 Problem number 57**

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + jx^5 + kx^6 + lx^7 + mx^8}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{-bc(aj + ce) + ab^2l + 2ac(-al + cg) - (2c^3e - c^2(2aj + bg) - b^3l + bc(3al + bj)) x^2}{4c^2(-4ac + b^2)(cx^4 + bx^2 + a)^2} \\
& - \frac{x(abc(ak + cf) - b^2(a^2m + c^2d) + 2ac(a^2m - ach + c^2d) + (ab^2ck + 2ac^2(-ak + cf) - ab^3m - bc(-3a^2m + a^2l + 2b(aj + 3ce) - 16a^2l - \frac{b^4l}{c^2} - b^2(3g - \frac{5al}{c})) + 2(-3abl + 2acj + b^2j - 3bcg + 6c^2e)) x^2}{4ac^2(-4ac + b^2)(cx^4 + bx^2 + a)^2} \\
& + \frac{\frac{b^3j}{c} + 2b(aj + 3ce) - 16a^2l - \frac{b^4l}{c^2} - b^2(3g - \frac{5al}{c}) + 2(-3abl + 2acj + b^2j - 3bcg + 6c^2e) x^2}{4(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\
& + \frac{x(4a^2bc^2(ak + 2cf) + ab^3c(2ak + cf) - ab^2c(-11a^2m + 7ach + 25c^2d) + 4a^2c^2(-9a^2m + ach + 7c^2d) + b^4(-2a^2l + 2b(aj + 3ce) - 16a^2l - \frac{b^4l}{c^2} - b^2(3g - \frac{5al}{c})) + 2(-3abl + 2acj + b^2j - 3bcg + 6c^2e))}{8a^2c^2(-4ac + b^2)^2(cx^4 + bx^2 + a)} \\
& - \frac{(-3abl + 2acj + b^2j - 3bcg + 6c^2e) \operatorname{arctanh}\left(\frac{2cx^2 + b}{\sqrt{-4ac + b^2}}\right)}{(-4ac + b^2)^{\frac{5}{2}}} \\
& + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}}\right) \left(a b^2c(3ak + cf) + 4a^2c^2(3ak + 5cf) + b^3(a^2m + 3c^2d) - 4abc(4a^2m + 3ach + 2a^2l + 2b(aj + 3ce) - 16a^2l - \frac{b^4l}{c^2} - b^2(3g - \frac{5al}{c})) + 2(-3abl + 2acj + b^2j - 3bcg + 6c^2e)\right)}{16a^2c^{\frac{3}{2}}(-4ac + b^2)^2\sqrt{b}} \\
& + \frac{\operatorname{arctan}\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}}\right) \left(a b^2c(3ak + cf) + 4a^2c^2(3ak + 5cf) + b^3(a^2m + 3c^2d) - 4abc(4a^2m + 3ach + 2a^2l + 2b(aj + 3ce) - 16a^2l - \frac{b^4l}{c^2} - b^2(3g - \frac{5al}{c})) + 2(-3abl + 2acj + b^2j - 3bcg + 6c^2e)\right)}{16a^2c^{\frac{3}{2}}(-4ac + b^2)^2\sqrt{b}}
\end{aligned}$$

command

```
integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**22.5 Problem number 58**

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^6 + kx^7}{(a + bx^2 + cx^4)^2} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{x \left( c \left( b^2 d - 2a(-ah + cd) - \frac{ab(aj+cf)}{c} \right) + (bc(ah + cd) - ab^2j - 2ac(-aj + cf)) x^2 \right)}{2ac(-4ac + b^2)(cx^4 + bx^2 + a)} \\
& + \frac{-bc(ai + ce) + ab^2k + 2ac(-ak + cg) - (2c^3e - c^2(2ai + bg) - b^3k + bc(3ak + bi)) x^2}{2c^2(-4ac + b^2)(cx^4 + bx^2 + a)} \\
& + \frac{(4c^3e - c^2(-4ai + 2bg) + b^3k - 6abck) \operatorname{arctanh} \left( \frac{2cx^2 + b}{\sqrt{-4ac + b^2}} \right) + \frac{k \ln(cx^4 + bx^2 + a)}{4c^2}}{2c^2(-4ac + b^2)^{\frac{3}{2}}} \\
& + \frac{\operatorname{arctan} \left( \frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \left( b(ah + cd) + \frac{ab^2j}{c} - 2a(3aj + cf) + \frac{b^2c(-ah+cd) - 4ac^2(ah+3cd) - ab^3j + 4abc(2aj+cf)}{c\sqrt{-4ac + b^2}} \right)}{4a(-4ac + b^2)\sqrt{c}\sqrt{b - \sqrt{-4ac + b^2}}} \\
& + \frac{\operatorname{arctan} \left( \frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \left( b(ah + cd) + \frac{ab^2j}{c} - 2a(3aj + cf) + \frac{-b^2c(-ah+cd) + 4ac^2(ah+3cd) + ab^3j - 4abc(2aj+cf)}{c\sqrt{-4ac + b^2}} \right)}{4a(-4ac + b^2)\sqrt{c}\sqrt{b + \sqrt{-4ac + b^2}}}
\end{aligned}$$

command

```
integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**22.6 Problem number 59**

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + ix^5 + jx^8 + kx^{11}}{(a + bx^2 + cx^4)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x \left( c^2 \left( abf - b^2 \left( d + \frac{a^2 j}{c^2} \right) + 2a \left( cd - ah + \frac{a^2 j}{c} \right) \right) + (2a c^3 f - a b^3 j - bc(-3a^2 j + ach + c^2 d)) x^2 \right)}{4a c^2 (-4ac + b^2) (c x^4 + b x^2 + a)^2} \\
+ & \frac{-b c^3 (ai + ce) + a b^4 k - 4a^2 b^2 ck + 2a c^2 (a^2 k + c^2 g) - (2c^5 e + b^2 c^3 i - c^4 (2ai + bg) - b^5 k + 5a b^3 ck - 5a^2 b c^2 k) x}{4c^4 (-4ac + b^2) (c x^4 + b x^2 + a)^2} \\
+ & \frac{x \left( c \left( a b^3 f + 8a^2 b c f + 4a^2 (-9a^2 j + ach + 7c^2 d) + b^4 \left( 3d - \frac{2a^2 j}{c^2} \right) - a b^2 \left( 25cd + 7ah - \frac{11a^2 j}{c} \right) \right) + (a b^2 c^2 f + 20a^2 b^3 c^2 i + 2b c^3 (ai + 3ce) + 11a b^4 k - \frac{b^6 k}{c} + 32a^3 c^2 k - 3b^2 (13a^2 ck + c^3 g) + 2(6c^5 e + b^2 c^3 i - c^4 (-2ai + 3bg) + 2b^5 k) x}{8a^2 c (-4ac + b^2)^2 (c x^4 + b x^2 + a)} \right)}{4c^3 (-4ac + b^2)^2 (c x^4 + b x^2 + a)} \\
+ & \frac{(12c^5 e + 2b^2 c^3 i - c^4 (-4ai + 6bg) - b^5 k + 10a b^3 ck - 30a^2 b c^2 k) \operatorname{arctanh} \left( \frac{2c x^2 + b}{\sqrt{-4ac + b^2}} \right)}{2c^3 (-4ac + b^2)^{\frac{5}{2}}} \\
+ & \frac{k \ln (c x^4 + b x^2 + a)}{4c^3} \\
+ & \frac{\operatorname{arctan} \left( \frac{x \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4ac + b^2}}} \right) \left( a b^2 c^2 f + 20a^2 c^3 f + b^3 (a^2 j + 3c^2 d) - 4abc(4a^2 j + 3ach + 6c^2 d) + \frac{a b^3 c^2 f - 52a^2 b^3 c^2 i}{16a^2 c^{\frac{3}{2}} (-4ac + b^2)^2 \sqrt{b - \sqrt{-4ac + b^2}}} \right)}{16a^2 c^{\frac{3}{2}} (-4ac + b^2)^2 \sqrt{b - \sqrt{-4ac + b^2}}} \\
+ & \frac{\operatorname{arctan} \left( \frac{x \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4ac + b^2}}} \right) \left( a b^2 c^2 f + 20a^2 c^3 f + b^3 (a^2 j + 3c^2 d) - 4abc(4a^2 j + 3ach + 6c^2 d) + \frac{-a b^3 c^2 f + 52a^2 b^3 c^2 i}{16a^2 c^{\frac{3}{2}} (-4ac + b^2)^2 \sqrt{b + \sqrt{-4ac + b^2}}} \right)}{16a^2 c^{\frac{3}{2}} (-4ac + b^2)^2 \sqrt{b + \sqrt{-4ac + b^2}}}
\end{aligned}$$

command

```
integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 23 Test file number 46

Test folder name:

test\_cases/1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.3\_General/46\_1.2.3.2-d\_x-  
 $\hat{m}+b_x\hat{n}+c_x\hat{-2}_n\hat{-p}$

### 23.1 Problem number 98

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{x^2}{9ab\sqrt{(bx^3+a)^2}} - \frac{x^2}{6b(bx^3+a)\sqrt{(bx^3+a)^2}} - \frac{(bx^3+a)\ln\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{27a^{\frac{4}{3}}b^{\frac{5}{3}}\sqrt{(bx^3+a)^2}} \\ & + \frac{(bx^3+a)\ln\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{4}{3}}b^{\frac{5}{3}}\sqrt{(bx^3+a)^2}} - \frac{(bx^3+a)\arctan\left(\frac{(a^{\frac{1}{3}}-2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{27a^{\frac{4}{3}}b^{\frac{5}{3}}\sqrt{(bx^3+a)^2}} \end{aligned}$$

command

`integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}\operatorname{absgn}(bx^3+a)} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{1}{3}}\operatorname{absgn}(bx^3+a)} \\ & - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2\operatorname{absgn}(bx^3+a)} + \frac{2bx^5-ax^2}{18(bx^3+a)^2\operatorname{absgn}(bx^3+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.2 Problem number 99

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x}{18ab\sqrt{(bx^3+a)^2}} - \frac{x}{6b(bx^3+a)\sqrt{(bx^3+a)^2}} + \frac{(bx^3+a)\ln\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{27a^{\frac{5}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}} - \frac{(bx^3+a)\ln\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{54a^{\frac{5}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}} - \frac{(bx^3+a)\arctan\left(\frac{(a^{\frac{1}{3}}-2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{27a^{\frac{5}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}}$$

command

```
integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{2}{3}}\operatorname{asgn}(bx^3+a)} - \frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54(-ab^2)^{\frac{2}{3}}\operatorname{asgn}(bx^3+a)} - \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2\operatorname{bsgn}(bx^3+a)} + \frac{bx^4-2ax}{18(bx^3+a)^2\operatorname{absgn}(bx^3+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.3 Problem number 100

$$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{6b(bx^3+a)\sqrt{(bx^3+a)^2}}$$

command

```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6(bx^3 + a)^2 b \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.4 Problem number 101

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x^2}{9a^2 \sqrt{(bx^3 + a)^2}} + \frac{x^2}{6a(bx^3 + a) \sqrt{(bx^3 + a)^2}} - \frac{2(bx^3 + a) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{27a^{\frac{7}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} \\ & + \frac{(bx^3 + a) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{27a^{\frac{7}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} - \frac{2(bx^3 + a) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{27a^{\frac{7}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27(-ab^2)^{\frac{1}{3}}a^2 \operatorname{sgn}(bx^3 + a)} - \frac{\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27(-ab^2)^{\frac{1}{3}}a^2 \operatorname{sgn}(bx^3 + a)} \\ & - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^3 \operatorname{sgn}(bx^3 + a)} + \frac{4bx^5 + 7ax^2}{18(bx^3 + a)^2 a^2 \operatorname{sgn}(bx^3 + a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.5 Problem number 102

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{x(bx^3 + a)}{6a(b^2x^6 + 2abx^3 + a^2)^{3/2}} + \frac{5x(bx^3 + a)^2}{18a^2(b^2x^6 + 2abx^3 + a^2)^{3/2}} + \frac{5(bx^3 + a)^3 \ln\left(a^{1/3} + b^{1/3}x\right)}{27a^{8/3}b^{1/3}(b^2x^6 + 2abx^3 + a^2)^{3/2}}$$

$$- \frac{5(bx^3 + a)^3 \ln\left(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2\right)}{54a^{8/3}b^{1/3}(b^2x^6 + 2abx^3 + a^2)^{3/2}} - \frac{5(bx^3 + a)^3 \arctan\left(\frac{(a^{1/3} - 2b^{1/3}x)\sqrt{3}}{3a^{1/3}}\right)\sqrt{3}}{27a^{8/3}b^{1/3}(b^2x^6 + 2abx^3 + a^2)^{3/2}}$$

command

```
integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{5\left(-\frac{a}{b}\right)^{1/3} \log\left(\left|x - \left(-\frac{a}{b}\right)^{1/3}\right|\right)}{27a^3 \operatorname{sgn}(bx^3 + a)} + \frac{5\sqrt{3}(-ab^2)^{1/3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{1/3}\right)}{3\left(-\frac{a}{b}\right)^{1/3}}\right)}{27a^3 b \operatorname{sgn}(bx^3 + a)}$$

$$+ \frac{5(-ab^2)^{1/3} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{1/3} + \left(-\frac{a}{b}\right)^{2/3}\right)}{54a^3 b \operatorname{sgn}(bx^3 + a)} + \frac{5bx^4 + 8ax}{18(bx^3 + a)^2 a^2 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.6 Problem number 103

$$\int \frac{1}{x(a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\frac{1}{3a^2\sqrt{(bx^3 + a)^2}} + \frac{1}{6a(bx^3 + a)\sqrt{(bx^3 + a)^2}} + \frac{(bx^3 + a)\ln(x)}{a^3\sqrt{(bx^3 + a)^2}} - \frac{(bx^3 + a)\ln(bx^3 + a)}{3a^3\sqrt{(bx^3 + a)^2}}$$

command

```
integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log(|bx^3 + a|)}{3a^3 \operatorname{sgn}(bx^3 + a)} + \frac{\log(|x|)}{a^3 \operatorname{sgn}(bx^3 + a)} + \frac{3b^2x^6 + 8abx^3 + 6a^2}{6(bx^3 + a)^2 a^3 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.7 Problem number 104

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7}{18a^2x\sqrt{(bx^3 + a)^2}} + \frac{1}{6ax(bx^3 + a)\sqrt{(bx^3 + a)^2}} \\ & - \frac{14(bx^3 + a)}{9a^3x\sqrt{(bx^3 + a)^2}} + \frac{14b^{\frac{1}{3}}(bx^3 + a)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{27a^{\frac{10}{3}}\sqrt{(bx^3 + a)^2}} \\ & - \frac{7b^{\frac{1}{3}}(bx^3 + a)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{27a^{\frac{10}{3}}\sqrt{(bx^3 + a)^2}} + \frac{14b^{\frac{1}{3}}(bx^3 + a)\arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{27a^{\frac{10}{3}}\sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{14b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^4\operatorname{sgn}(bx^3 + a)} + \frac{14\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4b\operatorname{sgn}(bx^3 + a)} \\ & - \frac{7(-ab^2)^{\frac{2}{3}}\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4b\operatorname{sgn}(bx^3 + a)} - \frac{10b^2x^5 + 13abx^2}{18(bx^3 + a)^2 a^3 \operatorname{sgn}(bx^3 + a)} - \frac{1}{a^3x\operatorname{sgn}(bx^3 + a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.8 Problem number 105

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4}{9a^2x^2\sqrt{(bx^3+a)^2}} + \frac{1}{6ax^2(bx^3+a)\sqrt{(bx^3+a)^2}} - \frac{10(bx^3+a)}{9a^3x^2\sqrt{(bx^3+a)^2}} \\ & - \frac{20b^{\frac{2}{3}}(bx^3+a)\ln\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{27a^{\frac{11}{3}}\sqrt{(bx^3+a)^2}} + \frac{10b^{\frac{2}{3}}(bx^3+a)\ln\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{27a^{\frac{11}{3}}\sqrt{(bx^3+a)^2}} \\ & + \frac{20b^{\frac{2}{3}}(bx^3+a)\arctan\left(\frac{\left(a^{\frac{1}{3}}-2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{27a^{\frac{11}{3}}\sqrt{(bx^3+a)^2}} \end{aligned}$$

command

`integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{20b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^4\operatorname{sgn}(bx^3+a)} - \frac{20\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^4\operatorname{sgn}(bx^3+a)} \\ & - \frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27a^4\operatorname{sgn}(bx^3+a)} - \frac{20b^2x^6+32abx^3+9a^2}{18(bx^4+ax)^2a^3\operatorname{sgn}(bx^3+a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.9 Problem number 106

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b}{3a^3\sqrt{(bx^3+a)^2}} - \frac{b}{6a^2(bx^3+a)\sqrt{(bx^3+a)^2}} + \frac{-bx^3-a}{3a^3x^3\sqrt{(bx^3+a)^2}} \\ & - \frac{3b(bx^3+a)\ln(x)}{a^4\sqrt{(bx^3+a)^2}} + \frac{b(bx^3+a)\ln(bx^3+a)}{a^4\sqrt{(bx^3+a)^2}} \end{aligned}$$



command

`integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \log(|bx^3 + a|)}{a^4 \operatorname{sgn}(bx^3 + a)} - \frac{3 b \log(|x|)}{a^4 \operatorname{sgn}(bx^3 + a)} - \frac{9 b^3 x^6 + 22 a b^2 x^3 + 14 a^2 b}{6 (bx^3 + a)^2 a^4 \operatorname{sgn}(bx^3 + a)} + \frac{3 b x^3 - a}{3 a^4 x^3 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.10 Problem number 107

$$\int \frac{x^6}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5x}{486a^2b^2 \sqrt{(bx^3 + a)^2}} - \frac{x^4}{12b(bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} - \frac{x}{27b^2(bx^3 + a)^2 \sqrt{(bx^3 + a)^2}} \\ & + \frac{x}{162ab^2(bx^3 + a) \sqrt{(bx^3 + a)^2}} + \frac{5(bx^3 + a) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{729a^{\frac{8}{3}}b^{\frac{7}{3}} \sqrt{(bx^3 + a)^2}} \\ & - \frac{5(bx^3 + a) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{1458a^{\frac{8}{3}}b^{\frac{7}{3}} \sqrt{(bx^3 + a)^2}} - \frac{5(bx^3 + a) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{729a^{\frac{8}{3}}b^{\frac{7}{3}} \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5 \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 (-ab^2)^{\frac{2}{3}} a^2 b \operatorname{sgn}(bx^3 + a)} - \frac{5 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 a^3 b^2 \operatorname{sgn}(bx^3 + a)} \\ & + \frac{5 \sqrt{3} (-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^3 b^3 \operatorname{sgn}(bx^3 + a)} + \frac{10 b^3 x^{10} + 36 a b^2 x^7 - 75 a^2 b x^4 - 20 a^3 x}{972 (bx^3 + a)^4 a^2 b^2 \operatorname{sgn}(bx^3 + a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.11 Problem number 108

$$\int \frac{x^5}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\frac{a}{12b^2 (bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} - \frac{1}{9b^2 (bx^3 + a)^2 \sqrt{(bx^3 + a)^2}}$$

command

```
integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4bx^3 + a}{36(bx^3 + a)^4 b^2 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.12 Problem number 109

$$\int \frac{x^4}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7x^2}{243a^3b\sqrt{(bx^3 + a)^2}} - \frac{x^2}{12b(bx^3 + a)^3\sqrt{(bx^3 + a)^2}} + \frac{x^2}{54ab(bx^3 + a)^2\sqrt{(bx^3 + a)^2}} \\ & + \frac{7x^2}{324a^2b(bx^3 + a)\sqrt{(bx^3 + a)^2}} - \frac{7(bx^3 + a)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{729a^{\frac{10}{3}}b^{\frac{5}{3}}\sqrt{(bx^3 + a)^2}} \\ & + \frac{7(bx^3 + a)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{1458a^{\frac{10}{3}}b^{\frac{5}{3}}\sqrt{(bx^3 + a)^2}} - \frac{7(bx^3 + a)\arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{729a^{\frac{10}{3}}b^{\frac{5}{3}}\sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

```
integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{7\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729(-ab^2)^{\frac{1}{3}}a^3\text{bsgn}(bx^3+a)} - \frac{7\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458(-ab^2)^{\frac{1}{3}}a^3\text{bsgn}(bx^3+a)} - \frac{7\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729a^4\text{bsgn}(bx^3+a)} + \frac{28b^3x^{11} + 105ab^2x^8 + 144a^2bx^5 - 14a^3x^2}{972(bx^3+a)^4a^3\text{bsgn}(bx^3+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.13 Problem number 110

$$\int \frac{x^3}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\frac{5x}{243a^3b\sqrt{(bx^3+a)^2}} - \frac{x}{12b(bx^3+a)^3\sqrt{(bx^3+a)^2}} + \frac{x}{108ab(bx^3+a)^2\sqrt{(bx^3+a)^2}} + \frac{x}{81a^2b(bx^3+a)\sqrt{(bx^3+a)^2}} + \frac{10(bx^3+a)\ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{729a^{\frac{11}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}} - \frac{5(bx^3+a)\ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{729a^{\frac{11}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}} - \frac{10(bx^3+a)\arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{729a^{\frac{11}{3}}b^{\frac{4}{3}}\sqrt{(bx^3+a)^2}}$$

command

`integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729(-ab^2)^{\frac{2}{3}}a^3\text{sgn}(bx^3+a)} - \frac{5\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729(-ab^2)^{\frac{2}{3}}a^3\text{sgn}(bx^3+a)} - \frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729a^4\text{bsgn}(bx^3+a)} + \frac{20b^3x^{10} + 72ab^2x^7 + 93a^2bx^4 - 40a^3x}{972(bx^3+a)^4a^3\text{bsgn}(bx^3+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.14 Problem number 111

$$\int \frac{x^2}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{12b(bx^3 + a)(b^2x^6 + 2abx^3 + a^2)^{\frac{3}{2}}}$$

command

```
integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{12(bx^3 + a)^4 b \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.15 Problem number 112

$$\int \frac{x}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35x^2}{243a^4 \sqrt{(bx^3 + a)^2}} + \frac{x^2}{12a(bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} + \frac{5x^2}{54a^2(bx^3 + a)^2 \sqrt{(bx^3 + a)^2}} \\ & + \frac{35x^2}{324a^3(bx^3 + a) \sqrt{(bx^3 + a)^2}} - \frac{35(bx^3 + a) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)}{729a^{\frac{13}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} \\ & + \frac{35(bx^3 + a) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)}{1458a^{\frac{13}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} - \frac{35(bx^3 + a) \arctan\left(\frac{(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}x)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{729a^{\frac{13}{3}}b^{\frac{2}{3}} \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

```
integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{35\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729(-ab^2)^{\frac{1}{3}}a^4\operatorname{sgn}(bx^3+a)} - \frac{35\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458(-ab^2)^{\frac{1}{3}}a^4\operatorname{sgn}(bx^3+a)} - \frac{35\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729a^5\operatorname{sgn}(bx^3+a)} + \frac{140b^3x^{11}+525ab^2x^8+720a^2bx^5+416a^3x^2}{972(bx^3+a)^4a^4\operatorname{sgn}(bx^3+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.16 Problem number 113

$$\int \frac{1}{(a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x(bx^3+a)}{12a(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} + \frac{11x(bx^3+a)^2}{108a^2(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} + \frac{11x(bx^3+a)^3}{81a^3(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} + \frac{55x(bx^3+a)^4}{243a^4(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} + \frac{110(bx^3+a)^5 \ln\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{729a^{\frac{14}{3}}b^{\frac{1}{3}}(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} - \frac{55(bx^3+a)^5 \ln\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{729a^{\frac{14}{3}}b^{\frac{1}{3}}(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}} - \frac{110(bx^3+a)^5 \arctan\left(\frac{\left(a^{\frac{1}{3}}-2b^{\frac{1}{3}}x\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{729a^{\frac{14}{3}}b^{\frac{1}{3}}(b^2x^6+2abx^3+a^2)^{\frac{5}{2}}}$$

command

`integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{110\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729a^5\operatorname{sgn}(bx^3+a)} + \frac{110\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729a^5b\operatorname{sgn}(bx^3+a)} + \frac{55(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729a^5b\operatorname{sgn}(bx^3+a)} + \frac{220b^3x^{10}+792ab^2x^7+1023a^2bx^4+532a^3x}{972(bx^3+a)^4a^4\operatorname{sgn}(bx^3+a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.17 Problem number 114

$$\int \frac{1}{x (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1}{3a^4 \sqrt{(bx^3 + a)^2}} + \frac{1}{12a (bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} + \frac{1}{9a^2 (bx^3 + a)^2 \sqrt{(bx^3 + a)^2}} \\ & + \frac{1}{6a^3 (bx^3 + a) \sqrt{(bx^3 + a)^2}} + \frac{(bx^3 + a) \ln(x)}{a^5 \sqrt{(bx^3 + a)^2}} - \frac{(bx^3 + a) \ln(bx^3 + a)}{3a^5 \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log(|bx^3 + a|)}{3a^5 \operatorname{sgn}(bx^3 + a)} + \frac{\log(|x|)}{a^5 \operatorname{sgn}(bx^3 + a)} + \frac{25b^4x^{12} + 112ab^3x^9 + 192a^2b^2x^6 + 152a^3bx^3 + 50a^4}{36(bx^3 + a)^4 a^5 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 23.18 Problem number 115

$$\int \frac{1}{x^2 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{455}{972a^4x \sqrt{(bx^3 + a)^2}} + \frac{1}{12ax (bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} + \frac{13}{108a^2x (bx^3 + a)^2 \sqrt{(bx^3 + a)^2}} \\ & + \frac{65}{324a^3x (bx^3 + a) \sqrt{(bx^3 + a)^2}} - \frac{455(bx^3 + a)}{243a^5x \sqrt{(bx^3 + a)^2}} + \frac{455b^{1/3}(bx^3 + a) \ln\left(a^{1/3} + b^{1/3}x\right)}{729a^{16/3} \sqrt{(bx^3 + a)^2}} \\ & - \frac{455b^{1/3}(bx^3 + a) \ln\left(a^{2/3} - a^{1/3}b^{1/3}x + b^{2/3}x^2\right)}{1458a^{16/3} \sqrt{(bx^3 + a)^2}} + \frac{455b^{1/3}(bx^3 + a) \arctan\left(\frac{(a^{1/3} - 2b^{1/3}x)\sqrt{3}}{3a^{1/3}}\right) \sqrt{3}}{729a^{16/3} \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{455 b \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 a^6 \operatorname{sgn}(bx^3 + a)} + \frac{455 \sqrt{3} (-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^6 b \operatorname{sgn}(bx^3 + a)}$$

$$- \frac{455 (-ab^2)^{\frac{2}{3}} \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 a^6 b \operatorname{sgn}(bx^3 + a)} - \frac{1}{a^5 x \operatorname{sgn}(bx^3 + a)}$$

$$- \frac{848 b^4 x^{11} + 2937 ab^3 x^8 + 3528 a^2 b^2 x^5 + 1520 a^3 b x^2}{972 (bx^3 + a)^4 a^5 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.19 Problem number 116

$$\int \frac{1}{x^3 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\frac{154}{243 a^4 x^2 \sqrt{(bx^3 + a)^2}} + \frac{1}{12 a x^2 (bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} + \frac{7}{54 a^2 x^2 (bx^3 + a)^2 \sqrt{(bx^3 + a)^2}}$$

$$+ \frac{77}{324 a^3 x^2 (bx^3 + a) \sqrt{(bx^3 + a)^2}} - \frac{385 (bx^3 + a)}{243 a^5 x^2 \sqrt{(bx^3 + a)^2}} - \frac{770 b^{\frac{2}{3}} (bx^3 + a) \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right)}{729 a^{\frac{17}{3}} \sqrt{(bx^3 + a)^2}}$$

$$+ \frac{385 b^{\frac{2}{3}} (bx^3 + a) \ln\left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right)}{729 a^{\frac{17}{3}} \sqrt{(bx^3 + a)^2}} + \frac{770 b^{\frac{2}{3}} (bx^3 + a) \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}} x\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) \sqrt{3}}{729 a^{\frac{17}{3}} \sqrt{(bx^3 + a)^2}}$$

command

`integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{770 b \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 a^6 \operatorname{sgn}(bx^3 + a)} - \frac{770 \sqrt{3} (-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^6 \operatorname{sgn}(bx^3 + a)}$$

$$- \frac{385 (-ab^2)^{\frac{1}{3}} \log\left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729 a^6 \operatorname{sgn}(bx^3 + a)} - \frac{1}{2 a^5 x^2 \operatorname{sgn}(bx^3 + a)}$$

$$- \frac{1054 b^4 x^{10} + 3600 ab^3 x^7 + 4245 a^2 b^2 x^4 + 1780 a^3 b x}{972 (bx^3 + a)^4 a^5 \operatorname{sgn}(bx^3 + a)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.20 Problem number 117

$$\int \frac{1}{x^4 (a^2 + 2abx^3 + b^2x^6)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4b}{3a^5 \sqrt{(bx^3 + a)^2}} - \frac{b}{12a^2 (bx^3 + a)^3 \sqrt{(bx^3 + a)^2}} \\ & -\frac{9a^3 (bx^3 + a)^2 \sqrt{(bx^3 + a)^2}}{2b} - \frac{2a^4 (bx^3 + a) \sqrt{(bx^3 + a)^2}}{b} \\ & + \frac{-bx^3 - a}{3a^5 x^3 \sqrt{(bx^3 + a)^2}} - \frac{5b(bx^3 + a) \ln(x)}{a^6 \sqrt{(bx^3 + a)^2}} + \frac{5b(bx^3 + a) \ln(bx^3 + a)}{3a^6 \sqrt{(bx^3 + a)^2}} \end{aligned}$$

command

`integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5b \log(|bx^3 + a|)}{3a^6 \operatorname{sgn}(bx^3 + a)} - \frac{5b \log(|x|)}{a^6 \operatorname{sgn}(bx^3 + a)} + \frac{5bx^3 - a}{3a^6 x^3 \operatorname{sgn}(bx^3 + a)} \\ & - \frac{125b^5 x^{12} + 548ab^4 x^9 + 912a^2 b^3 x^6 + 688a^3 b^2 x^3 + 202a^4 b}{36(bx^3 + a)^4 a^6 \operatorname{sgn}(bx^3 + a)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 23.21 Problem number 453

$$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b \operatorname{arctanh}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{2a^{\frac{5}{2}}} \\ & - \frac{2(b^2 - 2ac + \frac{bc}{x})x}{a(-4ac + b^2) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}} + \frac{(-8ac + 3b^2)x \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{a^2(-4ac + b^2)} \end{aligned}$$



command

```
integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & - \frac{\left(3 b^3 \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)-12 a b c \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)+6 \sqrt{a} b^2 \sqrt{c}-16 a^{\frac{3}{2}} c^{\frac{3}{2}}\right) \operatorname{sgn}(x)}{2\left(a^{\frac{5}{2}} b^2-4 a^{\frac{7}{2}} c\right)} \\
 & + \frac{\left(\frac{\left(a b^2-4 a^2 c\right) x}{a^2 b^2 \operatorname{sgn}(x)-4 a^3 c \operatorname{sgn}(x)}+\frac{3 b^3-10 a b c}{a^2 b^2 \operatorname{sgn}(x)-4 a^3 c \operatorname{sgn}(x)}\right) x+\frac{3 b^2 c-8 a c^2}{a^2 b^2 \operatorname{sgn}(x)-4 a^3 c \operatorname{sgn}(x)}}{\sqrt{a x^2+b x+c}} \\
 & + \frac{3 b \log \left(\left|-2\left(\sqrt{a} x-\sqrt{a x^2+b x+c}\right) \sqrt{a}-b\right|\right)}{2 a^{\frac{5}{2}} \operatorname{sgn}(x)}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 23.22 Problem number 454

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{2\left(b^2-2 a c+\frac{b c}{x}\right) x}{3 a\left(-4 a c+b^2\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{\frac{3}{2}}}-\frac{5 b \operatorname{arctanh}\left(\frac{2 a+\frac{b}{x}}{2 \sqrt{a} \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2 a^{\frac{7}{2}}} \\
 & - \frac{2\left(5 b^4-32 a b^2 c+32 a^2 c^2+\frac{b c(-28 a c+5 b^2)}{x}\right) x}{3 a^2\left(-4 a c+b^2\right)^2 \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} \\
 & + \frac{\left(128 a^2 c^2-100 a b^2 c+15 b^4\right) x \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{3 a^3\left(-4 a c+b^2\right)^2}
 \end{aligned}$$

command

```
integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{\left(15 b^5 \sqrt{c} \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)-120 a b^3 c^{\frac{3}{2}} \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)+240 a^2 b c^{\frac{5}{2}} \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)+30 \sqrt{a}\right)}{6\left(a^{\frac{7}{2}} b^4 \sqrt{c}-8 a^{\frac{9}{2}} b^2 c^{\frac{3}{2}}+16 a^{\frac{11}{2}} c^{\frac{5}{2}}\right)} \\
& + \frac{\left(\left(\frac{3\left(a^2 b^4-8 a^3 b^2 c+16 a^4 c^2\right) x}{a^3 b^4 \operatorname{sgn}(x)-8 a^4 b^2 c \operatorname{sgn}(x)+16 a^5 c^2 \operatorname{sgn}(x)}+\frac{4\left(5 a b^5-37 a^2 b^3 c+64 a^3 b c^2\right)}{a^3 b^4 \operatorname{sgn}(x)-8 a^4 b^2 c \operatorname{sgn}(x)+16 a^5 c^2 \operatorname{sgn}(x)}\right) x+\frac{3\left(5 b^6-30 a b^4 c+16 a^2 b^2 c^2+64 a^3 c^3\right)}{a^3 b^4 \operatorname{sgn}(x)-8 a^4 b^2 c \operatorname{sgn}(x)+16 a^5 c^2 \operatorname{sgn}(x)}\right)}{3\left(a x^2+b x+c\right)^{\frac{3}{2}}} \\
& + \frac{5 b \log \left(\left|-2\left(\sqrt{a} x-\sqrt{a x^2+b x+c}\right) \sqrt{a}-b\right|\right)}{2 a^{\frac{7}{2}} \operatorname{sgn}(x)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 23.23 Problem number 596

$$\int (dx)^m (a + bx^n + cx^{2n})^3 dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{3a^2 b x^{1+n} (dx)^m}{1+m+n} + \frac{3a(ac+b^2) x^{1+2n} (dx)^m}{1+m+2n} + \frac{b(6ac+b^2) x^{1+3n} (dx)^m}{1+m+3n} \\
& + \frac{3c(ac+b^2) x^{1+4n} (dx)^m}{1+m+4n} + \frac{3b c^2 x^{1+5n} (dx)^m}{1+m+5n} + \frac{c^3 x^{1+6n} (dx)^m}{1+m+6n} + \frac{a^3 (dx)^{1+m}}{d(1+m)}
\end{aligned}$$

command

```
integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 24 Test file number 50

Test folder name:

```
test_cases/1_Algebraic_functions/1.2_Trinomial_products/1.2.4_Improper/50_1.2.4.2-
d_x^-m-a_x^q+b_x^n+c_x^-2_n-q-^p
```

## 24.1 Problem number 49

$$\int \frac{x^3}{\sqrt{ax^2 + bx^3 + cx^4}} dx$$

Optimal antiderivative

$$\frac{(-4ac + 3b^2) x \operatorname{arctanh}\left(\frac{2cx+b}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) \sqrt{cx^2+bx+a}}{8c^{\frac{5}{2}}\sqrt{cx^4+bx^3+ax^2}} + \frac{\sqrt{cx^4+bx^3+ax^2}}{2c} - \frac{3b\sqrt{cx^4+bx^3+ax^2}}{4c^2x}$$

command

`integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} \sqrt{cx^2+bx+a} \left( \frac{2x}{c \operatorname{sgn}(x)} - \frac{3b}{c^2 \operatorname{sgn}(x)} \right) + \frac{(3b^2 \log(|-b+2\sqrt{a}\sqrt{c}|) - 4ac \log(|-b+2\sqrt{a}\sqrt{c}|) + 6\sqrt{a}b\sqrt{c}) \operatorname{sgn}(x)}{8c^{\frac{5}{2}}} - \frac{(3b^2 - 4ac) \log\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2+bx+a}\right)\sqrt{c} - b\right|\right)}{8c^{\frac{5}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

## 24.2 Problem number 55

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{2x^4(bx+2a)}{(-4ac+b^2)\sqrt{cx^4+bx^3+ax^2}} + \frac{3(-4ac+5b^2)x \operatorname{arctanh}\left(\frac{2cx+b}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) \sqrt{cx^2+bx+a}}{8c^{\frac{7}{2}}\sqrt{cx^4+bx^3+ax^2}} + \frac{(-12ac+5b^2)\sqrt{cx^4+bx^3+ax^2}}{2c^2(-4ac+b^2)} - \frac{b(-52ac+15b^2)\sqrt{cx^4+bx^3+ax^2}}{4c^3(-4ac+b^2)x} - \frac{2bx\sqrt{cx^4+bx^3+ax^2}}{c(-4ac+b^2)}$$

command

```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(15 b^4 \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)-72 a b^2 c \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)+48 a^2 c^2 \log \left(|-b+2 \sqrt{a} \sqrt{c}|\right)+30 \sqrt{a} b^3 \sqrt{c}-1\right)}{8\left(b^2 c^{\frac{7}{2}}-4 a c^{\frac{9}{2}}\right)} + \frac{\left(\left(\frac{2\left(b^2 c^2-4 a c^3\right) x}{b^2 c^3 \operatorname{sgn}(x)-4 a c^4 \operatorname{sgn}(x)}-\frac{5\left(b^3 c-4 a b c^2\right)}{b^2 c^3 \operatorname{sgn}(x)-4 a c^4 \operatorname{sgn}(x)}\right) x-\frac{15 b^4-62 a b^2 c+24 a^2 c^2}{b^2 c^3 \operatorname{sgn}(x)-4 a c^4 \operatorname{sgn}(x)}\right) x-\frac{15 a b^3-52 a^2 b c}{b^2 c^3 \operatorname{sgn}(x)-4 a c^4 \operatorname{sgn}(x)}}{4 \sqrt{c x^2+b x+a}}$$

$$-\frac{3\left(5 b^2-4 a c\right) \log \left(\left|-2\left(\sqrt{c} x-\sqrt{c x^2+b x+a}\right) \sqrt{c}-b\right|\right)}{8 c^{\frac{7}{2}} \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^7}{\left(c x^4+b x^3+a x^2\right)^{\frac{3}{2}}} d x$$

### 24.3 Problem number 60

$$\int \frac{x^2}{\left(a x^2+b x^3+c x^4\right)^{\frac{3}{2}}} d x$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{x(b x+2 a)}{2 \sqrt{a} \sqrt{c x^4+b x^3+a x^2}}\right)}{a^{\frac{3}{2}}} + \frac{2 x(b c x-2 a c+b^2)}{a(-4 a c+b^2) \sqrt{c x^4+b x^3+a x^2}}$$

command

```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(a b^2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4 a^2 c \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a} \sqrt{a} b^2-2 \sqrt{-a} a^{\frac{3}{2}} c\right) \operatorname{sgn}(x)}{\sqrt{-a} a^2 b^2-4 \sqrt{-a} a^3 c} + \frac{2\left(\frac{a b c x \operatorname{sgn}(x)}{a^2 b^2-4 a^3 c}+\frac{a b^2 \operatorname{sgn}(x)-2 a^2 c \operatorname{sgn}(x)}{a^2 b^2-4 a^3 c}\right)}{\sqrt{c x^2+b x+a}} + \frac{2 \arctan\left(\frac{-\sqrt{c} x-\sqrt{c x^2+b x+a}}{\sqrt{-a}}\right)}{\sqrt{-a} a \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 25 Test file number 51

Test folder name:

test\_cases/1\_Algebraic\_functions/1.3\_Miscellaneous/51\_1.3.1\_Rational\_functions

### 25.1 Problem number 43

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{4ex+d}{\sqrt{3d^2 - 2\sqrt{-64ae^3 + d^4}}} \right)}{\sqrt{-64ae^3 + d^4} \sqrt{3d^2 - 2\sqrt{-64ae^3 + d^4}}} - \frac{2 \operatorname{arctanh} \left( \frac{4ex+d}{\sqrt{3d^2 + 2\sqrt{-64ae^3 + d^4}}} \right)}{\sqrt{-64ae^3 + d^4} \sqrt{3d^2 + 2\sqrt{-64ae^3 + d^4}}}$$

command

`integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log \left( \frac{1}{4} de^{(-1)} + \frac{1}{4} \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} + x \right)}{\left( de^{(-1)} + \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^3 e^3 - 3 \left( de^{(-1)} + \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^2 de^2 + 2 de^2 + 2} \\ \frac{2 \log \left( \frac{1}{4} de^{(-1)} - \frac{1}{4} \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} + x \right)}{\left( de^{(-1)} - \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^3 e^3 - 3 \left( de^{(-1)} - \sqrt{3d^2e^2 + 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^2 de^2 + 2 de^2 + 2} \\ \frac{2 \log \left( \frac{1}{4} de^{(-1)} + \frac{1}{4} \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} + x \right)}{\left( de^{(-1)} + \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^3 e^3 - 3 \left( de^{(-1)} + \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^2 de^2 + 2 de^2 + 2} \\ \frac{2 \log \left( \frac{1}{4} de^{(-1)} - \frac{1}{4} \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} + x \right)}{\left( de^{(-1)} - \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^3 e^3 - 3 \left( de^{(-1)} - \sqrt{3d^2e^2 - 2\sqrt{d^4 - 64ae^3} e^2} e^{(-2)} \right)^2 de^2 + 2 de^2 + 2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2} dx$$

## 25.2 Problem number 44

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx$$

Optimal antiderivative

$$\frac{2e\left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e} + x\right)^2\right)}{(-16384a^2e^6 - 64ad^4e^3 + 5d^8)(8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2)}$$

$$- \frac{24e \operatorname{arctanh}\left(\frac{4ex+d}{\sqrt{3d^2 - 2\sqrt{-64ae^3 + d^4}}}\right) \left(d^4 + 128ae^3 - d^2\sqrt{-64ae^3 + d^4}\right)}{(-64ae^3 + d^4)^{\frac{3}{2}}(256ae^3 + 5d^4)\sqrt{3d^2 - 2\sqrt{-64ae^3 + d^4}}}$$

$$+ \frac{24e \operatorname{arctanh}\left(\frac{4ex+d}{\sqrt{3d^2 + 2\sqrt{-64ae^3 + d^4}}}\right) \left(d^4 + 128ae^3 + d^2\sqrt{-64ae^3 + d^4}\right)}{(-64ae^3 + d^4)^{\frac{3}{2}}(256ae^3 + 5d^4)\sqrt{3d^2 + 2\sqrt{-64ae^3 + d^4}}}$$

command

```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 25.3 Problem number 413

$$\int \frac{1}{(d + ex)^2 (a + cx^4)^3} dx$$

Optimal antiderivative

Expression too large to display

command

```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26 Test file number 52

Test folder name:

test\_cases/1\_Algebraic\_functions/1.3\_Miscellaneous/52\_1.3.2\_Algebraic\_functions

### 26.1 Problem number 276

$$\int x^5 \left( \frac{e(a + bx^2)}{c + dx^2} \right)^{3/2} dx$$

Optimal antiderivative

$$\frac{\left( \frac{e(bx^2+a)}{dx^2+c} \right)^{\frac{5}{2}} (dx^2+c)^3}{6bd^2e}$$

$$- \frac{(-ad+bc)(-a^2d^2-10abcd+35b^2c^2)e^{\frac{3}{2}} \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{b} \sqrt{e}} \right)}{16b^{\frac{3}{2}}d^{\frac{9}{2}}}$$

$$+ \frac{c^2(-ad+bc)e \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{d^4} + \frac{(-5a^2d^2-50abcd+79b^2c^2)e(dx^2+c) \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{48bd^4}$$

$$- \frac{(ad+11bc)e(dx^2+c)^2 \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{24d^4}$$

command

```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{96} \left( 2 \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \left( 2 \left( \frac{4bx^2 \operatorname{sgn}(dx^2+c)}{d^2} - \frac{11b^3cd^{10} \operatorname{sgn}(dx^2+c) - 7ab^2d^{11} \operatorname{sgn}(dx^2+c)}{b^2d^{13}} \right) x^2 + \right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.2 Problem number 277

$$\int x^3 \left( \frac{e(a + bx^2)}{c + dx^2} \right)^{3/2} dx$$

Optimal antiderivative

$$\frac{3(-ad + bc)(-ad + 5bc)e^{\frac{3}{2}} \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{\sqrt{b} \sqrt{e}} \right)}{8d^{\frac{7}{2}} \sqrt{b}} - \frac{c(-ad + bc)e \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{d^3} - \frac{(-5ad + 9bc)e(dx^2 + c) \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{8d^3} + \frac{be(dx^2 + c)^2 \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{4d^3}$$

command

```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{16} \left( 2 \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \left( \frac{2bx^2 \operatorname{sgn}(dx^2 + c)}{d^2} - \frac{7b^2cd^5 \operatorname{sgn}(dx^2 + c) - 5abd^6 \operatorname{sgn}(dx^2 + c)}{bd^8} \right) - \frac{16(b^2c^3 \operatorname{sgn}(dx^2 + c))}{((\sqrt{bdx^4 + bcx^2 + adx^2 + ac}))^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.3 Problem number 278

$$\int x \left( \frac{e(a + bx^2)}{c + dx^2} \right)^{3/2} dx$$

Optimal antiderivative

$$\frac{\left( \frac{e(bx^2 + a)}{dx^2 + c} \right)^{\frac{3}{2}} (dx^2 + c)}{2d} - \frac{3(-ad + bc)e^{\frac{3}{2}} \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{\sqrt{b} \sqrt{e}} \right) \sqrt{b}}{2d^{\frac{5}{2}}} + \frac{3(-ad + bc)e \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{2d^2}$$



command

```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} \left( \frac{2 \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \operatorname{bsgn}(dx^2 + c)}{d^2} + \frac{4(b^2c^2 \operatorname{sgn}(dx^2 + c) - 2abcd \operatorname{sgn}(dx^2 + c) + a^2d^2 \operatorname{sgn}(dx^2 + c))}{\left( (\sqrt{bd} x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac})d + \sqrt{bd} c \right) d^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.4 Problem number 296

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc) (5a^2d^2 + 2abcd + b^2c^2) \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{\sqrt{b} \sqrt{e}} \right)}{16b^{\frac{7}{2}}d^{\frac{5}{2}}\sqrt{e}} + \frac{(5a^2d^2 + 2abcd + b^2c^2) (dx^2 + c) \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{16b^3d^2e} - \frac{(5ad + 3bc) (dx^2 + c)^2 \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{24b^2d^2e} - \frac{(dx^2 + c)^3 \left( a - \frac{c(bx^2 + a)}{dx^2 + c} \right) \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{6bd(-ad + bc)e}$$

command

```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 2 \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \left( 2x^2 \left( \frac{4x^2}{b} + \frac{b^2cd - 5abd^2}{b^3d^2} \right) - \frac{3b^2c^2 + 4abcd - 15a^2d^2}{b^3d^2} \right) - \frac{3(b^3c^3 + ab^2c^2d + 3a^2bcd^2 - 5a^3d^3) \log\left( \left| \frac{(\sqrt{bd} x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac})d + \sqrt{bd} c}{\sqrt{bd} c} \right| \right)}{96 \operatorname{sgn}(dx^2 + c)} \right)}{96 \operatorname{sgn}(dx^2 + c)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.5 Problem number 297

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal antiderivative

$$\frac{(-ad+bc)(3ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{b} \sqrt{e}}\right)}{8b^{\frac{5}{2}}d^{\frac{3}{2}}\sqrt{e}} - \frac{(3ad+bc)(dx^2+c) \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8b^2de} + \frac{(dx^2+c)^2 \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4bde}$$

command

```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2 \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \left(\frac{2x^2}{b} + \frac{bc-3ad}{b^2d}\right) + \frac{(b^2c^2+2abcd-3a^2d^2) \log\left(\left| -bc-ad-2 \left(\sqrt{bd} x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac} \right)\right|}{\sqrt{bd} b^2d}\right)}{16 \operatorname{sgn}(dx^2+c)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.6 Problem number 298

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal antiderivative

$$\frac{(-ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{b} \sqrt{e}}\right)}{2b^{\frac{3}{2}}\sqrt{d} \sqrt{e}} + \frac{(dx^2+c) \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}$$

command

```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{2\sqrt{bdx^4 + bcx^2 + adx^2 + ac}}{b} - \frac{(bc-ad)\sqrt{bd} \log\left(\left| -2\left(\sqrt{bd}x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac}\right)bd - \sqrt{bd}bc - \sqrt{bd}ad \right|\right)}{b^2d} \right)}{4 \operatorname{sgn}(dx^2 + c)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.7 Problem number 300

$$\int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal antiderivative

$$\frac{(-ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{\sqrt{a} \sqrt{e}}\right)}{2a^{\frac{3}{2}} \sqrt{c} \sqrt{e}} + \frac{(-ad + bc) \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}}}{2a \left( ae - \frac{ce(bx^2 + a)}{dx^2 + c} \right)}$$

command

```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{(bc-ad) \operatorname{arctan}\left(-\frac{\sqrt{bd}x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac} a} - \frac{\left(\sqrt{bd}x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac}\right)bc + \left(\sqrt{bd}x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac}\right)ad}{\left(\left(\sqrt{bd}x^2 - \sqrt{bdx^4 + bcx^2 + adx^2 + ac}\right)^2 - bd\right)^2} \right)}{2 \operatorname{sgn}(dx^2 + c)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.8 Problem number 301

$$\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal antiderivative

$$\frac{(-ad+bc)(ad+3bc) \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{\sqrt{a} \sqrt{e}}\right)}{8a^{\frac{5}{2}}c^{\frac{3}{2}}\sqrt{e}} - \frac{(-ad+bc)^2 e \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4ac \left(ae - \frac{ce(bx^2+a)}{dx^2+c}\right)^2} - \frac{(-ad+bc)(ad+3bc) \sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8a^2c \left(ae - \frac{ce(bx^2+a)}{dx^2+c}\right)}$$

command

`integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(3b^2c^2-2abcd-a^2d^2) \operatorname{arctan}\left(-\frac{\sqrt{bd}x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2c} - 3\left(\sqrt{bd}x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^3 b^2c^2 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.9 Problem number 442

$$\int \frac{1}{\left(\sqrt{a+bx} + \sqrt{a+cx}\right)^3} dx$$

Optimal antiderivative

$$\frac{3bc \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{(b-c)^3 \sqrt{a}} + \frac{3bc \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{(b-c)^3 \sqrt{a}} - \frac{2a\sqrt{bx+a}}{(b-c)^3 x^2} - \frac{(2b+3c)\sqrt{bx+a}}{(b-c)^3 x} + \frac{2a\sqrt{cx+a}}{(b-c)^3 x^2} + \frac{(3b+2c)\sqrt{cx+a}}{(b-c)^3 x}$$

command

```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26.10 Problem number 457

$$\int \frac{1}{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{af^2 \ln\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1 + \frac{af^2}{d^2}\right) \ln\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e} \\ & - \frac{2de\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{af^2} \end{aligned}$$

command

```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(af^2 + d^2)e^{(-1)} \log\left(|-af^2 + 2dxe + d^2|\right)}{4d^2} - \frac{\sqrt{af^2 + x^2e^2} |f|e^{(-1)}}{2df} \\ & + \frac{x}{2d} + \frac{(af^2|f| + d^2|f|)e^{(-1)} \log\left(\left|af^2 - \left(xe - \sqrt{af^2 + x^2e^2}\right)d\right|\right)}{4d^2f} \\ & - \frac{(af^2|f| + d^2|f|)e^{(-1)} \log\left(\left|-xe - d + \sqrt{af^2 + x^2e^2}\right|\right)}{4d^2f} \\ & + \frac{(af^2|f| - d^2|f|)e^{(-1)} \log\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right)}{4d^2f} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

[undef, +∞, 1]

## 26.11 Problem number 458

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{af^2 \ln\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{d^3e} + \frac{af^2 \ln\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{d^3e} \\ & -\frac{af^2}{2d^2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{-1 - \frac{af^2}{d^2}}{2e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} \end{aligned}$$

command

`integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{af|f|e^{(-1)} \log\left(\left|af^2 - \left(xe - \sqrt{af^2 + x^2e^2}\right)d\right|\right)}{2d^3} + \frac{af^2e^{(-1)} \log\left(\left|-af^2 + 2dxe + d^2\right|\right)}{2d^3} \\ & - \frac{af|f|e^{(-1)} \log\left(\left|-xe - d + \sqrt{af^2 + x^2e^2}\right|\right)}{2d^3} + \frac{af|f|e^{(-1)} \log\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right)}{2d^3} \\ & - \frac{\sqrt{af^2 + x^2e^2}|f|e^{(-1)}}{2d^2f} + \frac{x}{2d^2} + \frac{(a^2f^4 + 2ad^2f^2 + d^4)e^{(-1)}}{4(af^2 - 2dxe - d^2)d^3} \\ & + \frac{\left(\left(xe - \sqrt{af^2 + x^2e^2}\right)a^2f^4|f| + 2a^2df^4|f| + 2ad^3f^2|f| - \left(xe - \sqrt{af^2 + x^2e^2}\right)d^4|f|\right)e^{(-1)}}{4\left(\left(xe - \sqrt{af^2 + x^2e^2}\right)af^2 + adf^2 - \left(xe - \sqrt{af^2 + x^2e^2}\right)^2d - \left(xe - \sqrt{af^2 + x^2e^2}\right)d^2\right)d^3f} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26.12 Problem number 459

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3af^2 \ln\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2d^4e} + \frac{3af^2 \ln\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2d^4e} \\ & -\frac{2d^3e}{af^2} \left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right) + \frac{-1 - \frac{af^2}{d^2}}{4e} \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2 \\ & -\frac{d^3e}{af^2} \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right) \end{aligned}$$

command

`integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{3af|f|e^{(-1)} \log\left(\left|af^2 - \left(xe - \sqrt{af^2 + x^2e^2}\right)d\right|\right)}{4d^4} + \frac{3af^2e^{(-1)} \log\left(\left|-af^2 + 2dxe + d^2\right|\right)}{4d^4} \\ & -\frac{3af|f|e^{(-1)} \log\left(\left|-xe - d + \sqrt{af^2 + x^2e^2}\right|\right)}{4d^4} + \frac{3af|f|e^{(-1)} \log\left(\left|-xe + \sqrt{af^2 + x^2e^2}\right|\right)}{4d^4} \\ & -\frac{\sqrt{af^2 + x^2e^2}|f|e^{(-1)}}{2d^3f} + \frac{x}{2d^3} + \frac{(5a^3f^6 - 3a^2d^2f^4 - 9ad^4f^2 - d^6 - 12(a^2df^4e + ad^3f^2e)x)e^{(-1)}}{8(af^2 - 2dxe - d^2)^2d^4} \\ & + \frac{\left(5\left(xe - \sqrt{af^2 + x^2e^2}\right)^2a^3f^6|f| + 14\left(xe - \sqrt{af^2 + x^2e^2}\right)a^3df^6|f| + 10a^3d^2f^6|f| - 6\left(xe - \sqrt{af^2 + x^2e^2}\right)\right)}{8(af^2 - 2dxe - d^2)^2d^4} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26.13 Problem number 476

$$\int \frac{1}{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(ae f^2 - bd f^2 + d^2 e) \ln \left( d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)}{(-b f^2 + 2de)^2} \\ & - \frac{f^2 (-b^2 f^2 + 4a e^2) \ln \left( b f^2 + 2e \left( ex + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right)}{2e (-b f^2 + 2de)^2} \\ & - \frac{2e (-b f^2 + 2de) \left( b f^2 + 2e \left( ex + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right)}{f^2 (-b^2 f^2 + 4a e^2)} \end{aligned}$$

command

```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{xe}{bf^2 - 2de} \\ & - \frac{(bdf^2|f| - af^2|f|e - d^2|f|e) \log \left( \left| - \left( xe - \sqrt{bf^2x + af^2 + x^2e^2} \right) bf^2 + bdf^2 - 2af^2e + 2 \left( xe - \sqrt{bf^2x + af^2 + x^2e^2} \right) \right. \right)}{b^2 f^5 - 4 bdf^3 e + 4 d^2 f e^2} \\ & - \frac{(bdf^2 - af^2e - d^2e) \log \left( \left| -bf^2x - af^2 + 2dxe + d^2 \right| \right)}{b^2 f^4 - 4 bdf^2 e + 4 d^2 e^2} \\ & - \frac{(b^2 f^4|f| - 2bdf^2|f|e - 2af^2|f|e^2 + 2d^2|f|e^2) \log \left( \left| bf^2 + 2 \left( xe - \sqrt{bf^2x + af^2 + x^2e^2} \right) e \right| \right)}{2 (b^2 f^5 e - 4 bdf^3 e^2 + 4 d^2 f e^3)} \\ & + \frac{(bdf^2|f| - af^2|f|e - d^2|f|e) \log \left( \left| xe + d - \sqrt{bf^2x + af^2 + x^2e^2} \right| \right)}{b^2 f^5 - 4 bdf^3 e + 4 d^2 f e^2} \\ & + \frac{(b^2 f^5|f| - 4 bdf^3|f|e + 4 d^2 f|f|e^2) \sqrt{bf^2x + af^2 + x^2e^2}}{b^3 f^8 - 6 b^2 df^6 e + 12 bd^2 f^4 e^2 - 8 d^3 f^2 e^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

[undef, +∞, 1]



## 26.14 Problem number 477

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2f^2(-b^2f^2 + 4ae^2) \ln\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)}{(-bf^2 + 2de)^3} \\ & - \frac{2f^2(-b^2f^2 + 4ae^2) \ln\left(bf^2 + 2e\left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)}{(-bf^2 + 2de)^3} \\ & - \frac{2(aef^2 - bdf^2 + d^2e)}{(-bf^2 + 2de)^2 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)} \\ & - \frac{f^2(-b^2f^2 + 4ae^2)}{(-bf^2 + 2de)^2 \left(bf^2 + 2e\left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)} \end{aligned}$$

command

`integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26.15 Problem number 478

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{6e f^2 (-b^2 f^2 + 4a e^2) \ln \left( d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)}{(-b f^2 + 2de)^4} \\
& - \frac{6e f^2 (-b^2 f^2 + 4a e^2) \ln \left( b f^2 + 2e \left( ex + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right)}{(-b f^2 + 2de)^4} \\
& + \frac{-ae f^2 + bd f^2 - d^2 e}{(-b f^2 + 2de)^2 \left( d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2} \\
& - \frac{2f^2 (-b^2 f^2 + 4a e^2)}{(-b f^2 + 2de)^3 \left( d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)} \\
& - \frac{2e f^2 (-b^2 f^2 + 4a e^2)}{(-b f^2 + 2de)^3 \left( b f^2 + 2e \left( ex + f \sqrt{a + \frac{x(b f^2 + e^2 x)}{f^2}} \right) \right)}
\end{aligned}$$

command

```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**26.16 Problem number 498**

$$\int \frac{\left(x + \sqrt{a + x^2}\right)^n}{\sqrt{a + x^2}} dx$$

Optimal antiderivative

$$\frac{\left(x + \sqrt{x^2 + a}\right)^n}{n}$$

command

```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(x + \sqrt{x^2 + a}\right)^n}{n}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(x + \sqrt{x^2 + a}\right)^n}{\sqrt{x^2 + a}} dx$$

### 26.17 Problem number 504

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\sqrt{a + x^2}} dx$$

Optimal antiderivative

$$-\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

command

`integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\sqrt{x^2 + a}} dx$$

### 26.18 Problem number 526

$$\int \frac{e - 4fx^3}{e^2 + 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2x\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-df} \log\left(\left|2fx^3 + 2\sqrt{-df}x + e\right|\right)}{4df} + \frac{\sqrt{-df} \log\left(\left|2fx^3 - 2\sqrt{-df}x + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{4fx^3 - e}{4f^2x^6 + 4efx^3 + 4dfx^2 + e^2} dx$$

## 26.19 Problem number 527

$$\int \frac{e - 4fx^3}{e^2 - 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{2x\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{df} \log\left(\left|2fx^3 + 2\sqrt{df}x + e\right|\right)}{4df} - \frac{\sqrt{df} \log\left(\left|2fx^3 - 2\sqrt{df}x + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{4fx^3 - e}{4f^2x^6 + 4efx^3 - 4dfx^2 + e^2} dx$$

### 26.20 Problem number 532

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^6} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2x^3\sqrt{d}\sqrt{f}}{2fx^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

`integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-df} \log\left(\left|2\sqrt{-df}x^3 + 2fx^2 + e\right|\right)}{4df} + \frac{\sqrt{-df} \log\left(\left|-2\sqrt{-df}x^3 + 2fx^2 + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(2fx^2 + 3e)x^2}{4dfx^6 + 4f^2x^4 + 4efx^2 + e^2} dx$$

### 26.21 Problem number 533

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{2x^3\sqrt{d}\sqrt{f}}{2fx^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

`integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{df} \log\left(\left|2\sqrt{df}x^3 + 2fx^2 + e\right|\right)}{4df} - \frac{\sqrt{df} \log\left(\left|-2\sqrt{df}x^3 + 2fx^2 + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{(2fx^2 + 3e)x^2}{4dfx^6 - 4f^2x^4 - 4efx^2 - e^2} dx$$

### 26.22 Problem number 536

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{2x^2\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-df} \log\left(\left|2fx^3 + 2\sqrt{-df}x^2 + e\right|\right)}{4df} + \frac{\sqrt{-df} \log\left(\left|2fx^3 - 2\sqrt{-df}x^2 + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2(fx^3 - e)x}{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2} dx$$

### 26.23 Problem number 537

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 - 4dfx^4 + 4f^2x^6} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{2x^2\sqrt{d}\sqrt{f}}{2fx^3+e}\right)}{2\sqrt{d}\sqrt{f}}$$

command

```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{df} \log\left(\left|2fx^3 + 2\sqrt{df}x^2 + e\right|\right)}{4df} - \frac{\sqrt{df} \log\left(\left|2fx^3 - 2\sqrt{df}x^2 + e\right|\right)}{4df}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2(fx^3 - e)x}{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2} dx$$

## 26.24 Problem number 717

$$\int \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{32\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{5}{2}}}{5} + \frac{48\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{7}{2}}}{7} \\ & + \frac{112\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{9}{2}}}{9} - \frac{320\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{11}{2}}}{11} \\ & + \frac{288\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{13}{2}}}{13} - \frac{112\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{15}{2}}}{15} \\ & + \frac{16\left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}\right)^{\frac{17}{2}}}{17} \end{aligned}$$

command

```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 26.25 Problem number 811

$$\int \frac{-1 + x^2}{\sqrt{a - b + \frac{b}{x^2}} x^3} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a - b\left(1 - \frac{1}{x^2}\right)}}{\sqrt{a - b}}\right)}{\sqrt{a - b}} + \frac{\sqrt{a - b\left(1 - \frac{1}{x^2}\right)}}{b}$$

command

```
integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log\left(\left(\sqrt{a-b}x - \sqrt{ax^2 - bx^2 + b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x - \sqrt{ax^2 - bx^2 + b}\right)^2 - b\right)\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.26 Problem number 812

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}$$

command

```
integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log\left(\left(\sqrt{a-b}x - \sqrt{ax^2 - bx^2 + b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x - \sqrt{ax^2 - bx^2 + b}\right)^2 - b\right)\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 26.27 Problem number 891

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} \sqrt{c + dx^2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{ax^2 + b}}{\sqrt{a} \sqrt{dx^2 + c}}\right) \sqrt{ax^2 + b}}{x \sqrt{a} \sqrt{d} \sqrt{a + \frac{b}{x^2}}}$$

command

```
integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(\left|-\sqrt{ad} \sqrt{b} + \sqrt{a^2c}\right|\right) \operatorname{sgn}(x)}{\sqrt{ad} |a|} - \frac{a \log\left(\left|-\sqrt{ax^2 + b} \sqrt{ad} + \sqrt{a^2c + (ax^2 + b)ad - abd}\right|\right)}{\sqrt{ad} |a| \operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 26.28 Problem number 899

$$\int \frac{1}{\left(1 + \frac{2x}{1+x^2}\right)^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 + \frac{3x}{2}}{\sqrt{1 + \frac{2x}{x^2 + 1}}} + \frac{-x^2 - 1}{2(1+x) \sqrt{1 + \frac{2x}{x^2 + 1}}} - \frac{3(1+x) \operatorname{arcsinh}(x)}{\sqrt{x^2 + 1} \sqrt{1 + \frac{2x}{x^2 + 1}}} - \frac{9(1+x) \operatorname{arctanh}\left(\frac{(1-x)\sqrt{2}}{2\sqrt{x^2 + 1}}\right) \sqrt{2}}{4\sqrt{x^2 + 1} \sqrt{1 + \frac{2x}{x^2 + 1}}}$$

command

```
integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9\sqrt{2} \log\left(\frac{\left| -2x-2\sqrt{2}+2\sqrt{x^2+1}-2 \right|}{\left| -2x+2\sqrt{2}+2\sqrt{x^2+1}-2 \right|}\right)}{4\operatorname{sgn}(x+1)} + \frac{3\log(-x+\sqrt{x^2+1})}{\operatorname{sgn}(x+1)} + \frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

$$+ \frac{7(x-\sqrt{x^2+1})^3 + 5(x-\sqrt{x^2+1})^2 - 13x + 13\sqrt{x^2+1} + 5}{\left((x-\sqrt{x^2+1})^2 + 2x - 2\sqrt{x^2+1} - 1\right)^2 \operatorname{sgn}(x+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{\frac{3}{2}}} dx$$

## 27 Test file number 53

Test folder name:

test\_cases/2\_Exponentials/53\_2.1\_u-F^-c-a+b\_x-^n

### 27.1 Problem number 82

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ad+bc)^4 e^{-bx-a}}{4d^5(dx+c)^4} + \frac{4b(-ad+bc)^3 e^{-bx-a}}{3d^5(dx+c)^3} + \frac{b(-ad+bc)^4 e^{-bx-a}}{12d^6(dx+c)^3} \\ & -\frac{3b^2(-ad+bc)^2 e^{-bx-a}}{d^5(dx+c)^2} - \frac{2b^2(-ad+bc)^3 e^{-bx-a}}{3d^6(dx+c)^2} - \frac{b^2(-ad+bc)^4 e^{-bx-a}}{24d^7(dx+c)^2} \\ & + \frac{4b^3(-ad+bc) e^{-bx-a}}{d^5(dx+c)} + \frac{3b^3(-ad+bc)^2 e^{-bx-a}}{d^6(dx+c)} + \frac{2b^3(-ad+bc)^3 e^{-bx-a}}{3d^7(dx+c)} \\ & + \frac{b^3(-ad+bc)^4 e^{-bx-a}}{24d^8(dx+c)} + \frac{b^4 e^{-a+\frac{bc}{d}} \operatorname{expIntegral}\left(-\frac{b(dx+c)}{d}\right)}{d^5} \\ & + \frac{4b^4(-ad+bc) e^{-a+\frac{bc}{d}} \operatorname{expIntegral}\left(-\frac{b(dx+c)}{d}\right)}{d^6} \\ & + \frac{3b^4(-ad+bc)^2 e^{-a+\frac{bc}{d}} \operatorname{expIntegral}\left(-\frac{b(dx+c)}{d}\right)}{d^7} \\ & + \frac{2b^4(-ad+bc)^3 e^{-a+\frac{bc}{d}} \operatorname{expIntegral}\left(-\frac{b(dx+c)}{d}\right)}{3d^8} \\ & + \frac{b^4(-ad+bc)^4 e^{-a+\frac{bc}{d}} \operatorname{expIntegral}\left(-\frac{b(dx+c)}{d}\right)}{24d^9} \end{aligned}$$

command

```
integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 28 Test file number 54

Test folder name:

test\_cases/2\_Exponentials/54\_2.2-c+d\_x-^m-F^-g-e+f\_x-^n-a+b-F^-g-e+f\_x-^n-^p

### 28.1 Problem number 32

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^2 (c + dx)^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(dx+c)^4}{4d} - \frac{12abd^3(F^{fgx+eg})^n}{f^4g^4n^4\ln(F)^4} - \frac{3b^2d^3(F^{fgx+eg})^{2n}}{8f^4g^4n^4\ln(F)^4} + \frac{12abd^2(F^{fgx+eg})^n(dx+c)}{f^3g^3n^3\ln(F)^3} \\ & + \frac{3b^2d^2(F^{fgx+eg})^{2n}(dx+c)}{4f^3g^3n^3\ln(F)^3} - \frac{6abd(F^{fgx+eg})^n(dx+c)^2}{f^2g^2n^2\ln(F)^2} \\ & - \frac{3b^2d(F^{fgx+eg})^{2n}(dx+c)^2}{4f^2g^2n^2\ln(F)^2} + \frac{2ab(F^{fgx+eg})^n(dx+c)^3}{fgn\ln(F)} + \frac{b^2(F^{fgx+eg})^{2n}(dx+c)^3}{2fgn\ln(F)} \end{aligned}$$

command

```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 28.2 Problem number 39

$$\int \left( a + b \left( F^{g(e+fx)} \right)^n \right)^3 (c + dx)^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(dx+c)^4}{4d} - \frac{18a^2bd^3(F^{fgx+eg})^n}{f^4g^4n^4\ln(F)^4} - \frac{9ab^2d^3(F^{fgx+eg})^{2n}}{8f^4g^4n^4\ln(F)^4} - \frac{2b^3d^3(F^{fgx+eg})^{3n}}{27f^4g^4n^4\ln(F)^4} \\ & + \frac{18a^2bd^2(F^{fgx+eg})^n(dx+c)}{f^3g^3n^3\ln(F)^3} + \frac{9ab^2d^2(F^{fgx+eg})^{2n}(dx+c)}{4f^3g^3n^3\ln(F)^3} + \frac{2b^3d^2(F^{fgx+eg})^{3n}(dx+c)}{9f^3g^3n^3\ln(F)^3} \\ & - \frac{9a^2bd(F^{fgx+eg})^n(dx+c)^2}{f^2g^2n^2\ln(F)^2} - \frac{9ab^2d(F^{fgx+eg})^{2n}(dx+c)^2}{4f^2g^2n^2\ln(F)^2} - \frac{b^3d(F^{fgx+eg})^{3n}(dx+c)^2}{3f^2g^2n^2\ln(F)^2} \\ & + \frac{3a^2b(F^{fgx+eg})^n(dx+c)^3}{fgn\ln(F)} + \frac{3ab^2(F^{fgx+eg})^{2n}(dx+c)^3}{2fgn\ln(F)} + \frac{b^3(F^{fgx+eg})^{3n}(dx+c)^3}{3fgn\ln(F)} \end{aligned}$$

command

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 29 Test file number 55

Test folder name:

test\_cases/2\_Exponentials/55\_2.3\_Exponential\_functions

### 29.1 Problem number 624

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2} dx$$

Optimal antiderivative

$$\text{expIntegral}(cx^2 + bx + a)$$

command

```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\text{Ei}(cx^2 + bx + a)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(2cx + b)e^{(cx^2 + bx + a)}}{cx^2 + bx + a} dx$$

### 30 Test file number 58

Test folder name:

test\_cases/3\_Logarithms/58\_3.1.5\_u-a+b\_log-c\_x^n~p

#### 30.1 Problem number 186

$$\int (d + ex^2) \sin^{-1}(ax) \log(cx^n) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2en(-a^2x^2 + 1)^{\frac{3}{2}}}{27a^3} - dx \arcsin(ax) - \frac{enx^3 \arcsin(ax)}{9} - \frac{en \operatorname{arctanh}\left(\sqrt{-a^2x^2 + 1}\right)}{9a^3} \\ & + \frac{(3a^2d + e)n \operatorname{arctanh}\left(\sqrt{-a^2x^2 + 1}\right)}{3a^3} - \frac{e(-a^2x^2 + 1)^{\frac{3}{2}} \ln(cx^n)}{9a^3} \\ & + dx \arcsin(ax) \ln(cx^n) + \frac{ex^3 \arcsin(ax) \ln(cx^n)}{3} - \frac{dn \sqrt{-a^2x^2 + 1}}{a} \\ & - \frac{(3a^2d + e)n \sqrt{-a^2x^2 + 1}}{3a^3} + \frac{(3a^2d + e) \ln(cx^n) \sqrt{-a^2x^2 + 1}}{3a^3} \end{aligned}$$

command

```
integrate((e*x^2+d)*arcsin(a*x)*log(c*x^n),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 30.2 Problem number 187

$$\int (d + ex^2) \cos^{-1}(ax) \log(cx^n) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2en(-a^2x^2+1)^{\frac{3}{2}}}{27a^3} - dnx \arccos(ax) - \frac{enx^3 \arccos(ax)}{9} + \frac{en \operatorname{arctanh}\left(\sqrt{-a^2x^2+1}\right)}{9a^3} \\ & - \frac{(3a^2d+e)n \operatorname{arctanh}\left(\sqrt{-a^2x^2+1}\right)}{3a^3} + \frac{e(-a^2x^2+1)^{\frac{3}{2}} \ln(cx^n)}{9a^3} \\ & + dx \arccos(ax) \ln(cx^n) + \frac{ex^3 \arccos(ax) \ln(cx^n)}{3} + \frac{dn\sqrt{-a^2x^2+1}}{a} \\ & + \frac{(3a^2d+e)n\sqrt{-a^2x^2+1}}{3a^3} - \frac{(3a^2d+e) \ln(cx^n) \sqrt{-a^2x^2+1}}{3a^3} \end{aligned}$$

command

```
integrate((e*x^2+d)*arccos(a*x)*log(c*x^n),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 31 Test file number 59

Test folder name:

```
test_cases/3_Logarithms/59_3.2.1-f+g_x^-m-A+B_log-e-a+b_x-over-c+d_x^-n-p
```

### 31.1 Problem number 33

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg + dgx} dx$$

Optimal antiderivative

$$-\frac{\left(A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) \ln\left(\frac{-ad+bc}{b(dx+c)}\right)}{dg} - \frac{Bn \operatorname{polylog}\left(2, \frac{d(bx+a)}{b(dx+c)}\right)}{dg}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \left( \frac{(Bb^3c^3n - 3Bab^2c^2dn + 3Ba^2bcd^2n - Ba^3d^3n) \log\left(\frac{bx+a}{dx+c}\right)}{b^2dg - \frac{2(bx+a)bd^2g}{dx+c} + \frac{(bx+a)^2d^3g}{(dx+c)^2}} - \frac{Bb^4c^3n - 3Bab^3c^2dn - \frac{(bx+a)Bb^3c^3dn}{dx+c} + 3Ba^2b^2c^2dn}{b^2dg - \frac{2(bx+a)bd^2g}{dx+c} + \frac{(bx+a)^2d^3g}{(dx+c)^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 31.2 Problem number 66

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(-ad+bc)n}{12(-ag+bf)(-cg+df)(gx+f)^3} - \frac{B(-ad+bc)(-adg-bcg+2bdf)n}{8(-ag+bf)^2(-cg+df)^2(gx+f)^2} \\ & -\frac{B(-ad+bc)(a^2d^2g^2-abdg(-cg+3df)+b^2(c^2g^2-3cdfg+3d^2f^2))n}{4(-ag+bf)^3(-cg+df)^3(gx+f)} \\ & + \frac{b^4Bn \ln(bx+a)}{4g(-ag+bf)^4} + \frac{-A - B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{4g(gx+f)^4} - \frac{Bd^4n \ln(dx+c)}{4g(-cg+df)^4} \\ & - \frac{B(-ad+bc)(-adg-bcg+2bdf)(2abd^2fg - a^2d^2g^2 - b^2(c^2g^2 - 2cdfg + 2d^2f^2))n \ln(gx+f)}{4(-ag+bf)^4(-cg+df)^4} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 31.3 Problem number 106

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx} dx$$

Optimal antiderivative

$$\frac{\text{polylog}\left(2, \frac{-ad+bc}{b(dx+c)}\right)}{df}$$

command

```
integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 31.4 Problem number 177

$$\int \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx} dx$$

Optimal antiderivative

$$-\frac{\ln\left(\frac{ad-bc}{d(bx+a)}\right)\left(A + B \ln\left(\frac{e(dx+c)}{bx+a}\right)\right)}{bg} - \frac{B \text{polylog}\left(2, 1 + \frac{-ad+bc}{d(bx+a)}\right)}{bg}$$

command

```
integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 31.5 Problem number 230

$$\int (f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B(-ad + bc)g(a^3d^3g^3 - a^2bd^2g^2(-cg + 5df) + ab^2dg(c^2g^2 - 5cdfg + 10d^2f^2) - b^3(-c^3g^3 + 5c^2dfg^2 - 10cd^2f^2g + 5b^4d^4))}{10b^3d^3} \\ & - \frac{B(-ad + bc)g^2(a^2d^2g^2 - abdg(-cg + 5df) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))x^2}{15b^2d^2} \\ & - \frac{B(-ad + bc)g^3(-adg - bcg + 5bdf)x^3}{20bd} - \frac{B(-ad + bc)g^4x^4}{5g} \\ & - \frac{B(-ag + bf)^5 \ln(bx + a)}{5b^5g} + \frac{(gx + f)^5 \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{5g} + \frac{B(-cg + df)^5 \ln(dx + c)}{5d^5g} \end{aligned}$$

command

```
integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 31.6 Problem number 238

$$\int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(f + gx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B(-ad + bc)}{6(-ag + bf)(-cg + df)(gx + f)^2} - \frac{B(-ad + bc)(-adg - bcg + 2bdf)}{3(-ag + bf)^2(-cg + df)^2(gx + f)} \\ & + \frac{b^3B \ln(bx + a)}{3g(-ag + bf)^3} + \frac{-A - B \ln \left( \frac{e(bx+a)}{dx+c} \right)}{3g(gx + f)^3} - \frac{Bd^3 \ln(dx + c)}{3g(-cg + df)^3} \\ & + \frac{B(-ad + bc)(a^2d^2g^2 - abdg(-cg + 3df) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \ln(gx + f)}{3(-ag + bf)^3(-cg + df)^3} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 31.7 Problem number 239

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B(-ad+bc)}{12(-ag+bf)(-cg+df)(gx+f)^3} - \frac{B(-ad+bc)(-adg-bcg+2bdf)}{8(-ag+bf)^2(-cg+df)^2(gx+f)^2} \\ & - \frac{B(-ad+bc)(a^2d^2g^2-abdg(-cg+3df)+b^2(c^2g^2-3cdfg+3d^2f^2))}{4(-ag+bf)^3(-cg+df)^3(gx+f)} \\ & + \frac{b^4B \ln(bx+a)}{4g(-ag+bf)^4} + \frac{-A-B \ln\left(\frac{e(bx+a)}{dx+c}\right)}{4g(gx+f)^4} - \frac{Bd^4 \ln(dx+c)}{4g(-cg+df)^4} \\ & - \frac{B(-ad+bc)(-adg-bcg+2bdf)(2abd^2fg-a^2d^2g^2-b^2(c^2g^2-2cdfg+2d^2f^2)) \ln(gx+f)}{4(-ag+bf)^4(-cg+df)^4} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32 Test file number 60

Test folder name:

```
test_cases/3_Logarithms/60_3.2.2-f+g_x^m-h+i_x^q-A+B_log-e-a+b_x-over-c+d_x^n^p
```

### 32.1 Problem number 31

$$\int \frac{(ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{ci + dix} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{g^3(bx+a)^3 \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{3di} - \frac{(-ad+bc) g^3(bx+a)^2 \left( 3A + B + 3B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{6d^2i} \\ & + \frac{(-ad+bc)^2 g^3(bx+a) \left( 6A + 5B + 6B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{6d^3i} \\ & + \frac{(-ad+bc)^3 g^3 \ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( 6A + 11B + 6B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{6d^4i} \\ & + \frac{B(-ad+bc)^3 g^3 \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^4i} \end{aligned}$$

command

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.2 Problem number 32

$$\int \frac{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{ci + dix} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{g^2(bx+a)^2 \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{2di} - \frac{(-ad+bc) g^2(bx+a) \left( 2A + B + 2B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{2d^2i} \\ & - \frac{(-ad+bc)^2 g^2 \ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( 2A + 3B + 2B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{2d^3i} \\ & - \frac{B(-ad+bc)^2 g^2 \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^3i} \end{aligned}$$

command

`integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.3 Problem number 33

$$\int \frac{(ag + bgx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{ci + dix} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{g(bx + a) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{di} + \frac{(-ad + bc) g \ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( A + B + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{d^2i} \\ & + \frac{B(-ad + bc) g \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^2i} \end{aligned}$$

command

`integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.4 Problem number 34

$$\int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{ci + dix} dx$$

Optimal antiderivative

$$-\frac{\ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{di} - \frac{B \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{di}$$

command

`integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.5 Problem number 37

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B(dx+c)^2\left(b - \frac{4d(bx+a)}{dx+c}\right)^2}{4(-ad+bc)^3 g^3 i (bx+a)^2} - \frac{B d^2 \ln\left(\frac{bx+a}{dx+c}\right)^2}{2(-ad+bc)^3 g^3 i} + \frac{2bd(dx+c)\left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^3 g^3 i (bx+a)} \\ & - \frac{b^2(dx+c)^2\left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^3 g^3 i (bx+a)^2} + \frac{d^2 \ln\left(\frac{bx+a}{dx+c}\right)\left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^3 g^3 i} \end{aligned}$$

command

`integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2i B e^3 \log\left(\frac{bx+ae}{dx+c}\right) + 2i A e^3 + i B e^3\right)(dx+c)^2 \left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)^2}{4(bxe+ae)^2 g^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.6 Problem number 38

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4(ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{3bB d^2(dx+c)}{(-ad+bc)^4 g^4 i (bx+a)} + \frac{3b^2 B d(dx+c)^2}{4(-ad+bc)^4 g^4 i (bx+a)^2} \\
 & -\frac{b^3 B(dx+c)^3}{9(-ad+bc)^4 g^4 i (bx+a)^3} + \frac{B d^3 \ln\left(\frac{bx+a}{dx+c}\right)^2}{2(-ad+bc)^4 g^4 i} \\
 & -\frac{3b d^2(dx+c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^4 g^4 i (bx+a)} + \frac{3b^2 d(dx+c)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^4 g^4 i (bx+a)^2} \\
 & -\frac{b^3(dx+c)^3 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{3(-ad+bc)^4 g^4 i (bx+a)^3} - \frac{d^3 \ln\left(\frac{bx+a}{dx+c}\right) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^4 g^4 i}
 \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-12i Bbe^4 \log\left(\frac{bx+ae}{dx+c}\right) + \frac{18i(bx+ae)Bde^3 \log\left(\frac{bx+ae}{dx+c}\right)}{dx+c} - 12i Abe^4 - 4i Bbe^4 + \frac{18i(bx+ae)Ade^3}{dx+c} + \frac{9i(bx+ae)Bde^3}{dx+c}\right)}{36 \left(\frac{(bx+ae)^3 b c g^4}{(dx+c)^3} - \frac{(bx+ae)^3 a d g^4}{(dx+c)^3}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.7 Problem number 39

$$\int \frac{(ag + bgx)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{3B(-ad+bc)^2 g^3 (bx+a)}{d^3 i^2 (dx+c)} - \frac{(6A+5B)(-ad+bc)^2 g^3 (bx+a)}{2d^3 i^2 (dx+c)} \\
 & -\frac{3B(-ad+bc)^2 g^3 (bx+a) \ln\left(\frac{e(bx+a)}{dx+c}\right)}{d^3 i^2 (dx+c)} + \frac{g^3 (bx+a)^3 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2d i^2 (dx+c)} \\
 & -\frac{(-ad+bc) g^3 (bx+a)^2 \left(3A + B + 3B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2d^2 i^2 (dx+c)} \\
 & -\frac{b(-ad+bc)^2 g^3 \ln\left(\frac{-ad+bc}{b(dx+c)}\right) \left(6A + 5B + 6B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2d^4 i^2} \\
 & -\frac{3bB(-ad+bc)^2 g^3 \operatorname{polylog}\left(2, \frac{d(bx+a)}{b(dx+c)}\right)}{d^4 i^2}
 \end{aligned}$$

command

```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.8 Problem number 40

$$\int \frac{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2B(-ad + bc) g^2 (bx + a)}{d^2 i^2 (dx + c)} + \frac{(2A + B) (-ad + bc) g^2 (bx + a)}{d^2 i^2 (dx + c)} \\ & + \frac{2B(-ad + bc) g^2 (bx + a) \ln \left( \frac{e(bx+a)}{dx+c} \right)}{d^2 i^2 (dx + c)} + \frac{g^2 (bx + a)^2 \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{d i^2 (dx + c)} \\ & + \frac{b(-ad + bc) g^2 \ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( 2A + B + 2B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{d^3 i^2} \\ & + \frac{2bB(-ad + bc) g^2 \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^3 i^2} \end{aligned}$$

command

```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.9 Problem number 41

$$\int \frac{(ag + bgx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{Ag(bx+a)}{d^2i^2(dx+c)} + \frac{Bg(bx+a)}{d^2i^2(dx+c)} - \frac{Bg(bx+a) \ln \left( \frac{e(bx+a)}{dx+c} \right)}{d^2i^2(dx+c)} \\ & - \frac{bg \ln \left( \frac{-ad+bc}{b(dx+c)} \right) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{d^2i^2} - \frac{bBg \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^2i^2} \end{aligned}$$

command

```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.10 Problem number 44

$$\int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(ag + bgx)^2 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B d^2 (bx+a)}{(-ad+bc)^3 g^2 i^2 (dx+c)} - \frac{b^2 B (dx+c)}{(-ad+bc)^3 g^2 i^2 (bx+a)} \\ & + \frac{b B d \ln \left( \frac{bx+a}{dx+c} \right)^2}{(-ad+bc)^3 g^2 i^2} + \frac{d^2 (bx+a) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad+bc)^3 g^2 i^2 (dx+c)} \\ & - \frac{b^2 (dx+c) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad+bc)^3 g^2 i^2 (bx+a)} - \frac{2bd \ln \left( \frac{bx+a}{dx+c} \right) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)}{(-ad+bc)^3 g^2 i^2} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{\left( Be^2 \log\left(\frac{bx+ae}{dx+c}\right) + Ae^2 + Be^2 \right) (dx+c) \left( \frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)} \right)^2}{(bx+ae)g^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.11 Problem number 45

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B d^3 (bx+a)}{(-ad+bc)^4 g^3 i^2 (dx+c)} + \frac{3b^2 B d (dx+c)}{(-ad+bc)^4 g^3 i^2 (bx+a)} \\ & - \frac{b^3 B (dx+c)^2}{4(-ad+bc)^4 g^3 i^2 (bx+a)^2} - \frac{3b B d^2 \ln\left(\frac{bx+a}{dx+c}\right)^2}{2(-ad+bc)^4 g^3 i^2} \\ & - \frac{d^3 (bx+a) \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)}{(-ad+bc)^4 g^3 i^2 (dx+c)} + \frac{3b^2 d (dx+c) \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)}{(-ad+bc)^4 g^3 i^2 (bx+a)} \\ & - \frac{b^3 (dx+c)^2 \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)}{2(-ad+bc)^4 g^3 i^2 (bx+a)^2} + \frac{3b d^2 \ln\left(\frac{bx+a}{dx+c}\right) \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)}{(-ad+bc)^4 g^3 i^2} \end{aligned}$$

command

`integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 2 B b e^3 \log\left(\frac{bx+ae}{dx+c}\right) - \frac{4(bx+ae)Bde^2 \log\left(\frac{bx+ae}{dx+c}\right)}{dx+c} + 2 A b e^3 + B b e^3 - \frac{4(bx+ae)Ade^2}{dx+c} - \frac{4(bx+ae)Bde^2}{dx+c} \right) \left( \frac{bc}{(bce-ade)(bc-ad)} \right)}{4 \left( \frac{(bx+ae)^2 bc g^3}{(dx+c)^2} - \frac{(bx+ae)^2 ad g^3}{(dx+c)^2} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.12 Problem number 46

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B d^4 (bx + a)}{(-ad + bc)^5 g^4 i^2 (dx + c)} - \frac{6b^2 B d^2 (dx + c)}{(-ad + bc)^5 g^4 i^2 (bx + a)} + \frac{b^3 B d (dx + c)^2}{(-ad + bc)^5 g^4 i^2 (bx + a)^2} \\ & - \frac{b^4 B (dx + c)^3}{9(-ad + bc)^5 g^4 i^2 (bx + a)^3} + \frac{2bB d^3 \ln\left(\frac{bx+a}{dx+c}\right)^2}{(-ad + bc)^5 g^4 i^2} + \frac{d^4 (bx + a) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^5 g^4 i^2 (dx + c)} \\ & - \frac{6b^2 d^2 (dx + c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^5 g^4 i^2 (bx + a)} + \frac{2b^3 d (dx + c)^2 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^5 g^4 i^2 (bx + a)^2} \\ & - \frac{b^4 (dx + c)^3 \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{3(-ad + bc)^5 g^4 i^2 (bx + a)^3} - \frac{4b d^3 \ln\left(\frac{bx+a}{dx+c}\right) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad + bc)^5 g^4 i^2} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(6 B b^2 e^4 \log\left(\frac{bx+ae}{dx+c}\right) - \frac{18 (bx+ae) B b d e^3 \log\left(\frac{bx+ae}{dx+c}\right)}{dx+c} + \frac{18 (bx+ae)^2 B d^2 e^2 \log\left(\frac{bx+ae}{dx+c}\right)}{(dx+c)^2} + 6 A b^2 e^4 + 2 B b^2 e^4 - \frac{18 (bx+ae) A}{dx+c}\right)}{18 \left(\frac{(bx+ae)^3 b^2 c^2 g^4}{(dx+c)^3} - \frac{2 (bx+ae)^3 a b c d g^4}{(dx+c)^3}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.13 Problem number 90

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3 (ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4bB^2d(dx+c)}{(-ad+bc)^3g^3i(bx+a)} - \frac{b^2B^2(dx+c)^2}{4(-ad+bc)^3g^3i(bx+a)^2} \\ & + \frac{4bBd(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^3g^3i(bx+a)} \\ & - \frac{b^2B(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^3g^3i(bx+a)^2} + \frac{2bd(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^3g^3i(bx+a)} \\ & - \frac{b^2(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{2(-ad+bc)^3g^3i(bx+a)^2} + \frac{d^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{3B(-ad+bc)^3g^3i} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2iB^2e^3\log\left(\frac{bx+ae}{dx+c}\right)^2 + 4iABe^3\log\left(\frac{bx+ae}{dx+c}\right) + 2iB^2e^3\log\left(\frac{bx+ae}{dx+c}\right) + 2iA^2e^3 + 2iABe^3 + iB^2e^3\right)(dx+c)^2}{4(bxe+ae)^2g^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.14 Problem number 91

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6bB^2d^2(dx+c)}{(-ad+bc)^4g^4i(bx+a)} + \frac{3b^2B^2d(dx+c)^2}{4(-ad+bc)^4g^4i(bx+a)^2} \\ & - \frac{2b^3B^2(dx+c)^3}{27(-ad+bc)^4g^4i(bx+a)^3} - \frac{6bBd^2(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^4g^4i(bx+a)} \\ & + \frac{3b^2Bd(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^4g^4i(bx+a)^2} - \frac{2b^3B(dx+c)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{9(-ad+bc)^4g^4i(bx+a)^3} \\ & - \frac{3bd^2(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^4g^4i(bx+a)} + \frac{3b^2d(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{2(-ad+bc)^4g^4i(bx+a)^2} \\ & - \frac{b^3(dx+c)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{3(-ad+bc)^4g^4i(bx+a)^3} - \frac{d^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{3B(-ad+bc)^4g^4i} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( -36i B^2 b e^4 \log\left(\frac{bxe+ae}{dx+c}\right)^2 + \frac{54i (bxe+ae) B^2 d e^3 \log\left(\frac{bxe+ae}{dx+c}\right)^2}{dx+c} - 72i AB b e^4 \log\left(\frac{bxe+ae}{dx+c}\right) - 24i B^2 b e^4 \log\left(\frac{bxe+ae}{dx+c}\right) + \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.15 Problem number 97

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2AB d^2 (bx+a)}{(-ad+bc)^3 g^2 i^2 (dx+c)} + \frac{2B^2 d^2 (bx+a)}{(-ad+bc)^3 g^2 i^2 (dx+c)} \\ & -\frac{2b^2 B^2 (dx+c)}{(-ad+bc)^3 g^2 i^2 (bx+a)} - \frac{2B^2 d^2 (bx+a) \ln\left(\frac{e(bx+a)}{dx+c}\right)}{(-ad+bc)^3 g^2 i^2 (dx+c)} \\ & -\frac{2b^2 B (dx+c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^3 g^2 i^2 (bx+a)} + \frac{d^2 (bx+a) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^3 g^2 i^2 (dx+c)} \\ & -\frac{b^2 (dx+c) \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^3 g^2 i^2 (bx+a)} - \frac{2bd \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{3B (-ad+bc)^3 g^2 i^2} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( B^2 e^2 \log\left(\frac{bxe+ae}{dx+c}\right)^2 + 2AB e^2 \log\left(\frac{bxe+ae}{dx+c}\right) + 2B^2 e^2 \log\left(\frac{bxe+ae}{dx+c}\right) + A^2 e^2 + 2AB e^2 + 2B^2 e^2 \right) (dx+c) \left( \frac{bc}{(bce-ade)} \right)}{(bxe+ae)g^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.16 Problem number 98

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ABd^3(bx+a)}{(-ad+bc)^4g^3i^2(dx+c)} - \frac{2B^2d^3(bx+a)}{(-ad+bc)^4g^3i^2(dx+c)} + \frac{6b^2B^2d(dx+c)}{(-ad+bc)^4g^3i^2(bx+a)} \\ & - \frac{b^3B^2(dx+c)^2}{4(-ad+bc)^4g^3i^2(bx+a)^2} + \frac{2B^2d^3(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(-ad+bc)^4g^3i^2(dx+c)} \\ & + \frac{6b^2Bd(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^4g^3i^2(bx+a)} - \frac{b^3B(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{2(-ad+bc)^4g^3i^2(bx+a)^2} \\ & - \frac{d^3(bx+a)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^4g^3i^2(dx+c)} + \frac{3b^2d(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^4g^3i^2(bx+a)} \\ & - \frac{b^3(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{2(-ad+bc)^4g^3i^2(bx+a)^2} + \frac{bd^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{B(-ad+bc)^4g^3i^2} \end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left(2B^2be^3\log\left(\frac{bx+ae}{dx+c}\right)^2 - \frac{4(bx+ae)B^2de^2\log\left(\frac{bx+ae}{dx+c}\right)}{dx+c} + 4ABbe^3\log\left(\frac{bx+ae}{dx+c}\right) + 2B^2be^3\log\left(\frac{bx+ae}{dx+c}\right) - \frac{8(bx+ae)AB}{dx+c}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.17 Problem number 99

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{2ABd^4(bx+a)}{(-ad+bc)^5g^4i^2(dx+c)} + \frac{2B^2d^4(bx+a)}{(-ad+bc)^5g^4i^2(dx+c)} \\
& -\frac{12b^2B^2d^2(dx+c)}{(-ad+bc)^5g^4i^2(bx+a)} + \frac{b^3B^2d(dx+c)^2}{(-ad+bc)^5g^4i^2(bx+a)^2} \\
& -\frac{2b^4B^2(dx+c)^3}{27(-ad+bc)^5g^4i^2(bx+a)^3} - \frac{2B^2d^4(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(-ad+bc)^5g^4i^2(dx+c)} \\
& -\frac{12b^2Bd^2(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^5g^4i^2(bx+a)} + \frac{2b^3Bd(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{(-ad+bc)^5g^4i^2(bx+a)^2} \\
& -\frac{2b^4B(dx+c)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)}{9(-ad+bc)^5g^4i^2(bx+a)^3} + \frac{d^4(bx+a)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^5g^4i^2(dx+c)} \\
& -\frac{6b^2d^2(dx+c)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^5g^4i^2(bx+a)} + \frac{2b^3d(dx+c)^2\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(-ad+bc)^5g^4i^2(bx+a)^2} \\
& -\frac{b^4(dx+c)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{3(-ad+bc)^5g^4i^2(bx+a)^3} - \frac{4bd^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^3}{3B(-ad+bc)^5g^4i^2}
\end{aligned}$$

command

```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(18B^2b^2e^4\log\left(\frac{bx+ae}{dx+c}\right)^2 - \frac{54(bxe+ae)B^2bde^3\log\left(\frac{bx+ae}{dx+c}\right)^2}{dx+c} + \frac{54(bxe+ae)^2B^2d^2e^2\log\left(\frac{bx+ae}{dx+c}\right)^2}{(dx+c)^2} + 36ABb^2e^4\log\left(\frac{bx+ae}{dx+c}\right) - \right)}{3B(-ad+bc)^5g^4i^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.18 Problem number 135

$$\int \frac{(ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{ci + dix} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{g^3(bx+a)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3di} \\
& - \frac{(-ad+bc) g^3(bx+a)^2 \left( 3A + Bn + 3B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{6d^2i} \\
& + \frac{(-ad+bc)^2 g^3(bx+a) \left( 6A + 5Bn + 6B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{6d^3i} \\
& + \frac{(-ad+bc)^3 g^3 \left( 6A + 11Bn + 6B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{-ad+bc}{b(dx+c)} \right)}{6d^4i} \\
& + \frac{B(-ad+bc)^3 g^3 n \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^4i}
\end{aligned}$$

command

```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**32.19 Problem number 136**

$$\int \frac{(ag+bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{ci+dir} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{g^2(bx+a)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{2di} \\
& - \frac{(-ad+bc) g^2(bx+a) \left( 2A + Bn + 2B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{2d^2i} \\
& - \frac{(-ad+bc)^2 g^2 \left( 2A + 3Bn + 2B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{-ad+bc}{b(dx+c)} \right)}{2d^3i} \\
& - \frac{B(-ad+bc)^2 g^2 n \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^3i}
\end{aligned}$$

command

`integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.20 Problem number 137

$$\int \frac{(ag + bgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{ci + dix} dx$$

Optimal antiderivative

$$\frac{g(bx + a) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{di} + \frac{(-ad + bc) g \left( A + Bn + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{-ad+bc}{b(dx+c)} \right)}{d^2i}$$

$$+ \frac{B(-ad + bc) gn \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^2i}$$

command

`integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.21 Problem number 138

$$\int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{ci + dix} dx$$

Optimal antiderivative

$$- \frac{\left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{-ad+bc}{b(dx+c)} \right)}{di} - \frac{Bn \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{di}$$

command



`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} \left( \frac{(i B b^3 c^3 n - 3i B a b^2 c^2 d n + 3i B a^2 b c d^2 n - i B a^3 d^3 n) \log\left(\frac{bx+a}{dx+c}\right)}{b^2 d - \frac{2(bx+a)bd^2}{dx+c} + \frac{(bx+a)^2 d^3}{(dx+c)^2}} + \frac{-i B b^4 c^3 n + 3i B a b^3 c^2 d n + \frac{(i b x + i a) B b^3 c^3}{dx+c}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.22 Problem number 141

$$\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3 (ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{Bn(dx+c)^2 \left(b - \frac{4d(bx+a)}{dx+c}\right)^2}{4(-ad+bc)^3 g^3 i (bx+a)^2} + \frac{2bd(dx+c) \left(A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(-ad+bc)^3 g^3 i (bx+a)} \\ & - \frac{b^2(dx+c)^2 \left(A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{2(-ad+bc)^3 g^3 i (bx+a)^2} \\ & + \frac{d^2 \left(A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) \ln\left(\frac{bx+a}{dx+c}\right)}{(-ad+bc)^3 g^3 i} - \frac{B d^2 n \ln\left(\frac{bx+a}{dx+c}\right)^2}{2(-ad+bc)^3 g^3 i} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( \frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2} \right)^2 \left( -\frac{2i(dx+c)^2 B n \log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2 g^3} + \frac{(-i B n - 2i A - 2i B)(dx+c)^2}{(bx+a)^2 g^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.23 Problem number 142

$$\int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{(ag + bgx)^4 (ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3bB d^2 n(dx+c)}{(-ad+bc)^4 g^4 i (bx+a)} + \frac{3b^2 B dn(dx+c)^2}{4(-ad+bc)^4 g^4 i (bx+a)^2} \\ & -\frac{b^3 B n(dx+c)^3}{9(-ad+bc)^4 g^4 i (bx+a)^3} - \frac{3b d^2(dx+c) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^4 g^4 i (bx+a)} \\ & + \frac{3b^2 d(dx+c)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{2(-ad+bc)^4 g^4 i (bx+a)^2} - \frac{b^3(dx+c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3(-ad+bc)^4 g^4 i (bx+a)^3} \\ & - \frac{d^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{bx+a}{dx+c} \right)}{(-ad+bc)^4 g^4 i} + \frac{B d^3 n \ln \left( \frac{bx+a}{dx+c} \right)^2}{2(-ad+bc)^4 g^4 i} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{36} \left( \frac{6 \left( -2i Bbn - \frac{3(-ibx-ia)Bdn}{dx+c} \right) \log \left( \frac{bx+a}{dx+c} \right)}{\frac{(bx+a)^3 bcg^4}{(dx+c)^3} - \frac{(bx+a)^3 adg^4}{(dx+c)^3}} + \frac{-4i Bbn - \frac{9(-ibx-ia)Bdn}{dx+c} - 12i Ab - 12i Bb - \frac{18(-ibx-ia)Ad}{dx+c}}{\frac{(bx+a)^3 bcg^4}{(dx+c)^3} - \frac{(bx+a)^3 adg^4}{(dx+c)^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.24 Problem number 143

$$\int \frac{(ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{3B(-ad+bc)^2 g^3 n (bx+a)}{d^3 i^2 (dx+c)} - \frac{(-ad+bc)^2 g^3 (5Bn+6A) (bx+a)}{2d^3 i^2 (dx+c)} \\
& - \frac{3B(-ad+bc)^2 g^3 (bx+a) \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{d^3 i^2 (dx+c)} + \frac{g^3 (bx+a)^3 \left(A+B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{2d i^2 (dx+c)} \\
& - \frac{(-ad+bc) g^3 (bx+a)^2 \left(3A+Bn+3B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{2d^2 i^2 (dx+c)} \\
& - \frac{b(-ad+bc)^2 g^3 \left(6A+5Bn+6B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) \ln\left(\frac{-ad+bc}{b(dx+c)}\right)}{2d^4 i^2} \\
& - \frac{3bB(-ad+bc)^2 g^3 n \operatorname{polylog}\left(2, \frac{d(bx+a)}{b(dx+c)}\right)}{d^4 i^2}
\end{aligned}$$

command

`integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.25 Problem number 144

$$\int \frac{(ag+bgx)^2 \left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2B(-ad+bc) g^2 n (bx+a)}{d^2 i^2 (dx+c)} + \frac{(-ad+bc) g^2 (Bn+2A) (bx+a)}{d^2 i^2 (dx+c)} \\
& + \frac{2B(-ad+bc) g^2 (bx+a) \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{d^2 i^2 (dx+c)} + \frac{g^2 (bx+a)^2 \left(A+B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{d i^2 (dx+c)} \\
& + \frac{b(-ad+bc) g^2 \left(2A+Bn+2B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right) \ln\left(\frac{-ad+bc}{b(dx+c)}\right)}{d^3 i^2} \\
& + \frac{2bB(-ad+bc) g^2 n \operatorname{polylog}\left(2, \frac{d(bx+a)}{b(dx+c)}\right)}{d^3 i^2}
\end{aligned}$$

command

`integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.26 Problem number 145

$$\int \frac{(ag + bgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{Ag(bx+a)}{d^2i^2(dx+c)} + \frac{Bgn(bx+a)}{d^2i^2(dx+c)} - \frac{Bg(bx+a) \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}{d^2i^2(dx+c)} \\ & - \frac{bg \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{-ad+bc}{b(dx+c)} \right)}{d^2i^2} - \frac{bBgn \operatorname{polylog} \left( 2, \frac{d(bx+a)}{b(dx+c)} \right)}{d^2i^2} \end{aligned}$$

command

`integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="giac")`  
Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \left( \frac{\left( Bb^4c^3gn - 3Bab^3c^2dgn - \frac{2(bx+a)Bb^3c^3dgn}{dx+c} + 3Ba^2b^2cd^2gn + \frac{6(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6(bx+a)Ba^2bcd^3g}{dx+c} \right)}{b^2d^2 - \frac{2(bx+a)bd^3}{dx+c} + \frac{(bx+a)^2d^4}{(dx+c)^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.27 Problem number 148

$$\int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{(ag + bgx)^2(ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B d^2 n(bx+a)}{(-ad+bc)^3 g^2 i^2 (dx+c)} - \frac{b^2 B n(dx+c)}{(-ad+bc)^3 g^2 i^2 (bx+a)} \\ & + \frac{d^2(bx+a) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^3 g^2 i^2 (dx+c)} - \frac{b^2(dx+c) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^3 g^2 i^2 (bx+a)} \\ & - \frac{2bd \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{bx+a}{dx+c} \right)}{(-ad+bc)^3 g^2 i^2} + \frac{bBdn \ln \left( \frac{bx+a}{dx+c} \right)^2}{(-ad+bc)^3 g^2 i^2} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(dx+c)Bn \log \left( \frac{bx+a}{dx+c} \right)}{(bx+a)g^2} + \frac{(Bn+A+B)(dx+c)}{(bx+a)g^2} \right) \left( \frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2} \right)^2$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.28 Problem number 149

$$\int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{(ag+bgx)^3 (ci+dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{B d^3 n(bx+a)}{(-ad+bc)^4 g^3 i^2 (dx+c)} + \frac{3b^2 B dn(dx+c)}{(-ad+bc)^4 g^3 i^2 (bx+a)} \\ & - \frac{b^3 B n(dx+c)^2}{4(-ad+bc)^4 g^3 i^2 (bx+a)^2} - \frac{d^3(bx+a) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^4 g^3 i^2 (dx+c)} \\ & + \frac{3b^2 d(dx+c) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^4 g^3 i^2 (bx+a)} - \frac{b^3(dx+c)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{2(-ad+bc)^4 g^3 i^2 (bx+a)^2} \\ & + \frac{3b d^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{bx+a}{dx+c} \right)}{(-ad+bc)^4 g^3 i^2} - \frac{3bB d^2 n \ln \left( \frac{bx+a}{dx+c} \right)^2}{2(-ad+bc)^4 g^3 i^2} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} \left( \frac{2 \left( Bbn - \frac{2(bx+a)Bdn}{dx+c} \right) \log \left( \frac{bx+a}{dx+c} \right)}{\frac{(bx+a)^2 bcg^3}{(dx+c)^2} - \frac{(bx+a)^2 adg^3}{(dx+c)^2}} + \frac{Bbn - \frac{4(bx+a)Bdn}{dx+c} + 2Ab + 2Bb - \frac{4(bx+a)Ad}{dx+c} - \frac{4(bx+a)Bd}{dx+c}}{\frac{(bx+a)^2 bcg^3}{(dx+c)^2} - \frac{(bx+a)^2 adg^3}{(dx+c)^2}} \right) \left( \frac{bc}{(bc-ad)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.29 Problem number 150

$$\int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{(ag + bgx)^4 (ci + dix)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{B d^4 n (bx+a)}{(-ad+bc)^5 g^4 i^2 (dx+c)} - \frac{6b^2 B d^2 n (dx+c)}{(-ad+bc)^5 g^4 i^2 (bx+a)} \\ & + \frac{b^3 B dn (dx+c)^2}{(-ad+bc)^5 g^4 i^2 (bx+a)^2} - \frac{b^4 B n (dx+c)^3}{9(-ad+bc)^5 g^4 i^2 (bx+a)^3} \\ & + \frac{d^4 (bx+a) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^5 g^4 i^2 (dx+c)} - \frac{6b^2 d^2 (dx+c) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^5 g^4 i^2 (bx+a)} \\ & + \frac{2b^3 d (dx+c)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{(-ad+bc)^5 g^4 i^2 (bx+a)^2} - \frac{b^4 (dx+c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{3(-ad+bc)^5 g^4 i^2 (bx+a)^3} \\ & - \frac{4b d^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) \ln \left( \frac{bx+a}{dx+c} \right)}{(-ad+bc)^5 g^4 i^2} + \frac{2b B d^3 n \ln \left( \frac{bx+a}{dx+c} \right)^2}{(-ad+bc)^5 g^4 i^2} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{18} \left( \frac{6 \left( Bb^2 n - \frac{3(bx+a)Bbdn}{dx+c} + \frac{3(bx+a)^2 Bd^2 n}{(dx+c)^2} \right) \log \left( \frac{bx+a}{dx+c} \right)}{\frac{(bx+a)^3 b^2 c^2 g^4}{(dx+c)^3} - \frac{2(bx+a)^3 abcdg^4}{(dx+c)^3} + \frac{(bx+a)^3 a^2 d^2 g^4}{(dx+c)^3}} + \frac{2Bb^2 n - \frac{9(bx+a)Bbdn}{dx+c} + \frac{18(bx+a)^2 Bd^2 n}{(dx+c)^2} + 6Ab^2 + 6Bb^2}{\frac{(bx+a)^3 b^2 c^2 g^4}{(dx+c)^3} - \frac{2(bx+a)^3 abcdg^4}{(dx+c)^3} + \frac{(bx+a)^3 a^2 d^2 g^4}{(dx+c)^3}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.30 Problem number 177

$$\int \frac{(ci + dix)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{(ag + bgx)^6} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2B^2 d^2 i^2 n^2 (dx+c)^3}{27(-ad+bc)^3 g^6 (bx+a)^3} + \frac{b B^2 d i^2 n^2 (dx+c)^4}{16(-ad+bc)^3 g^6 (bx+a)^4} - \frac{2b^2 B^2 i^2 n^2 (dx+c)^5}{125(-ad+bc)^3 g^6 (bx+a)^5} \\ & - \frac{2B d^2 i^2 n (dx+c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{9(-ad+bc)^3 g^6 (bx+a)^3} + \frac{b B d i^2 n (dx+c)^4 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{4(-ad+bc)^3 g^6 (bx+a)^4} \\ & - \frac{2b^2 B i^2 n (dx+c)^5 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)}{25(-ad+bc)^3 g^6 (bx+a)^5} - \frac{d^2 i^2 (dx+c)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^2}{3(-ad+bc)^3 g^6 (bx+a)^3} \\ & + \frac{b d i^2 (dx+c)^4 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^2}{2(-ad+bc)^3 g^6 (bx+a)^4} - \frac{b^2 i^2 (dx+c)^5 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^2}{5(-ad+bc)^3 g^6 (bx+a)^5} \end{aligned}$$

command

`integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{54000} \left( \frac{1800 \left( 6 B^2 b^2 n^2 - \frac{15 (bx+a) B^2 b d n^2}{dx+c} + \frac{10 (bx+a)^2 B^2 d^2 n^2}{(dx+c)^2} \right) \log \left( \frac{bx+a}{dx+c} \right)^2}{\frac{(bx+a)^5 b^2 c^2 g^6}{(dx+c)^5} - \frac{2 (bx+a)^5 a b c d g^6}{(dx+c)^5} + \frac{(bx+a)^5 a^2 d^2 g^6}{(dx+c)^5}} + \frac{60 \left( 72 B^2 b^2 n^2 - \frac{225 (bx+a) B^2 b d n^2}{dx+c} + \frac{200 (bx+a)^2 B^2 d^2 n^2}{(dx+c)^2} \right)}{\frac{(bx+a)^5 b^2 c^2 g^6}{(dx+c)^5} - \frac{2 (bx+a)^5 a b c d g^6}{(dx+c)^5} + \frac{(bx+a)^5 a^2 d^2 g^6}{(dx+c)^5}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 32.31 Problem number 192

$$\int \frac{\left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{(ag + bgx)^3 (ci + dix)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4bB^2dn^2(dx+c)}{(-ad+bc)^3g^3i(bx+a)} - \frac{b^2B^2n^2(dx+c)^2}{4(-ad+bc)^3g^3i(bx+a)^2} \\ & + \frac{4bBdn(dx+c)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(-ad+bc)^3g^3i(bx+a)} \\ & - \frac{b^2Bn(dx+c)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{2(-ad+bc)^3g^3i(bx+a)^2} + \frac{2bd(dx+c)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(-ad+bc)^3g^3i(bx+a)} \\ & - \frac{b^2(dx+c)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{2(-ad+bc)^3g^3i(bx+a)^2} + \frac{d^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}{3B(-ad+bc)^3g^3in} \end{aligned}$$

command

`integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( -\frac{2i(dx+c)^2B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3} + \frac{2(-iB^2n^2-2iABn-2iB^2n)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3} + \frac{(-iB^2n^2-2iABn-2iB^2n)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.32 Problem number 199

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ABd^2n(bx+a)}{(-ad+bc)^3g^2i^2(dx+c)} + \frac{2B^2d^2n^2(bx+a)}{(-ad+bc)^3g^2i^2(dx+c)} \\ & -\frac{2b^2B^2n^2(dx+c)}{(-ad+bc)^3g^2i^2(bx+a)} - \frac{2B^2d^2n(bx+a)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{(-ad+bc)^3g^2i^2(dx+c)} \\ & -\frac{2b^2Bn(dx+c)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(-ad+bc)^3g^2i^2(bx+a)} + \frac{d^2(bx+a)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(-ad+bc)^3g^2i^2(dx+c)} \\ & -\frac{b^2(dx+c)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(-ad+bc)^3g^2i^2(bx+a)} - \frac{2bd\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}{3B(-ad+bc)^3g^2i^2n} \end{aligned}$$



command

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(dx+c)B^2n^2 \log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2} + \frac{2(B^2n^2 + ABn + B^2n)(dx+c) \log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2} + \frac{(2B^2n^2 + 2ABn + 2B^2n + A^2 + 2A^2n)}{(bx+a)g^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 32.33 Problem number 224

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)} dx$$

Optimal antiderivative

$$\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^{1+p}}{(-ad+bc)n(1+p)}$$

command

```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(n \log\left(\frac{bx+a}{dx+c}\right) + 1\right)^{p+1}}{(bcn - adn)(p+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{(bx+a)(dx+c)} dx$$

### 32.34 Problem number 225

$$\int \frac{\log^p \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{ac + (bc + ad)x + bdx^2} dx$$

Optimal antiderivative

$$\frac{\ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^{1+p}}{(-ad + bc) n (1 + p)}$$

command

`integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( n \log \left( \frac{bx+a}{dx+c} \right) + 1 \right)^{p+1}}{(bcn - adn)(p + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^p}{bdx^2 + ac + (bc + ad)x} dx$$

### 32.35 Problem number 234

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^p}{(a + bx)(c + dx)} dx$$

Optimal antiderivative

$$\frac{(A + B \ln(e(bx + a)^n(dx + c)^{-n}))^{1+p}}{B(-ad + bc) n (1 + p)}$$

command

`integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bn \log(bx + a) - Bn \log(dx + c) + A + B)^{p+1}}{(Bbcn - Badn)(p + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^p}{(bx + a)(dx + c)} dx$$

### 32.36 Problem number 235

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^p}{(af + bfx)(cg + dgx)} dx$$

Optimal antiderivative

$$\frac{(A + B \ln(e(bx + a)^n(dx + c)^{-n}))^{1+p}}{B(-ad + bc)fgn(1 + p)}$$

command

`integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bn \log(bx + a) - Bn \log(dx + c) + A + B)^{p+1}}{(Bbcfgn - Badfgn)(p + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^p}{(bfx + af)(dgx + cg)} dx$$

### 32.37 Problem number 236

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^p}{acf + (bc + ad)fx + bdfx^2} dx$$

Optimal antiderivative

$$\frac{(A + B \ln(e(bx + a)^n(dx + c)^{-n}))^{1+p}}{B(-ad + bc)fn(1 + p)}$$

command

`integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(Bn \log(bx + a) - Bn \log(dx + c) + A + B)^{p+1}}{(Bbcfn - Badfn)(p + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^p}{bdfx^2 + acf + (bc + ad)fx} dx$$

### 33 Test file number 61

Test folder name:

test\_cases/3\_Logarithms/61\_3.2.3\_u\_log-e-f-a+b\_x-^p-c+d\_x-^q-^r-^s

#### 33.1 Problem number 28

$$\int (g + hx) \log(e(f(a + bx)^p(c + dx)^q)^r) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(-ah + bg) prx}{2b} - \frac{(-ch + dg) qrx}{2d} - \frac{pr(hx + g)^2}{4h} - \frac{qr(hx + g)^2}{4h} - \frac{(-ah + bg)^2 pr \ln(bx + a)}{2b^2h} \\ & - \frac{(-ch + dg)^2 qr \ln(dx + c)}{2d^2h} + \frac{(hx + g)^2 \ln(e(f(bx + a)^p(dx + c)^q)^r)}{2h} \end{aligned}$$

command

```
integrate((h*x+g)*log(e*(f*(b*x+a)^p*(d*x+c)^q)^r),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{1}{4}(hpr + hqr - 2hr \log(f) - 2h)x^2 \\ & + \frac{1}{2}(hprx^2 + 2gprx) \log(bx + a) + \frac{1}{2}(hqr x^2 + 2gqrx) \log(dx + c) \\ & - \frac{(2bdgpr - adhpr + 2bdgqr - bchqr - 2bdgr \log(f) - 2bdg)x}{2bd} \\ & + \frac{(2abgpr - a^2hpr) \log(-bx - a)}{2b^2} + \frac{(2cdgqr - c^2hqr) \log(dx + c)}{2d^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 33.2 Problem number 43

$$\int \frac{\left(a + b \log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1 - c^2x^2} dx$$

Optimal antiderivative

$$\frac{\left(a + b \ln\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^{1+n}}{bc(1+n)}$$

command

```
integrate((a+b*log((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(-\frac{1}{2} b \log (c x+1)+\frac{1}{2} b \log (-c x+1)+a\right)^{n+1}}{b c(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{\left(b \log \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)+a\right)^n}{c^2 x^2-1} d x$$

## 34 Test file number 62

Test folder name:

test\_cases/3\_Logarithms/62\_3.3\_u-a+b\_log-c-d+e\_x-~n-~p

### 34.1 Problem number 455

$$\int \frac{(g+hx)^2}{(a+b \log (c(d(e+fx)^p)^q))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-eh+fg)^2(fx+e) \operatorname{expIntegral}\left(\frac{a+b \ln (c(d(fx+e)^p)^q)}{bpq}\right) e^{-\frac{a}{bpq}}(c(d(fx+e)^p)^q)^{-\frac{1}{pq}}}{2b^3 f^3 p^3 q^3} \\ & + \frac{4h(-eh+fg)(fx+e)^2 \operatorname{expIntegral}\left(\frac{2a+2b \ln (c(d(fx+e)^p)^q)}{bpq}\right) e^{-\frac{2a}{bpq}}(c(d(fx+e)^p)^q)^{-\frac{2}{pq}}}{b^3 f^3 p^3 q^3} \\ & + \frac{9h^2(fx+e)^3 \operatorname{expIntegral}\left(\frac{3a+3b \ln (c(d(fx+e)^p)^q)}{bpq}\right) e^{-\frac{3a}{bpq}}(c(d(fx+e)^p)^q)^{-\frac{3}{pq}}}{2b^3 f^3 p^3 q^3} \\ & - \frac{(fx+e)(hx+g)^2}{2bfpq(a+b \ln (c(d(fx+e)^p)^q))^2} + \frac{(-eh+fg)(fx+e)(hx+g)}{b^2 f^2 p^2 q^2(a+b \ln (c(d(fx+e)^p)^q))} \\ & - \frac{3(fx+e)(hx+g)^2}{2b^2 f p^2 q^2(a+b \ln (c(d(fx+e)^p)^q))} \end{aligned}$$

command

```
integrate((h*x+g)^2/(a+b*log(c*(d*(f*x+e)^p)^q))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 35 Test file number 63

Test folder name:

test\_cases/3\_Logarithms/63\_3.4\_u-a+b\_log-c-d+e\_x^m-n-p

### 35.1 Problem number 634

$$\int \log \left( c \left( d + \frac{e}{(f + gx)^3} \right)^q \right) dx$$

Optimal antiderivative

$$\frac{(gx + f) \ln \left( c \left( d + \frac{e}{(gx+f)^3} \right)^q \right)}{g} + \frac{e^{\frac{1}{3}} q \ln \left( e^{\frac{1}{3}} + d^{\frac{1}{3}}(gx + f) \right)}{d^{\frac{1}{3}} g}$$

$$- \frac{e^{\frac{1}{3}} q \ln \left( e^{\frac{2}{3}} - d^{\frac{1}{3}} e^{\frac{1}{3}}(gx + f) + d^{\frac{2}{3}}(gx + f)^2 \right)}{2d^{\frac{1}{3}} g} - \frac{e^{\frac{1}{3}} q \arctan \left( \frac{\left( e^{\frac{1}{3}} - 2d^{\frac{1}{3}}(gx+f) \right) \sqrt{3}}{3e^{\frac{1}{3}}} \right) \sqrt{3}}{d^{\frac{1}{3}} g}$$

command

```
integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} dg^5 q \left( \frac{2 f e^{(-1)} \log(|dg^3 x^3 + 3 df g^2 x^2 + 3 df^2 gx + df^3 + e|)}{dg^6} - \frac{6 f e^{(-1)} \log(|gx + f|)}{dg^6} + \frac{\left( 2 \sqrt{3} (d^5 g^{21})^{\frac{1}{3}} \arctan \left( \frac{\left( e^{\frac{1}{3}} - 2d^{\frac{1}{3}}(gx+f) \right) \sqrt{3}}{3e^{\frac{1}{3}}} \right)}{d^{\frac{1}{3}} g} \right)}{d^{\frac{1}{3}} g} \right)$$

$$+ qx \log(dg^3 x^3 + 3 df g^2 x^2 + 3 df^2 gx + df^3 + e) - qx \log(g^3 x^3 + 3 f g^2 x^2 + 3 f^2 gx + f^3) + x \log(c)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 36 Test file number 64

Test folder name:

test\_cases/3\_Logarithms/64\_3.5\_Logarithm\_functions

### 36.1 Problem number 271

$$\int (\sin(x \log(x)) + \log(x) \sin(x \log(x))) dx$$

Optimal antiderivative

$$-\cos(x \ln(x))$$

command

```
integrate(sin(x*log(x))+log(x)*sin(x*log(x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\cos(x \log(x))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

### 37 Test file number 65

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/65\_4.1.0-a\_sin-<sup>m</sup>-b\_trg-<sup>n</sup>

#### 37.1 Problem number 340

$$\int \cos^5(a + bx)(c \sin(a + bx))^m dx$$

Optimal antiderivative

$$\frac{(c \sin(bx + a))^{1+m}}{bc(1+m)} - \frac{2(c \sin(bx + a))^{3+m}}{bc^3(3+m)} + \frac{(c \sin(bx + a))^{5+m}}{bc^5(5+m)}$$

command

```
integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(c \sin(bx + a))^m c^5 m^2 \sin(bx + a)^5 + 4(c \sin(bx + a))^m c^5 m \sin(bx + a)^5 - 2(c \sin(bx + a))^m c^5 m^2 \sin(bx + a)^3 + \dots}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 37.2 Problem number 355

$$\int (d \cos(a + bx))^n \sin^5(a + bx) dx$$

Optimal antiderivative

$$-\frac{(d \cos(bx + a))^{1+n}}{bd(1+n)} + \frac{2(d \cos(bx + a))^{3+n}}{bd^3(3+n)} - \frac{(d \cos(bx + a))^{5+n}}{bd^5(5+n)}$$

command

```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(d \cos(bx + a))^n d^5 n^2 \cos(bx + a)^5 + 4(d \cos(bx + a))^n d^5 n \cos(bx + a)^5 - 2(d \cos(bx + a))^n d^5 n^2 \cos(bx + a)^3}{195 b^6 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 37.3 Problem number 371

$$\int \sqrt{b \sec(e + fx)} \sin^7(e + fx) dx$$

Optimal antiderivative

$$\frac{2b^7}{13f(b \sec(fx + e))^{\frac{13}{2}}} - \frac{2b^5}{3f(b \sec(fx + e))^{\frac{9}{2}}} + \frac{6b^3}{5f(b \sec(fx + e))^{\frac{5}{2}}} - \frac{2b}{f\sqrt{b \sec(fx + e)}}$$

command

```
integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 15 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^6 - 65 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^4 + 117 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^2 \right)}{195 b^6 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 37.4 Problem number 372

$$\int \sqrt{b \sec(e + fx)} \sin^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{2b^5}{9f(b \sec(fx + e))^{\frac{9}{2}}} + \frac{4b^3}{5f(b \sec(fx + e))^{\frac{5}{2}}} - \frac{2b}{f\sqrt{b \sec(fx + e)}}$$

command

```
integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 5 \sqrt{b \cos(fx + e)} b^4 \cos(fx + e)^4 - 18 \sqrt{b \cos(fx + e)} b^4 \cos(fx + e)^2 + 45 \sqrt{b \cos(fx + e)} b^4 \right) \operatorname{sgn}(\cos(fx + e))}{45 b^4 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.5 Problem number 373

$$\int \sqrt{b \sec(e + fx)} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{2b^3}{5f(b \sec(fx + e))^{\frac{5}{2}}} - \frac{2b}{f\sqrt{b \sec(fx + e)}}$$

command

```
integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \sqrt{b \cos(fx + e)} b^2 \cos(fx + e)^2 - 5 \sqrt{b \cos(fx + e)} b^2 \right) \operatorname{sgn}(\cos(fx + e))}{5 b^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.6 Problem number 374

$$\int \sqrt{b \sec(e + fx)} \sin(e + fx) dx$$

Optimal antiderivative

$$-\frac{2b}{f \sqrt{b \sec(fx + e)}}$$

command

```
integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \sqrt{b \cos(fx + e)} \operatorname{sgn}(\cos(fx + e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.7 Problem number 375

$$\int \csc(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{f}$$

command

```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \left( \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b} - \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} \right) \operatorname{sgn}(\cos(fx + e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.8 Problem number 376

$$\int \csc^3(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$-\frac{(\cot^2(fx + e)) (b \sec(fx + e))^{\frac{3}{2}}}{2bf} + \frac{3 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{4f} - \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{4f}$$

command

```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b^4 \left( \frac{2 \sqrt{b \cos(fx + e)}}{(b^2 \cos(fx + e)^2 - b^2) b^2} + \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^3} - \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}} \right) \operatorname{sgn}(\cos(fx + e))$$


---

$4f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.9 Problem number 377

$$\int \csc^5(e + fx) \sqrt{b \sec(e + fx)} dx$$

Optimal antiderivative

$$-\frac{7(\cot^2(fx + e)) (b \sec(fx + e))^{\frac{3}{2}}}{16bf} - \frac{(\cot^4(fx + e)) (b \sec(fx + e))^{\frac{7}{2}}}{4b^3f} + \frac{21 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{32f} - \frac{21 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right) \sqrt{b}}{32f}$$

command

`integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^6 \left( \frac{21 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^5} - \frac{21 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}} + \frac{2 \left(7 \sqrt{b \cos(fx+e)} b^2 \cos(fx+e)^2 - 11 \sqrt{b \cos(fx+e)}\right)}{(b^2 \cos(fx+e)^2 - b^2)^2 b^4} \right) \frac{1}{32 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.10 Problem number 385

$$\int (b \sec(e + fx))^{3/2} \sin^7(e + fx) dx$$

Optimal antiderivative

$$\frac{2b^7}{11f (b \sec(fx+e))^{\frac{11}{2}}} - \frac{6b^5}{7f (b \sec(fx+e))^{\frac{7}{2}}} + \frac{2b^3}{f (b \sec(fx+e))^{\frac{3}{2}}} + \frac{2b \sqrt{b \sec(fx+e)}}{f}$$

command

`integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 7 \sqrt{b \cos(fx+e)} b^5 \cos(fx+e)^5 - 33 \sqrt{b \cos(fx+e)} b^5 \cos(fx+e)^3 + 77 \sqrt{b \cos(fx+e)} b^5 \cos(fx+e) \right)}{77 b^4 f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx+e))^{\frac{3}{2}} \sin(fx+e)^7 dx$$

### 37.11 Problem number 386

$$\int (b \sec(e + fx))^{3/2} \sin^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{2b^5}{7f(b \sec(fx + e))^{7/2}} + \frac{4b^3}{3f(b \sec(fx + e))^{3/2}} + \frac{2b\sqrt{b \sec(fx + e)}}{f}$$

command

`integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 3 \sqrt{b \cos(fx + e)} b^3 \cos(fx + e)^3 - 14 \sqrt{b \cos(fx + e)} b^3 \cos(fx + e) - \frac{21b^4}{\sqrt{b \cos(fx + e)}} \right) \operatorname{sgn}(\cos(fx + e))}{21b^2f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx + e))^{3/2} \sin(fx + e)^5 dx$$

### 37.12 Problem number 387

$$\int (b \sec(e + fx))^{3/2} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{2b^3}{3f(b \sec(fx + e))^{3/2}} + \frac{2b\sqrt{b \sec(fx + e)}}{f}$$

command

`integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \sqrt{b \cos(fx + e)} b \cos(fx + e) + \frac{3b^2}{\sqrt{b \cos(fx + e)}} \right) \operatorname{sgn}(\cos(fx + e))}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx + e))^{3/2} \sin(fx + e)^3 dx$$

### 37.13 Problem number 388

$$\int (b \sec(e + fx))^{3/2} \sin(e + fx) dx$$

Optimal antiderivative

$$\frac{2b\sqrt{b \sec(fx + e)}}{f}$$

command

```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2b^2 \operatorname{sgn}(\cos(fx + e))}{\sqrt{b \cos(fx + e)} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.14 Problem number 389

$$\int \csc(e + fx)(b \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{f} - \frac{b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b \sec(fx + e)}}{f}$$

command

```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^4 \left( \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2} + \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} + \frac{2}{\sqrt{b \cos(fx + e)} b^2} \right) \operatorname{sgn}(\cos(fx + e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.15 Problem number 390

$$\int \csc^3(e + fx)(b \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4f} \\ & - \frac{(\cot^2(fx + e))(b \sec(fx + e))^{\frac{5}{2}}}{2bf} + \frac{5b\sqrt{b \sec(fx + e)}}{2f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b^6 \left( \frac{5 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^4} + \frac{5 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}} + \frac{2(5b^2 \cos(fx + e)^2 - 4b^2)}{\left(\sqrt{b \cos(fx + e)} b^2 \cos(fx + e)^2 - \sqrt{b \cos(fx + e)} b^2\right)} \right) \frac{1}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.16 Problem number 397

$$\int (b \sec(e + fx))^{5/2} \sin^7(e + fx) dx$$

Optimal antiderivative

$$\frac{2b^7}{9f(b \sec(fx + e))^{\frac{9}{2}}} - \frac{6b^5}{5f(b \sec(fx + e))^{\frac{5}{2}}} + \frac{2b(b \sec(fx + e))^{\frac{3}{2}}}{3f} + \frac{6b^3}{f\sqrt{b \sec(fx + e)}}$$

command

```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( 5 \sqrt{b \cos(fx + e)} b^4 \cos(fx + e)^4 - 27 \sqrt{b \cos(fx + e)} b^4 \cos(fx + e)^2 + 135 \sqrt{b \cos(fx + e)} b^4 + \frac{1}{\sqrt{b \cos(fx + e)}} \right) \frac{1}{45b^2f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx + e))^{\frac{5}{2}} \sin(fx + e)^7 dx$$

### 37.17 Problem number 398

$$\int (b \sec(e + fx))^{5/2} \sin^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{2b^5}{5f (b \sec(fx + e))^{5/2}} + \frac{2b(b \sec(fx + e))^{3/2}}{3f} + \frac{4b^3}{f \sqrt{b \sec(fx + e)}}$$

command

`integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 3 \sqrt{b \cos(fx + e)} b^2 \cos(fx + e)^2 - 30 \sqrt{b \cos(fx + e)} b^2 - \frac{5b^3}{\sqrt{b \cos(fx + e)} \cos(fx + e)} \right) \operatorname{sgn}(\cos(fx + e))}{15f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx + e))^{5/2} \sin(fx + e)^5 dx$$

### 37.18 Problem number 399

$$\int (b \sec(e + fx))^{5/2} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{2b(b \sec(fx + e))^{3/2}}{3f} + \frac{2b^3}{f \sqrt{b \sec(fx + e)}}$$

command

`integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 3 \sqrt{b \cos(fx + e)} b + \frac{b^2}{\sqrt{b \cos(fx + e)} \cos(fx + e)} \right) b \operatorname{sgn}(\cos(fx + e))}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(fx + e))^{5/2} \sin(fx + e)^3 dx$$



## 37.19 Problem number 400

$$\int (b \sec(e + fx))^{5/2} \sin(e + fx) dx$$

Optimal antiderivative

$$\frac{2b(b \sec(fx + e))^{3/2}}{3f}$$

command

```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2b^3 \operatorname{sgn}(\cos(fx + e))}{3 \sqrt{b \cos(fx + e)} f \cos(fx + e)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.20 Problem number 401

$$\int \csc(e + fx)(b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{b^{5/2} \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{f} + \frac{2b(b \sec(fx + e))^{3/2}}{3f}$$

command

```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^6 \left( \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^3} - \frac{3 \operatorname{arctan}\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{7/2}} + \frac{2}{\sqrt{b \cos(fx + e)} b^3 \cos(fx + e)} \right) \operatorname{sgn}(\cos(fx + e))}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.21 Problem number 402

$$\int \csc^3(e + fx)(b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{7b^{5/2} \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b \sec(fx + e))^{3/2}}{6f} - \frac{(\cot^2(fx + e))(b \sec(fx + e))^{7/2}}{2bf}$$

command

```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^8 \left( \frac{6 \sqrt{b \cos(fx + e)}}{(b^2 \cos(fx + e)^2 - b^2) b^4} + \frac{21 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^5} - \frac{21 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{11/2}} + \frac{8}{\sqrt{b \cos(fx + e)} b^5 \cos(fx + e)} \right)}{12f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.22 Problem number 403

$$\int \csc^5(e + fx)(b \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{77b^{5/2} \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b \sec(fx + e))^{3/2}}{48f} - \frac{11(\cot^2(fx + e))(b \sec(fx + e))^{7/2}}{16bf} - \frac{(\cot^4(fx + e))(b \sec(fx + e))^{11/2}}{4b^3f}$$

command

```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b^{10} \left( \frac{231 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^7} - \frac{231 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{15}{2}}} + \frac{6 \left(15 \sqrt{b \cos(fx+e)} b^2 \cos(fx+e)^2 - 19 \sqrt{b \cos(fx+e)}\right)}{\left(b^2 \cos(fx+e)^2 - b^2\right)^2 b^6} \right) \frac{1}{96 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.23 Problem number 410

$$\int \frac{\sin^7(e+fx)}{\sqrt{b \sec(e+fx)}} dx$$

Optimal antiderivative

$$\frac{2b^7}{15f (b \sec(fx+e))^{\frac{15}{2}}} - \frac{6b^5}{11f (b \sec(fx+e))^{\frac{11}{2}}} + \frac{6b^3}{7f (b \sec(fx+e))^{\frac{7}{2}}} - \frac{2b}{3f (b \sec(fx+e))^{\frac{3}{2}}}$$

command

`integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 77 \sqrt{b \cos(fx+e)} b^7 \cos(fx+e)^7 - 315 \sqrt{b \cos(fx+e)} b^7 \cos(fx+e)^5 + 495 \sqrt{b \cos(fx+e)} b^7 \cos(fx+e)^3 - 19 \sqrt{b \cos(fx+e)} b^7 \cos(fx+e) \right)}{1155 b^8 f \operatorname{sgn}(\cos(fx+e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.24 Problem number 411

$$\int \frac{\sin^5(e+fx)}{\sqrt{b \sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{2b^5}{11f (b \sec(fx+e))^{\frac{11}{2}}} + \frac{4b^3}{7f (b \sec(fx+e))^{\frac{7}{2}}} - \frac{2b}{3f (b \sec(fx+e))^{\frac{3}{2}}}$$

command

```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 21 \sqrt{b \cos (fx + e)} b^5 \cos (fx + e)^5 - 66 \sqrt{b \cos (fx + e)} b^5 \cos (fx + e)^3 + 77 \sqrt{b \cos (fx + e)} b^5 \cos (fx + e) \right)}{231 b^6 f \operatorname{sgn}(\cos (fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.25 Problem number 413

$$\int \frac{\sin(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2b}{3f(b \sec (fx + e))^{\frac{3}{2}}}$$

command

```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \sqrt{b \cos (fx + e)} \cos (fx + e)}{3 b f \operatorname{sgn}(\cos (fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.26 Problem number 414

$$\int \frac{\csc(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\arctan \left( \frac{\sqrt{b \sec (fx + e)}}{\sqrt{b}} \right)}{f \sqrt{b}} - \frac{\operatorname{arctanh} \left( \frac{\sqrt{b \sec (fx + e)}}{\sqrt{b}} \right)}{f \sqrt{b}}$$

command

```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{f \operatorname{sgn}(\cos(fx+e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.27 Problem number 415

$$\int \frac{\csc^3(e+fx)}{\sqrt{b \sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{b \sec(fx+e)}}{\sqrt{b}}\right)}{4f\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx+e)}}{\sqrt{b}}\right)}{4f\sqrt{b}} - \frac{(\cot^2(fx+e))\sqrt{b \sec(fx+e)}}{2bf}$$

command

```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \left( \frac{2 \sqrt{b \cos(fx+e)} \cos(fx+e)}{(b^2 \cos(fx+e)^2 - b^2) b} + \frac{\arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2} + \frac{\arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} \right)}{4 f \operatorname{sgn}(\cos(fx+e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.28 Problem number 416

$$\int \frac{\csc^5(e + fx)}{\sqrt{b \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(\cot^4(fx + e)) (b \sec(fx + e))^{\frac{5}{2}}}{4b^3 f} - \frac{5 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32f\sqrt{b}} \\ & - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32f\sqrt{b}} - \frac{5(\cot^2(fx + e)) \sqrt{b \sec(fx + e)}}{16bf} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b^4 \left( \frac{5 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^4} + \frac{5 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}} + \frac{2 \left(5 \sqrt{b \cos(fx + e)} b^3 \cos(fx + e)^3 - 9 \sqrt{b \cos(fx + e)}\right)}{(b^2 \cos(fx + e)^2 - b^2)^2 b^4} \right) \frac{1}{32 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.29 Problem number 424

$$\int \frac{\sin^7(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2b^7}{17f (b \sec(fx + e))^{\frac{17}{2}}} - \frac{6b^5}{13f (b \sec(fx + e))^{\frac{13}{2}}} + \frac{2b^3}{3f (b \sec(fx + e))^{\frac{9}{2}}} - \frac{2b}{5f (b \sec(fx + e))^{\frac{5}{2}}}$$

command

```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 195 \sqrt{b \cos(fx + e)} b^8 \cos(fx + e)^8 - 765 \sqrt{b \cos(fx + e)} b^8 \cos(fx + e)^6 + 1105 \sqrt{b \cos(fx + e)} b^8 \cos(fx + e)^4 - 420 \sqrt{b \cos(fx + e)} b^8 \cos(fx + e)^2 + 105 \sqrt{b \cos(fx + e)} b^8 \right)}{3315 b^{10} f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.30 Problem number 425

$$\int \frac{\sin^5(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2b^5}{13f(b \sec(fx + e))^{13/2}} + \frac{4b^3}{9f(b \sec(fx + e))^{9/2}} - \frac{2b}{5f(b \sec(fx + e))^{5/2}}$$

command

```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 45 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^6 - 130 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^4 + 117 \sqrt{b \cos(fx + e)} b^6 \cos(fx + e)^2 - 585 b^8 f \operatorname{sgn}(\cos(fx + e)) \right)}{585 b^8 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.31 Problem number 428

$$\int \frac{\csc(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{b^{3/2} f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{b^{3/2} f} + \frac{2}{bf \sqrt{b \sec(fx + e)}}$$

command

```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right) - \sqrt{b} \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right) + 2 \sqrt{b \cos(fx + e)}}{b^2 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.32 Problem number 429

$$\int \frac{\csc^3(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4b^{\frac{3}{2}}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4b^{\frac{3}{2}}f} - \frac{(\cot^2(fx + e))(b \sec(fx + e))^{\frac{3}{2}}}{2b^3f}$$

command

```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2\sqrt{b \cos(fx + e)}}{b^2 \cos(fx + e)^2 - b^2} - \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}}{4f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.33 Problem number 430

$$\int \frac{\csc^5(e + fx)}{(b \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{3 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32b^{\frac{3}{2}}f} + \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32b^{\frac{3}{2}}f} - \frac{3(\cot^2(fx + e))(b \sec(fx + e))^{\frac{3}{2}}}{16b^3f} - \frac{(\cot^4(fx + e))(b \sec(fx + e))^{\frac{3}{2}}}{4b^3f}$$

command

```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$b^2 \left( \frac{3 \arctan \left( \frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}} \right)}{\sqrt{-b} b^3} - \frac{3 \arctan \left( \frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}} \right)}{b^{\frac{7}{2}}} + \frac{2 \left( \sqrt{b \cos(fx + e)} b^2 \cos(fx + e)^2 + 3 \sqrt{b \cos(fx + e)} \right)}{(b^2 \cos(fx + e)^2 - b^2)^2 b^2} \right)$$


---


$$32 f \operatorname{sgn}(\cos(fx + e))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.34 Problem number 437

$$\int \frac{\sin^7(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2b^7}{19f (b \sec(fx + e))^{\frac{19}{2}}} - \frac{2b^5}{5f (b \sec(fx + e))^{\frac{15}{2}}} + \frac{6b^3}{11f (b \sec(fx + e))^{\frac{11}{2}}} - \frac{2b}{7f (b \sec(fx + e))^{\frac{7}{2}}}$$

command

`integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 385 \sqrt{b \cos(fx + e)} b^9 \cos(fx + e)^9 - 1463 \sqrt{b \cos(fx + e)} b^9 \cos(fx + e)^7 + 1995 \sqrt{b \cos(fx + e)} b^9 \cos(fx + e)^5 - 1001 \sqrt{b \cos(fx + e)} b^9 \cos(fx + e)^3 + 220 \sqrt{b \cos(fx + e)} b^9 \cos(fx + e) \right)}{7315 b^{12} f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.35 Problem number 438

$$\int \frac{\sin^5(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2b^5}{15f (b \sec(fx + e))^{\frac{15}{2}}} + \frac{4b^3}{11f (b \sec(fx + e))^{\frac{11}{2}}} - \frac{2b}{7f (b \sec(fx + e))^{\frac{7}{2}}}$$

command

```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 77 \sqrt{b \cos (f x + e)} b^7 \cos (f x + e)^7 - 210 \sqrt{b \cos (f x + e)} b^7 \cos (f x + e)^5 + 165 \sqrt{b \cos (f x + e)} b^7 \cos (f x + e)^3 - 105 \sqrt{b \cos (f x + e)} b^7 \cos (f x + e) \right)}{1155 b^{10} f \operatorname{sgn}(\cos (f x + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 37.36 Problem number 441

$$\int \frac{\csc(e + f x)}{(b \sec(e + f x))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{b \sec(f x + e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}} f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b \sec(f x + e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}} f} + \frac{2}{3 b f (b \sec(f x + e))^{\frac{3}{2}}}$$

command

```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 b \arctan\left(\frac{\sqrt{b \cos (f x + e)}}{\sqrt{-b}}\right) + 3 \sqrt{b} \arctan\left(\frac{\sqrt{b \cos (f x + e)}}{\sqrt{b}}\right) + 2 \sqrt{b \cos (f x + e)} \cos (f x + e)}{3 b^3 f \operatorname{sgn}(\cos (f x + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.37 Problem number 442

$$\int \frac{\csc^3(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4b^{5/2}f} + \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{4b^{5/2}f} - \frac{(\cot^2(fx + e)) \sqrt{b \sec(fx + e)}}{2b^3f}$$

command

```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2 \sqrt{b \cos(fx + e)} b \cos(fx + e)}{b^2 \cos(fx + e)^2 - b^2} - \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx + e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{4b^2 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 37.38 Problem number 443

$$\int \frac{\csc^5(e + fx)}{(b \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3 \arctan\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32b^{5/2}f} + \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{b \sec(fx + e)}}{\sqrt{b}}\right)}{32b^{5/2}f} - \frac{(\cot^2(fx + e)) \sqrt{b \sec(fx + e)}}{16b^3f} - \frac{(\cot^4(fx + e)) \sqrt{b \sec(fx + e)}}{4b^3f}$$

command

```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b} b^2} + \frac{3 \arctan\left(\frac{\sqrt{b \cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} + \frac{2 \left(3 \sqrt{b \cos(fx+e)} b^3 \cos(fx+e)^3 + \sqrt{b \cos(fx+e)} b^3\right)}{(b^2 \cos(fx+e)^2 - b^2)^2 b^2}$$


---


$$32 f \operatorname{sgn}(\cos(fx+e))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 38 Test file number 66

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/66\_4.1.10-c+d\_x-~m-a+b\_sin-~n

### 38.1 Problem number 22

$$\int \frac{\sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b^2 \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} \\ & + \frac{9b^2 \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^3} - \frac{3b^2 \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^3} \\ & - \frac{3b \cos(bx+a) (\sin^2(bx+a))}{2d^2(dx+c)} - \frac{\sin^3(bx+a)}{2d(dx+c)^2} \end{aligned}$$

command

`integrate(sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.2 Problem number 106

$$\int \frac{(a + a \sin(e + fx))^2}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 f^2 \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \cos\left(-2e + \frac{2cf}{d}\right)}{d^3} \\ & - \frac{a^2 f^2 \cos\left(-e + \frac{cf}{d}\right) \operatorname{sinIntegral}\left(\frac{cf}{d} + fx\right)}{d^3} \\ & + \frac{a^2 f^2 \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{d^3} \\ & + \frac{a^2 f^2 \operatorname{cosineIntegral}\left(\frac{cf}{d} + fx\right) \sin\left(-e + \frac{cf}{d}\right)}{d^3} \\ & - \frac{4a^2 f \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \left(\sin^3\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)}{d^2 (dx + c)} - \frac{2a^2 \left(\sin^4\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)}{d (dx + c)^2} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.3 Problem number 128

$$\int x^3 (a + a \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{1280a\sqrt{a+a\sin(fx+e)}}{9f^4} + \frac{16ax^2\sqrt{a+a\sin(fx+e)}}{f^2} \\
& + \frac{640ax\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{9f^3} \\
& - \frac{8ax^3\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\
& + \frac{32ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{9f^3} \\
& - \frac{4ax^3\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\
& - \frac{64a\left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)\sqrt{a+a\sin(fx+e)}}{27f^4} \\
& + \frac{8ax^2\left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)\sqrt{a+a\sin(fx+e)}}{3f^2}
\end{aligned}$$

command

```
integrate(x^3*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**38.4 Problem number 129**

$$\int x^2(a+a\sin(e+fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{32ax\sqrt{a+a\sin(fx+e)}}{3f^2} + \frac{224a\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{9f^3} \\ & - \frac{8ax^2\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\ & - \frac{32a\left(\cos^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{27f^3} \\ & - \frac{4ax^2\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\ & + \frac{16ax\left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)\sqrt{a+a\sin(fx+e)}}{9f^2} \end{aligned}$$

command

```
integrate(x^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 38.5 Problem number 130

$$\int x(a+a\sin(e+fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{16a\sqrt{a+a\sin(fx+e)}}{3f^2} - \frac{8ax\cot\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\ & - \frac{4ax\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\sqrt{a+a\sin(fx+e)}}{3f} \\ & + \frac{8a\left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right)\sqrt{a+a\sin(fx+e)}}{9f^2} \end{aligned}$$

command

```
integrate(x*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}}{f} \left( \frac{54 a \operatorname{esgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{f} + \frac{6 a \operatorname{esgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(-\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e)}{f} - \frac{108 a \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 38.6 Problem number 131

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \operatorname{cosineIntegral}\left(\frac{3fx}{2}\right) \cos\left(\frac{3e}{2} + \frac{\pi}{4}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{2} \\ & + \frac{3a \cos\left(\frac{e}{2} + \frac{\pi}{4}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{2} \\ & + \frac{a \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{3fx}{2}\right) \sin\left(\frac{3e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{2} \\ & + \frac{3a \operatorname{cosineIntegral}\left(\frac{fx}{2}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{2} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}}{f} \left( af \cos\left(\frac{3}{4} \pi - \frac{3}{2} e\right) \operatorname{Ci}\left(\frac{3}{2} fx\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) + 3af \cos\left(\frac{1}{4} \pi - \frac{1}{2} e\right) \operatorname{Ci}\left(\frac{1}{2} fx\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 38.7 Problem number 132

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3af \cos\left(\frac{3e}{2} + \frac{\pi}{4}\right) \csc\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{3fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{4} \\ & + \frac{3af \operatorname{cosineIntegral}\left(\frac{fx}{2}\right) \csc\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \cos\left(\frac{e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{4} \\ & - \frac{3af \csc\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{4} \\ & + \frac{3af \operatorname{cosineIntegral}\left(\frac{3fx}{2}\right) \csc\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sin\left(\frac{3e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{4} \\ & - \frac{2a \left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right) \sqrt{a + a \sin(fx + e)}}{x} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 38.8 Problem number 133

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9a f^2 \operatorname{cosineIntegral}\left(\frac{3fx}{2}\right) \cos\left(\frac{3e}{2} + \frac{\pi}{4}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{16} \\ & - \frac{3a f^2 \cos\left(\frac{e}{2} + \frac{\pi}{4}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{16} \\ & - \frac{9a f^2 \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \operatorname{sinIntegral}\left(\frac{3fx}{2}\right) \sin\left(\frac{3e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{16} \\ & - \frac{3a f^2 \operatorname{cosineIntegral}\left(\frac{fx}{2}\right) \operatorname{csc}\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{\pi}{4}\right) \sqrt{a + a \sin(fx + e)}}{16} \\ & - \frac{3af \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sin\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right) \sqrt{a + a \sin(fx + e)}}{2x} \\ & - \frac{a \left(\sin^2\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)\right) \sqrt{a + a \sin(fx + e)}}{x^2} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 38.9 Problem number 162

$$\int \frac{(a + b \sin(e + fx))^2}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^2}{2d(dx+c)^2} + \frac{b^2 f^2 \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \cos\left(-2e + \frac{2cf}{d}\right)}{d^3} \\ & - \frac{abf \cos(fx + e)}{d^2(dx+c)} - \frac{ab f^2 \cos\left(-e + \frac{cf}{d}\right) \operatorname{sinIntegral}\left(\frac{cf}{d} + fx\right)}{d^3} \\ & + \frac{b^2 f^2 \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{d^3} \\ & + \frac{ab f^2 \operatorname{cosineIntegral}\left(\frac{cf}{d} + fx\right) \sin\left(-e + \frac{cf}{d}\right)}{d^3} - \frac{ab \sin(fx + e)}{d(dx+c)^2} \\ & - \frac{b^2 f \cos(fx + e) \sin(fx + e)}{d^2(dx+c)} - \frac{b^2 (\sin^2(fx + e))}{2d(dx+c)^2} \end{aligned}$$

command

```
integrate((a+b*sin(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.10 Problem number 187

$$\int \frac{(e + fx) \sin^2(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{ex}{a} - \frac{fx^2}{2a} - \frac{(fx + e) \cos(dx + c)}{ad} - \frac{(fx + e) \cot\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)}{ad} \\ & + \frac{2f \ln\left(\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} + \frac{f \sin(dx + c)}{ad^2} \end{aligned}$$

command

```
integrate((f*x+e)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.11 Problem number 193

$$\int \frac{(e + fx) \sin^3(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3ex}{2a} + \frac{3fx^2}{4a} + \frac{(fx + e) \cos(dx + c)}{ad} + \frac{(fx + e) \cot\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)}{ad} - \frac{2f \ln\left(\sin\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} \\ & - \frac{f \sin(dx + c)}{ad^2} - \frac{(fx + e) \cos(dx + c) \sin(dx + c)}{2ad} + \frac{f(\sin^2(dx + c))}{4ad^2} \end{aligned}$$

command

`integrate((f*x+e)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.12 Problem number 257

$$\int \frac{(e + fx)^3 \cos^2(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(fx + e)^4}{4af} - \frac{6f^2(fx + e) \cos(dx + c)}{ad^3} + \frac{(fx + e)^3 \cos(dx + c)}{ad} \\ & + \frac{6f^3 \sin(dx + c)}{ad^4} - \frac{3f(fx + e)^2 \sin(dx + c)}{ad^2} \end{aligned}$$

command

`integrate((f*x+e)^3*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.13 Problem number 258

$$\int \frac{(e + fx)^2 \cos^2(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{(fx + e)^3}{3af} - \frac{2f^2 \cos(dx + c)}{ad^3} + \frac{(fx + e)^2 \cos(dx + c)}{ad} - \frac{2f(fx + e) \sin(dx + c)}{ad^2}$$

command

`integrate((f*x+e)^2*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 38.14 Problem number 259

$$\int \frac{(e + fx) \cos^2(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{ex}{a} + \frac{f x^2}{2a} + \frac{(fx + e) \cos(dx + c)}{ad} - \frac{f \sin(dx + c)}{a d^2}$$

command

```
integrate((f*x+e)*cos(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 38.15 Problem number 263

$$\int \frac{(e + fx)^3 \cos^3(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3f^3x}{8ad^3} + \frac{(fx + e)^3}{4ad} - \frac{6f^3 \cos(dx + c)}{ad^4} + \frac{3f(fx + e)^2 \cos(dx + c)}{ad^2} \\ & - \frac{6f^2(fx + e) \sin(dx + c)}{ad^3} + \frac{(fx + e)^3 \sin(dx + c)}{ad} \\ & + \frac{3f^3 \cos(dx + c) \sin(dx + c)}{8ad^4} - \frac{3f(fx + e)^2 \cos(dx + c) \sin(dx + c)}{4ad^2} \\ & + \frac{3f^2(fx + e) (\sin^2(dx + c))}{4ad^3} - \frac{(fx + e)^3 (\sin^2(dx + c))}{2ad} \end{aligned}$$

command

```
integrate((f*x+e)^3*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.16 Problem number 264

$$\int \frac{(e + fx)^2 \cos^3(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{efx}{2ad} + \frac{f^2x^2}{4ad} + \frac{2f(fx + e) \cos(dx + c)}{ad^2} - \frac{2f^2 \sin(dx + c)}{ad^3} + \frac{(fx + e)^2 \sin(dx + c)}{ad} \\ & - \frac{f(fx + e) \cos(dx + c) \sin(dx + c)}{2ad^2} + \frac{f^2(\sin^2(dx + c))}{4ad^3} - \frac{(fx + e)^2(\sin^2(dx + c))}{2ad} \end{aligned}$$

command

```
integrate((f*x+e)^2*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.17 Problem number 265

$$\int \frac{(e + fx) \cos^3(c + dx)}{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{fx}{4ad} + \frac{f \cos(dx + c)}{ad^2} + \frac{(fx + e) \sin(dx + c)}{ad} - \frac{f \cos(dx + c) \sin(dx + c)}{4ad^2} - \frac{(fx + e) (\sin^2(dx + c))}{2ad}$$

command

```
integrate((f*x+e)*cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 38.18 Problem number 268

$$\int \frac{\cos^3(c + dx)}{(e + fx)^2(a + a \sin(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{d \operatorname{cosineIntegral}\left(\frac{2de}{f} + 2dx\right) \cos\left(2c - \frac{2de}{f}\right)}{a f^2} - \frac{\cos(dx + c)}{a f (fx + e)} \\ & - \frac{d \cos\left(c - \frac{de}{f}\right) \operatorname{sinIntegral}\left(\frac{de}{f} + dx\right)}{a f^2} + \frac{d \operatorname{sinIntegral}\left(\frac{2de}{f} + 2dx\right) \sin\left(2c - \frac{2de}{f}\right)}{a f^2} \\ & - \frac{d \operatorname{cosineIntegral}\left(\frac{de}{f} + dx\right) \sin\left(c - \frac{de}{f}\right)}{a f^2} + \frac{\sin(2dx + 2c)}{2af (fx + e)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^3/(f*x+e)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 39 Test file number 68

Test folder name:

```
test_cases/4_Trig_functions/4.1_Sine/68_4.1.11-e_x^-m-a+b_x^n-^p_sin
```

### 39.1 Problem number 33

$$\int \frac{x^3 \sin(c + dx)}{(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2 d \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{b^5} - \frac{\cos(dx + c)}{b^3 d} + \frac{a^3 d \cos(dx + c)}{2b^5 (bx + a)} \\ & - \frac{3a \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{a^3 d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2b^6} \\ & + \frac{3a \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{b^4} - \frac{a^3 d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2b^6} \\ & + \frac{3a^2 d \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{b^5} + \frac{a^3 \sin(dx + c)}{2b^4 (bx + a)^2} - \frac{3a^2 \sin(dx + c)}{b^4 (bx + a)} \end{aligned}$$

command

```
integrate(x^3*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 39.2 Problem number 34

$$\int \frac{x^2 \sin(c + dx)}{(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2ad \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{b^4} - \frac{a^2 d \cos(dx + c)}{2b^4 (bx + a)} \\ & + \frac{\cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{a^2 d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2b^5} \\ & - \frac{\operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{b^3} + \frac{a^2 d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2b^5} \\ & - \frac{2ad \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{b^4} - \frac{a^2 \sin(dx + c)}{2b^3 (bx + a)^2} + \frac{2a \sin(dx + c)}{b^3 (bx + a)} \end{aligned}$$

command

```
integrate(x^2*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 39.3 Problem number 35

$$\int \frac{x \sin(c + dx)}{(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{b^3} + \frac{ad \cos(dx + c)}{2b^3 (bx + a)} \\ & + \frac{a d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2b^4} - \frac{a d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2b^4} \\ & + \frac{d \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{b^3} + \frac{a \sin(dx + c)}{2b^2 (bx + a)^2} - \frac{\sin(dx + c)}{b^2 (bx + a)} \end{aligned}$$

command

```
integrate(x*sin(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 39.4 Problem number 37

$$\int \frac{\sin(c + dx)}{x(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{d \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{a^2 b} + \frac{d \cos(dx + c)}{2ab (bx + a)} \\ & + \frac{\cos(c) \operatorname{sinIntegral}(dx)}{a^3} - \frac{\cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{a^3} \\ & + \frac{d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2a b^2} + \frac{\operatorname{cosineIntegral}(dx) \sin(c)}{a^3} \\ & + \frac{\operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^3} - \frac{d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2a b^2} \\ & - \frac{d \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^2 b} + \frac{\sin(dx + c)}{2a (bx + a)^2} + \frac{\sin(dx + c)}{a^2 (bx + a)} \end{aligned}$$

command

`integrate(sin(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 39.5 Problem number 38

$$\int \frac{\sin(c + dx)}{x^2(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d \operatorname{cosineIntegral}(dx) \cos(c)}{a^3} + \frac{2d \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{a^3} \\ & - \frac{d \cos(dx + c)}{2a^2(bx + a)} - \frac{3b \cos(c) \operatorname{sinIntegral}(dx)}{a^4} + \frac{3b \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{a^4} \\ & - \frac{d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2a^2b} - \frac{3b \operatorname{cosineIntegral}(dx) \sin(c)}{a^4} \\ & - \frac{d \operatorname{sinIntegral}(dx) \sin(c)}{a^3} - \frac{3b \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^4} \\ & + \frac{d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2a^2b} + \frac{2d \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^3} \\ & - \frac{\sin(dx + c)}{a^3x} - \frac{b \sin(dx + c)}{2a^2(bx + a)^2} - \frac{2b \sin(dx + c)}{a^3(bx + a)} \end{aligned}$$

command

`integrate(sin(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 39.6 Problem number 39

$$\int \frac{\sin(c + dx)}{x^3(a + bx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3bd \operatorname{cosineIntegral}(dx) \cos(c)}{a^4} - \frac{3bd \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \cos\left(-c + \frac{ad}{b}\right)}{a^4} \\ & - \frac{d \cos(dx + c)}{2a^3x} + \frac{bd \cos(dx + c)}{2a^3(bx + a)} + \frac{6b^2 \cos(c) \operatorname{sinIntegral}(dx)}{a^5} \\ & - \frac{d^2 \cos(c) \operatorname{sinIntegral}(dx)}{2a^3} - \frac{6b^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{a^5} \\ & + \frac{d^2 \cos\left(-c + \frac{ad}{b}\right) \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right)}{2a^3} + \frac{6b^2 \operatorname{cosineIntegral}(dx) \sin(c)}{a^5} \\ & - \frac{d^2 \operatorname{cosineIntegral}(dx) \sin(c)}{2a^3} + \frac{3bd \operatorname{sinIntegral}(dx) \sin(c)}{a^4} \\ & + \frac{6b^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^5} \\ & - \frac{d^2 \operatorname{cosineIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{2a^3} - \frac{3bd \operatorname{sinIntegral}\left(\frac{ad}{b} + dx\right) \sin\left(-c + \frac{ad}{b}\right)}{a^4} \\ & - \frac{\sin(dx + c)}{2a^3x^2} + \frac{3b \sin(dx + c)}{a^4x} + \frac{b^2 \sin(dx + c)}{2a^3(bx + a)^2} + \frac{3b^2 \sin(dx + c)}{a^4(bx + a)} \end{aligned}$$

command

```
integrate(sin(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 40 Test file number 69

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/69\_4.1.12-e\_x~m-a+b\_sin-c+d\_x~n~p

## 40.1 Problem number 122

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x^3} dx$$

Optimal antiderivative

$$\frac{\cos\left(a + \frac{b}{x^2}\right)}{2b}$$

command

```
integrate(sin(a+b/x^2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\cos\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sin\left(a + \frac{b}{x^2}\right)}{x^3} dx$$

## 40.2 Problem number 217

$$\int (e + fx)^2 \sin \left( a + \frac{b}{\sqrt[3]{c + dx}} \right) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b^9 f^2 \operatorname{cosineIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) \cos(a)}{120960d^3} \\
& + \frac{b^3(-cf+de)^2 \operatorname{cosineIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) \cos(a)}{2d^3} \\
& + \frac{b^5 f(-cf+de)(dx+c)^{\frac{1}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{120d^3} \\
& - \frac{b^7 f^2(dx+c)^{\frac{2}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{120960d^3} \\
& + \frac{b(-cf+de)^2(dx+c)^{\frac{2}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{2d^3} \\
& - \frac{b^3 f(-cf+de)(dx+c) \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{60d^3} \\
& + \frac{b^5 f^2(dx+c)^{\frac{4}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{20160d^3} \\
& + \frac{bf(-cf+de)(dx+c)^{\frac{5}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{5d^3} \\
& - \frac{b^3 f^2(dx+c)^2 \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{1008d^3} + \frac{b f^2(dx+c)^{\frac{8}{3}} \cos\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{24d^3} \\
& + \frac{b^6 f(-cf+de) \cos(a) \operatorname{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{120d^3} \\
& + \frac{b^6 f(-cf+de) \operatorname{cosineIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) \sin(a)}{120d^3} \\
& + \frac{b^9 f^2 \operatorname{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) \sin(a)}{120960d^3} \\
& - \frac{b^3(-cf+de)^2 \operatorname{sinIntegral}\left(\frac{b}{(dx+c)^{\frac{1}{3}}}\right) \sin(a)}{2d^3} \\
& + \frac{b^8 f^2(dx+c)^{\frac{1}{3}} \sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{120960d^3} \\
& - \frac{b^2(-cf+de)^2(dx+c)^{\frac{1}{3}} \sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{2d^3} \\
& + \frac{b^4 f(-cf+de)(dx+c)^{\frac{2}{3}} \sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{120d^3} \\
& - \frac{b^6 f^2(dx+c) \sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{60480d^3} + \frac{(-cf+de)^2(dx+c) \sin\left(a + \frac{b}{(dx+c)^{\frac{1}{3}}}\right)}{d^3}
\end{aligned}$$

command

```
integrate((f*x+e)^2*sin(a+b/(d*x+c)^(1/3)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41 Test file number 70

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/70\_4.1.1.2-g\_cos-^p-a+b\_sin-^m

### 41.1 Problem number 108

$$\int \sec(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2} \sqrt{a}}{d}$$

command

```
integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} (\log(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1) - \log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{2d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 41.2 Problem number 123

$$\int \sec^3(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{(\sec^2(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{2d} + \frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{4d}$$

command

```
integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{3}{2}} \left( \frac{2 \cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)}{\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 - 1} - \log(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1) + \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1) \right) \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))}{8d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.3 Problem number 124

$$\int \sec^4(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{(\sec^3(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{3d} - \frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(dx + c)}}\right) \sqrt{2}}{4d} + \frac{a \sec(dx + c) \sqrt{a + a \sin(dx + c)}}{2d}$$

command

```
integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{3}{2}} \left( \frac{2 \left( 3 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 + 1 \right)}{\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^3} - 3 \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1) + 3 \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1) \right)}{24d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 41.4 Problem number 125

$$\int \sec^5(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{(\sec^4(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{4d} + \frac{15a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{64d} - \frac{15a^2}{32d\sqrt{a + a \sin(dx + c)}} + \frac{5a(\sec^2(dx + c)) \sqrt{a + a \sin(dx + c)}}{16d}$$

command

```
integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{3}{2}} \left( \frac{2 \left( 7 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^3 - 9 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^2} + \frac{16}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)} - 15 \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) \right)}{128 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.5 Problem number 126

$$\int \sec^6(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{7a^3 \cos(dx + c)}{16d(a + a \sin(dx + c))^{\frac{3}{2}}} + \frac{(\sec^5(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{5d} - \frac{7a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(dx + c)}}\right) \sqrt{2}}{32d} + \frac{7a^2 \sec(dx + c)}{12d\sqrt{a + a \sin(dx + c)}} + \frac{7a(\sec^3(dx + c)) \sqrt{a + a \sin(dx + c)}}{30d}$$

command

```
integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{3}{2}} \left( \frac{30 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)}{\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 - 1} + \frac{4 \left( 45 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^4 + 10 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 + 3 \right)}{\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^5} \right) - 105 \log \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)}{960 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.6 Problem number 132

$$\int \sec(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a(a + a \sin(dx + c))^{\frac{3}{2}}}{3d} + \frac{4a^{\frac{5}{2}} \operatorname{arctanh} \left( \frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}} \right) \sqrt{2}}{d} - \frac{4a^2 \sqrt{a + a \sin(dx + c)}}{d}$$

command

`integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2} \left( 2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right)^3 + 6 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) - 3 \log \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right) + 3 \log \left( -\cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{3d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.7 Problem number 134

$$\int \sec^3(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a(\sec^2(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{d} - \frac{a^{\frac{5}{2}} \operatorname{arctanh} \left( \frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}} \right) \sqrt{2}}{2d}$$

command

```
integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{5}{2}} \left( \frac{2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} + \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) - \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) \right) \operatorname{sgn}(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right))}{4 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.8 Problem number 135

$$\int \sec^4(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a(\sec^3(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{3d}$$

command

```
integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{2} a^{\frac{5}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}{6 d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.9 Problem number 136

$$\int \sec^5(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{3a(\sec^2(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{16d} + \frac{(\sec^4(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{4d} + \frac{3a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{32d}$$

command

```
integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{5}{2}} \left( \frac{2 \left( 3 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^3 - 5 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^2} - 3 \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) + 3 \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \right)}{64 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.10 Problem number 137

$$\int \sec^6(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a(\sec^3(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{6d} + \frac{(\sec^5(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{5d} - \frac{a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right)\sqrt{2}}{8d} + \frac{a^2 \sec(dx + c) \sqrt{a + a \sin(dx + c)}}{4d}$$

command

```
integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{5}{2}} \left( \frac{2 \left( 15 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^4 + 5 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 + 3 \right)}{\sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^5} - 15 \log\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) + 15 \log\left(-\sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \right)}{240 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.11 Problem number 138

$$\int \sec^7(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7a(\sec^4(dx + c))(a + a \sin(dx + c))^{3/2}}{48d} + \frac{(\sec^6(dx + c))(a + a \sin(dx + c))^{5/2}}{6d} \\ & + \frac{35a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{256d} \\ & - \frac{35a^3}{128d\sqrt{a + a \sin(dx + c)}} + \frac{35a^2(\sec^2(dx + c))\sqrt{a + a \sin(dx + c)}}{192d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{5/2} \left( \frac{96}{\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} + \frac{2(57 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^5 - 136 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^3 + 87 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^3} - 105 \log(\cos) \right)}{1536 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.12 Problem number 146

$$\int \sec(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4a^2(a + a \sin(dx + c))^{3/2}}{3d} - \frac{2a(a + a \sin(dx + c))^{5/2}}{5d} \\ & + \frac{8a^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{d} - \frac{8a^3\sqrt{a + a \sin(dx + c)}}{d} \end{aligned}$$

command

```
integrate(sec(d*x+c)*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}\left(6\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5 + 10\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^3 + 30\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) - 15\log\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)\right)}{15d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.13 Problem number 147

$$\int \sec^2(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{16a^2 \sec(dx + c)(a + a \sin(dx + c))^{\frac{3}{2}}}{3d} - \frac{2a \sec(dx + c)(a + a \sin(dx + c))^{\frac{5}{2}}}{3d} \\ & + \frac{64a^3 \sec(dx + c) \sqrt{a + a \sin(dx + c)}}{3d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}\left(a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^3 - 6a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}{3d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.14 Problem number 148

$$\int \sec^3(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(\sec^2(dx + c)(a + a \sin(dx + c))^{\frac{5}{2}})}{d} \\ & - \frac{3a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{d \sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{d} + \frac{3a^3 \sqrt{a + a \sin(dx + c)}}{d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{7}{2}} \left( \frac{2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} - 4 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 3 \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) - 3 \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \right)}{2 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.15 Problem number 149

$$\int \sec^4(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$-\frac{8a^2(\sec^3(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{3d} + \frac{2a(\sec^3(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{d}$$

command

```
integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \right) \sqrt{a}}{3 d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.16 Problem number 150

$$\int \sec^5(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$-\frac{a^2(\sec^2(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{8d} + \frac{a(\sec^4(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{2d} - \frac{a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{16d}$$

command

```
integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} a^{\frac{7}{2}} \left( \frac{4 \left( \frac{1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)} + \cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)}{\left( \frac{1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)} + \cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)^2} - \log \left( \left| \frac{1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)} + \cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) + 2 \right| \right) + \log \right)$$


---

$64d$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.17 Problem number 151

$$\int \sec^6(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{2a(\sec^5(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{5d}$$

command

```
integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{2} a^{\frac{7}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}{20d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.18 Problem number 152

$$\int \sec^7(c + dx)(a + a \sin(c + dx))^{7/2} dx$$



Optimal antiderivative

$$\frac{5a^2(\sec^2(dx+c))(a+a\sin(dx+c))^{\frac{3}{2}}}{64d} + \frac{5a(\sec^4(dx+c))(a+a\sin(dx+c))^{\frac{5}{2}}}{48d} \\ + \frac{(\sec^6(dx+c))(a+a\sin(dx+c))^{\frac{7}{2}}}{6d} + \frac{5a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}}{128d}$$

command

```
integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{7}{2}} \left( \frac{2 \left( 15 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^5 - 40 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^3 + 33 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^3} - 15 \log\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \dots \right)}{768 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 41.19 Problem number 153

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^{7/2} dx$$

Optimal antiderivative

$$\frac{a^2(\sec^3(dx+c))(a+a\sin(dx+c))^{\frac{3}{2}}}{12d} + \frac{a(\sec^5(dx+c))(a+a\sin(dx+c))^{\frac{5}{2}}}{10d} \\ + \frac{(\sec^7(dx+c))(a+a\sin(dx+c))^{\frac{7}{2}}}{7d} \\ - \frac{a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right)\sqrt{2}}{16d} + \frac{a^3 \sec(dx+c) \sqrt{a+a\sin(dx+c)}}{8d}$$

command

```
integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{7}{2}} \left( \frac{2 \left( 105 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^6 + 35 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^4 + 21 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2 + 15 \right)}{\sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^7} - 105 \log\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \dots \right)}{3360 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.20 Problem number 154

$$\int \sec^9(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{21a^2(\sec^4(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{256d} + \frac{3a(\sec^6(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{32d} \\ & + \frac{(\sec^8(dx + c))(a + a \sin(dx + c))^{\frac{7}{2}}}{8d} + \frac{315a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{4096d} \\ & - \frac{315a^4}{2048d\sqrt{a + a \sin(dx + c)}} + \frac{105a^3(\sec^2(dx + c))\sqrt{a + a \sin(dx + c)}}{1024d} \end{aligned}$$

command

`integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} a^{\frac{7}{2}} \left( \frac{256}{\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} + \frac{2(187 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^7 - 643 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^5 + 765 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^3 - 325 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.21 Problem number 155

$$\int \sec^{10}(c + dx)(a + a \sin(c + dx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{11a^5 \cos(dx + c)}{64d(a + a \sin(dx + c))^{\frac{3}{2}}} + \frac{11a^2(\sec^5(dx + c))(a + a \sin(dx + c))^{\frac{3}{2}}}{140d} \\ & + \frac{11a(\sec^7(dx + c))(a + a \sin(dx + c))^{\frac{5}{2}}}{126d} + \frac{(\sec^9(dx + c))(a + a \sin(dx + c))^{\frac{7}{2}}}{9d} \\ & - \frac{11a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(dx + c)}}\right) \sqrt{2}}{128d} \\ & + \frac{11a^4 \sec(dx + c)}{48d\sqrt{a + a \sin(dx + c)}} + \frac{11a^3(\sec^3(dx + c))\sqrt{a + a \sin(dx + c)}}{120d} \end{aligned}$$

command

```
integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} a^{\frac{7}{2}} \left( \frac{630 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)}{\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 - 1} + \frac{4 (1575 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^8 + 420 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^6 + 189 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^4 + 90 \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 + 15)}{\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^9} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 41.22 Problem number 167

$$\int \frac{\sec^5(c + dx)}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{21a}{64d(a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{9a(\sec^2(dx + c))}{40d(a + a \sin(dx + c))^{\frac{3}{2}}} \\ & + \frac{63 \operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{256d\sqrt{a}} - \frac{63}{128d\sqrt{a + a \sin(dx + c)}} \\ & + \frac{63(\sec^2(dx + c))}{160d\sqrt{a + a \sin(dx + c)}} + \frac{\sec^4(dx + c)}{4d\sqrt{a + a \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{a} \left( \frac{315 \sqrt{2} \log(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{315 \sqrt{2} \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{10 (15 \sqrt{2} \cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^3 - 17 \sqrt{2} \cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 5)}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 \operatorname{asgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} \right) / 2560 d$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



command

```
integrate(sec(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \left( \frac{105 \sqrt{2} \log(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{105 \sqrt{2} \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{30 \sqrt{2} \cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c)^2 - 1) a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} \right)}{960 d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 41.25 Problem number 181

$$\int \frac{\sec^6(c + dx)}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3003 \cos(dx + c)}{8192d (a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{1001 \sec(dx + c)}{5120d (a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{143(\sec^3(dx + c))}{960d (a + a \sin(dx + c))^{\frac{3}{2}}} \\ & - \frac{\sec^5(dx + c)}{8d (a + a \sin(dx + c))^{\frac{3}{2}}} - \frac{3003 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right)\sqrt{2}}{16384a^{\frac{3}{2}}d} \\ & + \frac{1001 \sec(dx + c)}{2048ad \sqrt{a + a \sin(dx + c)}} + \frac{143(\sec^3(dx + c))}{640ad \sqrt{a + a \sin(dx + c)}} + \frac{13(\sec^5(dx + c))}{80ad \sqrt{a + a \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \left( \frac{45045 \sqrt{2} \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{45045 \sqrt{2} \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c))} - \frac{10 \left( 3249 \sqrt{2} \sin(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c) \right)^7 - 1063}{\dots} \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 41.26 Problem number 192

$$\int \frac{\sec(c + dx)}{(a + a \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{5d(a + a \sin(dx + c))^{5/2}} - \frac{1}{6ad(a + a \sin(dx + c))^{3/2}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + a \sin(dx + c)} \sqrt{2}}{2\sqrt{a}}\right) \sqrt{2}}{8a^{5/2}d} - \frac{1}{4a^2d\sqrt{a + a \sin(dx + c)}} \end{aligned}$$

command

```
integrate(sec(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \left( \frac{15\sqrt{2} \log(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} - \frac{15\sqrt{2} \log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} - \frac{2\sqrt{2} (15 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^4 + 5 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^2)}{a^3 \cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^5 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} \right)}{240d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 41.27 Problem number 343

$$\int \cos^7(c + dx)(a + a \sin(c + dx))^m dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(a + a \sin(dx + c))^{4+m}}{a^4d(4 + m)} - \frac{12(a + a \sin(dx + c))^{5+m}}{a^5d(5 + m)} \\ & + \frac{6(a + a \sin(dx + c))^{6+m}}{a^6d(6 + m)} - \frac{(a + a \sin(dx + c))^{7+m}}{a^7d(7 + m)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a \sin(dx + c) + a)^7 (a \sin(dx + c) + a)^m m^3 - 6 (a \sin(dx + c) + a)^6 (a \sin(dx + c) + a)^m a m^3 + 12 (a \sin(dx + c) + a)^5 (a \sin(dx + c) + a)^m a^2 m^3 - 6 (a \sin(dx + c) + a)^4 (a \sin(dx + c) + a)^m a^3 m^3 + 12 (a \sin(dx + c) + a)^3 (a \sin(dx + c) + a)^m a^4 m^3 - 6 (a \sin(dx + c) + a)^2 (a \sin(dx + c) + a)^m a^5 m^3 + 12 (a \sin(dx + c) + a) (a \sin(dx + c) + a)^m a^6 m^3 - 12 (a \sin(dx + c) + a)^m a^7 m^3}{240d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.28 Problem number 344

$$\int \cos^5(c + dx)(a + a \sin(c + dx))^m dx$$

Optimal antiderivative

$$\frac{4(a + a \sin(dx + c))^{3+m}}{a^3 d (3 + m)} - \frac{4(a + a \sin(dx + c))^{4+m}}{a^4 d (4 + m)} + \frac{(a + a \sin(dx + c))^{5+m}}{a^5 d (5 + m)}$$

command

```
integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a \sin(dx + c) + a)^5 (a \sin(dx + c) + a)^m m^2 - 4(a \sin(dx + c) + a)^4 (a \sin(dx + c) + a)^m a m^2 + 4(a \sin(dx + c) + a)^3 (a \sin(dx + c) + a)^m a^2 m^2 - 4(a \sin(dx + c) + a)^2 (a \sin(dx + c) + a)^m a^3 m^2 + 4(a \sin(dx + c) + a) (a \sin(dx + c) + a)^m a^4 m^2 - 4(a \sin(dx + c) + a)^m a^5 m^2}{a^3 d (3 + m) - a^4 d (4 + m) + a^5 d (5 + m)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.29 Problem number 473

$$\int \cos^5(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(a^2 - b^2)^2 (a + b \sin(dx + c))^{\frac{3}{2}}}{3b^5 d} - \frac{8a(a^2 - b^2) (a + b \sin(dx + c))^{\frac{5}{2}}}{5b^5 d} + \frac{4(3a^2 - b^2) (a + b \sin(dx + c))^{\frac{7}{2}}}{7b^5 d} - \frac{8a(a + b \sin(dx + c))^{\frac{9}{2}}}{9b^5 d} + \frac{2(a + b \sin(dx + c))^{\frac{11}{2}}}{11b^5 d}$$

command

```
integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 315 (b \sin(dx + c) + a)^{\frac{11}{2}} - 1540 (b \sin(dx + c) + a)^{\frac{9}{2}} a + 2970 (b \sin(dx + c) + a)^{\frac{7}{2}} a^2 - 2772 (b \sin(dx + c) + a)^{\frac{5}{2}} a^3 + 1188 (b \sin(dx + c) + a)^{\frac{3}{2}} a^4 - 1188 (b \sin(dx + c) + a)^{\frac{1}{2}} a^5 \right)}{11b^5 d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{b \sin(dx + c) + a} \cos(dx + c)^5 dx$$

## 41.30 Problem number 476

$$\int \sec(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a - b}}\right) \sqrt{a - b}}{d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a + b}}\right) \sqrt{a + b}}{d}$$

command

```
integrate(sec(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b \left( \frac{(a-b) \operatorname{arctan}\left(\frac{\sqrt{b \sin(dx + c)} + a}{\sqrt{-a + b}}\right)}{\sqrt{-a + b} b} - \frac{(a+b) \operatorname{arctan}\left(\frac{\sqrt{b \sin(dx + c)} + a}{\sqrt{-a - b}}\right)}{\sqrt{-a - b} b} \right) / d$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{b \sin(dx + c) + a} \sec(dx + c) dx$$

## 41.31 Problem number 477

$$\int \sec^3(c + dx) \sqrt{a + b \sin(c + dx)} dx$$

Optimal antiderivative

$$-\frac{(2a - b) \operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a - b}}\right)}{4d\sqrt{a - b}} + \frac{(2a + b) \operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a + b}}\right)}{4d\sqrt{a + b}} + \frac{\sec(dx + c) \sqrt{a + b \sin(dx + c)} \tan(dx + c)}{2d}$$

command

```
integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$b^3 \left( \frac{(2a-b) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b} b^3} - \frac{(2a+b) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b} b^3} - \frac{2 \left( (b \sin(dx+c)+a)^{\frac{3}{2}} - \sqrt{b \sin(dx+c)+a} \right)}{\left( (b \sin(dx+c)+a)^2 - 2(b \sin(dx+c)+a)a + a^2 \right)} \right) \frac{1}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{b \sin(dx+c)+a} \sec(dx+c)^3 dx$$

### 41.32 Problem number 478

$$\int \sec^5(c+dx) \sqrt{a+b \sin(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(12a^2 - 18ab + 5b^2) \operatorname{arctanh}\left(\frac{\sqrt{a+b \sin(dx+c)}}{\sqrt{a-b}}\right)}{32(a-b)^{\frac{3}{2}} d} \\ & + \frac{(12a^2 + 18ab + 5b^2) \operatorname{arctanh}\left(\frac{\sqrt{a+b \sin(dx+c)}}{\sqrt{a+b}}\right)}{32(a+b)^{\frac{3}{2}} d} \\ & - \frac{(\sec^2(dx+c)) (ab - (6a^2 - 5b^2) \sin(dx+c)) \sqrt{a+b \sin(dx+c)}}{16(a^2 - b^2) d} \\ & + \frac{(\sec^3(dx+c)) \sqrt{a+b \sin(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

`integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^5 \left( \frac{(12a^2 - 18ab + 5b^2) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(ab^5 - b^6) \sqrt{-a+b}} - \frac{(12a^2 + 18ab + 5b^2) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(ab^5 + b^6) \sqrt{-a-b}} - \frac{2 \left( 6(b \sin(dx+c)+a)^{\frac{3}{2}} - \sqrt{b \sin(dx+c)+a} \right)}{\left( (b \sin(dx+c)+a)^2 - 2(b \sin(dx+c)+a)a + a^2 \right)} \right) \frac{1}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{b \sin(dx+c)+a} \sec(dx+c)^5 dx$$

## 41.33 Problem number 506

$$\int \frac{\cos^5(c + dx)}{\sqrt{a + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{8a(a^2 - b^2)(a + b \sin(dx + c))^{\frac{3}{2}}}{3b^5d} + \frac{4(3a^2 - b^2)(a + b \sin(dx + c))^{\frac{5}{2}}}{5b^5d} \\ - \frac{8a(a + b \sin(dx + c))^{\frac{7}{2}}}{7b^5d} + \frac{2(a + b \sin(dx + c))^{\frac{9}{2}}}{9b^5d} + \frac{2(a^2 - b^2)^2 \sqrt{a + b \sin(dx + c)}}{b^5d}$$

command

```
integrate(cos(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( 35 (b \sin(dx + c) + a)^{\frac{9}{2}} - 180 (b \sin(dx + c) + a)^{\frac{7}{2}} a + 378 (b \sin(dx + c) + a)^{\frac{5}{2}} a^2 - 420 (b \sin(dx + c) + a)^{\frac{3}{2}} a^3 - \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(dx + c)^5}{\sqrt{b \sin(dx + c) + a}} dx$$

## 41.34 Problem number 509

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \sin(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a - b}}\right)}{d\sqrt{a - b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sin(dx + c)}}{\sqrt{a + b}}\right)}{d\sqrt{a + b}}$$

command

```
integrate(sec(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b \left( \frac{\arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b} b} - \frac{\arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b} b} \right) \frac{1}{d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sec(dx+c)}{\sqrt{b \sin(dx+c)+a}} dx$$

**41.35 Problem number 510**

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b \sin(c+dx)}} dx$$

Optimal antiderivative

$$\frac{(2a-3b) \operatorname{arctanh}\left(\frac{\sqrt{a+b \sin(dx+c)}}{\sqrt{a-b}}\right)}{4(a-b)^{\frac{3}{2}} d} + \frac{(2a+3b) \operatorname{arctanh}\left(\frac{\sqrt{a+b \sin(dx+c)}}{\sqrt{a+b}}\right)}{4(a+b)^{\frac{3}{2}} d} - \frac{(\sec^2(dx+c))(b-a \sin(dx+c)) \sqrt{a+b \sin(dx+c)}}{2(a^2-b^2) d}$$

command

`integrate(sec(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^3 \left( \frac{(2a-3b) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(ab^3-b^4)\sqrt{-a+b}} - \frac{(2a+3b) \arctan\left(\frac{\sqrt{b \sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(ab^3+b^4)\sqrt{-a-b}} - \frac{2 \left( (b \sin(dx+c)+a)^{\frac{3}{2}} a - \sqrt{b \sin(dx+c)+a} \right)}{(a^2b^2-b^4)(b \sin(dx+c)+a)} \right) \frac{1}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sec(dx+c)^3}{\sqrt{b \sin(dx+c)+a}} dx$$

## 41.36 Problem number 511

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sin(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(4a^2 - 10ab + 7b^2) \operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a-b}}\right)}{32(a-b)^{\frac{5}{2}}d} \\ & + \frac{3(4a^2 + 10ab + 7b^2) \operatorname{arctanh}\left(\frac{\sqrt{a+b\sin(dx+c)}}{\sqrt{a+b}}\right)}{32(a+b)^{\frac{5}{2}}d} \\ & - \frac{(\sec^4(dx+c))(b-a\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{4(a^2-b^2)d} \\ & - \frac{(\sec^2(dx+c))(b(a^2-7b^2)-6a(a^2-2b^2)\sin(dx+c))\sqrt{a+b\sin(dx+c)}}{16(a^2-b^2)^2d} \end{aligned}$$

command

`integrate(sec(d*x+c)^5/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^5 \left( \frac{3(4a^2-10ab+7b^2) \operatorname{arctan}\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a+b}}\right)}{(a^2b^5-2ab^6+b^7)\sqrt{-a+b}} - \frac{3(4a^2+10ab+7b^2) \operatorname{arctan}\left(\frac{\sqrt{b\sin(dx+c)+a}}{\sqrt{-a-b}}\right)}{(a^2b^5+2ab^6+b^7)\sqrt{-a-b}} - \frac{2(6(b\sin(dx+c)))}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\sin(dx+c)+a}} dx$$

## 41.37 Problem number 630

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^m dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a^2-b^2)^3(a+b\sin(dx+c))^{1+m}}{b^7d(1+m)} + \frac{6a(a^2-b^2)^2(a+b\sin(dx+c))^{2+m}}{b^7d(2+m)} \\ & - \frac{3(5a^4-6a^2b^2+b^4)(a+b\sin(dx+c))^{3+m}}{b^7d(3+m)} + \frac{4a(5a^2-3b^2)(a+b\sin(dx+c))^{4+m}}{b^7d(4+m)} \\ & - \frac{3(5a^2-b^2)(a+b\sin(dx+c))^{5+m}}{b^7d(5+m)} + \frac{6a(a+b\sin(dx+c))^{6+m}}{b^7d(6+m)} - \frac{(a+b\sin(dx+c))^{7+m}}{b^7d(7+m)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 41.38 Problem number 631

$$\int \cos^5(c + dx)(a + b \sin(c + dx))^m dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a^2 - b^2)^2 (a + b \sin(dx + c))^{1+m}}{b^5 d (1 + m)} - \frac{4a(a^2 - b^2)(a + b \sin(dx + c))^{2+m}}{b^5 d (2 + m)} \\ & + \frac{2(3a^2 - b^2)(a + b \sin(dx + c))^{3+m}}{b^5 d (3 + m)} - \frac{4a(a + b \sin(dx + c))^{4+m}}{b^5 d (4 + m)} + \frac{(a + b \sin(dx + c))^{5+m}}{b^5 d (5 + m)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^m,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42 Test file number 72

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/72\_4.1.1.3-g\_tan^p-a+b\_sin^m

## 42.1 Problem number 2

$$\int (a + a \sin(c + dx)) \tan^3(c + dx) dx$$

Optimal antiderivative

$$\frac{5a \ln(1 - \sin(dx + c))}{4d} - \frac{a \ln(1 + \sin(dx + c))}{4d} + \frac{a \sin(dx + c)}{d} + \frac{a^2}{2d(a - a \sin(dx + c))}$$

command

```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.2 Problem number 27

$$\int (a + a \sin(c + dx))^3 \tan(c + dx) dx$$

Optimal antiderivative

$$-\frac{4a^3 \ln(1 - \sin(dx + c))}{d} - \frac{4a^3 \sin(dx + c)}{d} - \frac{3a^3(\sin^2(dx + c))}{2d} - \frac{a^3(\sin^3(dx + c))}{3d}$$

command

```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 42.3 Problem number 36

$$\int (a + a \sin(c + dx))^4 \tan(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{8a^4 \ln(1 - \sin(dx + c))}{d} - \frac{8a^4 \sin(dx + c)}{d} - \frac{7a^4 (\sin^2(dx + c))}{2d} \\ & - \frac{4a^4 (\sin^3(dx + c))}{3d} - \frac{a^4 (\sin^4(dx + c))}{4d} \end{aligned}$$

command

```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 42.4 Problem number 92

$$\int \sqrt{a + a \sin(e + fx)} \tan^2(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \sec(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{af} - \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{a}\sqrt{2}}{2f} \\ & + \frac{5 \sec(fx + e) \sqrt{a + a \sin(fx + e)}}{f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \log \left( \frac{2 \left( \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^2 + 2 \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right) + 1 \right)}{\tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^2 + 1} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right) \right)^3$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{a \sin(fx + e) + a} \tan(fx + e)^2 dx$$

### 42.5 Problem number 93

$$\int \cot^2(e + fx) \sqrt{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)\sqrt{a}}{f} + \frac{3a\cos(fx+e)}{f\sqrt{a+a\sin(fx+e)}} - \frac{\cot(fx+e)\sqrt{a+a\sin(fx+e)}}{f}$$

command

```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 42.6 Problem number 94

$$\int \cot^4(e + fx) \sqrt{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{11 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)\sqrt{a}}{8f} - \frac{2a\cos(fx+e)}{f\sqrt{a+a\sin(fx+e)}} + \frac{11a\cot(fx+e)}{8f\sqrt{a+a\sin(fx+e)}} - \frac{a\cot(fx+e)\csc(fx+e)}{12f\sqrt{a+a\sin(fx+e)}} - \frac{\cot(fx+e)(\csc^2(fx+e))\sqrt{a+a\sin(fx+e)}}{3f}$$

command

```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 33 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 192 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 42.7 Problem number 95

$$\int (a + a \sin(e + fx))^{3/2} \tan^4(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3 (\cos^3(fx + e))}{3f (a + a \sin(fx + e))^{3/2}} + \frac{(\sec^3(fx + e)) (a + a \sin(fx + e))^{3/2}}{3f} \\ & - \frac{a^{3/2} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{4f} \\ & - \frac{4a^2 \cos(fx + e)}{f \sqrt{a + a \sin(fx + e)}} - \frac{7a \sec(fx + e) \sqrt{a + a \sin(fx + e)}}{2f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.8 Problem number 96

$$\int (a + a \sin(e + fx))^{3/2} \tan^2(e + fx) dx$$

Optimal antiderivative

$$\frac{7 \sec(fx + e) (a + a \sin(fx + e))^{3/2}}{3f} - \frac{2 \sec(fx + e) (a + a \sin(fx + e))^{5/2}}{3af} + \frac{11a^2 \cos(fx + e)}{3f \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^8 + 60 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right) \right)}{6 \left( \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.9 Problem number 97

$$\int \cot^2(e + fx)(a + a \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{f} - \frac{\cot(fx+e)(a+a\sin(fx+e))^{3/2}}{f} + \frac{11a^2 \cos(fx+e)}{3f\sqrt{a+a\sin(fx+e)}} + \frac{5a \cos(fx+e) \sqrt{a+a\sin(fx+e)}}{3f}$$

command

```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^3 - 9\sqrt{2} a \log\left(\frac{\left| -2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|}{\left| 2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right|}\right) \right) \operatorname{sgn}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.10 Problem number 98

$$\int \cot^4(e + fx)(a + a \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{37a^{3/2} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{8f} - \frac{\cot(fx+e) (\csc^2(fx+e)) (a+a\sin(fx+e))^{3/2}}{3f} - \frac{8a^2 \cos(fx+e)}{3f\sqrt{a+a\sin(fx+e)}} + \frac{29a^2 \cot(fx+e)}{24f\sqrt{a+a\sin(fx+e)}} - \frac{2a \cos(fx+e) \sqrt{a+a\sin(fx+e)}}{3f} - \frac{a \cot(fx+e) \csc(fx+e) \sqrt{a+a\sin(fx+e)}}{4f}$$

command

`integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 128 a \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^3 - 111 \sqrt{2} a \log \left( \frac{-2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)}{2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 42.11 Problem number 99

$$\int (a + a \sin(e + fx))^{5/2} \tan^4(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^5(\cos^5(fx+e))}{5f(a+a\sin(fx+e))^{\frac{5}{2}}} + \frac{8a^4(\cos^3(fx+e))}{3f(a+a\sin(fx+e))^{\frac{3}{2}}} + \frac{2a(\sec^3(fx+e))(a+a\sin(fx+e))^{\frac{3}{2}}}{3f} \\ & -\frac{12a^3\cos(fx+e)}{f\sqrt{a+a\sin(fx+e)}} - \frac{8a^2\sec(fx+e)\sqrt{a+a\sin(fx+e)}}{f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 42.12 Problem number 100

$$\int (a + a \sin(e + fx))^{5/2} \tan^2(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9 \sec(fx+e)(a+a\sin(fx+e))^{\frac{5}{2}}}{5f} - \frac{2 \sec(fx+e)(a+a\sin(fx+e))^{\frac{7}{2}}}{5af} \\ & + \frac{124a^3\cos(fx+e)}{15f\sqrt{a+a\sin(fx+e)}} + \frac{31a^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 42.13 Problem number 101

$$\int \cot^2(e + fx)(a + a \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{f} + \frac{7a \cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{5f} \\ & -\frac{\cot(fx+e)(a+a\sin(fx+e))^{\frac{5}{2}}}{f} + \frac{49a^3 \cos(fx+e)}{15f\sqrt{a+a\sin(fx+e)}} \\ & + \frac{31a^2 \cos(fx+e)\sqrt{a+a\sin(fx+e)}}{15f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 96 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^5 - 320 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.14 Problem number 102

$$\int \cot^4(e + fx)(a + a \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{55a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{8f} - \frac{2a \cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{5f} \\ & - \frac{5a \cot(fx+e) \csc(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{12f} \\ & - \frac{\cot(fx+e)(\csc^2(fx+e))(a+a\sin(fx+e))^{\frac{5}{2}}}{3f} - \frac{9a^3 \cos(fx+e)}{40f \sqrt{a+a\sin(fx+e)}} \\ & - \frac{16a^2 \cos(fx+e) \sqrt{a+a\sin(fx+e)}}{15f} + \frac{17a^2 \cot(fx+e) \sqrt{a+a\sin(fx+e)}}{24f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 768 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^5 - 2560 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.15 Problem number 103

$$\int \frac{\tan^4(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{67 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{128f\sqrt{a}} - \frac{\sec(fx+e)(53+127\sin(fx+e))}{192f\sqrt{a+a\sin(fx+e)}} \\ & + \frac{a \sin(fx+e) \tan(fx+e)}{24f(a+a\sin(fx+e))^{\frac{3}{2}}} + \frac{\tan^3(fx+e)}{3f\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{201\sqrt{2}\log(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{201\sqrt{2}\log(-\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{6\sqrt{2}(21\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^3-19\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2-1)^2\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{768f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.16 Problem number 104

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{5\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{8f\sqrt{a}} - \frac{\sec(fx+e)}{2f\sqrt{a+a\sin(fx+e)}} + \frac{3\sec(fx+e)\sqrt{a+a\sin(fx+e)}}{4af}$$

command

```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{5\sqrt{2}\log(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{5\sqrt{2}\log(-\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{2\sqrt{2}(3\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2-2)}{(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^3-\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.17 Problem number 105

$$\int \frac{\cot^2(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{f\sqrt{a}} - \frac{\cot(fx+e)}{f\sqrt{a+a\sin(fx+e)}}$$

command

```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} \log\left(\frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}\right)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{(2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1) \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.18 Problem number 107

$$\int \frac{\tan^4(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7 \cos(fx + e)}{256f(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{\sec(fx + e)(65 + 87 \sin(fx + e))}{192f(a + a \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{7 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{512a^{\frac{3}{2}}f} \\ & + \frac{a \sin(fx + e) \tan(fx + e)}{12f(a + a \sin(fx + e))^{\frac{5}{2}}} + \frac{\tan^3(fx + e)}{3f(a + a \sin(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{21\sqrt{2}\log(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{21\sqrt{2}\log(-\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{2\sqrt{2}\left(21\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^8 - 312\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^6 + 1248\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^4 - 1248\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2 + 312\sqrt{a}\right)}{(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^3 - \sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

3072 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.19 Problem number 108

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)}{32f(a+a\sin(fx+e))^{\frac{3}{2}}} - \frac{\sec(fx+e)}{4f(a+a\sin(fx+e))^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{64a^{\frac{3}{2}}f} + \frac{5\sec(fx+e)}{8af\sqrt{a+a\sin(fx+e)}}$$

command

`integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\sqrt{2}}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)} - \frac{\sqrt{2}\left(9\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^3 - 7\sqrt{a}\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)\right)}{(\sin(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2 - 1)^2 a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 42.20 Problem number 109

$$\int \frac{\cot^2(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{a^{\frac{3}{2}}f} - \frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{a^{\frac{3}{2}}f} - \frac{\cot(fx+e)}{af\sqrt{a+a\sin(fx+e)}}$$

command

`integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2}\sqrt{a} \left( \frac{3\sqrt{2} \log\left(\left|\frac{-2\sqrt{2}+4\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{2\sqrt{2}+4\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}\right|\right)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{4 \log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{4 \log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{1}{2 \sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)} \right)$$


---


$$4f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.21 Problem number 110

$$\int \frac{\cot^4(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{8a^{\frac{3}{2}}f} - \frac{\cot(fx+e)}{8af\sqrt{a+a\sin(fx+e)}} + \frac{11 \cot(fx+e) \csc(fx+e)}{12af\sqrt{a+a\sin(fx+e)}} - \frac{\cot(fx+e) (\csc^2(fx+e)) \sqrt{a+a\sin(fx+e)}}{3a^2f}$$

command

```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{3 \sqrt{2} \log \left( \frac{|-2 \sqrt{2} + 4 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)|}{2 \sqrt{2} + 4 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)} \right)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{4 \left( 12 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^5 + 16 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^3 - 3 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \right)}{\left( 2 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 1 \right)^3 a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right)$$


---

$96 f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.22 Problem number 111

$$\int \frac{\tan^4(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{317 \cos(fx + e)}{3072 f (a + a \sin(fx + e))^{5/2}} - \frac{\sec(fx + e) (115 + 129 \sin(fx + e))}{384 f (a + a \sin(fx + e))^{5/2}} \\ & + \frac{317 \cos(fx + e)}{4096 a f (a + a \sin(fx + e))^{3/2}} + \frac{317 \operatorname{arctanh} \left( \frac{\cos(fx + e) \sqrt{a} \sqrt{2}}{2 \sqrt{a + a \sin(fx + e)}} \right) \sqrt{2}}{8192 a^{5/2} f} \\ & + \frac{5 a \sin(fx + e) \tan(fx + e)}{48 f (a + a \sin(fx + e))^{7/2}} + \frac{\tan^3(fx + e)}{3 f (a + a \sin(fx + e))^{5/2}} \end{aligned}$$

command

```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{128 \sqrt{2} \left( 9 \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 1 \right)}{a^{5/2} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^3} - \frac{\sqrt{2} \left( 201 \sqrt{a} \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^7 - 1249 \sqrt{a} \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^5 + 1567 \sqrt{a} \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^3 - 3 \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \right)}{\left( \sin(\frac{3}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 1 \right)^4 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$


---

$24576 f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.23 Problem number 112

$$\int \frac{\tan^2(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\sec(fx + e)}{6f(a + a \sin(fx + e))^{5/2}} - \frac{11 \cos(fx + e)}{128af(a + a \sin(fx + e))^{3/2}} + \frac{17 \sec(fx + e)}{48af(a + a \sin(fx + e))^{3/2}} \\ & - \frac{11 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{256a^{5/2}f} + \frac{11 \sec(fx + e)}{96a^2f\sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{48\sqrt{2}}{a^{5/2}\operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))\sin(\frac{3}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} - \frac{\sqrt{2}\left(15\sqrt{a}\sin(\frac{3}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^5 - 56\sqrt{a}\sin(\frac{3}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^3 + 33\sqrt{a}\sin(\frac{3}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)\right)}{\left(\sin(\frac{3}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1\right)^3 a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \\ \hline 768f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.24 Problem number 113

$$\int \frac{\cot^2(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{a^{5/2}f} - \frac{2 \cos(fx + e)}{af(a + a \sin(fx + e))^{3/2}} \\ & - \frac{\cot(fx + e)}{af(a + a \sin(fx + e))^{3/2}} - \frac{7 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{2a^{5/2}f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{a} \left( \frac{7\sqrt{2} \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{7\sqrt{2} \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{10 \log\left(\left|\sqrt{2} + 2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)\right|\right)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) + \frac{10 \log\left(\left|\sqrt{2} + 2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)\right|\right)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

---

4 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.25 Problem number 114

$$\int \frac{\cot^4(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{45 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{8a^{\frac{5}{2}}f} - \frac{4 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{a^{\frac{5}{2}}f} - \frac{19 \cot(fx+e)}{8a^2 f \sqrt{a+a\sin(fx+e)}} + \frac{13 \cot(fx+e) \csc(fx+e)}{12a^2 f \sqrt{a+a\sin(fx+e)}} - \frac{\cot(fx+e) (\csc^2(fx+e))}{3a^2 f \sqrt{a+a\sin(fx+e)}}$$

command

`integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{135\sqrt{2} \log\left(\frac{\left| -2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) \right|}{\left| 2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) \right|}\right)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{192 \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{192 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) - \frac{192 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

---

96 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 42.26 Problem number 140

$$\int (a + b \sin(c + dx)) \tan^3(c + dx) dx$$

Optimal antiderivative

$$\frac{(2a + 3b) \ln(1 - \sin(dx + c))}{4d} + \frac{(2a - 3b) \ln(1 + \sin(dx + c))}{4d} + \frac{3b \sin(dx + c)}{2d} + \frac{(a + b \sin(dx + c)) (\tan^2(dx + c))}{2d}$$

command

```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 42.27 Problem number 161

$$\int (a + b \sin(c + dx))^3 \tan(c + dx) dx$$

Optimal antiderivative

$$-\frac{(a + b)^3 \ln(1 - \sin(dx + c))}{2d} - \frac{(a - b)^3 \ln(1 + \sin(dx + c))}{2d} - \frac{b(3a^2 + b^2) \sin(dx + c)}{d} - \frac{3ab^2(\sin^2(dx + c))}{2d} - \frac{b^3(\sin^3(dx + c))}{3d}$$

command

```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43 Test file number 73

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/73\_4.1.2.1-a+b\_sin-^m-c+d\_sin-^n

#### 43.1 Problem number 49

$$\int \csc^2(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{3a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} - \frac{a^2 \cot(dx+c)}{d\sqrt{a+a\sin(dx+c)}}$$

command

```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \operatorname{asgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)}{2 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^2} \right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 43.2 Problem number 50

$$\int \csc^3(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{7a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{4d} - \frac{7a^2 \cot(dx+c)}{4d\sqrt{a+a\sin(dx+c)}} - \frac{a^2 \cot(dx+c) \csc(dx+c)}{2d\sqrt{a+a\sin(dx+c)}}$$

command

```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 7 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left(14 \operatorname{asgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}{16 d}$$

---

16 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.3 Problem number 51

$$\int \csc^4(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{11a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{8d} - \frac{11a^2 \cot(dx+c)}{8d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{11a^2 \cot(dx+c) \csc(dx+c)}{12d\sqrt{a+a\sin(dx+c)}} - \frac{a^2 \cot(dx+c) (\csc^2(dx+c))}{3d\sqrt{a+a\sin(dx+c)}} \end{aligned}$$

command

`integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 33 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left(132 \operatorname{asgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}{96 d}$$

---

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Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.4 Problem number 56

$$\int \csc(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} - \frac{14a^3 \cos(dx+c)}{3d\sqrt{a+a\sin(dx+c)}} - \frac{2a^2 \cos(dx+c)\sqrt{a+a\sin(dx+c)}}{3d}$$

command

```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 8a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3\sqrt{2}a^2 \log\left(\frac{\left| -2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}\right)}{6d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.5 Problem number 57

$$\int \csc^2(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{5a^{5/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} - \frac{a^3 \cos(dx+c)}{d\sqrt{a+a\sin(dx+c)}} - \frac{a^2 \cot(dx+c)\sqrt{a+a\sin(dx+c)}}{d}$$

command

```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 5\sqrt{2}a^2 \log\left(\frac{\left| -2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) - 8a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 43.6 Problem number 58

$$\int \csc^3(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{19a^{5/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{4d} - \frac{9a^3 \cot(dx+c)}{4d\sqrt{a+a\sin(dx+c)}} - \frac{a^2 \cot(dx+c) \csc(dx+c) \sqrt{a+a\sin(dx+c)}}{2d}$$

command

```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 19 \sqrt{2} a^2 \log \left( \left| \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} \right| \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) + \frac{4(22a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)))}{16d} \right)}{16d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.7 Problem number 59

$$\int \csc^4(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{25a^{5/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{8d} - \frac{25a^3 \cot(dx+c)}{8d\sqrt{a+a\sin(dx+c)}} - \frac{13a^3 \cot(dx+c) \csc(dx+c)}{12d\sqrt{a+a\sin(dx+c)}} - \frac{a^2 \cot(dx+c) (\csc^2(dx+c)) \sqrt{a+a\sin(dx+c)}}{3d}$$

command

```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 75 \sqrt{2} a^2 \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left(300 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)\right)}{\dots}$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.8 Problem number 60

$$\int \csc^5(c + dx)(a + a \sin(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{163a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{64d} - \frac{163a^3 \cot(dx+c)}{64d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{163a^3 \cot(dx+c) \csc(dx+c)}{96d\sqrt{a+a\sin(dx+c)}} - \frac{17a^3 \cot(dx+c) (\csc^2(dx+c))}{24d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{a^2 \cot(dx+c) (\csc^3(dx+c)) \sqrt{a+a\sin(dx+c)}}{4d} \end{aligned}$$

command

```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 489 \sqrt{2} a^2 \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left(3912 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)\right)}{\dots}$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.9 Problem number 65

$$\int \frac{\csc(c + dx)}{\sqrt{a + a \sin(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}}$$

command

```
integrate(csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{\sqrt{2} \log\left(\frac{-2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)}{2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)}\right)}{\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)} + \frac{\log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)}{\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{\log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)}{\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)} \right)}{2\sqrt{a}d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.10 Problem number 75

$$\int \frac{\csc^3(c + dx)}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{19 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{4a^{\frac{3}{2}}d} + \frac{\cot(dx+c) \csc(dx+c)}{2d(a+a\sin(dx+c))^{\frac{3}{2}}}$$

$$+ \frac{13 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}d}$$

$$+ \frac{7 \cot(dx+c)}{4ad\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) \csc(dx+c)}{ad\sqrt{a+a\sin(dx+c)}}$$

command

```
integrate(csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{19 \log \left( \frac{\left| -16 \sqrt{2} - 32 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 16 \sqrt{2} - 32 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right)}{a^{\frac{3}{2}} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)} + \frac{2 \sqrt{2} \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right)}{\left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 - 1} a^{\frac{3}{2}} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)} + \frac{2 \sqrt{2} \left( 10 \sqrt{a} \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left( 2 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 - 1} \frac{1}{8d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 43.11 Problem number 86

$$\int \frac{\sqrt{a - a \sin(e + fx)}}{\sqrt{-\sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \arcsin \left( \frac{\cos(fx+e) \sqrt{a}}{\sqrt{a - a \sin(fx + e)}} \right) \sqrt{a}}{f}$$

command

```
integrate((a-a*sin(f*x+e))^(1/2)/(-sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \sqrt{a} \arctan \left( -\frac{1}{2} \sqrt{2} \left( \sqrt{2} + \frac{2 \left( 2 \sqrt{2} - \sqrt{-\tan \left( -\frac{1}{8} \pi + \frac{1}{4} fx + \frac{1}{4} e \right)^4 + 6 \tan \left( -\frac{1}{8} \pi + \frac{1}{4} fx + \frac{1}{4} e \right)^2 - 1} \right)}{\tan \left( -\frac{1}{8} \pi + \frac{1}{4} fx + \frac{1}{4} e \right)^2 - 3} \right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.12 Problem number 89

$$\int \frac{1}{\sqrt{1-\sin(x)} \sqrt{\sin(x)}} dx$$

Optimal antiderivative

$$\operatorname{arctanh} \left( \frac{\cos(x) \sqrt{2}}{2\sqrt{1-\sin(x)} \sqrt{\sin(x)}} \right) \sqrt{2}$$

command

```
integrate(1/(1-sin(x))^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \log \left( \tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^2 - \sqrt{\tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^4 - 6 \tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^2 + 1} + 1 \right) - \log \left( \left| -\tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right) \right| \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-\sin(x)+1} \sqrt{\sin(x)}} dx$$

## 43.13 Problem number 90

$$\int \frac{1}{\sqrt{\sin(x)} \sqrt{a-a\sin(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \frac{\cos(x) \sqrt{a} \sqrt{2}}{2\sqrt{\sin(x)} \sqrt{a-a\sin(x)}} \right) \sqrt{2}}{\sqrt{a}}$$

command

```
integrate(1/sin(x)^(1/2)/(a-a*sin(x))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \log \left( \tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^2 - \sqrt{\tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^4 - 6 \tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right)^2 + 1} + 1 \right) - \log \left( \left| -\tan \left( -\frac{1}{8} \pi + \frac{1}{4} x \right) \right| \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-a\sin(x)+a} \sqrt{\sin(x)}} dx$$

### 43.14 Problem number 153

$$\int \csc(e + fx)(a + b \sin(e + fx)) dx$$

Optimal antiderivative

$$bx - \frac{a \operatorname{arctanh}(\cos(fx + e))}{f}$$

command

```
integrate(csc(f*x+e)*(a+b*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(fx + e)b + a \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e)|)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.15 Problem number 154

$$\int \csc^2(e + fx)(a + b \sin(e + fx)) dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arctanh}(\cos(fx + e))}{f} - \frac{a \cot(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2b \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e)|) + a \tan(\frac{1}{2}fx + \frac{1}{2}e) - \frac{2b \tan(\frac{1}{2}fx + \frac{1}{2}e) + a}{\tan(\frac{1}{2}fx + \frac{1}{2}e)}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.16 Problem number 155

$$\int \csc^3(e + fx)(a + b \sin(e + fx)) dx$$

Optimal antiderivative

$$-\frac{a \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{b \cot(fx + e)}{f} - \frac{a \cot(fx + e) \csc(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 4a \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + 4b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{6a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 4b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.17 Problem number 156

$$\int \csc^4(e + fx)(a + b \sin(e + fx)) dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{a \cot(fx + e)}{f} - \frac{a(\cot^3(fx + e))}{3f} - \frac{b \cot(fx + e) \csc(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 3b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 12b \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + 9a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{22b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3}{24f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.18 Problem number 161

$$\int \csc(e + fx)(a + b \sin(e + fx))^2 dx$$

Optimal antiderivative

$$2abx - \frac{a^2 \operatorname{arctanh}(\cos(fx + e))}{f} - \frac{b^2 \cos(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(fx + e)ab + a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - \frac{2b^2}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.19 Problem number 162

$$\int \csc^2(e + fx)(a + b \sin(e + fx))^2 dx$$

Optimal antiderivative

$$b^2x - \frac{2ab \operatorname{arctanh}(\cos(fx + e))}{f} - \frac{a^2 \cot(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(fx + e)b^2 + 4ab \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{4ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a^2}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 43.20 Problem number 163

$$\int \csc^3(e + fx)(a + b \sin(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{(a^2 + 2b^2) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{2ab \cot(fx + e)}{f} - \frac{a^2 \cot(fx + e) \csc(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 8ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 4(a^2 + 2b^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - \frac{6a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 12b^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} + \dots}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.21 Problem number 164

$$\int \csc^4(e + fx)(a + b \sin(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{ab \operatorname{arctanh}(\cos(fx + e))}{f} - \frac{(2a^2 + 3b^2) \cot(fx + e)}{3f} - \frac{ab \cot(fx + e) \csc(fx + e)}{f} - \frac{a^2 \cot(fx + e) (\csc^2(fx + e))}{3f}$$

command

```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 6ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 24ab \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + 9a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 12b^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.22 Problem number 165

$$\int \csc^5(e + fx)(a + b \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{(3a^2 + 4b^2) \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{2ab \cot(fx + e)}{f} - \frac{2ab(\cot^3(fx + e))}{3f} - \frac{(3a^2 + 4b^2) \cot(fx + e) \csc(fx + e)}{8f} - \frac{a^2 \cot(fx + e) (\csc^3(fx + e))}{4f}$$

command

```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 16ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 24a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 24b^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 144ab \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.23 Problem number 170

$$\int \csc(e + fx)(a + b \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{b(6a^2 + b^2)x}{2} - \frac{a^3 \operatorname{arctanh}(\cos(fx + e))}{f} - \frac{5ab^2 \cos(fx + e)}{2f} - \frac{b^2 \cos(fx + e)(a + b \sin(fx + e))}{2f}$$

command

```
integrate(csc(f*x+e)*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2a^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + (6a^2b + b^3)(fx + e) + \frac{2\left(b^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 6ab^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - b^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 6ab^2\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)^2}$$


---

$2f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.24 Problem number 171

$$\int \csc^2(e + fx)(a + b \sin(e + fx))^3 dx$$

Optimal antiderivative

$$3ab^2x - \frac{3a^2b \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{b(a^2 - b^2) \cos(fx + e)}{f} - \frac{a^2 \cot(fx + e)(a + b \sin(fx + e))}{f}$$

command

```
integrate(csc(f*x+e)^2*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(fx + e)ab^2 + 6a^2b \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) + a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \frac{2a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} + \frac{2a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{2f}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.25 Problem number 172

$$\int \csc^3(e + fx)(a + b \sin(e + fx))^3 dx$$

Optimal antiderivative

$$b^3x - \frac{a(a^2 + 6b^2) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{5a^2b \cot(fx + e)}{2f} - \frac{a^2 \cot(fx + e) \csc(fx + e)(a + b \sin(fx + e))}{2f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 8(fx + e)b^3 + 12a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 4(a^3 + 6ab^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - \frac{6a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.26 Problem number 173

$$\int \csc^4(e + fx)(a + b \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b(3a^2 + 2b^2) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{a(2a^2 + 9b^2) \cot(fx + e)}{3f} \\ & - \frac{7a^2b \cot(fx + e) \csc(fx + e)}{6f} - \frac{a^2 \cot(fx + e) (\csc^2(fx + e)) (a + b \sin(fx + e))}{3f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^4*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 9a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 9a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 36ab^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 12(3a^2b + 2b^3) \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.27 Problem number 174

$$\int \csc^5(e + fx)(a + b \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3a(a^2 + 4b^2) \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{b(2a^2 + b^2) \cot(fx + e)}{f} \\ & - \frac{3a(a^2 + 4b^2) \cot(fx + e) \csc(fx + e)}{8f} - \frac{3a^2b \cot(fx + e) (\csc^2(fx + e))}{4f} \\ & - \frac{a^2 \cot(fx + e) (\csc^3(fx + e)) (a + b \sin(fx + e))}{4f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5*(a+b*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 8a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 8a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 24ab^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 72a^2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.28 Problem number 290**

$$\int (a + a \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{256a c^5 (\cos^3 (fx + e))}{315f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{2a c^2 (\cos^3 (fx + e)) (c - c \sin (fx + e))^{\frac{3}{2}}}{9f}$$

$$+ \frac{64a c^4 (\cos^3 (fx + e))}{105f \sqrt{c - c \sin (fx + e)}} + \frac{8a c^3 (\cos^3 (fx + e)) \sqrt{c - c \sin (fx + e)}}{21f}$$

command

```
integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

---


$$\sqrt{2} (4410 a c^3 \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) - 504 a c^3 \cos(-\frac{5}{4} \pi + \frac{5}{2} f x + \frac{5}{2} e) \operatorname{sgn}(\sin(-\frac{5}{4} \pi + \frac{5}{2} f x + \frac{5}{2} e)))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.29 Problem number 291**

$$\int (a + a \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{64a c^4 (\cos^3 (fx + e))}{105f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{16a c^3 (\cos^3 (fx + e))}{35f \sqrt{c - c \sin (fx + e)}} + \frac{2a c^2 (\cos^3 (fx + e)) \sqrt{c - c \sin (fx + e)}}{7f}$$

command

```
integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

---


$$\sqrt{2} (525 a c^2 \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 35 a c^2 \cos(-\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e) \operatorname{sgn}(\sin(-\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e)))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.30 Problem number 292**

$$\int (a + a \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a c^3 (\cos^3 (fx + e))}{15f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{2a c^2 (\cos^3 (fx + e))}{5f \sqrt{c - c \sin (fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (30 a c \cos (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \operatorname{sgn}(\sin (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 5 a c \cos (-\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e) \operatorname{sgn}(\sin (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{30 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.31 Problem number 293**

$$\int (a + a \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a c^2 (\cos^3 (fx + e))}{3f (c - c \sin (fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (3 a \cos (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) \operatorname{sgn}(\sin (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + a \cos (-\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e) \operatorname{sgn}(\sin (-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.32 Problem number 294

$$\int \frac{a + a \sin(e + fx)}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2a \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{c}} - \frac{2a \cos(fx+e)}{f\sqrt{c-c\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a \log\left(-\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{\sqrt{c} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{4\sqrt{2} a}{\sqrt{c} \left(\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.33 Problem number 295

$$\int \frac{a + a \sin(e + fx)}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{2c^{\frac{3}{2}}f}$$

command

`integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2} a \log\left(-\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{c^{\frac{3}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{\sqrt{2} a (\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1)}{c^{\frac{3}{2}} (\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2} \left(a\sqrt{c} + \frac{2a\sqrt{c} (\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}\right)}{c^2 (\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.34 Problem number 296

$$\int \frac{a + a \sin(e + fx)}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{2f(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a \cos(fx + e)}{8cf(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{16c^{\frac{5}{2}}f}$$

command

`integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}a \log\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c^{\frac{5}{2}} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2}a(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)^2}{c^{\frac{5}{2}}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\sqrt{2}\left(a\sqrt{c} - \frac{2a\sqrt{c}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right) \sqrt{2}}{128f c^3(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.35 Problem number 297

$$\int \frac{a + a \sin(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{3f(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{a \cos(fx + e)}{24cf(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a \cos(fx + e)}{32c^2f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{64c^{\frac{7}{2}}f}$$

command

`integrate((a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{12\sqrt{2} a \log\left(-\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{c^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(a\sqrt{c} - \frac{3a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} - \frac{3a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2} + \frac{22a\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}\right)}{c^4\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 43.36 Problem number 298

$$\int (a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{256a^2c^6(\cos^5(fx + e))}{1155f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{64a^2c^5(\cos^5(fx + e))}{231f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{8a^2c^4(\cos^5(fx + e))}{33f\sqrt{c - c \sin(fx + e)}} + \frac{2a^2c^3(\cos^5(fx + e))\sqrt{c - c \sin(fx + e)}}{11f}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\left(16170a^2c^3\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 2310a^2c^3\cos\left(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e\right)\right)\right)}{c^4\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.37 Problem number 299

$$\int (a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{64a^2c^5(\cos^5(fx + e))}{315f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{16a^2c^4(\cos^5(fx + e))}{63f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{2a^2c^3(\cos^5(fx + e))}{9f\sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 1890 a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 420 a^2 c^2 \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.38 Problem number 300

$$\int (a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a^2c^4(\cos^5(fx + e))}{35f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{2a^2c^3(\cos^5(fx + e))}{7f(c - c \sin(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 105 a^2 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 35 a^2 c \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.39 Problem number 301

$$\int (a + a \sin(e + fx))^2 \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^2c^3(\cos^5(fx + e))}{5f(c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^2*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 10 a^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 5 a^2 \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right) \right)}{10 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.40 Problem number 302

$$\int \frac{(a + a \sin(e + fx))^2}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2a^2c(\cos^3(fx+e))}{3f(c-c\sin(fx+e))^{\frac{3}{2}}} + \frac{4a^2 \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}} \right) \sqrt{2}}{f\sqrt{c}} - \frac{4a^2 \cos(fx+e)}{f\sqrt{c-c\sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{3\sqrt{2} a^2 \log \left( -\frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}{\sqrt{c} \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)} - \frac{8\sqrt{2} \left( 2a^2\sqrt{c} - \frac{3a^2\sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} + \frac{3a^2\sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right)^2}{\left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right)^2} \right)}{c \left( \frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} - 1 \right)^3 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)} \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.41 Problem number 303

$$\int \frac{(a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a^2 c (\cos^3(fx + e))}{f (c - c \sin(fx + e))^{5/2}} - \frac{3a^2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{c^2 f} + \frac{3a^2 \cos(fx + e)}{cf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6\sqrt{2}a^2 \log\left(\frac{-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\sqrt{2}a^2(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{c^2(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)\operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2}\left(a^2\sqrt{c} - \frac{14a^2\sqrt{c}\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}\right)}{c^2\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} - \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}\right)}$$


---

$4f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.42 Problem number 304

$$\int \frac{(a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a^2 c (\cos^3(fx + e))}{2f (c - c \sin(fx + e))^{7/2}} - \frac{3a^2 \cos(fx + e)}{4cf (c - c \sin(fx + e))^{3/2}} + \frac{3a^2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{8c^2 f}$$

command

`integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12\sqrt{2}a^2 \log\left(\frac{-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2}\left(a^2\sqrt{c} + \frac{8a^2\sqrt{c}\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + \frac{18a^2\sqrt{c}\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)^2}\right)}{c^3(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.43 Problem number 305

$$\int \frac{(a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 c (\cos^3 (fx + e))}{3f (c - c \sin (fx + e))^{\frac{9}{2}}} - \frac{a^2 \cos (fx + e)}{4cf (c - c \sin (fx + e))^{\frac{5}{2}}} \\ & + \frac{a^2 \cos (fx + e)}{16c^2 f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{a^2 \operatorname{arctanh} \left( \frac{\cos (fx + e) \sqrt{c} \sqrt{2}}{2\sqrt{c - c \sin (fx + e)}} \right) \sqrt{2}}{32c^{\frac{7}{2}} f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12\sqrt{2} a^2 \log \left( -\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)}{c^{\frac{7}{2}} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\sqrt{2} \left( a^2 \sqrt{c} + \frac{3a^2 \sqrt{c} (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} - \frac{3a^2 \sqrt{c} (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)^2}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)^2} - \frac{22a^2 \sqrt{c}}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)^3} \right)}{c^4 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.44 Problem number 306

$$\int \frac{(a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 c (\cos^3 (fx + e))}{4f (c - c \sin (fx + e))^{\frac{11}{2}}} - \frac{a^2 \cos (fx + e)}{8cf (c - c \sin (fx + e))^{\frac{7}{2}}} + \frac{a^2 \cos (fx + e)}{64c^2 f (c - c \sin (fx + e))^{\frac{5}{2}}} \\ & + \frac{3a^2 \cos (fx + e)}{256c^3 f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{3a^2 \operatorname{arctanh} \left( \frac{\cos (fx + e) \sqrt{c} \sqrt{2}}{2\sqrt{c - c \sin (fx + e)}} \right) \sqrt{2}}{512c^{\frac{9}{2}} f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12\sqrt{2} a^2 \log\left(\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)^2}{c^{\frac{9}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(a^2\sqrt{c} - \frac{8a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2} + \frac{18a^2\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^4}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^4}\right)\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{c^5\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.45 Problem number 307

$$\int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{256a^3c^7(\cos^7(fx + e))}{3003f(c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{64a^3c^6(\cos^7(fx + e))}{429f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{24a^3c^5(\cos^7(fx + e))}{143f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{2a^3c^4(\cos^7(fx + e))}{13f\sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\left(60060a^3c^3\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 15015a^3c^3\cos\left(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e\right)\right)\right)}{c^5\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.46 Problem number 308

$$\int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{64a^3c^6(\cos^7(fx + e))}{693f(c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{16a^3c^5(\cos^7(fx + e))}{99f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{2a^3c^4(\cos^7(fx + e))}{11f(c - c \sin(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 6930 a^3 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 2310 a^3 c^2 \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 43.47 Problem number 309

$$\int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a^3c^5(\cos^7(fx + e))}{63f(c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{2a^3c^4(\cos^7(fx + e))}{9f(c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 378 a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 168 a^3 c \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 43.48 Problem number 310

$$\int (a + a \sin(e + fx))^3 \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^3c^4(\cos^7(fx + e))}{7f(c - c \sin(fx + e))^{\frac{7}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 35 a^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 21 a^3 \cos \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{15 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.49 Problem number 311

$$\int \frac{(a + a \sin(e + fx))^3}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^3c^2(\cos^5(fx+e))}{5f(c-c\sin(fx+e))^{\frac{5}{2}}} - \frac{4a^3c(\cos^3(fx+e))}{3f(c-c\sin(fx+e))^{\frac{3}{2}}} \\ & + \frac{8a^3 \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}} \right) \sqrt{2}}{f\sqrt{c}} - \frac{8a^3 \cos(fx+e)}{f\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( \frac{15 \sqrt{2} a^3 \log \left( -\frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}{\sqrt{c} \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)} - \frac{4 \sqrt{2} \left( 23 a^3 \sqrt{c} - \frac{70 a^3 \sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} + \frac{140 a^3 \sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right)^2}{\left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right)^2} \right)}{15 f} + \frac{c \left( \frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} - 1 \right)^5 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{15 f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 43.50 Problem number 312

$$\int \frac{(a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a^3 c^2 (\cos^5 (fx + e))}{f (c - c \sin (fx + e))^{\frac{7}{2}}} + \frac{5a^3 (\cos^3 (fx + e))}{3f (c - c \sin (fx + e))^{\frac{3}{2}}} - \frac{10a^3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{c^{\frac{3}{2}}f} + \frac{10a^3 \cos (fx + e)}{cf \sqrt{c - c \sin (fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{30\sqrt{2}a^3 \log\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c^{\frac{3}{2}} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{3\sqrt{2}a^3 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{c^{\frac{3}{2}} (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{3\sqrt{2}\left(a^3 + \frac{10a^3 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c^{\frac{3}{2}} (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}$$

6f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.51 Problem number 313

$$\int \frac{(a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a^3 c^2 (\cos^5 (fx + e))}{2f (c - c \sin (fx + e))^{\frac{9}{2}}} - \frac{5a^3 (\cos^3 (fx + e))}{4f (c - c \sin (fx + e))^{\frac{5}{2}}} + \frac{15a^3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{4c^{\frac{5}{2}}f} - \frac{15a^3 \cos (fx + e)}{4c^2 f \sqrt{c - c \sin (fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{60\sqrt{2}a^3\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{c^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{128\sqrt{2}a^3}{c^{\frac{5}{2}}\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(a^3\sqrt{c} + \frac{16a^3\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}\right)}{c^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.52 Problem number 315

$$\int \frac{(a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{a^3c^2(\cos^5(fx+e))}{4f(c-c\sin(fx+e))^{\frac{13}{2}}} - \frac{5a^3(\cos^3(fx+e))}{24f(c-c\sin(fx+e))^{\frac{9}{2}}} + \frac{5a^3\cos(fx+e)}{32c^2f(c-c\sin(fx+e))^{\frac{5}{2}}} - \frac{5a^3\cos(fx+e)}{128c^3f(c-c\sin(fx+e))^{\frac{3}{2}}} - \frac{5a^3\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{256c^{\frac{9}{2}}f}$$

command

`integrate((a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.53 Problem number 317

$$\int \frac{(c - c \sin(e + fx))^{7/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{64c^2 \sec(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{5af} + \frac{8c \sec(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{5af}$$

$$+ \frac{2 \sec(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{5af} - \frac{256c^3 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{5af}$$

command

```
integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16\sqrt{2} \left( \frac{5c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{a \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + 1 \right)} - \frac{11c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - \frac{50c^3 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + \frac{80c^3 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.54 Problem number 318

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{16c \sec(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{3af} + \frac{2 \sec(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{3af}$$

$$- \frac{64c^2 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{3af}$$

command

```
integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$8\sqrt{2}\sqrt{c} \left( \frac{3c^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a \left( \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} + 1 \right)} - \frac{5c^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \frac{12c^2 \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} + \frac{3c^2}{a \left( \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} - 1 \right)^3} \right)$$


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$3f$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.55 Problem number 319

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \sec(fx + e) (c - c \sin(fx + e))^{3/2}}{af} - \frac{8c \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{af}$$

command

`integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\sqrt{2}c^{3/2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{af \left( \frac{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2} - 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.56 Problem number 320

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{af}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{c}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{af\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.57 Problem number 321

$$\int \frac{1}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{2af\sqrt{c}} - \frac{\sec(fx+e)\sqrt{c-c\sin(fx+e)}}{acf}$$

command

`integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a\sqrt{c}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\sqrt{2}}{af\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.58 Problem number 322

$$\int \frac{1}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 \cos(fx + e)}{4af(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{8ac^{\frac{3}{2}}f} - \frac{\sec(fx + e)}{acf\sqrt{c - c \sin(fx + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6\sqrt{2} \log\left(-\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{ac^{\frac{3}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2} \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)}{ac^{\frac{3}{2}} \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{\sqrt{2} \left(\sqrt{c} + \frac{14\sqrt{c} \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{ac^2 \left(\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} + \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}\right)}$$


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32 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.59 Problem number 323

$$\int \frac{1}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{15 \cos(fx + e)}{32acf(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{\sec(fx + e)}{4acf(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{15 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{64ac^{\frac{5}{2}}f} - \frac{5 \sec(fx + e)}{8ac^2f\sqrt{c - c \sin(fx + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{60\sqrt{2}\log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^{\frac{5}{2}}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(\sqrt{c}-\frac{16\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{90\sqrt{c}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{ac^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.60 Problem number 324

$$\int \frac{(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4096c^3(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{15a^2f} \\ & - \frac{1024c^2(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{5a^2f} + \frac{128c(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{5a^2f} \\ & + \frac{32(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{9}{2}}}{15a^2f} + \frac{2(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{11}{2}}}{5a^2cf} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$16\sqrt{2}\sqrt{c}\left(\frac{5\left(11c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{24c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+\frac{9c^4\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}+1\right)^3}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.61 Problem number 325

$$\int \frac{(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{256c^2(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{3a^2f} - \frac{64c(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{a^2f} + \frac{8(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{a^2f} + \frac{2(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{9}{2}}}{3a^2cf}$$

command

`integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{128\sqrt{2}\left(c^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \frac{3c^3\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2}\right)\sqrt{c}}{3a^2f\left(\frac{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2} - 1\right)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.62 Problem number 326

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{64c(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{3a^2f} - \frac{16(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{a^2f} + \frac{2(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{a^2cf}$$

command

`integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output



$$4\sqrt{2}\sqrt{c} \left( \frac{3c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{a^2 \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} - 1 \right)} - \frac{5c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{12c^2(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + \frac{3c^2}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}}{a^2 \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + 1 \right)^3} \right)$$


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$3f$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.63 Problem number 327

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{8(\sec^3(fx + e))(c - c \sin(fx + e))^{3/2}}{3a^2f} - \frac{2(\sec^3(fx + e))(c - c \sin(fx + e))^{5/2}}{a^2cf}$$

command

`integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2} \left( c \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{3c(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right) \sqrt{c}}{3a^2f \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + 1 \right)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.64 Problem number 328

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{2(\sec^3(fx + e))(c - c \sin(fx + e))^{3/2}}{3a^2cf}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c} \left( \frac{3 (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^2} + \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \right)}{3 a^2 f \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + 1 \right)^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.65 Problem number 329

$$\int \frac{1}{(a + a \sin(e + f x))^2 \sqrt{c - c \sin(e + f x)}} dx$$

Optimal antiderivative

$$-\frac{(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{3a^2c^2f} + \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{4a^2f\sqrt{c}} - \frac{\sec(fx + e)\sqrt{c - c \sin(fx + e)}}{2a^2cf}$$

command

`integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \sqrt{2} \log\left(-\frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1}\right)}{a^2 \sqrt{c} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{8 \left( 2 \sqrt{2} + \frac{3 \sqrt{2} (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + \frac{3 \sqrt{2} (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^2}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^2} \right)}{a^2 \sqrt{c} \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + 1 \right)^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.66 Problem number 330

$$\int \frac{1}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{5 \cos(fx + e)}{8a^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{5 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{16a^2 c^{\frac{3}{2}} f}$$

$$- \frac{5 \sec(fx + e)}{6a^2 c f \sqrt{c - c \sin(fx + e)}} - \frac{(\sec^3(fx + e) \sqrt{c - c \sin(fx + e)})}{3a^2 c^2 f}$$

command

`integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2} \left( \frac{10 \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} - 1 \right) \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)}{a^2 c^{\frac{3}{2}} \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{30\sqrt{2} \log\left(-\frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1}\right)}{a^2 c^{\frac{3}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{3\sqrt{2} \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right)}{a^2 c^{\frac{3}{2}} \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

192 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.67 Problem number 331

$$\int \frac{1}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{35 \cos(fx + e)}{64a^2 c f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{7 \sec(fx + e)}{24a^2 c f (c - c \sin(fx + e))^{\frac{3}{2}}}$$

$$+ \frac{35 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{128a^2 c^{\frac{5}{2}} f}$$

$$- \frac{35 \sec(fx + e)}{48a^2 c^2 f \sqrt{c - c \sin(fx + e)}} - \frac{\sec^3(fx + e)}{3a^2 c^2 f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{420 \sqrt{2} \log\left(-\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{a^2 c^{\frac{5}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3 \sqrt{2} \left( \sqrt{c} - \frac{24 \sqrt{c} \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1} + \frac{210 \sqrt{c} \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2} \right) \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2 c^3 \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.68 Problem number 332

$$\int \frac{(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4096c^2(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{15a^3f} \\ & + \frac{1024c(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{3a^3f} - \frac{128(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{9}{2}}}{a^3f} \\ & + \frac{32(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{11}{2}}}{3a^3cf} + \frac{2(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{13}{2}}}{3a^3c^2f} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$8 \sqrt{2} \sqrt{c} \left( \frac{5 \left( 11 c^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) - \frac{24 c^4 \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1} + \frac{9 c^4 \left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2} \right)}{a^3 \left( \frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1} - 1 \right)^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.69 Problem number 333

$$\int \frac{(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{256c(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{5a^3f} + \frac{64(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{a^3f} \\ & - \frac{24(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{9}{2}}}{a^3cf} + \frac{2(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{11}{2}}}{a^3c^2f} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4\sqrt{2}\sqrt{c}\left(\frac{5c^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)} - \frac{11c^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + \frac{50c^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}}{\dots}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.70 Problem number 334

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{64(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{15a^3f} + \frac{16(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{7}{2}}}{3a^3cf} \\ & - \frac{2(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{9}{2}}}{a^3c^2f} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16\sqrt{2} \left( c^2 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + \frac{5c^2 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + \frac{10c^2 (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)}{15a^3 f \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.71 Problem number 335

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{8(\sec^5(fx + e))(c - c \sin(fx + e))^{5/2}}{15a^3 c f} - \frac{2(\sec^5(fx + e))(c - c \sin(fx + e))^{7/2}}{3a^3 c^2 f}$$

command

`integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2} \left( c \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + \frac{5c (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} - \frac{5c (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)}{15a^3 f \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.72 Problem number 336

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$-\frac{2(\sec^5(fx + e))(c - c \sin(fx + e))^{5/2}}{5a^3 c^2 f}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c} \left( \frac{10 (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^2} + \frac{5 (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^4 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^4} + \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \right)}{10 a^3 f \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + 1 \right)^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.73 Problem number 337

$$\int \frac{1}{(a + a \sin(e + fx))^3 \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(\sec^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{6a^3 c^2 f} - \frac{(\sec^5(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{5a^3 c^3 f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\cos(fx + e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right)\sqrt{2}}{8a^3 f \sqrt{c}} - \frac{\sec(fx + e)\sqrt{c - c \sin(fx + e)}}{4a^3 c f} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 \sqrt{2} \log\left(\frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1}\right)}{a^3 \sqrt{c} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{4 \sqrt{2} \left( 23 \sqrt{c} + \frac{70 \sqrt{c} (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + \frac{140 \sqrt{c} (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^2}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^2} + \frac{90 \sqrt{c} (\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)^3}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)^3} \right)}{a^3 c \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1} + 1 \right)^5 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

240 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.74 Problem number 338

$$\int \frac{1}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7 \cos(fx + e)}{16a^3 f (c - c \sin(fx + e))^{3/2}} - \frac{(\sec^5(fx + e)) (c - c \sin(fx + e))^{3/2}}{5a^3 c^3 f} \\ & + \frac{7 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right) \sqrt{2}}{32a^3 c^{3/2} f} - \frac{7 \sec(fx + e)}{12a^3 c f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{7(\sec^3(fx + e)) \sqrt{c - c \sin(fx + e)}}{30a^3 c^2 f} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 \sqrt{2} \left( \frac{14 \left( \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) - 1\right)}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) + 1} - 1 \right) \left( \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) + 1 \right)}{a^3 c^{3/2} \left( \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) - 1 \right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)} - \frac{210 \sqrt{2} \log\left(-\frac{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) - 1}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) + 1}\right)}{a^3 c^{3/2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)} + \frac{15 \sqrt{2} \left( \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) + 1 \right)}{a^3 c^{3/2} \left( \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right) + 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.75 Problem number 339

$$\int \frac{1}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{63 \cos(fx + e)}{128a^3 c f (c - c \sin(fx + e))^{3/2}} + \frac{21 \sec(fx + e)}{80a^3 c f (c - c \sin(fx + e))^{3/2}} \\ & + \frac{63 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right) \sqrt{2}}{256a^3 c^{5/2} f} - \frac{21 \sec(fx + e)}{32a^3 c^2 f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{3(\sec^3(fx + e))}{10a^3 c^2 f \sqrt{c - c \sin(fx + e)}} - \frac{(\sec^5(fx + e)) \sqrt{c - c \sin(fx + e)}}{5a^3 c^3 f} \end{aligned}$$



command

```
integrate(1/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 43.76 Problem number 340

$$\int \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$-\frac{a \cos(fx + e) (c - c \sin(fx + e))^{7/2}}{4f \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \sqrt{a} c^{7/2} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^8}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.77 Problem number 341

$$\int \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$-\frac{a \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{3f \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\sqrt{a}c^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.78 Problem number 342

$$\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}dx$$

Optimal antiderivative

$$-\frac{a\cos(fx+e)(c-c\sin(fx+e))^{\frac{3}{2}}}{2f\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a}c^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.79 Problem number 343

$$\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}dx$$

Optimal antiderivative

$$-\frac{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}}{f\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a}\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.80 Problem number 344

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{a \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a} \log\left(\left|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{c} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.81 Problem number 345

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{f (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{a} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{2c^{\frac{3}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.82 Problem number 346

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{2f (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{a} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{8c^{\frac{5}{2}}f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.83 Problem number 347

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)}{3f (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{a} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{24c^{\frac{7}{2}}f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.84 Problem number 348

$$\int (a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{10f \sqrt{a + a \sin(fx + e)}} - \frac{a \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{5f}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 4ac^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{10} - 5ac^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)}{5f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.85 Problem number 349

$$\int (a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{6f \sqrt{a + a \sin(fx + e)}} - \frac{a \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{4f}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3ac^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 8ac^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.86 Problem number 350**

$$\int (a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{3f \sqrt{a + a \sin(fx + e)}} - \frac{a \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}{3f}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 2ac \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 3ac \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.87 Problem number 351**

$$\int (a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2f \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^{\frac{3}{2}} \sqrt{c} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.88 Problem number 352

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{2a^2 \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^{\frac{3}{2}} \sqrt{c} \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{\operatorname{csgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{\operatorname{csgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.89 Problem number 353

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{a^2 \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 2a \log(|\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2} \right) \sqrt{a}}{c^{\frac{3}{2}} f \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.90 Problem number 354

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{4f (c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right) \sqrt{a}}{4c^3 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.91 Problem number 355

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{6f (c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{24cf (c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(3a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 2a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right) \sqrt{a}}{24c^{\frac{7}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 43.92 Problem number 356

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$-\frac{a^2 \cos(fx + e)}{12cf(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{4f(c - c \sin(fx + e))^{\frac{9}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 3a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right) \sqrt{a}}{96c^5 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.93 Problem number 357

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$-\frac{a^2 \cos(fx + e)}{20cf(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{5f(c - c \sin(fx + e))^{\frac{11}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(5a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 4a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right) \sqrt{a}}{320c^{\frac{11}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{10}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.94 Problem number 358

$$\int (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{6f} \\ & - \frac{a^3 \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16 \left( 10 a^2 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - 36 a^2 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.95 Problem number 359

$$\int (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{5f} \\ & - \frac{2a^3 \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 6 a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 15 a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.96 Problem number 360

$$\int (a + a \sin(e + f x))^{5/2} (c - c \sin(e + f x))^{3/2} dx$$

Optimal antiderivative

$$\frac{c^2 \cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}}}{6 f \sqrt{c - c \sin(f x + e)}} + \frac{c \cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}} \sqrt{c - c \sin(f x + e)}}{4 f}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3 a^2 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^8 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 4 a^2 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.97 Problem number 361

$$\int (a + a \sin(e + f x))^{5/2} \sqrt{c - c \sin(e + f x)} dx$$

Optimal antiderivative

$$\frac{c \cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}}}{3 f \sqrt{c - c \sin(f x + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 a^{\frac{5}{2}} \sqrt{c} \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^6 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.98 Problem number 362

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2f \sqrt{c - c \sin(fx + e)}} - \frac{4a^3 \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{2a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 a^{\frac{5}{2}} \sqrt{c} \left( \frac{c \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^4 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 2 c \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{c^2} + \frac{2 \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{c \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right) \frac{1}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.99 Problem number 363

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{4a^3 \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{2a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 a^{\frac{5}{2}} \sqrt{c} \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} + \frac{2 \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 1)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \frac{1}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 - 1) c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right) \frac{1}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.100 Problem number 364

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2f (c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf (c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a^3 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4 a^2 \log \left(\left|\sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right|\right) \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + \frac{4 a^2 \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2}{\sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^4}\right)}{2 c^{\frac{5}{2}} f \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.101 Problem number 365

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{6f (c - c \sin(fx + e))^{\frac{7}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(3 a^2 \sqrt{c} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^4 - 3 a^2 \sqrt{c} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{6 c^4 f \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin \left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.102 Problem number 366

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{8f(c - c \sin(fx + e))^{9/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{48cf(c - c \sin(fx + e))^{7/2}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(6a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 8a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{48c^2 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.103 Problem number 367

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{10f(c - c \sin(fx + e))^{11/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{40cf(c - c \sin(fx + e))^{9/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{240c^2 f(c - c \sin(fx + e))^{7/2}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(10a^2 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15a^2 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{240c^6 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.104 Problem number 368

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{6f(c - c \sin(fx + e))^{\frac{13}{2}}} + \frac{a^3 \cos(fx + e)}{60c^2 f (c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}} - \frac{a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15cf (c - c \sin(fx + e))^{\frac{11}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(15 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^4 - 24 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{960 c^{\frac{13}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.105 Problem number 369

$$\int (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{9/2} dx$$

Optimal antiderivative

$$\frac{3a^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{9}{2}}}{28f} - \frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{9}{2}}}{8f} - \frac{a^4 \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}}}{35f \sqrt{a + a \sin(fx + e)}} - \frac{a^3 \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}}{14f}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

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$$32 \left( 35 a^3 c^4 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{16} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 160 a^3 c^4 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)$$


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Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.106 Problem number 370

$$\int (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{7f} \\ & - \frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{7f} \\ & - \frac{2a^4 \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{35f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^3 \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{35f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

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$$32 \left( 20 a^3 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 70 a^3 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)$$


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Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



**43.107 Problem number 371**

$$\int (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \cos(fx + e) (a + a \sin(fx + e))^{7/2} (c - c \sin(fx + e))^{3/2}}{6f} \\ & + \frac{c^3 \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{15f \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2c^2 \cos(fx + e) (a + a \sin(fx + e))^{7/2} \sqrt{c - c \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 10 a^3 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - 24 a^3 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**43.108 Problem number 372**

$$\int (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{c^2 \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{10f \sqrt{c - c \sin(fx + e)}} + \frac{c \cos(fx + e) (a + a \sin(fx + e))^{7/2} \sqrt{c - c \sin(fx + e)}}{5f}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 4 a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - 5 a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)}{5f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.109 Problem number 373

$$\int (a + a \sin(e + fx))^{7/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{4f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^{7/2} \sqrt{c} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.110 Problem number 374

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a^2 \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{f \sqrt{c - c \sin(fx + e)}} - \frac{a \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{3f \sqrt{c - c \sin(fx + e)}} - \frac{8a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{4a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^{7/2} \sqrt{c} \left( \frac{6 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{2c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 + 3c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 + 6c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}{c^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.111 Problem number 375

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{f (c - c \sin(fx + e))^{3/2}} + \frac{3a^2 \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2cf \sqrt{c - c \sin(fx + e)}} \\ + \frac{12a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{6a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2a^{7/2} \sqrt{c} \left( \frac{6 \log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{c^2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^4 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 4c^2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c^4} \right) f$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.112 Problem number 376

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{2f (c - c \sin(fx + e))^{5/2}} - \frac{3a^2 \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2cf (c - c \sin(fx + e))^{3/2}} \\ - \frac{6a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{3a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a^{7/2} \sqrt{c} \left( \frac{2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{6 \log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{6 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 5}{(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1)^2 c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) f$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.113 Problem number 377

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{3f (c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{a^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2cf (c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{\frac{7}{2}} \sqrt{c} \left( \frac{6 \sqrt{2} \log(-2 \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 2)}{c^4 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \frac{\sqrt{2} (18 \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^4 - 27 \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 11)}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 - 1)^3 c^4 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right) \operatorname{sgn}(\cos(fx + e))}{12 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.114 Problem number 378

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{7}{2}}}{8f (c - c \sin(fx + e))^{\frac{9}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^6 - 6 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^4 + 4 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 - 4 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{8 c^{\frac{9}{2}} f \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.115 Problem number 379

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{10f(c - c \sin(fx + e))^{11/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{80cf(c - c \sin(fx + e))^{9/2}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(10 a^3 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 - 20 a^3 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6\right)}{80 c^6 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.116 Problem number 380

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{12f(c - c \sin(fx + e))^{13/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{60cf(c - c \sin(fx + e))^{11/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{480c^2f(c - c \sin(fx + e))^{9/2}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(20 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 - 45 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6\right)}{480 c^{\frac{13}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.117 Problem number 381

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{15/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{14f(c - c \sin(fx + e))^{15/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{56cf(c - c \sin(fx + e))^{13/2}} \\ + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{280c^2f(c - c \sin(fx + e))^{11/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{2240c^3f(c - c \sin(fx + e))^{9/2}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(35 a^3 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 - 84 a^3 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^5\right)}{2240 c^8 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.118 Problem number 382

$$\int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{17/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e)(a + a \sin(fx + e))^{5/2}}{8f(c - c \sin(fx + e))^{17/2}} - \frac{3a^2 \cos(fx + e)(a + a \sin(fx + e))^{3/2}}{56cf(c - c \sin(fx + e))^{15/2}} \\ - \frac{a^4 \cos(fx + e)}{280c^3f(c - c \sin(fx + e))^{11/2} \sqrt{a + a \sin(fx + e)}} + \frac{a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{56c^2f(c - c \sin(fx + e))^{13/2}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(56 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 - 140 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^5\right)}{8960 c^{17/2} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.119 Problem number 383

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2f \sqrt{a + a \sin(fx + e)}} + \frac{4c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{2c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \sqrt{a} c^{\frac{5}{2}} \left( \frac{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^4 + 2 \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{a^2} + \frac{2 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.120 Problem number 384

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2c^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \sqrt{a} c^{\frac{3}{2}} \left( \frac{\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.121 Problem number 385

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c} \left( \frac{\sqrt{2} \log\left(\left| \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1} - \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right| \right) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sqrt{a} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2} \log\left(\left| \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right| \right)}{\sqrt{c}} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.122 Problem number 387

$$\int \frac{1}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{2f (c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} + \frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \left( \frac{\log(-64 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 64)}{\sqrt{a} c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2 \log(|\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|)}{\sqrt{a} c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\sqrt{c}}{2f} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{3/2}} dx$$



## 43.123 Problem number 388

$$\int \frac{1}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{4f(c - c \sin(fx + e))^{5/2} \sqrt{a + a \sin(fx + e)}} + \frac{\cos(fx + e)}{4cf(c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} \\ + \frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{4c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \left( \frac{2 \log(-256 \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 256)}{\sqrt{a} c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \frac{4 \log(|\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)|)}{\sqrt{a} c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} + \frac{\sqrt{c}}{16f} \right)}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{5/2}} dx$$

## 43.124 Problem number 389

$$\int \frac{(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{f (a + a \sin(fx + e))^{3/2}} - \frac{3c^2 \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2af \sqrt{a + a \sin(fx + e)}} \\ - \frac{12c^4 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{6c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \sqrt{a} c^{\frac{7}{2}} \left( \frac{6 \log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 + 4a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{a^4} \right)$$


---

$f$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.125 Problem number 390

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{f (a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{4c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \sqrt{a} c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 4 \sqrt{a} c^2 \log\left(\left|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)}{a^2 f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.126 Problem number 391

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f (a + a \sin(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} c^{\frac{3}{2}} \left( \frac{\sqrt{2} \log\left(-2 \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 + 2\right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{\sqrt{2}}{\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)^2 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} \right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.127 Problem number 392

$$\int \frac{\sqrt{c - c \sin(e + f x)}}{(a + a \sin(e + f x))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c \cos(f x + e)}{f (a + a \sin(f x + e))^{\frac{3}{2}} \sqrt{c - c \sin(f x + e)}}$$

command

```
integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{2 a^{\frac{3}{2}} f \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.128 Problem number 393

$$\int \frac{1}{(a + a \sin(e + f x))^{3/2} \sqrt{c - c \sin(e + f x)}} dx$$

Optimal antiderivative

$$-\frac{\cos(f x + e)}{2 f (a + a \sin(f x + e))^{\frac{3}{2}} \sqrt{c - c \sin(f x + e)}} + \frac{\operatorname{arctanh}(\sin(f x + e)) \cos(f x + e)}{2 a f \sqrt{a + a \sin(f x + e)} \sqrt{c - c \sin(f x + e)}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{a} \sqrt{c} \left( \frac{\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^2 \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^2 \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{1}{a^2 c \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)} \right) \frac{1}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(a \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

### 43.129 Problem number 395

$$\int \frac{1}{(a + a \sin(e + fx))^{\frac{3}{2}} (c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)}{2f(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{3 \cos(fx + e)}{8af(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{3 \cos(fx + e)}{8acf(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{3 \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8a^2 c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{a} \sqrt{c} \left( \frac{6 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^2 c^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{12 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^2 c^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)} \right) \frac{1}{32f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

## 43.130 Problem number 396

$$\int \frac{(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c^2 \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{af (a + a \sin(fx + e))^{3/2}} - \frac{c \cos(fx + e) (c - c \sin(fx + e))^{7/2}}{2f (a + a \sin(fx + e))^{5/2}} \\ & + \frac{3c^3 \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{a^2 f \sqrt{a + a \sin(fx + e)}} + \frac{24c^5 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{12c^4 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{a}c^{\frac{9}{2}}\left(\frac{12\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+6a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^6}\right)$$


---

*f*

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.131 Problem number 397

$$\int \frac{(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3c^2 \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2af (a + a \sin(fx + e))^{3/2}} - \frac{c \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{2f (a + a \sin(fx + e))^{5/2}} \\ & + \frac{6c^4 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{3c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} c^{\frac{7}{2}} \left( \frac{2 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{6 \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} - \frac{6 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 5}{(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 1)^2 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.132 Problem number 398

$$\int \frac{(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{2f (a + a \sin(fx + e))^{\frac{5}{2}}} + \frac{c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af (a + a \sin(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} c^{\frac{5}{2}} \left( \frac{2 \sqrt{2} \log(-2 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 + 2)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} - \frac{\sqrt{2} (4 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 3)}{(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 - 1)^2 a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.133 Problem number 399

$$\int \frac{(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)(c - c \sin(fx + e))^{\frac{3}{2}}}{4f(a + a \sin(fx + e))^{\frac{5}{2}}}$$

command

`integrate((c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(2\sqrt{a}c\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \sqrt{a}c\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)\sqrt{c}}{4a^3f\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.134 Problem number 400

$$\int \frac{\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{c \cos(fx + e)}{2f(a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{8a^{\frac{5}{2}}f\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.135 Problem number 401

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)}{4f(a + a \sin(fx + e))^{5/2} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{\cos(fx + e)}{4af(a + a \sin(fx + e))^{3/2} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{4a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \left( \frac{2 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^{5/2} \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{4 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^{5/2} \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{5}{a^{5/2} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)} \right)}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(a \sin(fx + e) + a)^{5/2} \sqrt{-c \sin(fx + e) + c}} dx$$

## 43.136 Problem number 402

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)}{4f(a + a \sin(fx + e))^{5/2} (c - c \sin(fx + e))^{3/2}} \\ & - \frac{3 \cos(fx + e)}{8af(a + a \sin(fx + e))^{3/2} (c - c \sin(fx + e))^{3/2}} \\ & + \frac{3 \cos(fx + e)}{8a^2 f (c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{3 \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8a^2 c f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$



command

```
integrate(1/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{c} \left( \frac{6 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^{\frac{5}{2}}c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{12 \log\left(\left|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)}{a^{\frac{5}{2}}c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)} \right) \frac{1}{32f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

### 43.137 Problem number 522

$$\int \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{12d^2(c+d) \cos(fx+e) (a+a \sin(fx+e))^{\frac{3}{2}}}{35af} - \frac{4a(c+d) (15c^2+10cd+7d^2) \cos(fx+e)}{35f \sqrt{a+a \sin(fx+e)}} \\ & - \frac{2a \cos(fx+e) (c+d \sin(fx+e))^3}{7f \sqrt{a+a \sin(fx+e)}} - \frac{8(5c-d)d(c+d) \cos(fx+e) \sqrt{a+a \sin(fx+e)}}{35f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 5d^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{7}{4}\pi + \frac{7}{2}fx + \frac{7}{2}e\right) + 35 \left( 8c^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 12c^2 d \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right) \right) \frac{1}{32f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.138 Problem number 523

$$\int \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{2d^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{5af} - \frac{2a(15c^2 + 10cd + 7d^2) \cos(fx + e)}{15f \sqrt{a + a \sin(fx + e)}} - \frac{4(5c - d) d \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15f}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{5}{4} \pi + \frac{5}{2} fx + \frac{5}{2} e) + 30 (2 c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) + 2 cd \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \right)}{15f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.139 Problem number 524

$$\int \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{2a(3c + d) \cos(fx + e)}{3f \sqrt{a + a \sin(fx + e)}} - \frac{2d \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3f}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{3}{4} \pi + \frac{3}{2} fx + \frac{3}{2} e) + 3 (2 c \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) + d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.140 Problem number 525

$$\int \sqrt{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2a \cos(fx + e)}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{2} \sqrt{a} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.141 Problem number 526

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}}\right) \sqrt{a}}{f \sqrt{d} \sqrt{c+d}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \sqrt{a} \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{-cd - d^2} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.142 Problem number 527

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)\sqrt{a}}{(c+d)^{\frac{3}{2}}f\sqrt{d}} - \frac{a\cos(fx+e)}{(c+d)f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-cd-d^2}(c+d)} + \frac{2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(2d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c+d)}\right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.143 Problem number 528

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{3\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)\sqrt{a}}{4(c+d)^{\frac{5}{2}}f\sqrt{d}} - \frac{a\cos(fx+e)}{2(c+d)f(c+d\sin(fx+e))^2\sqrt{a+a\sin(fx+e)}} - \frac{3a\cos(fx+e)}{4(c+d)^2f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{3 \sqrt{2} \arctan \left( \frac{\sqrt{2} d \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)}{\sqrt{-cd - d^2}} \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{(c^2 + 2cd + d^2) \sqrt{-cd - d^2}} + \frac{2 \left( 6 d \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{8f} \right)$$

8f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 43.144 Problem number 529

$$\int (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(c-17d)d(c+d)\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{105f} \\ & + \frac{4a^2(c-17d)(c+d)(15c^2+10cd+7d^2)\cos(fx+e)}{315df\sqrt{a+a\sin(fx+e)}} \\ & + \frac{2a^2(c-17d)\cos(fx+e)(c+d\sin(fx+e))^3}{63df\sqrt{a+a\sin(fx+e)}} - \frac{2a^2\cos(fx+e)(c+d\sin(fx+e))^4}{9df\sqrt{a+a\sin(fx+e)}} \\ & + \frac{8a(c-17d)(5c-d)(c+d)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{315f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 35 ad^3 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{9}{4} \pi + \frac{9}{2} f x + \frac{9}{2} e \right) + 1890 \left( 4 ac^3 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 8 a \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.145 Problem number 530

$$\int (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(7c - d) d \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{35f} \\ & - \frac{2d^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{7af} - \frac{8a^2(35c^2 + 42cd + 19d^2) \cos(fx + e)}{105f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(35c^2 + 42cd + 19d^2) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (15 ad^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{7}{4} \pi + \frac{7}{2} fx + \frac{7}{2} e) + 105 (12 ac^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 16$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.146 Problem number 531

$$\int (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2d \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{5f} - \frac{8a^2(5c + 3d) \cos(fx + e)}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(5c + 3d) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (3 ad \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{5}{4} \pi + \frac{5}{2} fx + \frac{5}{2} e) + 30 (3 ac \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 2 ad \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.147 Problem number 532

$$\int (a + a \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{8a^2 \cos(fx + e)}{3f \sqrt{a + a \sin(fx + e)}} - \frac{2a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3f}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 9 \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) + \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \sin \left( -\frac{3}{4} \pi + \frac{3}{2} fx + \frac{3}{2} e \right) \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.148 Problem number 533

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^{3/2}(c-d) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{d^{3/2} f \sqrt{c+d}} - \frac{2a^2 \cos(fx+e)}{df \sqrt{a+a \sin(fx+e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2 \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)}{d} + \frac{\sqrt{2} \left( \operatorname{acsgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - \operatorname{adsgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \right) \operatorname{arctan} \left( \frac{\sqrt{2} \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)}{\sqrt{-cd - d^2}} \right)}{\sqrt{-cd - d^2}} \right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.149 Problem number 534

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^{\frac{3}{2}}(c + 3d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{d^{\frac{3}{2}}(c+d)^{\frac{3}{2}}f} \\ & + \frac{a^2(c-d)\cos(fx+e)}{d(c+d)f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2}\sqrt{a} \left( \frac{\sqrt{2} \left( \operatorname{acsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 3 \operatorname{adsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right) \operatorname{arctan}\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right)}{(cd+d^2)\sqrt{-cd - d^2}} \right) - \frac{2 \operatorname{acsgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.150 Problem number 535

$$\int \frac{(a + a \sin(e + fx))^{3/2}}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^{\frac{3}{2}}(c + 7d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{4d^{\frac{3}{2}}(c+d)^{\frac{5}{2}}f} \\ & + \frac{a^2(c-d)\cos(fx+e)}{2d(c+d)f(c+d\sin(fx+e))^2\sqrt{a+a\sin(fx+e)}} \\ & - \frac{a^2(c+7d)\cos(fx+e)}{4d(c+d)^2f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}} \end{aligned}$$



command

`integrate((a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} (a \operatorname{csgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 7 a d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))) \arctan\left(\frac{\sqrt{2} d \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{\sqrt{-c d - d^2}}\right)}{(c^2 d + 2 c d^2 + d^3) \sqrt{-c d - d^2}} \right) + \frac{2 (2 a c d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{(c^2 d + 2 c d^2 + d^3) \sqrt{-c d - d^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.151 Problem number 536

$$\int (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(c+d)(3c^2 - 38cd + 355d^2) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{1155f} \\ & - \frac{4a^3(c+d)(15c^2 + 10cd + 7d^2)(3c^2 - 38cd + 355d^2) \cos(fx + e)}{3465d^2 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a^3(3c^2 - 38cd + 355d^2) \cos(fx + e) (c + d \sin(fx + e))^3}{693d^2 f \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2a^3(3c - 23d) \cos(fx + e) (c + d \sin(fx + e))^4}{99d^2 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{8a^2(5c - d)(c + d)(3c^2 - 38cd + 355d^2) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3465df} \\ & - \frac{2a^2 \cos(fx + e) (c + d \sin(fx + e))^4 \sqrt{a + a \sin(fx + e)}}{11df} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (315 a^2 d^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(-\frac{11}{4} \pi + \frac{11}{2} f x + \frac{11}{2} e) + 6930 (40 a^2 c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.152 Problem number 537

$$\int (a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(21c^2 + 30cd + 13d^2) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{105f} \\ & - \frac{4(9c - d) d \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{63f} \\ & - \frac{2d^2 \cos(fx + e) (a + a \sin(fx + e))^{\frac{7}{2}}}{9af} - \frac{64a^3(21c^2 + 30cd + 13d^2) \cos(fx + e)}{315f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{16a^2(21c^2 + 30cd + 13d^2) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{315f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (35 a^2 d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{9}{4} \pi + \frac{9}{2} fx + \frac{9}{2} e) + 630 (20 a^2 c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 3$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.153 Problem number 538

$$\int (a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7c + 5d) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{35f} - \frac{2d \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{7f} \\ & - \frac{64a^3(7c + 5d) \cos(fx + e)}{105f \sqrt{a + a \sin(fx + e)}} - \frac{16a^2(7c + 5d) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 15 a^2 \operatorname{dsgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{7}{4} \pi + \frac{7}{2} f x + \frac{7}{2} e \right) + 525 \left( 4 a^2 \operatorname{csgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 3 a^2 \right) \right)}{30 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 43.154 Problem number 539

$$\int (a + a \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{5f} - \frac{64a^3 \cos(fx + e)}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{16a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 150 a^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 25 a^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) \right)}{30 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 43.155 Problem number 540

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^{\frac{5}{2}} (c - d)^2 \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx + e)}} \right)}{d^{\frac{5}{2}} f \sqrt{c+d}} \\ & + \frac{2a^3 (3c - 7d) \cos(fx + e)}{3d^2 f \sqrt{a + a \sin(fx + e)}} - \frac{2a^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3df} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{3 \sqrt{2} (a^2 c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) - 2 a^2 c d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + a^2 d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))) \arctan\left(\frac{\sqrt{2} d \sin(\dots)}{\sqrt{-\dots}}\right)}{\sqrt{-cd - d^2} a^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 43.156 Problem number 541

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^{\frac{5}{2}}(c-d)(3c+5d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{d^{\frac{5}{2}}(c+d)^{\frac{3}{2}}f} - \frac{a^3(3c+d)\cos(fx+e)}{d^2(c+d)f\sqrt{a+a\sin(fx+e)}} + \frac{a^2(c-d)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{d(c+d)f(c+d\sin(fx+e))}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{4 a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{d^2} + \frac{\sqrt{2} (3 a^2 c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 2 a^2 c d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) - 5 a^2 d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))) \arctan\left(\frac{\sqrt{2} d \sin(\dots)}{\sqrt{-\dots}}\right)}{(cd^2+d^3)\sqrt{-cd}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.157 Problem number 542

$$\int \frac{(a + a \sin(e + fx))^{5/2}}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{5}{2}}(3c^2 + 10cd + 19d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{4d^{\frac{5}{2}}(c+d)^{\frac{5}{2}}f} \\ & + \frac{3a^3(c-d)(c+3d)\cos(fx+e)}{4d^2(c+d)^2 f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}} \\ & + \frac{a^2(c-d)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{2d(c+d)f(c+d\sin(fx+e))^2} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2}\sqrt{a} \left( \frac{\sqrt{2}(3a^2c^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+10a^2cd\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+19a^2d^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\operatorname{arctan}\left(\frac{\sqrt{2}d}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{(c^2d^2+2cd^3+d^4)\sqrt{-cd-d^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 43.158 Problem number 543

$$\int \frac{(c + d \sin(e + fx))^3}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(c-d)^3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}} - \frac{4d(21c^2-12cd+7d^2)\cos(fx+e)}{15f\sqrt{a+a\sin(fx+e)}} \\ & - \frac{2d\cos(fx+e)(c+d\sin(fx+e))^2}{5f\sqrt{a+a\sin(fx+e)}} - \frac{2(9c-d)d^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{15af} \end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15\sqrt{2}\left(\sqrt{a}c^3-3\sqrt{a}c^2d+3\sqrt{a}cd^2-\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{15\sqrt{2}\left(\sqrt{a}c^3-3\sqrt{a}c^2d+3\sqrt{a}cd^2-\sqrt{a}d^3\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.159 Problem number 544

$$\int \frac{(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{(c-d)^2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}} - \frac{4(3c-d)d\cos(fx+e)}{3f\sqrt{a+a\sin(fx+e)}} - \frac{2d^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{3af}$$

command

```
integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2}\left(\sqrt{a}c^2-2\sqrt{a}cd+\sqrt{a}d^2\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(\sqrt{a}c^2-2\sqrt{a}cd+\sqrt{a}d^2\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{8}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.160 Problem number 545

$$\int \frac{c + d \sin(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{(c - d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}} - \frac{2d \cos(fx+e)}{f\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}d\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}(\sqrt{a}c-\sqrt{a}d)\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(\sqrt{a}c-\sqrt{a}d)\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.161 Problem number 546

$$\int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}+\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+2\right|\right)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}+\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-2\right|\right)}{\sqrt{a}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.162 Problem number 547

$$\int \frac{1}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{(c-d)f\sqrt{a}} + \frac{2\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)\sqrt{d}}{(c-d)f\sqrt{a}\sqrt{c+d}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} d \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right)}{\sqrt{-cd - d^2} (\operatorname{csgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - d \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))} + \frac{\log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{\operatorname{csgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - d \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)}{2\sqrt{a}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.163 Problem number 548

$$\int \frac{1}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{(c-d)^2 f\sqrt{a}} \\ & + \frac{(3c+d)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)\sqrt{d}}{(c-d)^2 (c+d)^{\frac{3}{2}} f\sqrt{a}} \\ & + \frac{d \cos(fx+e)}{(c^2 - d^2) f (c + d \sin(fx+e)) \sqrt{a + a \sin(fx+e)}} \end{aligned}$$

command



`integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{\sqrt{2} (3cd+d^2) \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(c^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - c^2 d \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - cd^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + d^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))) \sqrt{-cd}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 43.164 Problem number 549

$$\int \frac{1}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a \sin(fx+e)}}\right) \sqrt{2}}{(c-d)^3 f \sqrt{a}} \\ & + \frac{(15c^2 + 10cd + 7d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a \sin(fx+e)}}\right) \sqrt{d}}{4(c-d)^3 (c+d)^{\frac{5}{2}} f \sqrt{a}} \\ & + \frac{d \cos(fx+e)}{2(c^2-d^2) f (c+d \sin(fx+e))^2 \sqrt{a+a \sin(fx+e)}} \\ & + \frac{d(7c+d) \cos(fx+e)}{4(c^2-d^2)^2 f (c+d \sin(fx+e)) \sqrt{a+a \sin(fx+e)}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.165 Problem number 550

$$\int \frac{(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(c - d) \cos(fx + e) (c + d \sin(fx + e))^2}{2f (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(c - d)^2 (c + 11d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}f} \\ & + \frac{d(3c^2 - 24cd + 13d^2) \cos(fx + e)}{3af \sqrt{a + a \sin(fx + e)}} + \frac{(3c - 7d) d^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{6a^2 f} \end{aligned}$$

command

`integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2}\left(\sqrt{a}c^3+9\sqrt{a}c^2d-21\sqrt{a}cd^2+11\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(\sqrt{a}c^3+9\sqrt{a}c^2d-21\sqrt{a}cd^2+11\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.166 Problem number 551

$$\int \frac{(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(c - d) \cos(fx + e) (c + d \sin(fx + e))}{2f (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(c - d) (c + 7d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}f} + \frac{(c - 5d) d \cos(fx + e)}{2af \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16\sqrt{2}d^2\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)}{a^{\frac{3}{2}}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}(\sqrt{a}c^2+6\sqrt{a}cd-7\sqrt{a}d^2)\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(\sqrt{a}c^2+6\sqrt{a}cd-7\sqrt{a}d^2)}{a^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

8 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.167 Problem number 552

$$\int \frac{c + d \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(c-d)\cos(fx+e)}{2f(a+a\sin(fx+e))^{\frac{3}{2}}} - \frac{(c+3d)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}f}$$

command

`integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}(\sqrt{a}c+3\sqrt{a}d)\log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2}(\sqrt{a}c+3\sqrt{a}d)\log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{2\sqrt{2}(\sqrt{a}c\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

8 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.168 Problem number 553

$$\int \frac{1}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{2f(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}f}$$

command

`integrate(1/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{\log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1) \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)}{8\sqrt{a}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.169 Problem number 554

$$\int \frac{1}{(a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{2(c - d)f(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{(c - 5d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}(c - d)^2 f} - \frac{2d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{a^{\frac{3}{2}}(c - d)^2 f \sqrt{c+d}}$$

command

`integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\sqrt{a}d^2 \arctan\left(\frac{\sqrt{2}d \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-cd - d^2}}\right)}{(a^2c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 2a^2cd \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + a^2d^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))) \sqrt{-cd - d^2}} - \frac{\sqrt{2}a^2c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sqrt{-cd - d^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.170 Problem number 556

$$\int \frac{1}{(a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{3}{2}} (35c^2 + 42cd + 19d^2) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{4a^{\frac{3}{2}} (c-d)^4 (c+d)^{\frac{5}{2}} f} \\ & - \frac{\cos(fx+e)}{2(c-d) f (a+a \sin(fx+e))^{\frac{3}{2}} (c+d \sin(fx+e))^2} \\ & - \frac{(c-13d) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{2}}{2\sqrt{a+a \sin(fx+e)}} \right) \sqrt{2}}{4a^{\frac{3}{2}} (c-d)^4 f} \\ & - \frac{d(c+2d) \cos(fx+e)}{2a(c-d)^2 (c+d) f (c+d \sin(fx+e))^2 \sqrt{a+a \sin(fx+e)}} \\ & - \frac{d(2c+d)(c+7d) \cos(fx+e)}{4a(c-d)^3 (c+d)^2 f (c+d \sin(fx+e)) \sqrt{a+a \sin(fx+e)}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.171 Problem number 557

$$\int \frac{(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(c-d)^2 (3c+13d) \cos(fx+e)}{16af (a+a \sin(fx+e))^{\frac{3}{2}}} - \frac{(c-d) \cos(fx+e) (c+d \sin(fx+e))^2}{4f (a+a \sin(fx+e))^{\frac{5}{2}}} \\ & - \frac{3(c-d) (c^2+6cd+25d^2) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{2}}{2\sqrt{a+a \sin(fx+e)}} \right) \sqrt{2}}{32a^{\frac{5}{2}} f} \\ & + \frac{(c-9d) d^2 \cos(fx+e)}{4a^2 f \sqrt{a+a \sin(fx+e)}} \end{aligned}$$

command

```
integrate((c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{128 \sqrt{2} d^3 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{3 \sqrt{2} (\sqrt{a} c^3 + 5 \sqrt{a} c^2 d + 19 \sqrt{a} c d^2 - 25 \sqrt{a} d^3) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} - \frac{3 \sqrt{2} (\sqrt{a} c^3 + 5 \sqrt{a} c^2 d + 19 \sqrt{a} c d^2 - 25 \sqrt{a} d^3) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.172 Problem number 558

$$\int \frac{(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3(c-d)(c+3d) \cos(fx+e)}{16af(a+a \sin(fx+e))^{\frac{3}{2}}} - \frac{(c-d) \cos(fx+e)(c+d \sin(fx+e))}{4f(a+a \sin(fx+e))^{\frac{5}{2}}} - \frac{(3c^2+10cd+19d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a \sin(fx+e)}}\right) \sqrt{2}}{32a^{\frac{5}{2}}f}$$

command

```
integrate((c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (3 \sqrt{a} c^2 + 10 \sqrt{a} c d + 19 \sqrt{a} d^2) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} - \frac{\sqrt{2} (3 \sqrt{a} c^2 + 10 \sqrt{a} c d + 19 \sqrt{a} d^2) \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) - 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.173 Problem number 559

$$\int \frac{c + d \sin(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(c-d) \cos(fx+e)}{4f(a+a \sin(fx+e))^{5/2}} - \frac{(3c+5d) \cos(fx+e)}{16af(a+a \sin(fx+e))^{3/2}} - \frac{(3c+5d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a \sin(fx+e)}}\right) \sqrt{2}}{32a^{5/2}f}$$

command

```
integrate((c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (3\sqrt{a}c+5\sqrt{a}d) \log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{\sqrt{2} (3\sqrt{a}c+5\sqrt{a}d) \log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{2(3\sqrt{2}\sqrt{a}c \sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{64f}$$

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.174 Problem number 560

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)}{4f(a+a \sin(fx+e))^{5/2}} - \frac{3 \cos(fx+e)}{16af(a+a \sin(fx+e))^{3/2}} - \frac{3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a \sin(fx+e)}}\right) \sqrt{2}}{32a^{5/2}f}$$

command

```
integrate(1/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{3 \log(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{3 \log(-\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{2(3 \sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^3 - 5 \sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2 - 1)^2 a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} \right)}{64\sqrt{a}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.175 Problem number 561

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)}{4(c - d)f(a + a \sin(fx + e))^{5/2}} - \frac{(3c - 11d)\cos(fx + e)}{16a(c - d)^2 f(a + a \sin(fx + e))^{3/2}} \\ & - \frac{(3c^2 - 14cd + 43d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{32a^{5/2}(c - d)^3 f} \\ & + \frac{2d^{5/2} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right)}{a^{5/2}(c - d)^3 f \sqrt{c+d}} \end{aligned}$$

command

`integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.176 Problem number 562

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{5/2}(7c + 5d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right)}{a^{5/2}(c - d)^4 (c + d)^{3/2} f} \\ & - \frac{\cos(fx + e)}{4(c - d)f(a + a \sin(fx + e))^{5/2}(c + d \sin(fx + e))} \\ & - \frac{3(c - 5d)\cos(fx + e)}{16a(c - d)^2 f(a + a \sin(fx + e))^{3/2}(c + d \sin(fx + e))} \\ & - \frac{(3c^2 - 22cd + 115d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{32a^{5/2}(c - d)^4 f} \\ & - \frac{(c - 7d)d(3c + 5d)\cos(fx + e)}{16a^2(c - d)^3 (c + d)f(c + d \sin(fx + e))\sqrt{a + a \sin(fx + e)}} \end{aligned}$$



command

```
integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 43.177 Problem number 563

$$\int \frac{1}{(a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3d^{\frac{5}{2}}(21c^2 + 30cd + 13d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{4a^{\frac{5}{2}}(c-d)^5(c+d)^{\frac{5}{2}}f} \\ & - \frac{\cos(fx+e)}{4(c-d)f(a+a\sin(fx+e))^{\frac{5}{2}}(c+d\sin(fx+e))^2} \\ & - \frac{(3c-19d)\cos(fx+e)}{16a(c-d)^2f(a+a\sin(fx+e))^{\frac{3}{2}}(c+d\sin(fx+e))^2} \\ & - \frac{3(c^2-10cd+73d^2) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{32a^{\frac{5}{2}}(c-d)^5f} \\ & - \frac{d(3c^2-20cd-31d^2)\cos(fx+e)}{16a^2(c-d)^3(c+d)f(c+d\sin(fx+e))^2\sqrt{a+a\sin(fx+e)}} \\ & - \frac{3d(c+3d)(c^2-10cd-7d^2)\cos(fx+e)}{16a^2(c-d)^4(c+d)^2f(c+d\sin(fx+e))\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

```
integrate(1/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 43.178 Problem number 568

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2a \cos(fx + e)}{(c + d) f \sqrt{a + a \sin(fx + e)} \sqrt{c + d \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}(c^2d^2 - 2cd^3 + d^4)\sqrt{a} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)}{(c^3d^2 - c^2d^3 - cd^4 + d^5)\sqrt{c \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^4 + d \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^4 + 2c \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 43.179 Problem number 569

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2a \cos(fx + e)}{3(c + d) f (c + d \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} - \frac{4a \cos(fx + e)}{3(c + d)^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c + d \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2} \left( \left( \frac{3(c^6d^4 - 2c^5d^5 - c^4d^6 + 4c^3d^7 - c^2d^8 - 2cd^9 + d^{10}) \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^2}{c^7d^4 - c^6d^5 - 3c^5d^6 + 3c^4d^7 + 3c^3d^8 - 3c^2d^9 - cd^{10} + d^{11}} + \frac{2(3c^6d^4 - 14c^5d^5 + 21c^4d^6 - 4c^3d^7 - 19c^2d^8 + 18cd^9 - 5d^{10})}{c^7d^4 - c^6d^5 - 3c^5d^6 + 3c^4d^7 + 3c^3d^8 - 3c^2d^9 - cd^{10} + d^{11}} \right)}{3 \left( c \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^4 + d \tan\left(-\frac{1}{8}\pi + \frac{1}{4}fx + \frac{1}{4}e\right)^4 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 43.180 Problem number 570

$$\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a \cos(fx + e)}{5(c + d) f (c + d \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{8a \cos(fx + e)}{15(c + d)^2 f (c + d \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & - \frac{16a \cos(fx + e)}{15(c + d)^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c + d \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44 Test file number 74

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/74\_4.1.2.2-g\_cos-^p-a+b\_sin-^m-c+d\_sin-^n

### 44.1 Problem number 1

$$\int \cos^2(e + fx) \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}}}{15cf \sqrt{a + a \sin(fx + e)}} - \frac{\cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}}{6cf}$$

command

```
integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \left( 5 c^3 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} - 6 c^3 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{15 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.2 Problem number 2

$$\int \cos^2(e + fx) \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (c - c \sin(fx + e))^{7/2}}{10cf \sqrt{a + a \sin(fx + e)}} - \frac{\cos(fx + e) (c - c \sin(fx + e))^{7/2} \sqrt{a + a \sin(fx + e)}}{5cf}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 4 c^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} - 5 c^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{5 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.3 Problem number 3

$$\int \cos^2(e + fx) \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{6cf \sqrt{a + a \sin(fx + e)}} - \frac{\cos(fx + e) (c - c \sin(fx + e))^{5/2} \sqrt{a + a \sin(fx + e)}}{4cf}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^8 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 8 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{5 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.4 Problem number 4

$$\int \cos^2(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{a \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{3cf \sqrt{a + a \sin(fx + e)}} - \frac{\cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}{3cf}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - 3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.5 Problem number 5

$$\int \frac{\cos^2(e + fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2af \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{a} \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^4 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{\sqrt{c} f \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.6 Problem number 6

$$\int \frac{\cos^2(e + fx) \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2a \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{\cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a}\sqrt{c}\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}\right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.7 Problem number 7

$$\int \frac{\cos^2(e + fx) \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{a \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2 \log\left(\left|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{\operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}\right) \sqrt{a}}{c^{\frac{5}{2}} f \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.8 Problem number 8

$$\int \frac{\cos^2(e + fx) \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{4acf (c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right) \sqrt{a}}{4c^4 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.9 Problem number 9

$$\int \cos^2(e + fx) (a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{9}{2}}}{7cf} \\ & - \frac{4a^2 \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}}}{105cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}}{21cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$128 \left( 15ac^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{14} - 35ac^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.10 Problem number 10

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{7}{2}}}{6cf} \\ & - \frac{a^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}}}{15cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{15cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16 \left( 10 ac^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{12} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 36 ac^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.11 Problem number 11

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{5}{2}}}{5cf} \\ & - \frac{2a^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}}}{15cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```



Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 6 a c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 15 a c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.12 Problem number 12

$$\int \cos^2(e + f x) (a + a \sin(e + f x))^{3/2} \sqrt{c - c \sin(e + f x)} dx$$

Optimal antiderivative

$$\frac{c \cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}}}{6 a f \sqrt{c - c \sin(f x + e)}} + \frac{\cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}} \sqrt{c - c \sin(f x + e)}}{4 a f}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3 a \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^8 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 4 a \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.13 Problem number 13

$$\int \frac{\cos^2(e + f x) (a + a \sin(e + f x))^{3/2}}{\sqrt{c - c \sin(e + f x)}} dx$$

Optimal antiderivative

$$\frac{\cos(f x + e) (a + a \sin(f x + e))^{\frac{5}{2}}}{3 a f \sqrt{c - c \sin(f x + e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8a^{\frac{3}{2}}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{3\sqrt{c}f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.14 Problem number 14

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{\cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{2cf\sqrt{c-c\sin(fx+e)}} - \frac{4a^2\cos(fx+e)\ln(1-\sin(fx+e))}{cf\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ &-\frac{2a\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{cf\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2a^{\frac{3}{2}}\sqrt{c}\left(\frac{2\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+2c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{c^4}\right)$$


---

*f*

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.15 Problem number 15

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}}{cf(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{4a^2 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{2a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^{\frac{3}{2}}\sqrt{c}\left(\frac{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2}{c^3\text{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{2\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1)}{c^3\text{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{1}{(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2-1)c^3\text{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.16 Problem number 16

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}}{2cf(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a^2 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4a \log\left(\left|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{4a \text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - a \text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{2c^{\frac{7}{2}}f \text{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}\right)}{2c^{\frac{7}{2}}f \text{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.17 Problem number 17

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{6acf(c - c \sin(fx + e))^{7/2}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(3a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)^4 - 3a\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{6c^5 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.18 Problem number 18

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{8acf(c - c \sin(fx + e))^{9/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{5/2}}{48a^2 c^2 f(c - c \sin(fx + e))^{7/2}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(6a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)^4 - 8a \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{48c^{\frac{11}{2}} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.19 Problem number 19

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{5/2}(c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3a \cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{9}{2}}}{28cf} \\ & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{5}{2}}(c - c \sin(fx + e))^{\frac{9}{2}}}{8cf} \\ & - \frac{a^3 \cos(fx + e)(c - c \sin(fx + e))^{\frac{9}{2}}}{35cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}}{14cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \left( 35 a^2 c^3 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^{16} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) - 160 a^2 c^3 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^{16} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.20 Problem number 20

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{5/2}(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a \cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{7}{2}}}{7cf} \\ & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{5}{2}}(c - c \sin(fx + e))^{\frac{7}{2}}}{7cf} \\ & - \frac{2a^3 \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}}}{35cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{35cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$32 \left( 20 a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 70 a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.21 Problem number 21

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{5/2}(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}(c - c \sin(fx + e))^{3/2}}{6af} + \frac{c^2 \cos(fx + e)(a + a \sin(fx + e))^{7/2}}{15af \sqrt{c - c \sin(fx + e)}} + \frac{2c \cos(fx + e)(a + a \sin(fx + e))^{7/2} \sqrt{c - c \sin(fx + e)}}{15af}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16 \left( 10 a^2 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 24 a^2 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.22 Problem number 22

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}}}{10af \sqrt{c - c \sin(fx + e)}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}} \sqrt{c - c \sin(fx + e)}}{5af}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 4a^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^{10} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 5a^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}e\right) \right)}{5f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.23 Problem number 23

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}}}{4af \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4a^{\frac{5}{2}} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{c} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.24 Problem number 24

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cos(fx+e)(a+a\sin(fx+e))^{3/2}}{cf\sqrt{c-c\sin(fx+e)}} - \frac{\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{3cf\sqrt{c-c\sin(fx+e)}} - \frac{8a^3 \cos(fx+e) \ln(1-\sin(fx+e))}{cf\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} - \frac{4a^2 \cos(fx+e)\sqrt{a+a\sin(fx+e)}}{cf\sqrt{c-c\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^{5/2}\sqrt{c}\left(\frac{6\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1)}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{2c^4\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^6+3c^4\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^4+6c^4\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2}{c^6\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}\right)\operatorname{sgn}}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.25 Problem number 25

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{cf(c-c\sin(fx+e))^{3/2}} + \frac{3a\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{2c^2f\sqrt{c-c\sin(fx+e)}} + \frac{12a^3\cos(fx+e)\ln(1-\sin(fx+e))}{c^2f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} + \frac{6a^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{c^2f\sqrt{c-c\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^{5/2}\sqrt{c}\left(\frac{6\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1)}{c^3\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{c^3\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^4\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+4c^3\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{c^6}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 44.26 Problem number 26

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{2cf(c-c\sin(fx+e))^{5/2}} - \frac{3a\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{2c^2f(c-c\sin(fx+e))^{3/2}} \\ - \frac{6a^3\cos(fx+e)\ln(1-\sin(fx+e))}{c^3f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} - \frac{3a^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{c^3f\sqrt{c-c\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^{5/2}\sqrt{c}\left(\frac{2\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2}{c^4\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{6\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1)}{c^4\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} - \frac{6\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2-5}{(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2-1)^2c^4\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.27 Problem number 27

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{3cf(c-c\sin(fx+e))^{7/2}} - \frac{a\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{2c^2f(c-c\sin(fx+e))^{5/2}} \\ + \frac{a^2\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{c^3f(c-c\sin(fx+e))^{3/2}} + \frac{a^3\cos(fx+e)\ln(1-\sin(fx+e))}{c^4f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(12a^2\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right) + \frac{18a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{6c^2f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{6c^2f\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.28 Problem number 28

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{8acf(c - c \sin(fx + e))^{9/2}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 - 6 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 + 4 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right) a^{5/2} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{8 \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^4 c^{11/2} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.29 Problem number 29

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{10acf(c - c \sin(fx + e))^{11/2}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{80a^2c^2f(c - c \sin(fx + e))^{9/2}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(10 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 - 20 a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4\right) a^{5/2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{80 c^{13/2} f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.30 Problem number 30

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{9/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^2 \cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{11}{2}}}{15cf} \\ & - \frac{4a \cos(fx + e)(a + a \sin(fx + e))^{\frac{5}{2}}(c - c \sin(fx + e))^{\frac{11}{2}}}{45cf} \\ & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}}(c - c \sin(fx + e))^{\frac{11}{2}}}{10cf} \\ & - \frac{4a^4 \cos(fx + e)(c - c \sin(fx + e))^{\frac{11}{2}}}{315cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^3 \cos(fx + e)(c - c \sin(fx + e))^{\frac{11}{2}} \sqrt{a + a \sin(fx + e)}}{105cf} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$256 \left( 126 a^3 c^4 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^{20} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) - 700 a^3 c^4 \cos\right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.31 Problem number 31

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^2 \cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}(c - c \sin(fx + e))^{\frac{9}{2}}}{21cf} \\ & - \frac{a \cos(fx + e)(a + a \sin(fx + e))^{\frac{5}{2}}(c - c \sin(fx + e))^{\frac{9}{2}}}{9cf} \\ & - \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}}(c - c \sin(fx + e))^{\frac{9}{2}}}{9cf} \\ & - \frac{8a^4 \cos(fx + e)(c - c \sin(fx + e))^{\frac{9}{2}}}{315cf \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^3 \cos(fx + e)(c - c \sin(fx + e))^{\frac{9}{2}} \sqrt{a + a \sin(fx + e)}}{63cf} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{256 \left( 70 a^3 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{18} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 315 a^3 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{16} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 44.32 Problem number 32

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3c \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}(c - c \sin(fx + e))^{\frac{3}{2}}}{28af} \\ & + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}(c - c \sin(fx + e))^{\frac{5}{2}}}{8af} \\ & + \frac{c^3 \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{35af \sqrt{c - c \sin(fx + e)}} \\ & + \frac{c^2 \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}} \sqrt{c - c \sin(fx + e)}}{14af} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \left( 35 a^3 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{16} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 120 a^3 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 44.33 Problem number 33

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}(c - c \sin(fx + e))^{\frac{3}{2}}}{7af} + \frac{4c^2 \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{105af \sqrt{c - c \sin(fx + e)}} + \frac{2c \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}} \sqrt{c - c \sin(fx + e)}}{21af}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{128 \left( 15 a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 35 a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{15 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 44.34 Problem number 34

$$\int \cos^2(e + fx)(a + a \sin(e + fx))^{7/2} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{c \cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{15af \sqrt{c - c \sin(fx + e)}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}} \sqrt{c - c \sin(fx + e)}}{6af}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \left( 5 a^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 6 a^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{15 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.35 Problem number 35

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(a+a\sin(fx+e))^{9/2}}{5af\sqrt{c-c\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{32a^{7/2}\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^{10}\operatorname{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{5\sqrt{c}f\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.36 Problem number 36

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{2a^2\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{cf\sqrt{c-c\sin(fx+e)}} - \frac{2a\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{3cf\sqrt{c-c\sin(fx+e)}} \\ &-\frac{\cos(fx+e)(a+a\sin(fx+e))^{7/2}}{4cf\sqrt{c-c\sin(fx+e)}} - \frac{16a^4\cos(fx+e)\ln(1-\sin(fx+e))}{cf\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ &-\frac{8a^3\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{cf\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4a^{7/2}\sqrt{c}\left(\frac{12\log(-\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^2+1)}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{3c^6\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^8\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+4c^6\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)^6\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.37 Problem number 37

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{cf(c - c \sin(fx + e))^{3/2}} + \frac{4a^2 \cos(fx + e)(a + a \sin(fx + e))^{3/2}}{c^2 f \sqrt{c - c \sin(fx + e)}} \\ & + \frac{4a \cos(fx + e)(a + a \sin(fx + e))^{5/2}}{3c^2 f \sqrt{c - c \sin(fx + e)}} + \frac{32a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{16a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8a^{7/2} \sqrt{c} \left( \frac{12 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{c^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{3}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right) c^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{c^6 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6}{3f} \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.38 Problem number 38

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos(fx + e)(a + a \sin(fx + e))^{7/2}}{2cf(c - c \sin(fx + e))^{5/2}} - \frac{2a \cos(fx + e)(a + a \sin(fx + e))^{5/2}}{c^2 f (c - c \sin(fx + e))^{3/2}} \\ & - \frac{3a^2 \cos(fx + e)(a + a \sin(fx + e))^{3/2}}{c^3 f \sqrt{c - c \sin(fx + e)}} - \frac{24a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{12a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^3 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 a^{\frac{7}{2}} \sqrt{c} \left( \frac{12 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{c^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{8 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 7}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^2 c^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{c^4 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 44.39 Problem number 39

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{7}{2}}}{3cf(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{2a \cos(fx + e)(a + a \sin(fx + e))^{\frac{5}{2}}}{3c^2 f(c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{2a^2 \cos(fx + e)(a + a \sin(fx + e))^{\frac{3}{2}}}{c^3 f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{8a^4 \cos(fx + e) \ln(1 - \sin(fx + e))}{c^4 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{4a^3 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^4 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 a^{\frac{7}{2}} \sqrt{c} \left( \frac{3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}{c^5 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{12 \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{c^5 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{18 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 30 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^3 c^5 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right) \frac{1}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 44.40 Problem number 40

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos(fx+e)(a+a\sin(fx+e))^{7/2}}{4cf(c-c\sin(fx+e))^{9/2}} - \frac{a\cos(fx+e)(a+a\sin(fx+e))^{5/2}}{3c^2f(c-c\sin(fx+e))^{7/2}} \\ & + \frac{a^2\cos(fx+e)(a+a\sin(fx+e))^{3/2}}{2c^3f(c-c\sin(fx+e))^{5/2}} - \frac{a^3\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{c^4f(c-c\sin(fx+e))^{3/2}} \\ & - \frac{a^4\cos(fx+e)\ln(1-\sin(fx+e))}{c^5f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} a^{7/2} \sqrt{c} \left( \frac{12 \sqrt{2} \log\left(-2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^2 + 2\right)}{c^6 \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)} - \frac{\sqrt{2} \left(48 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^6 - 108 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^4 + 88 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^2 - 1\right)^4 c^6 \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)}{24 f} \right)}{24 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.41 Problem number 41

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(a+a\sin(fx+e))^{9/2}}{10acf(c-c\sin(fx+e))^{11/2}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(5 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^8 - 10 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^6 + 10 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^4 - 10 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^2 + 10 a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^0\right) \sqrt{c-c\sin(fx+e)}}{10 a c f (c-c\sin(fx+e))^{11/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.42 Problem number 42

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{15/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{12acf(c - c \sin(fx + e))^{\frac{13}{2}}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{120ac^2f(c - c \sin(fx + e))^{\frac{11}{2}}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

---


$$\left(15a^3\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 - 40a^3\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^7\right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.43 Problem number 43

$$\int \frac{\cos^2(e + fx)(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{17/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{14acf(c - c \sin(fx + e))^{\frac{15}{2}}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{84ac^2f(c - c \sin(fx + e))^{\frac{13}{2}}} + \frac{\cos(fx + e)(a + a \sin(fx + e))^{\frac{9}{2}}}{840ac^3f(c - c \sin(fx + e))^{\frac{11}{2}}}$$

command

```
integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

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$$\left(35a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8 - 105a^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^7\right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.44 Problem number 44

$$\int \frac{\cos^2(e + fx)(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)(c - c \sin(fx + e))^{7/2}}{4cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4c^{5/2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^8}{\sqrt{a} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.45 Problem number 45

$$\int \frac{\cos^2(e + fx)(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)(c - c \sin(fx + e))^{5/2}}{3cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8c^{3/2} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6}{3\sqrt{a} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.46 Problem number 46

$$\int \frac{\cos^2(e + fx) \sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{2cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{c} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4}{\sqrt{a} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.47 Problem number 47

$$\int \frac{\cos^2(e + fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) \sqrt{c - c \sin(fx + e)}}{cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}{\sqrt{a} \sqrt{c} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{\sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}} dx$$

## 44.48 Problem number 48

$$\int \frac{\cos^2(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(\left|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)}{\sqrt{a} c^{\frac{3}{2}} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 44.49 Problem number 49

$$\int \frac{\cos^2(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{cf (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2 \sqrt{a} c^{\frac{5}{2}} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

## 44.50 Problem number 50

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c^2 \cos(fx+e)(c-c\sin(fx+e))^{\frac{3}{2}}}{af\sqrt{a+a\sin(fx+e)}} + \frac{2c \cos(fx+e)(c-c\sin(fx+e))^{\frac{5}{2}}}{3af\sqrt{a+a\sin(fx+e)}} \\ & + \frac{\cos(fx+e)(c-c\sin(fx+e))^{\frac{7}{2}}}{4af\sqrt{a+a\sin(fx+e)}} + \frac{16c^4 \cos(fx+e) \ln(1+\sin(fx+e))}{af\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & + \frac{8c^3 \cos(fx+e)\sqrt{c-c\sin(fx+e)}}{af\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{2}\sqrt{a}c^{\frac{7}{2}}\left(\frac{12\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{3\sqrt{2}a^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^8+4\sqrt{2}a^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.51 Problem number 51

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \cos(fx+e)(c-c\sin(fx+e))^{\frac{3}{2}}}{af\sqrt{a+a\sin(fx+e)}} + \frac{\cos(fx+e)(c-c\sin(fx+e))^{\frac{5}{2}}}{3af\sqrt{a+a\sin(fx+e)}} \\ & + \frac{8c^3 \cos(fx+e) \ln(1+\sin(fx+e))}{af\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} + \frac{4c^2 \cos(fx+e)\sqrt{c-c\sin(fx+e)}}{af\sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{a}c^{\frac{5}{2}}\left(\frac{6\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{\sqrt{2}\left(2a^4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6+3a^4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+6a^4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^6\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.52 Problem number 52

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(c-c\sin(fx+e))^{\frac{3}{2}}}{2af\sqrt{a+a\sin(fx+e)}} + \frac{4c^2\cos(fx+e)\ln(1+\sin(fx+e))}{af\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} + \frac{2c\cos(fx+e)\sqrt{c-c\sin(fx+e)}}{af\sqrt{a+a\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\sqrt{a}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\sqrt{a}c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{a^2f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.53 Problem number 53

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2c\cos(fx+e)\ln(1+\sin(fx+e))}{af\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} + \frac{\cos(fx+e)\sqrt{c-c\sin(fx+e)}}{af\sqrt{a+a\sin(fx+e)}}$$

command

```
integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \sqrt{a} \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^2 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) - 2 \sqrt{2} \sqrt{a} \log \left( \left| \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right| \right) \right)}{a^2 f \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 44.54 Problem number 54

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \log \left( \left| \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right| \right)}{a^{\frac{3}{2}} \sqrt{c} f \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{(a \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

#### 44.55 Problem number 55

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{acf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$



command

`integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^2 c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^2 c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos^2(fx + e)^2}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

#### 44.56 Problem number 56

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{\frac{3}{2}} (c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e)}{2acf(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} + \frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2a^2 c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{c} \left( \frac{\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^{\frac{3}{2}} c^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^{\frac{3}{2}} c^3 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos^2(fx + e)^2}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

## 44.57 Problem number 57

$$\int \frac{\cos^2(e + fx)(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)(c - c \sin(fx + e))^{\frac{9}{2}}}{af(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{10c^3 \cos(fx + e)(c - c \sin(fx + e))^{\frac{3}{2}}}{a^2 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{10c^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}}}{3a^2 f \sqrt{a + a \sin(fx + e)}} - \frac{5c \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}}}{4a^2 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{80c^5 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{40c^4 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{2}\sqrt{a}c^{\frac{9}{2}}\left(\frac{60\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{12\sqrt{2}}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)+\frac{3\sqrt{2}a^9\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2 f \sqrt{a + a \sin(fx + e)}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.58 Problem number 58

$$\int \frac{\cos^2(e + fx)(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}}}{af(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{4c^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{3}{2}}}{a^2 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4c \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}}}{3a^2 f \sqrt{a + a \sin(fx + e)}} - \frac{32c^4 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{16c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}\sqrt{a}c^{\frac{7}{2}}\left(\frac{12\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{3\sqrt{2}}{\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)+\frac{\sqrt{2}\left(a^6\sin\left(-\frac{1}{4}\pi\right)\right)}{3f}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.59 Problem number 59

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e)(c-c\sin(fx+e))^{\frac{5}{2}}}{af(a+a\sin(fx+e))^{\frac{3}{2}}}-\frac{3c\cos(fx+e)(c-c\sin(fx+e))^{\frac{3}{2}}}{2a^2f\sqrt{a+a\sin(fx+e)}}-\frac{12c^3\cos(fx+e)\ln(1+\sin(fx+e))}{a^2f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}}-\frac{6c^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}}{a^2f\sqrt{a+a\sin(fx+e)}}$$

command

`integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-6\sqrt{a}c^2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi\right)\right)\right)}{a^3f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.60 Problem number 60

$$\int \frac{\cos^2(e + fx)(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)(c - c \sin(fx + e))^{3/2}}{af(a + a \sin(fx + e))^{3/2}} - \frac{4c^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \sqrt{a} c \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 4\sqrt{2} \sqrt{a} c \log\left(\left|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \right)}{a^3 f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.61 Problem number 61

$$\int \frac{\cos^2(e + fx) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{\cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af(a + a \sin(fx + e))^{3/2}} \end{aligned}$$

command

```
integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 2\sqrt{a} \log\left(\left|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{\sqrt{a} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2} \right) \sqrt{c}}{a^3 f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.62 Problem number 62

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)}{af(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2a^{\frac{5}{2}}\sqrt{c}f\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{(a \sin(fx + e) + a)^{\frac{5}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

## 44.63 Problem number 63

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)}{2acf(a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} + \frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2a^2cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{\log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^3 c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2 \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^3 c^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{1}{a^3 c^2} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(fx + e)^2}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 44.64 Problem number 174

$$\int (g \cos(e + fx))^{1-2m} (a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1+m} dx$$

Optimal antiderivative

$$\frac{g \ln(1 - \sin(fx + e)) (a + a \sin(fx + e))^m (c - c \sin(fx + e))^m (g \cos(fx + e))^{-2m}}{cf}$$

command

`integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+m),x, algorithm="gia`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.65 Problem number 175

$$\int (g \cos(e + fx))^{5-2m} (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8a^3 (g \cos(fx + e))^{6-2m} (a + a \sin(fx + e))^{-3+m} (c - c \sin(fx + e))^n}{fg(3 - m + n)(4 - m + n)(5 - m + n)} \\ & - \frac{4a^2 (g \cos(fx + e))^{6-2m} (a + a \sin(fx + e))^{-2+m} (c - c \sin(fx + e))^n}{fg(4 - m + n)(5 - m + n)} \\ & - \frac{a (g \cos(fx + e))^{6-2m} (a + a \sin(fx + e))^{-1+m} (c - c \sin(fx + e))^n}{fg(5 - m + n)} \end{aligned}$$

command

`integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.66 Problem number 176

$$\int (g \cos(e + fx))^{3-2m} (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n dx$$

Optimal antiderivative

$$\frac{2a^2 (g \cos(fx + e))^{4-2m} (a + a \sin(fx + e))^{-2+m} (c - c \sin(fx + e))^n}{fg(2 - m + n)(3 - m + n)} - \frac{a(g \cos(fx + e))^{4-2m} (a + a \sin(fx + e))^{-1+m} (c - c \sin(fx + e))^n}{fg(3 - m + n)}$$

command

```
integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.67 Problem number 177

$$\int (g \cos(e + fx))^{1-2m} (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n dx$$

Optimal antiderivative

$$\frac{a(g \cos(fx + e))^{2-2m} (a + a \sin(fx + e))^{-1+m} (c - c \sin(fx + e))^n}{fg(1 - m + n)}$$

command

```
integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.68 Problem number 325

$$\int \cos(c + dx) \cot(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{d} + \frac{2a \cos(dx+c)}{3d\sqrt{a+a\sin(dx+c)}} + \frac{2 \cos(dx+c) \sqrt{a+a\sin(dx+c)}}{3d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^3 - 3\sqrt{2} \log\left(\frac{\left| -2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}\right) \operatorname{sgn}(\cos)}{6d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.69 Problem number 326

$$\int \cot^2(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{d} + \frac{3a \cos(dx+c)}{d\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) \sqrt{a+a\sin(dx+c)}}{d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \log\left(\frac{\left| -2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4\sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + 8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 44.70 Problem number 327

$$\int \cot^2(c + dx) \csc(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{5 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{4d} - \frac{a \cot(dx+c)}{4d\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) \csc(dx+c) \sqrt{a+a\sin(dx+c)}}{2d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 5 \sqrt{2} \log \left( \left| \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} \right| \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) - \frac{4(6 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{(2s)} \right)}{16d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.71 Problem number 328

$$\int \cot^2(c + dx) \csc^2(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{8d} + \frac{3a \cot(dx+c)}{8d\sqrt{a+a\sin(dx+c)}} - \frac{a \cot(dx+c) \csc(dx+c)}{12d\sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) (\csc^2(dx+c)) \sqrt{a+a\sin(dx+c)}}{3d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 9 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4 \left( 36 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)^2}{96d} \right)$$

96 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.72 Problem number 332

$$\int \cos(c + dx) \cot(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^{3/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} + \frac{2\cos(dx+c)(a+a\sin(dx+c))^{3/2}}{5d} \\ & -\frac{2a^2\cos(dx+c)}{5d\sqrt{a+a\sin(dx+c)}} + \frac{2a\cos(dx+c)\sqrt{a+a\sin(dx+c)}}{5d} \end{aligned}$$

command

`integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.73 Problem number 333

$$\int \cot^2(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} - \frac{\cot(dx+c)(a+a\sin(dx+c))^{3/2}}{d} \\ & + \frac{11a^2\cos(dx+c)}{3d\sqrt{a+a\sin(dx+c)}} + \frac{5a\cos(dx+c)\sqrt{a+a\sin(dx+c)}}{3d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 a \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right)^3 - 9 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \operatorname{sgn} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.74 Problem number 334

$$\int \cot^2(c + dx) \csc(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a^{3/2} \operatorname{arctanh} \left( \frac{\cos(dx+c) \sqrt{a}}{\sqrt{a + a \sin(dx+c)}} \right)}{4d} - \frac{\cot(dx+c) \csc(dx+c) (a + a \sin(dx+c))^{3/2}}{2d} + \frac{13a^2 \cos(dx+c)}{4d \sqrt{a + a \sin(dx+c)}} - \frac{3a \cot(dx+c) \sqrt{a + a \sin(dx+c)}}{4d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) - 32 a \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.75 Problem number 335

$$\int \cot^2(c + dx) \csc^2(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{13a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{8d} - \frac{\cot(dx+c) (\csc^2(dx+c)) (a+a\sin(dx+c))^{\frac{3}{2}}}{3d}$$

$$+ \frac{5a^2 \cot(dx+c)}{24d\sqrt{a+a\sin(dx+c)}} - \frac{a \cot(dx+c) \csc(dx+c) \sqrt{a+a\sin(dx+c)}}{4d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 39 \sqrt{2} a \log \left( \left| \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} \right| \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) - \frac{4(36 \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{96d} \right)$$

96 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.76 Problem number 346

$$\int \frac{\cos(c + dx) \cot(c + dx)}{(a + a \sin(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{a^{\frac{3}{2}}d} + \frac{2 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right) \sqrt{2}}{a^{\frac{3}{2}}d}$$

command

```
integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} \log \left( \frac{|-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)|}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} \right)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} + \frac{2 \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} - \frac{2 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} \right)$$


---

$2d$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 44.77 Problem number 445

$$\int \cos^3(c + dx) \cot(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{12 \cos(dx + c) (a + a \sin(dx + c))^{\frac{3}{2}}}{35ad} - \frac{2 \operatorname{arctanh} \left( \frac{\cos(dx+c)\sqrt{a}}{\sqrt{a + a \sin(dx + c)}} \right) \sqrt{a}}{d} \\ & + \frac{8a \cos(dx + c)}{15d \sqrt{a + a \sin(dx + c)}} - \frac{2a \cos(dx + c) (\sin^3(dx + c))}{7d \sqrt{a + a \sin(dx + c)}} \\ & + \frac{164 \cos(dx + c) \sqrt{a + a \sin(dx + c)}}{105d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 480 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)^7 - 1008 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)) \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.78 Problem number 446

$$\int \cos^2(c + dx) \cot^2(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cos(dx + c) (a + a \sin(dx + c))^{\frac{3}{2}}}{5ad} - \frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a + a \sin(dx + c)}}\right) \sqrt{a}}{d} \\ & + \frac{61a \cos(dx + c)}{15d \sqrt{a + a \sin(dx + c)}} + \frac{4 \cos(dx + c) \sqrt{a + a \sin(dx + c)}}{15d} \\ & - \frac{\cot(dx + c) \sqrt{a + a \sin(dx + c)}}{d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 96 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5 - 160 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}c\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.79 Problem number 447

$$\int \cos(c + dx) \cot^3(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{13 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a + a \sin(dx + c)}}\right) \sqrt{a}}{4d} - \frac{2a \cos(dx + c)}{3d \sqrt{a + a \sin(dx + c)}} - \frac{a \cot(dx + c)}{4d \sqrt{a + a \sin(dx + c)}} \\ & - \frac{2 \cos(dx + c) \sqrt{a + a \sin(dx + c)}}{3d} - \frac{\cot(dx + c) \csc(dx + c) \sqrt{a + a \sin(dx + c)}}{2d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 64 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right)^3 - 39 \sqrt{2} \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.80 Problem number 448

$$\int \cot^4(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{11 \operatorname{arctanh} \left( \frac{\cos(dx+c) \sqrt{a}}{\sqrt{a + a \sin(dx+c)}} \right) \sqrt{a}}{8d} - \frac{2a \cos(dx+c)}{d \sqrt{a + a \sin(dx+c)}} + \frac{11a \cot(dx+c)}{8d \sqrt{a + a \sin(dx+c)}} - \frac{a \cot(dx+c) \csc(dx+c)}{12d \sqrt{a + a \sin(dx+c)}} - \frac{\cot(dx+c) (\csc^2(dx+c)) \sqrt{a + a \sin(dx+c)}}{3d}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 33 \sqrt{2} \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 192 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.81 Problem number 449

$$\int \cot^4(c+dx) \csc(c+dx) \sqrt{a+a \sin(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{67 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a \sin(dx+c)}}\right) \sqrt{a}}{64d} + \frac{61a \cot(dx+c)}{64d \sqrt{a+a \sin(dx+c)}} \\ & + \frac{61a \cot(dx+c) \csc(dx+c)}{96d \sqrt{a+a \sin(dx+c)}} - \frac{a \cot(dx+c) (\csc^2(dx+c))}{24d \sqrt{a+a \sin(dx+c)}} \\ & - \frac{\cot(dx+c) (\csc^3(dx+c)) \sqrt{a+a \sin(dx+c)}}{4d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 201 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) - \frac{4 \left(1464 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\sqrt{a+a \sin(dx+c)}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.82 Problem number 450

$$\int \cot^4(c+dx) \csc^2(c+dx) \sqrt{a+a \sin(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{31 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a \sin(dx+c)}}\right) \sqrt{a}}{128d} - \frac{31a \cot(dx+c)}{128d \sqrt{a+a \sin(dx+c)}} \\ & + \frac{97a \cot(dx+c) \csc(dx+c)}{192d \sqrt{a+a \sin(dx+c)}} + \frac{97a \cot(dx+c) (\csc^2(dx+c))}{240d \sqrt{a+a \sin(dx+c)}} \\ & - \frac{a \cot(dx+c) (\csc^3(dx+c))}{40d \sqrt{a+a \sin(dx+c)}} - \frac{\cot(dx+c) (\csc^4(dx+c)) \sqrt{a+a \sin(dx+c)}}{5d} \end{aligned}$$



command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 465 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4 \left( 7440 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 44.83 Problem number 451

$$\int \cot^4(c + dx) \csc^3(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{55 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{512d} - \frac{55a \cot(dx+c)}{512d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{55a \cot(dx+c) \csc(dx+c)}{768d\sqrt{a+a\sin(dx+c)}} + \frac{329a \cot(dx+c) (\csc^2(dx+c))}{960d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{47a \cot(dx+c) (\csc^3(dx+c))}{160d\sqrt{a+a\sin(dx+c)}} - \frac{a \cot(dx+c) (\csc^4(dx+c))}{60d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{\cot(dx+c) (\csc^5(dx+c)) \sqrt{a+a\sin(dx+c)}}{6d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 825 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4 \left( 26400 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.84 Problem number 452

$$\int \cot^4(c + dx) \csc^4(c + dx) \sqrt{a + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{61 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right) \sqrt{a}}{1024d} - \frac{61a \cot(dx+c)}{1024d \sqrt{a+a\sin(dx+c)}} \\ & - \frac{61a \cot(dx+c) \csc(dx+c)}{1536d \sqrt{a+a\sin(dx+c)}} - \frac{61a \cot(dx+c) (\csc^2(dx+c))}{1920d \sqrt{a+a\sin(dx+c)}} \\ & + \frac{579a \cot(dx+c) (\csc^3(dx+c))}{2240d \sqrt{a+a\sin(dx+c)}} + \frac{193a \cot(dx+c) (\csc^4(dx+c))}{840d \sqrt{a+a\sin(dx+c)}} \\ & - \frac{a \cot(dx+c) (\csc^5(dx+c))}{84d \sqrt{a+a\sin(dx+c)}} - \frac{\cot(dx+c) (\csc^6(dx+c)) \sqrt{a+a\sin(dx+c)}}{7d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 6405 \sqrt{2} \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4(409920 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)))}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.85 Problem number 455

$$\int \cos^3(c + dx) \cot(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} + \frac{16 \cos(dx+c) (a + a \sin(dx+c))^{\frac{3}{2}}}{105d} \\ & - \frac{14a^2 \cos(dx+c)}{45d \sqrt{a+a\sin(dx+c)}} - \frac{34a^2 \cos(dx+c) (\sin^3(dx+c))}{63d \sqrt{a+a\sin(dx+c)}} \\ & - \frac{2a^2 \cos(dx+c) (\sin^4(dx+c))}{9d \sqrt{a+a\sin(dx+c)}} + \frac{388a \cos(dx+c) \sqrt{a+a\sin(dx+c)}}{315d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 2240 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^9 - 7200 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^7 \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.86 Problem number 456

$$\int \cos^2(c + dx) \cot^2(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{d} + \frac{4\cos(dx+c)(a+a\sin(dx+c))^{3/2}}{35d} \\ & -\frac{\cot(dx+c)(a+a\sin(dx+c))^{3/2}}{d} - \frac{2\cos(dx+c)(a+a\sin(dx+c))^{5/2}}{7ad} \\ & + \frac{171a^2\cos(dx+c)}{35d\sqrt{a+a\sin(dx+c)}} + \frac{69a\cos(dx+c)\sqrt{a+a\sin(dx+c)}}{35d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 320 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^7 - 896 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5 \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.87 Problem number 457

$$\int \cos(c + dx) \cot^3(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{4d} - \frac{2\cos(dx+c)(a+a\sin(dx+c))^{\frac{3}{2}}}{5d} \\ & - \frac{\cot(dx+c)\csc(dx+c)(a+a\sin(dx+c))^{\frac{3}{2}}}{2d} + \frac{73a^2\cos(dx+c)}{20d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{2a\cos(dx+c)\sqrt{a+a\sin(dx+c)}}{5d} - \frac{3a\cot(dx+c)\sqrt{a+a\sin(dx+c)}}{4d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 128 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^5 - 320 \operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.88 Problem number 458

$$\int \cot^4(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{37a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{8d} - \frac{\cot(dx+c)(\csc^2(dx+c))(a+a\sin(dx+c))^{\frac{3}{2}}}{3d} \\ & - \frac{8a^2\cos(dx+c)}{3d\sqrt{a+a\sin(dx+c)}} + \frac{29a^2\cot(dx+c)}{24d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{2a\cos(dx+c)\sqrt{a+a\sin(dx+c)}}{3d} - \frac{a\cot(dx+c)\csc(dx+c)\sqrt{a+a\sin(dx+c)}}{4d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 128 a \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right)^3 - 111 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.89 Problem number 459

$$\int \cot^4(c + dx) \csc(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{21a^{\frac{3}{2}} \operatorname{arctanh} \left( \frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}} \right)}{64d} - \frac{\cot(dx+c) (\csc^3(dx+c)) (a+a\sin(dx+c))^{\frac{3}{2}}}{4d} \\ & - \frac{2a^2 \cos(dx+c)}{d\sqrt{a+a\sin(dx+c)}} + \frac{149a^2 \cot(dx+c)}{64d\sqrt{a+a\sin(dx+c)}} + \frac{19a^2 \cot(dx+c) \csc(dx+c)}{32d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{a \cot(dx+c) (\csc^2(dx+c)) \sqrt{a+a\sin(dx+c)}}{8d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 21 \sqrt{2} a \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 512 a \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.90 Problem number 460

$$\int \cot^4(c + dx) \csc^2(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{165a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{128d} - \frac{\cot(dx+c)(\csc^4(dx+c))(a+a\sin(dx+c))^{\frac{3}{2}}}{5d} \\ & + \frac{91a^2 \cot(dx+c)}{128d\sqrt{a+a\sin(dx+c)}} + \frac{73a^2 \cot(dx+c) \csc(dx+c)}{64d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{31a^2 \cot(dx+c)(\csc^2(dx+c))}{80d\sqrt{a+a\sin(dx+c)}} - \frac{3a \cot(dx+c)(\csc^3(dx+c))\sqrt{a+a\sin(dx+c)}}{40d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 825 \sqrt{2} a \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) - \frac{4(7280 a \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)))}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.91 Problem number 461

$$\int \cot^4(c + dx) \csc^3(c + dx)(a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{179a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{512d} \\ & - \frac{\cot(dx+c)(\csc^5(dx+c))(a+a\sin(dx+c))^{\frac{3}{2}}}{6d} - \frac{179a^2 \cot(dx+c)}{512d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{111a^2 \cot(dx+c) \csc(dx+c)}{256d\sqrt{a+a\sin(dx+c)}} + \frac{239a^2 \cot(dx+c)(\csc^2(dx+c))}{320d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{137a^2 \cot(dx+c)(\csc^3(dx+c))}{480d\sqrt{a+a\sin(dx+c)}} - \frac{a \cot(dx+c)(\csc^4(dx+c))\sqrt{a+a\sin(dx+c)}}{20d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 2685 \sqrt{2} a \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4(85920 \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)))}{\dots} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 44.92 Problem number 462

$$\int \cot^4(c + dx) \csc^4(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{171a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{1024d} \\ & - \frac{\cot(dx+c) (\csc^6(dx+c)) (a+a\sin(dx+c))^{\frac{3}{2}}}{7d} - \frac{171a^2 \cot(dx+c)}{1024d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{57a^2 \cot(dx+c) \csc(dx+c)}{512d\sqrt{a+a\sin(dx+c)}} + \frac{199a^2 \cot(dx+c) (\csc^2(dx+c))}{640d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{1237a^2 \cot(dx+c) (\csc^3(dx+c))}{2240d\sqrt{a+a\sin(dx+c)}} + \frac{9a^2 \cot(dx+c) (\csc^4(dx+c))}{40d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{a \cot(dx+c) (\csc^5(dx+c)) \sqrt{a+a\sin(dx+c)}}{28d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 5985 \sqrt{2} a \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) + \frac{4(383040 \operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)))}{\dots} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.93 Problem number 463

$$\int \cot^4(c + dx) \csc^5(c + dx) (a + a \sin(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{1587a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{16384d} \\ & - \frac{\cot(dx+c) (\csc^7(dx+c)) (a+a\sin(dx+c))^{\frac{3}{2}}}{8d} - \frac{1587a^2 \cot(dx+c)}{16384d\sqrt{a+a\sin(dx+c)}} \\ & - \frac{529a^2 \cot(dx+c) \csc(dx+c)}{8192d\sqrt{a+a\sin(dx+c)}} - \frac{529a^2 \cot(dx+c) (\csc^2(dx+c))}{10240d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{8653a^2 \cot(dx+c) (\csc^3(dx+c))}{35840d\sqrt{a+a\sin(dx+c)}} + \frac{1957a^2 \cot(dx+c) (\csc^4(dx+c))}{4480d\sqrt{a+a\sin(dx+c)}} \\ & + \frac{83a^2 \cot(dx+c) (\csc^5(dx+c))}{448d\sqrt{a+a\sin(dx+c)}} - \frac{3a \cot(dx+c) (\csc^6(dx+c)) \sqrt{a+a\sin(dx+c)}}{112d} \end{aligned}$$

command

`integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 55545 \sqrt{2} a \log \left( \left| \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)} \right| \right) \operatorname{sgn} \left( \cos \left( -\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c \right) \right) + \frac{4(7109760 a \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.94 Problem number 485

$$\int \frac{\cos^3(c + dx) \cot(c + dx)}{(a + a \sin(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}}{\sqrt{a+a\sin(dx+c)}}\right)}{a^{\frac{5}{2}}d} \\ & + \frac{4 \operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(dx+c)}}\right) \sqrt{2}}{a^{\frac{5}{2}}d} - \frac{2 \cos(dx+c)}{a^2d\sqrt{a+a\sin(dx+c)}} \end{aligned}$$



command

```
integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} \log \left( \frac{|-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)|}{|2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c)|} \right)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} + \frac{4 \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} - \frac{4 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} - \frac{4 \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) - 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} + \frac{4 \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c) - 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c))} \right)$$

---

$2d$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 44.95 Problem number 565

$$\int \cos^5(c + dx) \sin^n(c + dx) (a + a \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{a^3 (\sin^{1+n}(dx + c))}{d(1+n)} + \frac{3a^3 (\sin^{2+n}(dx + c))}{d(2+n)} + \frac{a^3 (\sin^{3+n}(dx + c))}{d(3+n)} - \frac{5a^3 (\sin^{4+n}(dx + c))}{d(4+n)} - \frac{5a^3 (\sin^{5+n}(dx + c))}{d(5+n)} + \frac{a^3 (\sin^{6+n}(dx + c))}{d(6+n)} + \frac{3a^3 (\sin^{7+n}(dx + c))}{d(7+n)} + \frac{a^3 (\sin^{8+n}(dx + c))}{d(8+n)}$$

command

```
integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^2 \sin(dx+c)^n \sin(dx+c)^8 + 10n \sin(dx+c)^n \sin(dx+c)^8 - 2n^2 \sin(dx+c)^n \sin(dx+c)^6 + 24 \sin(dx+c)^n \sin(dx+c)^8 - 24n \sin(dx+c)^n \sin(dx+c)^6 + n^3 + 18n^2 + 104n + 192)}{n^3 + 18n^2 + 104n + 192}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.96 Problem number 566

$$\int \cos^5(c + dx) \sin^n(c + dx) (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{a^2(\sin^{1+n}(dx+c))}{d(1+n)} + \frac{2a^2(\sin^{2+n}(dx+c))}{d(2+n)} - \frac{a^2(\sin^{3+n}(dx+c))}{d(3+n)} - \frac{4a^2(\sin^{4+n}(dx+c))}{d(4+n)} \\ - \frac{a^2(\sin^{5+n}(dx+c))}{d(5+n)} + \frac{2a^2(\sin^{6+n}(dx+c))}{d(6+n)} + \frac{a^2(\sin^{7+n}(dx+c))}{d(7+n)}$$

command

```
integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^2 \sin(dx+c)^n \sin(dx+c)^7 + 8n \sin(dx+c)^n \sin(dx+c)^7 - 2n^2 \sin(dx+c)^n \sin(dx+c)^5 + 15 \sin(dx+c)^n \sin(dx+c)^7 - 20n \sin(dx+c)^n \sin(dx+c)^5 + n^2 \sin(dx+c)^n \sin(dx+c)^7) dx}{n^3 + 15n^2 + 71n + 105}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.97 Problem number 567

$$\int \cos^5(c + dx) \sin^n(c + dx) (a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$\frac{a(\sin^{1+n}(dx+c))}{d(1+n)} + \frac{a(\sin^{2+n}(dx+c))}{d(2+n)} - \frac{2a(\sin^{3+n}(dx+c))}{d(3+n)} \\ - \frac{2a(\sin^{4+n}(dx+c))}{d(4+n)} + \frac{a(\sin^{5+n}(dx+c))}{d(5+n)} + \frac{a(\sin^{6+n}(dx+c))}{d(6+n)}$$

command

```
integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^2 \sin(dx+c)^n \sin(dx+c)^6 + 6n \sin(dx+c)^n \sin(dx+c)^6 - 2n^2 \sin(dx+c)^n \sin(dx+c)^4 + 8 \sin(dx+c)^n \sin(dx+c)^6 - 16n \sin(dx+c)^n \sin(dx+c)^4 + n^2 \sin(dx+c)^n \sin(dx+c)^6) dx}{n^3 + 12n^2 + 44n + 48}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.98 Problem number 697

$$\int \cos^7(c + dx) \sin^n(c + dx) (a + a \sin(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(\sin^{1+n}(dx + c))}{d(1+n)} + \frac{3a^3(\sin^{2+n}(dx + c))}{d(2+n)} - \frac{8a^3(\sin^{4+n}(dx + c))}{d(4+n)} - \frac{6a^3(\sin^{5+n}(dx + c))}{d(5+n)} \\ & + \frac{6a^3(\sin^{6+n}(dx + c))}{d(6+n)} + \frac{8a^3(\sin^{7+n}(dx + c))}{d(7+n)} - \frac{3a^3(\sin^{9+n}(dx + c))}{d(9+n)} - \frac{a^3(\sin^{10+n}(dx + c))}{d(10+n)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 44.99 Problem number 698

$$\int \cos^7(c + dx) \sin^n(c + dx) (a + a \sin(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(\sin^{1+n}(dx + c))}{d(1+n)} + \frac{2a^2(\sin^{2+n}(dx + c))}{d(2+n)} - \frac{2a^2(\sin^{3+n}(dx + c))}{d(3+n)} - \frac{6a^2(\sin^{4+n}(dx + c))}{d(4+n)} \\ & + \frac{6a^2(\sin^{6+n}(dx + c))}{d(6+n)} + \frac{2a^2(\sin^{7+n}(dx + c))}{d(7+n)} - \frac{2a^2(\sin^{8+n}(dx + c))}{d(8+n)} - \frac{a^2(\sin^{9+n}(dx + c))}{d(9+n)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.100 Problem number 699

$$\int \cos^7(c + dx) \sin^n(c + dx)(a + a \sin(c + dx)) dx$$

Optimal antiderivative

$$\frac{a(\sin^{1+n}(dx + c))}{d(1+n)} + \frac{a(\sin^{2+n}(dx + c))}{d(2+n)} - \frac{3a(\sin^{3+n}(dx + c))}{d(3+n)} - \frac{3a(\sin^{4+n}(dx + c))}{d(4+n)} \\ + \frac{3a(\sin^{5+n}(dx + c))}{d(5+n)} + \frac{3a(\sin^{6+n}(dx + c))}{d(6+n)} - \frac{a(\sin^{7+n}(dx + c))}{d(7+n)} - \frac{a(\sin^{8+n}(dx + c))}{d(8+n)}$$

command

```
integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^3 \sin(dx+c)^n \sin(dx+c)^8 + 12n^2 \sin(dx+c)^n \sin(dx+c)^8 - 3n^3 \sin(dx+c)^n \sin(dx+c)^6 + 44n \sin(dx+c)^n \sin(dx+c)^8 - 42n^2 \sin(dx+c)^n \sin(dx+c)^8 + \dots)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.101 Problem number 936

$$\int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{a^{\frac{3}{2}}(c-d)f} + \frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)\sqrt{c+d}}{a^{\frac{3}{2}}(c-d)f\sqrt{d}}$$

command

```
integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(c+d)\operatorname{arctan}\left(\frac{\sqrt{2}d\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-cd-d^2}}\right)}{(a^2\operatorname{csgn}(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right))-a^2\operatorname{dsgn}(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)))\sqrt{-cd-d^2}} + \frac{\log(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1)}{a^2\operatorname{csgn}(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right))-a^2\operatorname{dsgn}(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right))}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**44.102 Problem number 1040**

$$\int (g \cos(e + fx))^p (a + a \sin(e + fx))^m (Am - A(1 + m + p) \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{A(g \cos (fx + e))^{1+p} (a + a \sin (fx + e))^m}{fg}$$

command

```
integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**44.103 Problem number 1041**

$$\int (g \cos(e + fx))^p (a - a \sin(e + fx))^m (Am + A(1 + m + p) \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{A(g \cos (fx + e))^{1+p} (a - a \sin (fx + e))^m}{fg}$$

command

```
integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 44.104 Problem number 1235

$$\int \cos^5(c+dx) \sin^n(c+dx) (a+b \sin(c+dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(\sin^{1+n}(dx+c))}{d(1+n)} + \frac{2ab(\sin^{2+n}(dx+c))}{d(2+n)} - \frac{(2a^2-b^2)(\sin^{3+n}(dx+c))}{d(3+n)} \\ & - \frac{4ab(\sin^{4+n}(dx+c))}{d(4+n)} + \frac{(a^2-2b^2)(\sin^{5+n}(dx+c))}{d(5+n)} \\ & + \frac{2ab(\sin^{6+n}(dx+c))}{d(6+n)} + \frac{b^2(\sin^{7+n}(dx+c))}{d(7+n)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^2 \sin(dx+c)^n \sin(dx+c)^5 + 4n \sin(dx+c)^n \sin(dx+c)^5 - 2n^2 \sin(dx+c)^n \sin(dx+c)^3 + 3 \sin(dx+c)^n \sin(dx+c)^5 - 12n \sin(dx+c)^n \sin(dx+c)^3 + n^2 \sin(dx+c)^n \sin(dx+c)^5)}{n^3 + 9n^2 + 23n + 15}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 44.105 Problem number 1236

$$\int \cos^5(c+dx) \sin^n(c+dx) (a+b \sin(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(\sin^{1+n}(dx+c))}{d(1+n)} + \frac{b(\sin^{2+n}(dx+c))}{d(2+n)} - \frac{2a(\sin^{3+n}(dx+c))}{d(3+n)} \\ & - \frac{2b(\sin^{4+n}(dx+c))}{d(4+n)} + \frac{a(\sin^{5+n}(dx+c))}{d(5+n)} + \frac{b(\sin^{6+n}(dx+c))}{d(6+n)} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(n^2 \sin(dx+c)^n \sin(dx+c)^5 + 4n \sin(dx+c)^n \sin(dx+c)^5 - 2n^2 \sin(dx+c)^n \sin(dx+c)^3 + 3 \sin(dx+c)^n \sin(dx+c)^5 - 12n \sin(dx+c)^n \sin(dx+c)^3 + n^2 \sin(dx+c)^n \sin(dx+c)^5)}{n^3 + 9n^2 + 23n + 15}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 45 Test file number 75

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/75\_4.1.2.3-g\_sin-^p-a+b\_sin-^m-c+d\_sin-^n

### 45.1 Problem number 5

$$\int \csc(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2 cx}{2} - \frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{a^2 c \cos(fx + e)}{f} + \frac{a^2 c \cos(fx + e) \sin(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(fx + e)a^2c + 2a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - \frac{2\left(a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 2a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 2a^2c\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)^2}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.2 Problem number 6

$$\int \csc^2(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$-a^2 cx - \frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{a^2 c \cos(fx + e)}{f} - \frac{a^2 c \cot(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)^2*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(fx + e)a^2c - 6a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) - 3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + \frac{2a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 10a^2c}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.3 Problem number 7

$$\int \csc^3(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$-a^2 cx + \frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{a^2 c \cot(fx + e)}{f} - \frac{a^2 c \cot(fx + e) \csc(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)^3*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 8(fx + e)a^2 c - 4a^2 c \log\left(\left|\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right|\right) + 4a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + \frac{6a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}{\tan\left(\frac{1}{2} e\right)}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.4 Problem number 8

$$\int \csc^4(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{a^2 c (\cot^3(fx + e))}{3f} - \frac{a^2 c \cot(fx + e) \csc(fx + e)}{2f}$$

command

```
integrate(csc(f*x+e)^4*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + 3a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 12a^2 c \log\left(\left|\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right|\right) - 3a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + \frac{22a^2 c \tan\left(\frac{1}{2} e\right)}{24f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 45.5 Problem number 9

$$\int \csc^5(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{a^2 c (\cot^3(fx + e))}{3f} + \frac{a^2 c \cot(fx + e) \csc(fx + e)}{8f} - \frac{a^2 c \cot(fx + e) (\csc^3(fx + e))}{4f}$$

command

```
integrate(csc(f*x+e)^5*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 8 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 24 a^2 c \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right|\right) - 24 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + \frac{50 a^2 c}{192 f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.6 Problem number 10

$$\int \csc^6(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{a^2 c (\cot^3(fx + e))}{3f} - \frac{a^2 c (\cot^5(fx + e))}{5f} + \frac{a^2 c \cot(fx + e) \csc(fx + e)}{8f} - \frac{a^2 c \cot(fx + e) (\csc^3(fx + e))}{4f}$$

command

```
integrate(csc(f*x+e)^6*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 15 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 10 a^2 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 120 a^2 c \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right|\right) - 60 a^2 c}{960 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.7 Problem number 11

$$\int \csc^7(e + fx)(a + a \sin(e + fx))^2(c - c \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2 c \operatorname{arctanh}(\cos(fx + e))}{16f} - \frac{a^2 c (\cot^3(fx + e))}{3f} \\ & - \frac{a^2 c (\cot^5(fx + e))}{5f} + \frac{a^2 c \cot(fx + e) \csc(fx + e)}{16f} \\ & + \frac{a^2 c \cot(fx + e) (\csc^3(fx + e))}{24f} - \frac{a^2 c \cot(fx + e) (\csc^5(fx + e))}{6f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^7*(a+a*sin(f*x+e))^2*(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$5 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 + 12 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 + 15 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 20 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 15 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 6 a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a^2 c$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 45.8 Problem number 13

$$\int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)\sqrt{a}}{cf} + \frac{2 \sec(fx+e) \sqrt{a+a\sin(fx+e)}}{cf}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{\sqrt{2} \log \left( \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c} + \frac{2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right) \sqrt{a}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 45.9 Problem number 14

$$\int \frac{\csc(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a}}{\sqrt{a + a \sin(fx + e)}} \right)}{cf\sqrt{a}} + \frac{\operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(fx + e)}} \right) \sqrt{2}}{2cf\sqrt{a}} + \frac{\sec(fx + e) \sqrt{a + a \sin(fx + e)}}{acf}$$

command

`integrate(1/sin(f*x+e)/(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} \log \left( \frac{-2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{2\sqrt{2} + 4 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right)}{c} + \frac{\log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c} - \frac{\log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{c} + \frac{2}{c \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)} \right) \sqrt{a}}{4\sqrt{a} f \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 45.10 Problem number 19

$$\int \csc(e + fx) \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{\ln(\sin(fx + e)) \sec(fx + e) \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}{f}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2)/sin(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \sqrt{c} \log\left(\left|2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^2 - 1\right|\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 45.11 Problem number 20

$$\int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{a \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{\ln(\sin(fx + e)) \sec(fx + e) \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}{cf}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

0

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 45.12 Problem number 21

$$\int \frac{\csc(e+fx) \sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{c \cos(fx+e) \ln(1+\sin(fx+e))}{f \sqrt{a+a\sin(fx+e)} \sqrt{c-c\sin(fx+e)}} + \frac{\ln(\sin(fx+e)) \sec(fx+e) \sqrt{a+a\sin(fx+e)} \sqrt{c-c\sin(fx+e)}}{af}$$

command

`integrate((c-c*sin(f*x+e))^(1/2)/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} \sqrt{c} \left( \frac{\sqrt{2} \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2} \log(|2 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1|)}{\operatorname{asgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-c\sin(fx+e)+c}}{\sqrt{a\sin(fx+e)+a}\sin(fx+e)} dx$$

## 45.13 Problem number 22

$$\int \frac{\csc(e+fx)}{\sqrt{a+a\sin(e+fx)} \sqrt{c-c\sin(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\cos(fx+e) \ln(\tan(fx+e))}{f \sqrt{a+a\sin(fx+e)} \sqrt{c-c\sin(fx+e)}}$$

command

`integrate(1/sin(f*x+e)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a} \sqrt{c} \left( \frac{\log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{\operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2 \log(|2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 - 1|)}{\operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 45.14 Problem number 23

$$\int \frac{\csc(e + fx) \sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right) \sqrt{a}}{cf} + \frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right) \sqrt{a}\sqrt{d}}{cf\sqrt{c+d}}$$

command

```
integrate((a+a*sin(f*x+e))^(1/2)/sin(f*x+e)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{2 \sqrt{2} d \operatorname{arctan}\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{-cd - d^2} c} - \frac{\sqrt{2} \log\left(\frac{-2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{c} \right) \frac{1}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 45.15 Problem number 24

$$\int \frac{\csc(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{a+a\sin(fx+e)}}\right)}{cf\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{(c-d)f\sqrt{a}} - \frac{2d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right)}{c(c-d)f\sqrt{a}\sqrt{c+d}}$$

command

```
integrate(1/sin(f*x+e)/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-cd - d^2}}\right)}{(c^2 - cd)\sqrt{-cd - d^2}} + \frac{\sqrt{2} \log\left(\frac{|-2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|}{2\sqrt{2} + 4 \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)}\right)}{c} + \frac{\log(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1)}{c - d} \right)}{2\sqrt{a} f \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46 Test file number 76

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/76\_4.1.3.1-a+b\_sin<sup>-m</sup>-c+d\_sin<sup>-n</sup>-A+B\_sin-

### 46.1 Problem number 14

$$\int \sin^n(c + dx) (a + a \sin(c + dx))^{-2-n} (-1 - n - (-2 - n) \sin(c + dx)) dx$$

Optimal antiderivative

$$\frac{\cos(dx + c) (\sin^{1+n}(dx + c)) (a + a \sin(dx + c))^{-2-n}}{d}$$

command

```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.2 Problem number 81

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256a(11A - 5B)c^5(\cos^3(fx + e))}{3465f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{2a(11A - 5B)c^2(\cos^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{99f} \\ & - \frac{2aBc(\cos^3(fx + e))(c - c \sin(fx + e))^{\frac{5}{2}}}{11f} + \frac{64a(11A - 5B)c^4(\cos^3(fx + e))}{1155f\sqrt{c - c \sin(fx + e)}} \\ & + \frac{8a(11A - 5B)c^3(\cos^3(fx + e))\sqrt{c - c \sin(fx + e)}}{231f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (6930 Bac^3 \cos(-\frac{3}{4}\pi + \frac{3}{2}fx + \frac{3}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 315 Bac^3 \cos(-\frac{11}{4}\pi + \frac{11}{2}fx + \frac{11}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.3 Problem number 82

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{64a(3A - B)c^4(\cos^3(fx + e))}{315f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{2aBc(\cos^3(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{9f} \\ & + \frac{16a(3A - B)c^3(\cos^3(fx + e))}{105f\sqrt{c - c \sin(fx + e)}} + \frac{2a(3A - B)c^2(\cos^3(fx + e))\sqrt{c - c \sin(fx + e)}}{21f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (35 Bac^2 \cos(-\frac{9}{4}\pi + \frac{9}{2}fx + \frac{9}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 630 (5 Aac^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



#### 46.4 Problem number 83

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a(7A - B)c^3(\cos^3(fx + e))}{105f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{2a(7A - B)c^2(\cos^3(fx + e))}{35f\sqrt{c - c \sin(fx + e)}} - \frac{2aBc(\cos^3(fx + e))\sqrt{c - c \sin(fx + e)}}{7f}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (15 Bac \cos(-\frac{7}{4}\pi + \frac{7}{2}fx + \frac{7}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 105(4Aac \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - Ba$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.5 Problem number 84

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a(5A + B)c^2(\cos^3(fx + e))}{15f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{2aBc(\cos^3(fx + e))}{5f\sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (30Aa \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 3Ba \cos(-\frac{5}{4}\pi + \frac{5}{2}fx + \frac{5}{2}e) \operatorname{sgn}(\sin(-\frac{1}{4}\pi +$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.6 Problem number 85

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2a(A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{f\sqrt{c}} - \frac{2a(3A + 5B) \cos(fx + e)}{3f\sqrt{c - c \sin(fx + e)}} + \frac{2aB \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{3cf}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2}(Aa\sqrt{c} + Ba\sqrt{c}) \log\left(\frac{-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{\operatorname{csgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{4\sqrt{2}\left(3Aa\sqrt{c} + 5Ba\sqrt{c} - \frac{6Aa\sqrt{c}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} - \frac{6Ba\sqrt{c}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{c\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)} \cdot \frac{1}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.7 Problem number 86

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a(A + B) \cos(fx + e)}{f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{a(A + 5B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{2c^{\frac{3}{2}}f} + \frac{2aB \cos(fx + e)}{cf\sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}(Aa\sqrt{C}+5Ba\sqrt{C})\log\left(\frac{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}\right)}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}\left(\frac{Aa\sqrt{C}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1} + \frac{Ba\sqrt{C}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}\right)}{c^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

8

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.8 Problem number 87

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a(A+B)\cos(fx+e)}{2f(c-c\sin(fx+e))^{5/2}} - \frac{a(A+9B)\cos(fx+e)}{8cf(c-c\sin(fx+e))^{3/2}} - \frac{a(A-7B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{16c^{5/2}f}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}(Aa-7Ba)\log\left(\frac{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}\right)}{c^{5/2}\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{\sqrt{2}\left(Aa\sqrt{C}+Ba\sqrt{C} + \frac{16Ba\sqrt{C}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1} - \frac{6Aa\sqrt{C}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1)}\right)}{c^3(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)^2\operatorname{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.9 Problem number 88

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(A + B) \cos(fx + e)}{3f(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{a(A + 13B) \cos(fx + e)}{24cf(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a(A - 3B) \cos(fx + e)}{32c^2f(c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{a(A - 3B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{64c^{\frac{7}{2}}f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.10 Problem number 89

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256a^2(13A - 3B)c^6(\cos^5(fx + e))}{15015f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{64a^2(13A - 3B)c^5(\cos^5(fx + e))}{3003f(c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{2a^2Bc^2(\cos^5(fx + e))(c - c \sin(fx + e))^{\frac{3}{2}}}{13f} + \frac{8a^2(13A - 3B)c^4(\cos^5(fx + e))}{429f\sqrt{c - c \sin(fx + e)}} \\ & + \frac{2a^2(13A - 3B)c^3(\cos^5(fx + e))\sqrt{c - c \sin(fx + e)}}{143f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 10010 A a^2 c^3 \cos\left(-\frac{9}{4} \pi + \frac{9}{2} f x + \frac{9}{2} e\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 1155 B a^2 c^3 \cos\left(-\frac{13}{4} \pi + \frac{13}{2} f x + \frac{13}{2} e\right) \right)}{143 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.11 Problem number 90

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{64a^2(11A - B)c^5(\cos^5(fx + e))}{3465f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{16a^2(11A - B)c^4(\cos^5(fx + e))}{693f(c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{2a^2(11A - B)c^3(\cos^5(fx + e))}{99f\sqrt{c - c \sin(fx + e)}} - \frac{2a^2Bc^2(\cos^5(fx + e))\sqrt{c - c \sin(fx + e)}}{11f}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (315 Ba^2 c^2 \cos(-\frac{11}{4} \pi + \frac{11}{2} fx + \frac{11}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 6930 (6 Aa^2 c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{11f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.12 Problem number 91

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a^2(9A + B)c^4(\cos^5(fx + e))}{315f(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{2a^2(9A + B)c^3(\cos^5(fx + e))}{63f(c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{2a^2Bc^2(\cos^5(fx + e))}{9f\sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (1890 Aa^2 c \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 35 Ba^2 c \cos(-\frac{9}{4} \pi + \frac{9}{2} fx + \frac{9}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{9f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.13 Problem number 92

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^2(7A + 3B)c^3(\cos^5(fx + e))}{35f(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{2a^2Bc^2(\cos^5(fx + e))}{7f(c - c \sin(fx + e))^{\frac{3}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 5Ba^2 \cos\left(-\frac{7}{4}\pi + \frac{7}{2}fx + \frac{7}{2}e\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 35 \left( 4Aa^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + Ba \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.14 Problem number 93

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^2Bc^2(\cos^5(fx + e))}{5f(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{2a^2(A + B)c(\cos^3(fx + e))}{3f(c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{4a^2(A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{f\sqrt{c}} - \frac{4a^2(A + B) \cos(fx + e)}{f\sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{15\sqrt{2} \left( Aa^2\sqrt{c} + Ba^2\sqrt{c} \right) \log\left( \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} \right)}{\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{8\sqrt{2} \left( 10Aa^2\sqrt{c} + 13Ba^2\sqrt{c} - \frac{35Aa^2\sqrt{c} \left( \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} \right)}{\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.15 Problem number 94

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(A+B)c^2(\cos^5(fx+e))}{2f(c-c\sin(fx+e))^{\frac{7}{2}}} + \frac{a^2(3A+7B)(\cos^3(fx+e))}{6f(c-c\sin(fx+e))^{\frac{3}{2}}} \\ & - \frac{a^2(3A+7B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{c^{\frac{3}{2}}f} + \frac{a^2(3A+7B)\cos(fx+e)}{cf\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.16 Problem number 95

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(A+B)c^2(\cos^5(fx+e))}{4f(c-c\sin(fx+e))^{\frac{9}{2}}} - \frac{a^2(A+9B)(\cos^3(fx+e))}{8f(c-c\sin(fx+e))^{\frac{5}{2}}} \\ & + \frac{3a^2(A+9B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{8c^{\frac{5}{2}}f} - \frac{3a^2(A+9B)\cos(fx+e)}{8c^2f\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.17 Problem number 96

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(A + B) c^2 (\cos^5 (fx + e))}{6f (c - c \sin (fx + e))^{\frac{11}{2}}} + \frac{a^2(A - 11B) (\cos^3 (fx + e))}{24f (c - c \sin (fx + e))^{\frac{7}{2}}} \\ & - \frac{a^2(A - 11B) \cos (fx + e)}{16c^2 f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{a^2(A - 11B) \operatorname{arctanh} \left( \frac{\cos (fx + e) \sqrt{c} \sqrt{2}}{2\sqrt{c - c \sin (fx + e)}} \right) \sqrt{2}}{32c^{\frac{7}{2}} f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.18 Problem number 97

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(A + B) c^2 (\cos^5 (fx + e))}{8f (c - c \sin (fx + e))^{\frac{13}{2}}} + \frac{a^2(3A - 13B) (\cos^3 (fx + e))}{48f (c - c \sin (fx + e))^{\frac{9}{2}}} - \frac{a^2(3A - 13B) \cos (fx + e)}{64c^2 f (c - c \sin (fx + e))^{\frac{5}{2}}} \\ & + \frac{a^2(3A - 13B) \cos (fx + e)}{256c^3 f (c - c \sin (fx + e))^{\frac{3}{2}}} + \frac{a^2(3A - 13B) \operatorname{arctanh} \left( \frac{\cos (fx + e) \sqrt{c} \sqrt{2}}{2\sqrt{c - c \sin (fx + e)}} \right) \sqrt{2}}{512c^{\frac{9}{2}} f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 46.19 Problem number 98

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{256a^3(15A - B)c^7(\cos^7(fx + e))}{45045f(c - c\sin(fx + e))^{\frac{7}{2}}} + \frac{64a^3(15A - B)c^6(\cos^7(fx + e))}{6435f(c - c\sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{8a^3(15A - B)c^5(\cos^7(fx + e))}{715f(c - c\sin(fx + e))^{\frac{3}{2}}} + \frac{2a^3(15A - B)c^4(\cos^7(fx + e))}{195f\sqrt{c - c\sin(fx + e)}} \\ & - \frac{2a^3Bc^3(\cos^7(fx + e))\sqrt{c - c\sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3003 Ba^3 c^3 \cos\left(-\frac{15}{4}\pi + \frac{15}{2}fx + \frac{15}{2}e\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 225225 \left( 8 Aa^3 c^3 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right. \right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.20 Problem number 99

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{64a^3(13A + B)c^6(\cos^7(fx + e))}{9009f(c - c\sin(fx + e))^{\frac{7}{2}}} + \frac{16a^3(13A + B)c^5(\cos^7(fx + e))}{1287f(c - c\sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{2a^3(13A + B)c^4(\cos^7(fx + e))}{143f(c - c\sin(fx + e))^{\frac{3}{2}}} - \frac{2a^3Bc^3(\cos^7(fx + e))}{13f\sqrt{c - c\sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 180180 Aa^3 c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 693 Ba^3 c^2 \cos\left(-\frac{13}{4}\pi + \frac{13}{2}fx + \frac{13}{2}e\right) \right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.21 Problem number 100

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{8a^3(11A + 3B)c^5(\cos^7(fx + e))}{693f(c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{2a^3(11A + 3B)c^4(\cos^7(fx + e))}{99f(c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{2a^3Bc^3(\cos^7(fx + e))}{11f(c - c \sin(fx + e))^{\frac{3}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (693 Ba^3 c \cos(-\frac{5}{4} \pi + \frac{5}{2} fx + \frac{5}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 63 Ba^3 c \cos(-\frac{11}{4} \pi + \frac{11}{2} fx + \frac{11}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{11 f (c - c \sin(fx + e))^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.22 Problem number 101

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^3(9A + 5B)c^4(\cos^7(fx + e))}{63f(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{2a^3Bc^3(\cos^7(fx + e))}{9f(c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (7 Ba^3 \cos(-\frac{9}{4} \pi + \frac{9}{2} fx + \frac{9}{2} e) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 126 (5 Aa^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 2 Bc^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))))}{11 f (c - c \sin(fx + e))^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.23 Problem number 102

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2a^3 B c^3 (\cos^7 (fx + e))}{7f (c - c \sin (fx + e))^{\frac{7}{2}}} - \frac{2a^3 (A + B) c^2 (\cos^5 (fx + e))}{5f (c - c \sin (fx + e))^{\frac{5}{2}}} - \frac{4a^3 (A + B) c (\cos^3 (fx + e))}{3f (c - c \sin (fx + e))^{\frac{3}{2}}} \\ & + \frac{8a^3 (A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{c}} - \frac{8a^3 (A + B) \cos (fx + e)}{f\sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.24 Problem number 103

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3 (A + B) c^3 (\cos^7 (fx + e))}{2f (c - c \sin (fx + e))^{\frac{9}{2}}} + \frac{a^3 (5A + 9B) c (\cos^5 (fx + e))}{10f (c - c \sin (fx + e))^{\frac{5}{2}}} + \frac{a^3 (5A + 9B) (\cos^3 (fx + e))}{3f (c - c \sin (fx + e))^{\frac{3}{2}}} \\ & - \frac{2a^3 (5A + 9B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{c^{\frac{3}{2}}f} + \frac{2a^3 (5A + 9B) \cos (fx + e)}{cf\sqrt{c - c \sin (fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.25 Problem number 104

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(A+B)c^3(\cos^7(fx+e))}{4f(c-c\sin(fx+e))^{\frac{11}{2}}} - \frac{a^3(3A+11B)c(\cos^5(fx+e))}{8f(c-c\sin(fx+e))^{\frac{7}{2}}} \\ & - \frac{5a^3(3A+11B)(\cos^3(fx+e))}{24cf(c-c\sin(fx+e))^{\frac{3}{2}}} \\ & + \frac{5a^3(3A+11B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{4c^{\frac{5}{2}}f} - \frac{5a^3(3A+11B)\cos(fx+e)}{4c^2f\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.26 Problem number 105

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(A+B)c^3(\cos^7(fx+e))}{6f(c-c\sin(fx+e))^{\frac{13}{2}}} - \frac{a^3(A+13B)c(\cos^5(fx+e))}{24f(c-c\sin(fx+e))^{\frac{9}{2}}} + \frac{5a^3(A+13B)(\cos^3(fx+e))}{48cf(c-c\sin(fx+e))^{\frac{5}{2}}} \\ & - \frac{5a^3(A+13B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{16c^{\frac{7}{2}}f} + \frac{5a^3(A+13B)\cos(fx+e)}{16c^3f\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46.27 Problem number 106

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(A+B)c^3(\cos^7(fx+e))}{8f(c-c\sin(fx+e))^{\frac{15}{2}}} + \frac{a^3(A-15B)c(\cos^5(fx+e))}{48f(c-c\sin(fx+e))^{\frac{11}{2}}} - \frac{5a^3(A-15B)(\cos^3(fx+e))}{192cf(c-c\sin(fx+e))^{\frac{7}{2}}} \\ & + \frac{5a^3(A-15B)\cos(fx+e)}{128c^3f(c-c\sin(fx+e))^{\frac{3}{2}}} - \frac{5a^3(A-15B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{256c^{\frac{9}{2}}f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.28 Problem number 107

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(A+B)c^3(\cos^7(fx+e))}{10f(c-c\sin(fx+e))^{\frac{17}{2}}} + \frac{a^3(3A-17B)c(\cos^5(fx+e))}{80f(c-c\sin(fx+e))^{\frac{13}{2}}} \\ & - \frac{a^3(3A-17B)(\cos^3(fx+e))}{96cf(c-c\sin(fx+e))^{\frac{9}{2}}} + \frac{a^3(3A-17B)\cos(fx+e)}{128c^3f(c-c\sin(fx+e))^{\frac{5}{2}}} \\ & - \frac{a^3(3A-17B)\cos(fx+e)}{512c^4f(c-c\sin(fx+e))^{\frac{3}{2}}} - \frac{a^3(3A-17B)\operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{1024c^{\frac{11}{2}}f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.29 Problem number 108

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12(7A - 9B) c^2 \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{35af} \\ & - \frac{(7A - 9B) c \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{7af} - \frac{(A - B) \sec(fx + e) (c - c \sin(fx + e))^{9/2}}{acf} \\ & - \frac{128(7A - 9B) c^4 \cos(fx + e)}{35af \sqrt{c - c \sin(fx + e)}} - \frac{32(7A - 9B) c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{35af} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
Giac 1.9.0-11 via sagemath 9.6 output
```

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.30 Problem number 109

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A - 7B) c \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{5af} - \frac{(A - B) \sec(fx + e) (c - c \sin(fx + e))^{7/2}}{acf} \\ & - \frac{32(5A - 7B) c^3 \cos(fx + e)}{15af \sqrt{c - c \sin(fx + e)}} - \frac{8(5A - 7B) c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{15af} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
Giac 1.9.0-11 via sagemath 9.6 output
```

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.31 Problem number 110

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sec(fx + e) (c - c \sin(fx + e))^{5/2}}{acf} - \frac{4(3A - 5B) c^2 \cos(fx + e)}{3af \sqrt{c - c \sin(fx + e)}} \\ & - \frac{(3A - 5B) c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{3af} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \sqrt{2} \sqrt{c} \left( \frac{3 (\text{Acsgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - \text{Bcsgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{a \left( \frac{\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) - 1}{\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) + 1} + 1 \right)} - \frac{3 \text{Acsgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 7 \text{Bcsgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.32 Problem number 111

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{a + a \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \sec(fx + e) (c - c \sin(fx + e))^{3/2}}{acf} - \frac{(A - 3B) c \cos(fx + e)}{af \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2 \sqrt{2} \left( \text{Asgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 3 \text{Bsgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - \frac{A(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) - 1) \text{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) + 1} \right)}{af \left( \frac{(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) - 1)^2}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) + 1)^2} - 1 \right)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.33 Problem number 112

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx)) \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A + B) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{c} \sqrt{2}}{2 \sqrt{c - c \sin(fx + e)}} \right) \sqrt{2}}{2af \sqrt{c}} - \frac{(A - B) \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{acf}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (A\sqrt{c} + B\sqrt{c}) \log \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)}{\operatorname{acsgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{4\sqrt{2} (A\sqrt{c} - B\sqrt{c})}{ac \left( \frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$


---


$$4f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46.34 Problem number 113

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(3A - B) \cos(fx + e)}{4af (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{(3A - B) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{c} \sqrt{2}}{2 \sqrt{c - c \sin(fx + e)}} \right) \sqrt{2}}{8ac^{\frac{3}{2}}f} - \frac{(A - B) \sec(fx + e)}{acf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output



$$\frac{2\sqrt{2}\left(3A\sqrt{C}-B\sqrt{C}\right)\log\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(\frac{A\sqrt{C}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1} + \frac{B\sqrt{C}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \dots$$

32 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.35 Problem number 114

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3(5A - 3B) \cos(fx + e)}{32acf(c - c \sin(fx + e))^{3/2}} + \frac{(A + B) \sec(fx + e)}{4acf(c - c \sin(fx + e))^{3/2}} + \frac{3(5A - 3B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{64ac^2 f} - \frac{(5A - 3B) \sec(fx + e)}{8ac^2 f \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12\sqrt{2}\left(5A\sqrt{C}-3B\sqrt{C}\right)\log\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^3\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{C}+B\sqrt{C}-\frac{16A\sqrt{C}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1} + \frac{90A\sqrt{C}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)}{ac^3\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \dots$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.36 Problem number 115

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{512(7A - 13B) c^3 \sec(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{105a^2 f} \\ & - \frac{64(7A - 13B) c^2 \sec(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{105a^2 f} \\ & - \frac{16(7A - 13B) c \sec(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{105a^2 f} \\ & - \frac{(7A - 13B) \sec(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}}}{21a^2 f} \\ & - \frac{(A - B) (\sec^3(fx + e) (c - c \sin(fx + e))^{\frac{13}{2}})}{3a^2 c^2 f} \\ & + \frac{2048(7A - 13B) c^4 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{105a^2 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.37 Problem number 116

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{32(5A - 11B) c^2 \sec(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{15a^2 f} \\
 & - \frac{4(5A - 11B) c \sec(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{15a^2 f} \\
 & - \frac{(5A - 11B) \sec(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{15a^2 f} \\
 & - \frac{(A - B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{\frac{11}{2}}}{3a^2 c^2 f} \\
 & + \frac{128(5A - 11B) c^3 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{15a^2 f}
 \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.38 Problem number 117

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{8(A - 3B) c \sec(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{3a^2 f} \\
 & - \frac{(A - 3B) \sec(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{3a^2 f} - \frac{(A - B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{\frac{9}{2}}}{3a^2 c^2 f} \\
 & + \frac{32(A - 3B) c^2 \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{3a^2 f}
 \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$32\sqrt{2} \left( Ac^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 3Bc^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - \frac{3Ac^2 \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.39 Problem number 118

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - 7B) \sec(fx + e) (c - c \sin(fx + e))^{3/2}}{3a^2 f} - \frac{(A - B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{7/2}}{3a^2 c^2 f} \\ & + \frac{4(A - 7B) c \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{3a^2 f} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4\sqrt{2}\sqrt{c} \left( \frac{3Bc \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^2 \left( \frac{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} - 1 \right)} + \frac{Ac \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 4Bc \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{3Ac \left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.40 Problem number 119

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{5/2}}{3a^2 c^2 f} - \frac{(A + 5B) \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{3a^2 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( A \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 5 B \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + \frac{12 B \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}{3 a^2 f \left( \frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.41 Problem number 120

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{\frac{3}{2}}}{3a^2c^2f} \\ & + \frac{(A + B) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}} \right) \sqrt{2}}{4a^2f\sqrt{c}} - \frac{(A + B) \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{2a^2cf} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \sqrt{2} \left( A \sqrt{c} + B \sqrt{c} \right) \log \left( -\frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}{a^2 c \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)} + \frac{8 \sqrt{2} \left( 2 A \sqrt{c} + B \sqrt{c} + \frac{3 A \sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} + \frac{3 B \sqrt{c} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} \right)}{a^2 c \left( \frac{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) - 1}{\cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) + 1} + 1 \right)^3 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}$$


---

24 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46.42 Problem number 121

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5A + B) \cos(fx + e)}{8a^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{(5A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{16a^2 c^{\frac{3}{2}} f} \\ & - \frac{(5A + B) \sec(fx + e)}{6a^2 c f \sqrt{c - c \sin(fx + e)}} - \frac{(A - B) (\sec^3(fx + e)) \sqrt{c - c \sin(fx + e)}}{3a^2 c^2 f} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6\sqrt{2}(5A\sqrt{c} + B\sqrt{c}) \log\left(\frac{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{a^2 c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{3\sqrt{2}\left(\frac{A\sqrt{c}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} + \frac{B\sqrt{c}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1)}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1}\right)}{a^2 c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} +$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.43 Problem number 122

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(7A - B) \cos(fx + e)}{64a^2 c f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{(7A - B) \sec(fx + e)}{24a^2 c f (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{5(7A - B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c - c \sin(fx + e)}}\right) \sqrt{2}}{128a^2 c^{\frac{5}{2}} f} \\ & - \frac{5(7A - B) \sec(fx + e)}{48a^2 c^2 f \sqrt{c - c \sin(fx + e)}} - \frac{(A - B) (\sec^3(fx + e))}{3a^2 c^2 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.44 Problem number 123

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2048(A - 3B) c^3 (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{3}{2}}}{15a^3 f} \\ & + \frac{512(A - 3B) c^2 (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{5}{2}}}{5a^3 f} \\ & - \frac{64(A - 3B) c (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{7}{2}}}{5a^3 f} \\ & - \frac{16(A - 3B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{9}{2}}}{15a^3 f} \\ & - \frac{(A - 3B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{11}{2}}}{5a^3 c f} \\ & - \frac{(A - B) (\sec^5 (fx + e)) (c - c \sin (fx + e))^{\frac{15}{2}}}{5a^3 c^3 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1024 \sqrt{2} \left( Ac^4 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - 3 Bc^4 \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) - \frac{5 Ac^4 (\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e) - 1)^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}{(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.45 Problem number 124

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{128(3A - 13B) c^2 (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{3}{2}}}{15a^3 f} \\ & + \frac{32(3A - 13B) c (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{5}{2}}}{5a^3 f} \\ & - \frac{4(3A - 13B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{7}{2}}}{5a^3 f} \\ & - \frac{(3A - 13B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{9}{2}}}{15a^3 c f} \\ & - \frac{(A - B) (\sec^5 (fx + e)) (c - c \sin (fx + e))^{\frac{13}{2}}}{5a^3 c^3 f} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.46 Problem number 125

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{32(A - 11B) c (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{3}{2}}}{15a^3 f} \\ & + \frac{8(A - 11B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{5}{2}}}{5a^3 f} \\ & - \frac{(A - 11B) (\sec^3 (fx + e)) (c - c \sin (fx + e))^{\frac{7}{2}}}{5a^3 c f} \\ & - \frac{(A - B) (\sec^5 (fx + e)) (c - c \sin (fx + e))^{\frac{11}{2}}}{5a^3 c^3 f} \end{aligned}$$



command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4\sqrt{2}\sqrt{c}\left(\frac{15Bc^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^3\left(\frac{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}-1\right)}+\frac{4Ac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-29Bc^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{20Ac^2\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.47 Problem number 126

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3} dx$$

Optimal antiderivative

$$\frac{4(A+9B)(\sec^3(fx+e))(c-c\sin(fx+e))^{3/2}}{15a^3f} - \frac{(A+9B)(\sec^3(fx+e))(c-c\sin(fx+e))^{5/2}}{5a^3cf} - \frac{(A-B)(\sec^5(fx+e))(c-c\sin(fx+e))^{9/2}}{5a^3c^3f}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{2}\left(A\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+9B\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{5Ac\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.48 Problem number 127

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{(3A + 7B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{\frac{3}{2}}}{15a^3cf} - \frac{(A - B) (\sec^5(fx + e)) (c - c \sin(fx + e))^{\frac{7}{2}}}{5a^3c^3f}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 A \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 7 B \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + \frac{20 B (\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) - 1) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.49 Problem number 128

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A + B) (\sec^3(fx + e)) (c - c \sin(fx + e))^{\frac{3}{2}}}{6a^3c^2f} - \frac{(A - B) (\sec^5(fx + e)) (c - c \sin(fx + e))^{\frac{5}{2}}}{5a^3c^3f} + \frac{(A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right)\sqrt{2}}{8a^3f\sqrt{c}} - \frac{(A + B) \sec(fx + e) \sqrt{c - c \sin(fx + e)}}{4a^3cf}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15\sqrt{2}(A\sqrt{c}+B\sqrt{c})\log\left(\frac{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}\right)}{a^3\operatorname{csgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))} + \frac{4\sqrt{2}\left(23A\sqrt{c}+17B\sqrt{c}+\frac{70A\sqrt{c}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}+\frac{70B\sqrt{c}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)-1)}{\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)+1}\right)}{a^3\operatorname{csgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 46.50 Problem number 129

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A + 3B) \cos(fx + e)}{16a^3 f (c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{(A - B) (\sec^5(fx + e)) (c - c \sin(fx + e))^{\frac{3}{2}}}{5a^3 c^3 f} \\ & + \frac{(7A + 3B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right) \sqrt{2}}{32a^3 c^{\frac{3}{2}} f} - \frac{(7A + 3B) \sec(fx + e)}{12a^3 c f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{(7A + 3B) (\sec^3(fx + e)) \sqrt{c - c \sin(fx + e)}}{30a^3 c^2 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.51 Problem number 130

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7(9A + B) \cos(fx + e)}{128a^3 c f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{7(9A + B) \sec(fx + e)}{240a^3 c f (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{7(9A + B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{c}\sqrt{2}}{2\sqrt{c-c\sin(fx+e)}}\right) \sqrt{2}}{256a^3 c^{\frac{5}{2}} f} - \frac{7(9A + B) \sec(fx + e)}{96a^3 c^2 f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{(9A + B) (\sec^3(fx + e))}{30a^3 c^2 f \sqrt{c - c \sin(fx + e)}} - \frac{(A - B) (\sec^5(fx + e)) \sqrt{c - c \sin(fx + e)}}{5a^3 c^3 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.52 Problem number 131

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$-\frac{a(A + B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{4f \sqrt{a + a \sin(fx + e)}} + \frac{aB \cos(fx + e) (c - c \sin(fx + e))^{\frac{9}{2}}}{5cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 8 B c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^{10} - 5 A c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^{10} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.53 Problem number 132

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$-\frac{a(A + B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{3f \sqrt{a + a \sin(fx + e)}} + \frac{aB \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{4cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 3 B c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^8 - 2 A c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^8 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.54 Problem number 133

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$-\frac{a(A + B) \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2f \sqrt{a + a \sin(fx + e)}} + \frac{aB \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{3cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 4 B c \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^6 - 3 A c \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{f \sqrt{a + a \sin(fx + e)}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.55 Problem number 134

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{aB \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2cf \sqrt{a + a \sin(fx + e)}} - \frac{a(A + B) \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( B \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^4 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + A \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{f \sqrt{a + a \sin(fx + e)}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.56 Problem number 135

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{a(A + B) \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{aB \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{cf \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2} B \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sqrt{c} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{\sqrt{2} (A\sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + B\sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{\operatorname{csgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.57 Problem number 136

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a(A + B) \cos(fx + e)}{f (c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} + \frac{aB \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 4B\sqrt{c} \log(|\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{A\sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + B\sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2} \right)}{2c^2 f \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.58 Problem number 137

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a(A + B) \cos(fx + e)}{2f(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} - \frac{aB \cos(fx + e)}{cf(c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4B\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - A\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - B\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)}{8c^3 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.59 Problem number 138

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a(A + B) \cos(fx + e)}{3f(c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} - \frac{aB \cos(fx + e)}{2cf(c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(3B\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 - A\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - B\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)}{24c^4 f \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.60 Problem number 139

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{6f} \\ & - \frac{a^2(3A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{30f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a(3A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$8 \left( 20 Bac^3 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^{12} - 12 Aac^3 \operatorname{sgn} \left( \cos \right. \right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.61 Problem number 140

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{5f} \\ & - \frac{a^2(5A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{30f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a(5A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{20f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`



Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 24 Bac^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) + 15 Aac^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.62 Problem number 141

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}}{4f} \\ & - \frac{a^2 A \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{3f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{aA \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}}{3f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 3 Bac \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)^8 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) + 2 Aac \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.63 Problem number 142

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2f \sqrt{c - c \sin(fx + e)}} + \frac{Bc \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{3af \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 4Ba \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 3Aa \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.64 Problem number 143

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{B \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2f \sqrt{c - c \sin(fx + e)}} - \frac{2a^2(A + B) \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{a(A + B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} \left( Aa \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + Ba \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right) \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{\sqrt{2} Bac^{\frac{3}{2}}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.65 Problem number 144

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2f (c - c \sin(fx + e))^{3/2}} + \frac{a^2 (A + 3B) \cos(fx + e) \ln(1 - \sin(fx + e))}{cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ + \frac{a(A + 3B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{2cf \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2 \sqrt{2} B a \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{c^{\frac{3}{2}} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{\sqrt{2} (A a \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 3 B a \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right)}{2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.66 Problem number 145

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{4f (c - c \sin(fx + e))^{5/2}} - \frac{aB \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{cf (c - c \sin(fx + e))^{3/2}} \\ - \frac{a^2 B \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{a} \left( \frac{4 \sqrt{2} B a \log(-2 \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 + 2) \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{c^{\frac{5}{2}} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{\sqrt{2} (A a \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 5 B a \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{8 f} \right)}{8 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.67 Problem number 146

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{6f (c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{(A - 5B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{24cf (c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(12Ba\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 3Aa\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.68 Problem number 147

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{8f (c - c \sin(fx + e))^{\frac{9}{2}}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{24cf (c - c \sin(fx + e))^{\frac{7}{2}}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{96c^2 f (c - c \sin(fx + e))^{\frac{5}{2}}}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(12Ba\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 4Aa\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.69 Problem number 148

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{10f (c - c \sin(fx + e))^{\frac{11}{2}}} \\ & - \frac{a^2(3A - 7B) \cos(fx + e)}{120c^2 f (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{a(3A - 7B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{40cf (c - c \sin(fx + e))^{\frac{9}{2}}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(40Ba\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15Aa\sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.70 Problem number 149

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a(7A - B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{42f} \\ & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{7f} \\ & - \frac{a^3(7A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{105f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a^2(7A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 120 B a^2 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 70 A a^2 c^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{14} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.71 Problem number 150

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{aA \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{5f} \\ & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}}{6f} \\ & - \frac{2a^3 A \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}}}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{a^2 A \cos(fx + e) (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}}{5f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 10 B a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 6 A a^2 c^2 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.72 Problem number 151

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{5/2} (c - c \sin(fx + e))^{3/2}}{5f} \\ & + \frac{(5A + B) c^2 \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{30f \sqrt{c - c \sin(fx + e)}} \\ & + \frac{(5A + B) c \cos(fx + e) (a + a \sin(fx + e))^{5/2} \sqrt{c - c \sin(fx + e)}}{20f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 24 B a^2 c \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 15 A a^2 c \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.73 Problem number 152

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{3f \sqrt{c - c \sin(fx + e)}} + \frac{B c \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{4a f \sqrt{c - c \sin(fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( 3 B a^2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 2 A a^2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.74 Problem number 153

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a(A + B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{2f \sqrt{c - c \sin(fx + e)}} - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{3f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{4a^3(A + B) \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2a^2(A + B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{6 \left( \sqrt{2} A a^2 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + \sqrt{2} B a^2 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \right) \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 + 1\right)}{\operatorname{csgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{\sqrt{2} a^3 (A + B) \cos(fx + e) \ln(1 - \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.75 Problem number 155

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{4f (c - c \sin(fx + e))^{5/2}} - \frac{a(A + 5B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{4cf (c - c \sin(fx + e))^{3/2}} \\ & - \frac{a^3(A + 5B) \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{a^2(A + 5B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{2c^2 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$



command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{4\sqrt{2} Ba^2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c^{\frac{5}{2}} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{2\sqrt{2} (Aa^2\sqrt{C} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 5Ba^2\sqrt{C} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.76 Problem number 156

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{6f(c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{aB \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2cf(c - c \sin(fx + e))^{\frac{5}{2}}} + \frac{a^2 B \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} + \frac{a^3 B \cos(fx + e) \ln(1 - \sin(fx + e))}{c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{6\sqrt{2} Ba^2 \log(-2 \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 2) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{c^{\frac{7}{2}} \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2} (3(Aa^2\sqrt{C} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 5Ba^2\sqrt{C} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))))}{c^3 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.77 Problem number 157

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{8f (c - c \sin(fx + e))^{9/2}} + \frac{(A - 7B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{48cf (c - c \sin(fx + e))^{7/2}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(24 Ba^2 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) + 6 Aa^2 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)\right)}{8f (c - c \sin(fx + e))^{9/2}} + \frac{(A - 7B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{48cf (c - c \sin(fx + e))^{7/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.78 Problem number 158

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{10f (c - c \sin(fx + e))^{11/2}} + \frac{(A - 4B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{40cf (c - c \sin(fx + e))^{9/2}} + \frac{(A - 4B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{240c^2 f (c - c \sin(fx + e))^{7/2}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(30 Ba^2 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) + 10 Aa^2 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)\right)}{10f (c - c \sin(fx + e))^{11/2}} + \frac{(A - 4B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{40cf (c - c \sin(fx + e))^{9/2}} + \frac{(A - 4B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{240c^2 f (c - c \sin(fx + e))^{7/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.79 Problem number 159

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{12f (c - c \sin(fx + e))^{13/2}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{40cf (c - c \sin(fx + e))^{11/2}}$$

$$+ \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{160c^2 f (c - c \sin(fx + e))^{9/2}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{960c^3 f (c - c \sin(fx + e))^{7/2}}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(40 Ba^2 \sqrt{c} \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^6 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 15 Aa^2 \sqrt{c} \cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^4 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{12f (c - c \sin(fx + e))^{13/2}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{40cf (c - c \sin(fx + e))^{11/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.80 Problem number 160

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{9/2} dx$$

Optimal antiderivative

$$\frac{a^2(9A - B) \cos(fx + e) (a + a \sin(fx + e))^{3/2} (c - c \sin(fx + e))^{9/2}}{84f}$$

$$- \frac{a(9A - B) \cos(fx + e) (a + a \sin(fx + e))^{5/2} (c - c \sin(fx + e))^{9/2}}{72f}$$

$$- \frac{B \cos(fx + e) (a + a \sin(fx + e))^{7/2} (c - c \sin(fx + e))^{9/2}}{9f}$$

$$- \frac{a^4(9A - B) \cos(fx + e) (c - c \sin(fx + e))^{9/2}}{315f \sqrt{a + a \sin(fx + e)}}$$

$$- \frac{a^3(9A - B) \cos(fx + e) (c - c \sin(fx + e))^{9/2} \sqrt{a + a \sin(fx + e)}}{126f}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.81 Problem number 161

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^2 A \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{7f} \\ & - \frac{a A \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{7f} \\ & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{\frac{7}{2}} (c - c \sin(fx + e))^{\frac{7}{2}}}{8f} \\ & - \frac{2a^4 A \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}}}{35f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^3 A \cos(fx + e) (c - c \sin(fx + e))^{\frac{7}{2}} \sqrt{a + a \sin(fx + e)}}{35f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$32 \left( 35 B a^3 c^3 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{16} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 20 A a^3 c^3 \cos\right.$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.82 Problem number 162

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(7A + B) c \cos(fx + e) (a + a \sin(fx + e))^{7/2} (c - c \sin(fx + e))^{3/2}}{42f} \\ & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{7/2} (c - c \sin(fx + e))^{5/2}}{7f} \\ & + \frac{(7A + B) c^3 \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{105f \sqrt{c - c \sin(fx + e)}} \\ & + \frac{2(7A + B) c^2 \cos(fx + e) (a + a \sin(fx + e))^{7/2} \sqrt{c - c \sin(fx + e)}}{105f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 120 B a^3 c^2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{14} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 70 A a^3 c^2 \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.83 Problem number 163

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{B \cos(fx + e) (a + a \sin(fx + e))^{7/2} (c - c \sin(fx + e))^{3/2}}{6f} \\ & + \frac{(3A + B) c^2 \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{30f \sqrt{c - c \sin(fx + e)}} \\ & + \frac{(3A + B) c \cos(fx + e) (a + a \sin(fx + e))^{7/2} \sqrt{c - c \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 20 B a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{12} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 12 A a^3 c \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.84 Problem number 164

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos (fx + e) (a + a \sin (fx + e))^{\frac{7}{2}}}{4 f \sqrt{c - c \sin (fx + e)}} + \frac{B c \cos (fx + e) (a + a \sin (fx + e))^{\frac{9}{2}}}{5 a f \sqrt{c - c \sin (fx + e)}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 8 B a^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right)^{10} \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 5 A a^3 \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.85 Problem number 165

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^2(A+B)\cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{f\sqrt{c-c\sin(fx+e)}} \\ & - \frac{a(A+B)\cos(fx+e)(a+a\sin(fx+e))^{\frac{5}{2}}}{3f\sqrt{c-c\sin(fx+e)}} - \frac{B\cos(fx+e)(a+a\sin(fx+e))^{\frac{7}{2}}}{4f\sqrt{c-c\sin(fx+e)}} \\ & - \frac{8a^4(A+B)\cos(fx+e)\ln(1-\sin(fx+e))}{f\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & - \frac{4a^3(A+B)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{f\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{2}\sqrt{a}\left(\frac{6\left(\sqrt{2}Aa^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\sqrt{2}Ba^3\sqrt{c}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{\operatorname{csgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+3\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.86 Problem number 166

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A+B)\cos(fx+e)(a+a\sin(fx+e))^{\frac{7}{2}}}{2f(c-c\sin(fx+e))^{\frac{3}{2}}} + \frac{a^2(3A+5B)\cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{2cf\sqrt{c-c\sin(fx+e)}} \\ & + \frac{a(3A+5B)\cos(fx+e)(a+a\sin(fx+e))^{\frac{5}{2}}}{6cf\sqrt{c-c\sin(fx+e)}} \\ & + \frac{4a^4(3A+5B)\cos(fx+e)\ln(1-\sin(fx+e))}{cf\sqrt{a+a\sin(fx+e)}\sqrt{c-c\sin(fx+e)}} \\ & + \frac{2a^3(3A+5B)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{cf\sqrt{c-c\sin(fx+e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{6 \left( 3 \sqrt{2} A a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 5 \sqrt{2} B a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \right) \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 + 1)}{c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.87 Problem number 167

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{4f (c - c \sin(fx + e))^{5/2}} - \frac{a(A + 3B) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{2cf (c - c \sin(fx + e))^{3/2}} \\ & - \frac{3a^2(A + 3B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{4c^2 f \sqrt{c - c \sin(fx + e)}} \\ & - \frac{6a^4(A + 3B) \cos(fx + e) \ln(1 - \sin(fx + e))}{c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{3a^3(A + 3B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^2 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{6 \sqrt{2} \left( A a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 3 B a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \right) \log(-\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 + 1)}{c^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right) + \frac{2 \left( \sqrt{2} \right)}{2}$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 46.88 Problem number 168

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{7}{2}}}{6f (c - c \sin(fx + e))^{\frac{7}{2}}} - \frac{a(A + 7B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{12cf (c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{a^2(A + 7B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{4c^2 f (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{a^4(A + 7B) \cos(fx + e) \ln(1 - \sin(fx + e))}{c^3 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{a^3(A + 7B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{2c^3 f \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{12 \sqrt{2} B a^3 \cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{c^{\frac{7}{2}} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{6 \sqrt{2} (A a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 7 B a^3 \sqrt{c} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{c^4 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.89 Problem number 169

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{\frac{7}{2}}}{8f (c - c \sin(fx + e))^{\frac{9}{2}}} - \frac{aB \cos(fx + e) (a + a \sin(fx + e))^{\frac{5}{2}}}{3cf (c - c \sin(fx + e))^{\frac{7}{2}}} \\ & + \frac{a^2 B \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{2c^2 f (c - c \sin(fx + e))^{\frac{5}{2}}} - \frac{a^3 B \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{c^3 f (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{a^4 B \cos(fx + e) \ln(1 - \sin(fx + e))}{c^4 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{24 \sqrt{2} B a^3 \log\left(-2 \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 + 2\right) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{c^{\frac{9}{2}} \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{\sqrt{2}}{12} \left(12 A a^3 \sqrt{c} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 7 B\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.90 Problem number 170

$$\int \frac{(a + a \sin(e + f x))^{7/2} (A + B \sin(e + f x))}{(c - c \sin(e + f x))^{11/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(f x + e) (a + a \sin(f x + e))^{\frac{7}{2}}}{10 f (c - c \sin(f x + e))^{\frac{11}{2}}} + \frac{(A - 9 B) \cos(f x + e) (a + a \sin(f x + e))^{\frac{7}{2}}}{80 c f (c - c \sin(f x + e))^{\frac{9}{2}}}$$

command

```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left(40 B a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 10 A a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.91 Problem number 171

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{13/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{12f (c - c \sin(fx + e))^{13/2}} + \frac{(A - 5B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{60cf (c - c \sin(fx + e))^{11/2}} \\ + \frac{(A - 5B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{480c^2 f (c - c \sin(fx + e))^{9/2}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(60Ba^3\sqrt{c} \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))^8 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 20Aa^3\sqrt{c} \cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^6 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{(c - c \sin(fx + e))^{13/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.92 Problem number 172

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{15/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{14f (c - c \sin(fx + e))^{15/2}} + \frac{(3A - 11B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{168cf (c - c \sin(fx + e))^{13/2}} \\ + \frac{(3A - 11B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{840c^2 f (c - c \sin(fx + e))^{11/2}} \\ + \frac{(3A - 11B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{6720c^3 f (c - c \sin(fx + e))^{9/2}}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x, algorithm="giac"`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left(280 B a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 105 A a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.93 Problem number 173

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{17/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A + B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{16f (c - c \sin(fx + e))^{17/2}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{56cf (c - c \sin(fx + e))^{15/2}} \\ & + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{224c^2 f (c - c \sin(fx + e))^{13/2}} + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{1120c^3 f (c - c \sin(fx + e))^{11/2}} \\ & + \frac{(A - 3B) \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{8960c^4 f (c - c \sin(fx + e))^{9/2}} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left(140 B a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)^8 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 56 A a^3 \sqrt{c} \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^6 \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.94 Problem number 174

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2f \sqrt{a + a \sin(fx + e)}} - \frac{B \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{3f \sqrt{a + a \sin(fx + e)}} \\ + \frac{4(A - B) c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{2(A - B) c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{c} \left( \frac{6 \sqrt{2} (A \sqrt{a} c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - B \sqrt{a} c^2 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) \log(-2 \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 2)}{\operatorname{asgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \sqrt{2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.95 Problem number 175

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$- \frac{B \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2f \sqrt{a + a \sin(fx + e)}} + \frac{2(A - B) c^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ + \frac{(A - B) c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{c} \left( \frac{\sqrt{2} (A \sqrt{a} c \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - B \sqrt{a} c \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 1)}{\operatorname{asgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \sqrt{2} B a^{3/2} c \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.96 Problem number 176

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos(fx + e) \ln(1 + \sin(fx + e))}{f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{B \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{f \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2 \sqrt{2} B \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2}{\sqrt{a} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \frac{\sqrt{2} (A \sqrt{a} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - B \sqrt{a} \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{a \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.97 Problem number 177

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{(A + B) \cos(fx + e) \ln(1 - \sin(fx + e))}{2f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{(A - B) \cos(fx + e) \ln(1 + \sin(fx + e))}{2f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 (A \sqrt{c} + B \sqrt{c}) \log(|\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)|)}{\sqrt{a} \operatorname{csgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - \frac{(A \sqrt{a} \sqrt{c} - B \sqrt{a} \sqrt{c}) \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 1)}{a \operatorname{csgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}} dx$$

## 46.98 Problem number 178

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e)}{2f (c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} + \frac{(A - B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c})\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)|\right)}{ac^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\dots}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{3/2}} dx$$

## 46.99 Problem number 179

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(A + B) \cos(fx + e)}{4f (c - c \sin(fx + e))^{5/2} \sqrt{a + a \sin(fx + e)}} + \frac{(A - B) \cos(fx + e)}{4cf (c - c \sin(fx + e))^{3/2} \sqrt{a + a \sin(fx + e)}} + \frac{(A - B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{4c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)|\right)}{ac^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{2\left(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)|\right)}{ac^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\dots}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{5/2}} dx$$

## 46.100 Problem number 180

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \cos(fx + e) (c - c \sin(fx + e))^{7/2}}{2f (a + a \sin(fx + e))^{3/2}} \\ & - \frac{(3A - 5B) c^2 \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2af \sqrt{a + a \sin(fx + e)}} \\ & - \frac{(3A - 5B) c \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{6af \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4(3A - 5B) c^4 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2(3A - 5B) c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{c} \left( \frac{6 \left( 3 \sqrt{2} A \sqrt{a} c^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 5 \sqrt{2} B \sqrt{a} c^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \right) \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 1)}{a^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} \right) -$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.101 Problem number 181

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \cos(fx + e) (c - c \sin(fx + e))^{5/2}}{2f (a + a \sin(fx + e))^{3/2}} - \frac{(A - 2B) c \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2af \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4(A - 2B) c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{2(A - 2B) c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af \sqrt{a + a \sin(fx + e)}} \end{aligned}$$



command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( B\sqrt{a} c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + A\sqrt{a} c^2 \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.102 Problem number 182

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A - B) \cos(fx + e) (c - c \sin(fx + e))^{3/2}}{2f (a + a \sin(fx + e))^{3/2}} - \frac{(A - 3B) c^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{(A - 3B) c \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{2af \sqrt{a + a \sin(fx + e)}}$$

command

```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{2\sqrt{2} B c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2}{a^{3/2} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2} \left( A\sqrt{a} c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 3B\sqrt{a} c \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \right)$$

2f

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.103 Problem number 183

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(A - B) c \cos(fx + e)}{f (a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} + \frac{B c \cos(fx + e) \ln(1 + \sin(fx + e))}{af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4 B \sqrt{a} \log\left(\left|\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right|\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - \frac{A \sqrt{a} \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - B \sqrt{a} \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}{\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2}}{2 a^2 f \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.104 Problem number 184

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{(A - B) \cos(fx + e)}{2f (a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} + \frac{(A + B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2af \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(A\sqrt{c} + B\sqrt{c}) \log\left(\left|\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right|\right)}{a^{\frac{3}{2}} c \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{(A\sqrt{a}\sqrt{c} + B\sqrt{a}\sqrt{c}) \log\left(-\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 + 1\right)}{a^2 c \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{a^2 c \cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}{4 f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

## 46.105 Problem number 185

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{2f (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{A \cos(fx + e)}{2af (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{A \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2acf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2A \log\left(-\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{4A \log\left(|\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^{\frac{3}{2}}c^{\frac{3}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right))}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 46.106 Problem number 186

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{2f (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{(3A - B) \cos(fx + e)}{8af (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(3A - B) \cos(fx + e)}{8acf (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(3A - B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8a c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^2c^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{4\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)\log\left(|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)|\right)}{a^2c^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{2\left(3A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}\right)}{a^2c^3}$$

32 f

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin (f x+e)+A}{\left(a \sin (f x+e)+a\right)^{\frac{3}{2}}\left(-c \sin (f x+e)+c\right)^{\frac{5}{2}}} d x$$

#### 46.107 Problem number 187

$$\int \frac{(A+B \sin (e+f x))(c-c \sin (e+f x))^{9 / 2}}{\left(a+a \sin (e+f x)\right)^{5 / 2}} d x$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3 A-7 B) c \cos (f x+e)\left(c-c \sin (f x+e)\right)^{\frac{7}{2}}}{4 a f\left(a+a \sin (f x+e)\right)^{\frac{3}{2}}}-\frac{(A-B) \cos (f x+e)\left(c-c \sin (f x+e)\right)^{\frac{9}{2}}}{4 f\left(a+a \sin (f x+e)\right)^{\frac{5}{2}}} \\ & +\frac{(3 A-7 B) c^3 \cos (f x+e)\left(c-c \sin (f x+e)\right)^{\frac{3}{2}}}{a^2 f \sqrt{a+a \sin (f x+e)}} \\ & +\frac{(3 A-7 B) c^2 \cos (f x+e)\left(c-c \sin (f x+e)\right)^{\frac{5}{2}}}{3 a^2 f \sqrt{a+a \sin (f x+e)}} \\ & +\frac{8(3 A-7 B) c^5 \cos (f x+e) \ln \left(1+\sin (f x+e)\right)}{a^2 f \sqrt{a+a \sin (f x+e)} \sqrt{c-c \sin (f x+e)}} \\ & +\frac{4(3 A-7 B) c^4 \cos (f x+e) \sqrt{c-c \sin (f x+e)}}{a^2 f \sqrt{a+a \sin (f x+e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2}\sqrt{c}\left(\frac{12\left(3\sqrt{2}A\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-7\sqrt{2}B\sqrt{a}c^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{a^3\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.108 Problem number 188

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 3B)c \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}}}{2af(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{(A - B) \cos(fx + e)(c - c \sin(fx + e))^{\frac{7}{2}}}{4f(a + a \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{3(A - 3B)c^2 \cos(fx + e)(c - c \sin(fx + e))^{\frac{3}{2}}}{4a^2 f \sqrt{a + a \sin(fx + e)}} \\ & + \frac{6(A - 3B)c^4 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{3(A - 3B)c^3 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{c} \left( \frac{6 \sqrt{2} (A \sqrt{a} c^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - 3B \sqrt{a} c^3 \operatorname{sgn}(\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) \log(-\sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)^2 + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))} - 2 \left( \sqrt{\dots} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.109 Problem number 189

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - 5B)c \cos(fx + e)(c - c \sin(fx + e))^{\frac{3}{2}}}{4af(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{(A - B) \cos(fx + e)(c - c \sin(fx + e))^{\frac{5}{2}}}{4f(a + a \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{(A - 5B)c^3 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{(A - 5B)c^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{2a^2 f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{4\sqrt{2} Bc^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2\sqrt{2} (A\sqrt{a} c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 5B\sqrt{a} c^2 \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.110 Problem number 190

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e) (c - c \sin(fx + e))^{\frac{3}{2}}}{4f (a + a \sin(fx + e))^{\frac{5}{2}}} \\ & - \frac{Bc^2 \cos(fx + e) \ln(1 + \sin(fx + e))}{a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} - \frac{Bc \cos(fx + e) \sqrt{c - c \sin(fx + e)}}{af (a + a \sin(fx + e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 8B\sqrt{a} c \log(|\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \frac{A\sqrt{a} c \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - B\sqrt{a} c \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{4a^3 f \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.111 Problem number 191

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(A - B) c \cos(fx + e)}{2f (a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}} - \frac{Bc \cos(fx + e)}{af (a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(4B\sqrt{a} \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + A\sqrt{a} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - B\sqrt{a} \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\right)}{8a^3 f \cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^4 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.112 Problem number 192

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{4f (a + a \sin(fx + e))^{\frac{5}{2}} \sqrt{c - c \sin(fx + e)}} \\ & - \frac{(A + B) \cos(fx + e)}{4af (a + a \sin(fx + e))^{\frac{3}{2}} \sqrt{c - c \sin(fx + e)}} \\ & + \frac{(A + B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{4a^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(A\sqrt{c} + B\sqrt{c}) \log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)}{a^{\frac{5}{2}} \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{4(A\sqrt{a} \sqrt{c} + B\sqrt{a} \sqrt{c}) \log\left(|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)|\right)}{a^3 \operatorname{csgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} + \frac{2(A\sqrt{a} \sqrt{c})}{\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} \cdot 16f$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

## 46.113 Problem number 193

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{4f (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(3A + B) \cos(fx + e)}{8af (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{3}{2}}} \\ & + \frac{(3A + B) \cos(fx + e)}{8a^2 f (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(3A + B) \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8a^2 c f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(3A\sqrt{a}\sqrt{c} + B\sqrt{a}\sqrt{c}) \log(-\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 1)}{a^3 c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{4(3A\sqrt{a}\sqrt{c} + B\sqrt{a}\sqrt{c}) \log(|\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|)}{a^3 c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{2(3A\sqrt{a}\sqrt{c})}{32f}$$

32 f

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 46.114 Problem number 194

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{4f (a + a \sin(fx + e))^{\frac{5}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}} \\ & - \frac{A \cos(fx + e)}{2af (a + a \sin(fx + e))^{\frac{3}{2}} (c - c \sin(fx + e))^{\frac{5}{2}}} \\ & + \frac{3A \cos(fx + e)}{8a^2 f (c - c \sin(fx + e))^{\frac{5}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{3A \cos(fx + e)}{8a^2 c f (c - c \sin(fx + e))^{\frac{3}{2}} \sqrt{a + a \sin(fx + e)}} \\ & + \frac{3A \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8a^2 c^2 f \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} \end{aligned}$$



command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12 A \log\left(-\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)^2 + 1\right)}{a^{\frac{5}{2}} c^{\frac{5}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} - \frac{24 A \log\left(|\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)|\right)}{a^{\frac{5}{2}} c^{\frac{5}{2}} \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)} + \frac{12 A \sqrt{a} \sqrt{c}}{64 f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

#### 46.115 Problem number 222

$$\int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n (B(m - n) - B(1 + m + n) \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{B \cos(fx + e) (a + a \sin(fx + e))^m (c - c \sin(fx + e))^n}{f}$$

command

`integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.116 Problem number 223

$$\int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^n (B(m - n) + B(1 + m + n) \sin(e + fx)) dx$$

Optimal antiderivative

$$\frac{B \cos(fx + e) (a - a \sin(fx + e))^m (c + c \sin(fx + e))^n}{f}$$

command

```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x, algorithm="
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.117 Problem number 286

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4d(c+d)(-9Ad+Bc-8Bd)\cos(fx+e)(a+a\sin(fx+e))^{\frac{3}{2}}}{105af} \\ & + \frac{4a(c+d)(-9Ad+Bc-8Bd)(15c^2+10cd+7d^2)\cos(fx+e)}{315df\sqrt{a+a\sin(fx+e)}} \\ & + \frac{2a(-9Ad+Bc-8Bd)\cos(fx+e)(c+d\sin(fx+e))^3}{63df\sqrt{a+a\sin(fx+e)}} \\ & - \frac{2aB\cos(fx+e)(c+d\sin(fx+e))^4}{9df\sqrt{a+a\sin(fx+e)}} \\ & + \frac{8(5c-d)(c+d)(-9Ad+Bc-8Bd)\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{315f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.118 Problem number 287

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2d(-7Ad + Bc - 6Bd) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{35af} \\ & + \frac{2a(-7Ad + Bc - 6Bd) (15c^2 + 10cd + 7d^2) \cos(fx + e)}{105df \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2aB \cos(fx + e) (c + d \sin(fx + e))^3}{7df \sqrt{a + a \sin(fx + e)}} \\ & + \frac{4(5c - d) (-7Ad + Bc - 6Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} (15 B d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) \sin(-\frac{7}{4} \pi + \frac{7}{2} f x + \frac{7}{2} e) + 105 (8 A c^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 4 B$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.119 Problem number 288

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2Bd \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{5af} - \frac{2a(15Ac + 5Ad + 5Bc + 7Bd) \cos(fx + e)}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2(5Ad + 5Bc - 2Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 B \operatorname{dsgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{5}{4} \pi + \frac{5}{2} f x + \frac{5}{2} e \right) + 30 \left( 2 A \operatorname{csgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + B \operatorname{csgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.120 Problem number 289

$$\int \sqrt{a + a \sin(e + f x)} (A + B \sin(e + f x)) dx$$

Optimal antiderivative

$$\frac{2a(3A + B) \cos(fx + e)}{3f \sqrt{a + a \sin(fx + e)}} - \frac{2B \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3f}$$

command

```
integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( B \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{3}{4} \pi + \frac{3}{2} f x + \frac{3}{2} e \right) + 3 \left( 2 A \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + B \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.121 Problem number 290

$$\int \frac{\sqrt{a + a \sin(e + f x)} (A + B \sin(e + f x))}{c + d \sin(e + f x)} dx$$

Optimal antiderivative

$$\frac{2(-Ad + Bc) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx + e)}} \right) \sqrt{a}}{d^{\frac{3}{2}} f \sqrt{c+d}} - \frac{2aB \cos(fx + e)}{df \sqrt{a + a \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{2 B \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}{d} + \frac{\sqrt{2} \left( B \operatorname{csgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - A d \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \right) \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}{\sqrt{-c d - d^2}}\right)}{\sqrt{-c d - d^2} d} \right)$$


---

$f$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.122 Problem number 291

$$\int \frac{\sqrt{a + a \sin(e + f x)} (A + B \sin(e + f x))}{(c + d \sin(e + f x))^2} dx$$

Optimal antiderivative

$$\frac{(A d + B(c + 2 d)) \operatorname{arctanh}\left(\frac{\cos(f x + e) \sqrt{a} \sqrt{d}}{\sqrt{c + d} \sqrt{a + a \sin(f x + e)}}\right) \sqrt{a}}{d^{\frac{3}{2}} (c + d)^{\frac{3}{2}} f} + \frac{a(-A d + B c) \cos(f x + e)}{d(c + d) f (c + d \sin(f x + e)) \sqrt{a + a \sin(f x + e)}}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} \left( B \operatorname{csgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + A d \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 2 B d \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \right) \arctan\left(\frac{\sqrt{2} d \sin\left(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e\right)}{\sqrt{-c d - d^2}}\right)}{(c d + d^2) \sqrt{-c d - d^2}} \right)$$


---

$2 f$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.123 Problem number 292

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3Ad + B(c + 4d)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a+a\sin(fx+e)}}\right) \sqrt{a}}{4d^{\frac{3}{2}}(c+d)^{\frac{5}{2}}f} \\ & + \frac{a(-Ad + Bc) \cos(fx + e)}{2d(c+d)f(c+d\sin(fx+e))^2 \sqrt{a+a\sin(fx+e)}} \\ & - \frac{a(3Ad + B(c + 4d)) \cos(fx + e)}{4d(c+d)^2 f(c+d\sin(fx+e)) \sqrt{a+a\sin(fx+e)}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{\sqrt{2} (B \operatorname{csgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 3A \operatorname{dsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 4B \operatorname{dsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))) \operatorname{arctan}\left(\frac{\sqrt{2} d \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-cd - d^2}}\right)}{(c^2d + 2cd^2 + d^3) \sqrt{-cd - d^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.124 Problem number 293

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{4(c+d)(11A(c-17d)d - 3B(c^2 - 9cd + 56d^2)) \cos(fx+e) (a + a \sin(fx+e))^{\frac{3}{2}}}{1155f} \\
& + \frac{4a^2(c+d)(15c^2 + 10cd + 7d^2)(11A(c-17d)d - 3B(c^2 - 9cd + 56d^2)) \cos(fx+e)}{3465d^2 f \sqrt{a + a \sin(fx+e)}} \\
& + \frac{2a^2(11A(c-17d)d - 3B(c^2 - 9cd + 56d^2)) \cos(fx+e) (c + d \sin(fx+e))^3}{693d^2 f \sqrt{a + a \sin(fx+e)}} \\
& + \frac{2a^2(3B(c-4d) - 11Ad) \cos(fx+e) (c + d \sin(fx+e))^4}{99d^2 f \sqrt{a + a \sin(fx+e)}} \\
& + \frac{8a(5c-d)(c+d)(11A(c-17d)d - 3B(c^2 - 9cd + 56d^2)) \cos(fx+e) \sqrt{a + a \sin(fx+e)}}{3465df} \\
& - \frac{2aB \cos(fx+e) (c + d \sin(fx+e))^4 \sqrt{a + a \sin(fx+e)}}{11df}
\end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.125 Problem number 294

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{2(3A(c-13d)d - B(c^2 - 7cd + 34d^2)) \cos(fx+e) (a + a \sin(fx+e))^{\frac{3}{2}}}{105f} \\
& + \frac{2a^2(15c^2 + 10cd + 7d^2)(3A(c-13d)d - B(c^2 - 7cd + 34d^2)) \cos(fx+e)}{315d^2 f \sqrt{a + a \sin(fx+e)}} \\
& + \frac{2a^2(-9Ad + 3Bc - 10Bd) \cos(fx+e) (c + d \sin(fx+e))^3}{63d^2 f \sqrt{a + a \sin(fx+e)}} \\
& + \frac{4a(5c-d)(3A(c-13d)d - B(c^2 - 7cd + 34d^2)) \cos(fx+e) \sqrt{a + a \sin(fx+e)}}{315df} \\
& - \frac{2aB \cos(fx+e) (c + d \sin(fx+e))^3 \sqrt{a + a \sin(fx+e)}}{9df}
\end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.126 Problem number 295

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(7Ad + 7Bc - 2Bd) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{35f} \\ & - \frac{2Bd \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{7af} - \frac{8a^2(35Ac + 21Ad + 21Bc + 19Bd) \cos(fx + e)}{105f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(35Ac + 21Ad + 21Bc + 19Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (15 Badsn(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \sin(-\frac{7}{4}\pi + \frac{7}{2}fx + \frac{7}{2}e) + 105 (12 Aacsngn(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + 8$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 46.127 Problem number 296

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2B \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{5f} - \frac{8a^2(5A + 3B) \cos(fx + e)}{15f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{2a(5A + 3B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 B \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \sin \left( -\frac{5}{4} \pi + \frac{5}{2} fx + \frac{5}{2} e \right) + 30 \left( 3 A \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) + 2 B \operatorname{asgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.128 Problem number 297

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a^{3/2} (c - d) (-Ad + Bc) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx + e)}} \right)}{d^{5/2} f \sqrt{c+d}} \\ & + \frac{2a^2 (-3Ad + 3Bc - 4Bd) \cos(fx + e)}{3d^2 f \sqrt{a + a \sin(fx + e)}} - \frac{2aB \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3df} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{3 \sqrt{2} (Bac^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - Aacd \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - Bacd \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + Aad^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{\sqrt{-cd - d^2} a^2} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.129 Problem number 298

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^{3/2} (Ad(c + 3d) - B(3c^2 + 3cd - 2d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{d^{5/2} (c+d)^{3/2} f} - \frac{a^2 (-Ad + 3Bc + 2Bd) \cos(fx+e)}{d^2 (c+d) f \sqrt{a+a \sin(fx+e)}} + \frac{a(-Ad + Bc) \cos(fx+e) \sqrt{a+a \sin(fx+e)}}{d(c+d) f (c+d \sin(fx+e))}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{4B a \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) \sin(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)}{d^2} + \frac{\sqrt{2} (3Bac^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - Aacd \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + 3Bcd \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - Aad^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)))}{\sqrt{-cd - d^2} a^2} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.130 Problem number 299

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2} (Ad(c + 7d) + 3B(c^2 + 3cd + 4d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx+e)}} \right)}{4d^{5/2} (c+d)^{5/2} f} \\ & + \frac{a^2 (A(c-5d)d + B(3c^2 + 5cd - 4d^2)) \cos(fx+e)}{4d^2 (c+d)^2 f (c+d \sin(fx+e)) \sqrt{a + a \sin(fx+e)}} \\ & + \frac{a(-Ad + Bc) \cos(fx+e) \sqrt{a + a \sin(fx+e)}}{2d(c+d) f (c+d \sin(fx+e))^2} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.131 Problem number 300

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a(c+d)(13Ad(3c^2 - 38cd + 355d^2) - B(15c^3 - 150c^2d + 799cd^2 - 4184d^3)) \cos(fx+e) (a + a \sin(fx+e))^{3/2}}{15015df} \\ & - \frac{2aB \cos(fx+e) (a + a \sin(fx+e))^{3/2} (c + d \sin(fx+e))^4}{13df} \\ & - \frac{4a^3(c+d)(15c^2 + 10cd + 7d^2)(13Ad(3c^2 - 38cd + 355d^2) - B(15c^3 - 150c^2d + 799cd^2 - 4184d^3)) \cos(fx+e)}{45045d^3 f \sqrt{a + a \sin(fx+e)}} \\ & - \frac{2a^3(13Ad(3c^2 - 38cd + 355d^2) - B(15c^3 - 150c^2d + 799cd^2 - 4184d^3)) \cos(fx+e) (c + d \sin(fx+e))^3}{9009d^3 f \sqrt{a + a \sin(fx+e)}} \\ & - \frac{2a^3(-39Acd + 299A^2d^2 + 15Bc^2 - 75Bcd + 280Bd^2) \cos(fx+e) (c + d \sin(fx+e))^4}{1287d^3 f \sqrt{a + a \sin(fx+e)}} \\ & - \frac{8a^2(5c-d)(c+d)(13Ad(3c^2 - 38cd + 355d^2) - B(15c^3 - 150c^2d + 799cd^2 - 4184d^3)) \cos(fx+e) \sqrt{a + a \sin(fx+e)}}{45045d^2 f} \\ & + \frac{2a^2(-13Ad + 5Bc - 16Bd) \cos(fx+e) (c + d \sin(fx+e))^4 \sqrt{a + a \sin(fx+e)}}{143d^2 f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.132 Problem number 301

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2a(11Ad(c^2 - 10cd + 73d^2) - B(5c^3 - 40c^2d + 169cd^2 - 710d^3)) \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}}}{1155df} \\ & - \frac{2aB \cos(fx + e) (a + a \sin(fx + e))^{\frac{3}{2}} (c + d \sin(fx + e))^3}{11df} \\ & - \frac{2a^3(15c^2 + 10cd + 7d^2) (11Ad(c^2 - 10cd + 73d^2) - B(5c^3 - 40c^2d + 169cd^2 - 710d^3)) \cos(fx + e)}{3465d^3 f \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2a^3(11A(3c - 19d)d - B(15c^2 - 65cd + 194d^2)) \cos(fx + e) (c + d \sin(fx + e))^3}{693d^3 f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{4a^2(5c - d) (11Ad(c^2 - 10cd + 73d^2) - B(5c^3 - 40c^2d + 169cd^2 - 710d^3)) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3465d^2 f} \\ & + \frac{2a^2(-11Ad + 5Bc - 14Bd) \cos(fx + e) (c + d \sin(fx + e))^3 \sqrt{a + a \sin(fx + e)}}{99d^2 f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.133 Problem number 302

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c + d \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(21Ac + 15Ad + 15Bc + 13Bd) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{105f} \\ & - \frac{2(9Ad + 9Bc - 2Bd) \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{63f} \\ & - \frac{2Bd \cos(fx + e) (a + a \sin(fx + e))^{7/2}}{9af} \\ & - \frac{64a^3(21Ac + 15Ad + 15Bc + 13Bd) \cos(fx + e)}{315f \sqrt{a + a \sin(fx + e)}} \\ & - \frac{16a^2(21Ac + 15Ad + 15Bc + 13Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{315f} \end{aligned}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 35 B a^2 \operatorname{dsgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{9}{4} \pi + \frac{9}{2} f x + \frac{9}{2} e \right) + 630 \left( 20 A a^2 \operatorname{csgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + \right. \right.}{\left. \left. \right. \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.134 Problem number 303

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a(7A + 5B) \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{35f} - \frac{2B \cos(fx + e) (a + a \sin(fx + e))^{5/2}}{7f} \\ & - \frac{64a^3(7A + 5B) \cos(fx + e)}{105f \sqrt{a + a \sin(fx + e)}} - \frac{16a^2(7A + 5B) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{105f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 B a^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) \sin \left( -\frac{7}{4} \pi + \frac{7}{2} f x + \frac{7}{2} e \right) + 525 \left( 4 A a^2 \operatorname{sgn} \left( \cos \left( -\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e \right) \right) + 3 B \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.135 Problem number 304

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2aB \cos(fx + e) (a + a \sin(fx + e))^{3/2}}{5df} \\ & + \frac{2a^{5/2} (c - d)^2 (-Ad + Bc) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx + e)}} \right)}{d^{7/2} f \sqrt{c+d}} \\ & + \frac{2a^3 (5A(3c - 7d) d - B(15c^2 - 35cd + 32d^2)) \cos(fx + e)}{15d^3 f \sqrt{a + a \sin(fx + e)}} \\ & + \frac{2a^2 (-5Ad + 5Bc - 8Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15d^2 f} \end{aligned}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{a} \left( \frac{15 \sqrt{2} (Ba^2 c^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) - Aa^2 c^2 d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) - 2Ba^2 c^2 d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)) + 2Aa^2 c d^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)))}{\sqrt{-c d}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.136 Problem number 305

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^{5/2} (c - d) (Ad(3c + 5d) - B(5c^2 + 5cd - 2d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{d^{7/2} (c+d)^{3/2} f} + \frac{a(-Ad + Bc) \cos(fx+e) (a + a \sin(fx+e))^{3/2}}{d(c+d) f (c+d \sin(fx+e))} - \frac{a^3 (3Ad(3c+d) - B(15c^2 - 5cd - 14d^2)) \cos(fx+e)}{3d^3 (c+d) f \sqrt{a+a \sin(fx+e)}} - \frac{a^2 (-3Ad + 5Bc + 2Bd) \cos(fx+e) \sqrt{a+a \sin(fx+e)}}{3d^2 (c+d) f}$$

command

`integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 46.137 Problem number 306

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{a^{5/2} (Ad(3c^2 + 10cd + 19d^2) - B(15c^3 + 30c^2d + 7cd^2 - 20d^3)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{4d^{7/2} (c+d)^{5/2} f} + \frac{a(-Ad + Bc) \cos(fx+e) (a + a \sin(fx+e))^{3/2}}{2d(c+d) f (c+d \sin(fx+e))^2} + \frac{a^3 (3Ad(c+3d) - B(15c^2 + 25cd + 4d^2)) \cos(fx+e)}{4d^3 (c+d)^2 f \sqrt{a+a \sin(fx+e)}} - \frac{a^2 (Ad(c+7d) - B(5c^2 + 7cd - 4d^2)) \cos(fx+e) \sqrt{a+a \sin(fx+e)}}{4d^2 (c+d)^2 f (c+d \sin(fx+e))}$$

command

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 46.138 Problem number 307

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B)(c - d)^3 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}} \\ & - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 37d^3))\cos(fx+e)}{105f\sqrt{a+a\sin(fx+e)}} \\ & - \frac{2(7Ad + 6Bc - Bd)\cos(fx+e)(c + d\sin(fx+e))^2}{35f\sqrt{a+a\sin(fx+e)}} \\ & - \frac{2B\cos(fx+e)(c + d\sin(fx+e))^3}{7f\sqrt{a+a\sin(fx+e)}} \\ & - \frac{2d(7A(9c-d)d + B(24c^2 - 15cd + 31d^2))\cos(fx+e)\sqrt{a+a\sin(fx+e)}}{105af} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105\sqrt{2}\left(A\sqrt{a}c^3 - B\sqrt{a}c^3 - 3A\sqrt{a}c^2d + 3B\sqrt{a}c^2d + 3A\sqrt{a}cd^2 - 3B\sqrt{a}cd^2 - A\sqrt{a}d^3 + B\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 46.139 Problem number 308

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A - B)(c - d)^2 \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{f\sqrt{a}} - \frac{4(5A(3c - d)d + B(6c^2 - 7cd + 7d^2)) \cos(fx + e)}{15f\sqrt{a + a \sin(fx + e)}} - \frac{2B \cos(fx + e)(c + d \sin(fx + e))^2}{5f\sqrt{a + a \sin(fx + e)}} - \frac{2d(5Ad + 4Bc - Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{15af}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15\sqrt{2}\left(A\sqrt{a}c^2 - B\sqrt{a}c^2 - 2A\sqrt{a}cd + 2B\sqrt{a}cd + A\sqrt{a}d^2 - B\sqrt{a}d^2\right) \log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{15\sqrt{2}\left(A\sqrt{a}c^2 - B\sqrt{a}c^2 - 2A\sqrt{a}cd + 2B\sqrt{a}cd + A\sqrt{a}d^2 - B\sqrt{a}d^2\right)}{15af}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.140 Problem number 309

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(A - B)(c - d) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{f\sqrt{a}} - \frac{2(3Ad + 3Bc - 2Bd) \cos(fx + e)}{3f\sqrt{a + a \sin(fx + e)}} - \frac{2Bd \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{3af}$$

command

`integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2}\left(A\sqrt{a}c-B\sqrt{a}c-A\sqrt{a}d+B\sqrt{a}d\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{3\sqrt{2}\left(A\sqrt{a}c-B\sqrt{a}c-A\sqrt{a}d+B\sqrt{a}d\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} \frac{1}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.141 Problem number 310

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{(A - B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right)\sqrt{2}}{f\sqrt{a}} - \frac{2B \cos(fx+e)}{f\sqrt{a+a\sin(fx+e)}}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}B\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{a}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\operatorname{asgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} \frac{1}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.142 Problem number 311

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{(A - B) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{2}}{2\sqrt{a+a\sin(fx+e)}} \right) \sqrt{2}}{(c-d) f \sqrt{a}} - \frac{2(-Ad + Bc) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a\sin(fx+e)}} \right)}{(c-d) f \sqrt{a} \sqrt{d} \sqrt{c+d}}$$

command

`integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2} (B\sqrt{a}c - A\sqrt{a}d) \operatorname{arctan} \left( \frac{\sqrt{2} d \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-cd - d^2}} \right)}{\left( \sqrt{2} \operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - \sqrt{2} \operatorname{adsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \right) \sqrt{-cd - d^2}} - \frac{(A\sqrt{a} - B\sqrt{a}) \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{\left( \sqrt{2} \operatorname{acsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - \sqrt{2} \operatorname{adsgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \right) f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 46.143 Problem number 312

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{(A - B) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{2}}{2\sqrt{a+a\sin(fx+e)}} \right) \sqrt{2}}{(c-d)^2 f \sqrt{a}} + \frac{(Ad(3c+d) - B(c^2 + cd + 2d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a\sin(fx+e)}} \right)}{(c-d)^2 (c+d)^{\frac{3}{2}} f \sqrt{a} \sqrt{d}} - \frac{(-Ad + Bc) \cos(fx+e)}{(c^2 - d^2) f (c + d \sin(fx+e)) \sqrt{a+a\sin(fx+e)}}$$

command

`integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} B \sqrt{a} c^2 - 3 \sqrt{2} A \sqrt{a} cd + \sqrt{2} B \sqrt{a} cd - \sqrt{2} A \sqrt{a} d^2 + 2 \sqrt{2} B \sqrt{a} d^2 \right) \arctan \left( \frac{\sqrt{2} d \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right)}{\sqrt{-cd - d^2}} \right)}{(ac^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - ac^2 d \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) - acd^2 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e)) + ad^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e))) \sqrt{-cd - d^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 46.144 Problem number 313

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(A - B) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{2}}{2 \sqrt{a + a \sin(fx + e)}} \right) \sqrt{2}}{(c - d)^3 f \sqrt{a}} \\ & + \frac{(Ad(15c^2 + 10cd + 7d^2) - B(3c^3 + 6c^2d + 19cd^2 + 4d^3)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a + a \sin(fx + e)}} \right)}{4(c - d)^3 (c + d)^{\frac{5}{2}} f \sqrt{a} \sqrt{d}} \\ & - \frac{(-Ad + Bc) \cos(fx + e)}{2(c^2 - d^2) f (c + d \sin(fx + e))^2 \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(Ad(7c + d) - B(3c^2 + cd + 4d^2)) \cos(fx + e)}{4(c^2 - d^2)^2 f (c + d \sin(fx + e)) \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46.145 Problem number 314

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e) (c + d \sin(fx + e))^3}{2f (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(c - d)^2 (3B(c - 5d) + A(c + 11d)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}f} \\ & + \frac{d(15Ac^2 - 120Acd + 65Ad^2 - 99Bc^2 + 168Bcd - 93Bd^2) \cos(fx + e)}{15af \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(5A - 9B) d \cos(fx + e) (c + d \sin(fx + e))^2}{10af \sqrt{a + a \sin(fx + e)}} \\ & + \frac{d^2(15Ac - 35Ad - 51Bc + 39Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{30a^2f} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15\sqrt{2}\left(A\sqrt{a}c^3+3B\sqrt{a}c^3+9A\sqrt{a}c^2d-21B\sqrt{a}c^2d-21A\sqrt{a}cd^2+33B\sqrt{a}cd^2+11A\sqrt{a}d^3-15B\sqrt{a}d^3\right)\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.146 Problem number 315

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e) (c + d \sin(fx + e))^2}{2f (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(c - d) (Ac + 7Ad + 3Bc - 11Bd) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(fx + e)}} \right) \sqrt{2}}{4a^{\frac{3}{2}} f} \\ & + \frac{d(3Ac - 9Ad - 15Bc + 13Bd) \cos(fx + e)}{3af \sqrt{a + a \sin(fx + e)}} \\ & + \frac{(3A - 7B) d^2 \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{6a^2 f} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{2} \left( A\sqrt{a} c^2 + 3B\sqrt{a} c^2 + 6A\sqrt{a} cd - 14B\sqrt{a} cd - 7A\sqrt{a} d^2 + 11B\sqrt{a} d^2 \right) \log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{3\sqrt{2} \left( A\sqrt{a} c^2 + 3B\sqrt{a} c^2 \right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 46.147 Problem number 317

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$- \frac{(A - B) \cos(fx + e)}{2f (a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{(A + 3B) \operatorname{arctanh} \left( \frac{\cos(fx+e)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \sin(fx + e)}} \right) \sqrt{2}}{4a^{\frac{3}{2}} f}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( A\sqrt{a} + 3B\sqrt{a} \right) \log\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{\sqrt{2} \left( A\sqrt{a} + 3B\sqrt{a} \right) \log\left(-\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)}{a^2 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)} - \frac{2\sqrt{2} \left( A\sqrt{a} \sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right) \right)}{\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

$8f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.148 Problem number 318

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{2(c - d) f (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(A(c - 5d) + B(3c + d)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}(c - d)^2 f} \\ & + \frac{2(-Ad + Bc) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right) \sqrt{d}}{a^{\frac{3}{2}}(c - d)^2 f \sqrt{c + d}} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\sqrt{2}(Bcd - Ad^2) \operatorname{arctan}\left(\frac{\sqrt{2} d \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-cd - d^2}}\right)}{\left(\sqrt{2} a^{\frac{3}{2}} c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 2\sqrt{2} a^{\frac{3}{2}} cd \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) + \sqrt{2} a^{\frac{3}{2}} d^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))\right) \sqrt{-cd - d^2}} + \sqrt{2} \dots$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 46.149 Problem number 319

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{2(c - d) f (a + a \sin(fx + e))^{\frac{3}{2}} (c + d \sin(fx + e))} \\ & - \frac{(Ac - 9Ad + 3Bc + 5Bd) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}(c - d)^3 f} \\ & - \frac{(Ad(5c + 3d) - B(3c^2 + 3cd + 2d^2)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right) \sqrt{d}}{a^{\frac{3}{2}}(c - d)^3 (c + d)^{\frac{3}{2}} f} \\ & + \frac{d(B(3c + d) - A(c + 3d)) \cos(fx + e)}{2a(c - d)^2 (c + d) f (c + d \sin(fx + e)) \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 46.150 Problem number 320

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - B) \cos(fx + e)}{2(c - d) f (a + a \sin(fx + e))^{\frac{3}{2}} (c + d \sin(fx + e))^2} \\ & - \frac{(A(c - 13d) + 3B(c + 3d)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{4a^{\frac{3}{2}} (c - d)^4 f} \\ & - \frac{(Ad(35c^2 + 42cd + 19d^2) - 3B(5c^3 + 10c^2d + 13cd^2 + 4d^3)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right) \sqrt{d}}{4a^{\frac{3}{2}} (c - d)^4 (c + d)^{\frac{5}{2}} f} \\ & + \frac{d(B(2c + d) - A(c + 2d)) \cos(fx + e)}{2a(c - d)^2 (c + d) f (c + d \sin(fx + e))^2 \sqrt{a + a \sin(fx + e)}} \\ & + \frac{d(3B(3c^2 + 3cd + 2d^2) - A(2c^2 + 15cd + 7d^2)) \cos(fx + e)}{4a(c - d)^3 (c + d)^2 f (c + d \sin(fx + e)) \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 46.151 Problem number 321

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3Ac + 9Ad + 5Bc - 17Bd) \cos(fx + e) (c + d \sin(fx + e))^2}{16af (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(A - B) \cos(fx + e) (c + d \sin(fx + e))^3}{4f (a + a \sin(fx + e))^{\frac{5}{2}}} \\ & - \frac{(c - d) (B(5c^2 + 62cd - 163d^2) + 3A(c^2 + 6cd + 25d^2)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{32a^{\frac{5}{2}}f} \\ & + \frac{d(A(9c^2 + 36cd - 93d^2) + B(15c^2 - 228cd + 197d^2)) \cos(fx + e)}{24a^2f \sqrt{a + a \sin(fx + e)}} \\ & + \frac{d^2(9Ac + 39Ad + 15Bc - 95Bd) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{48a^3f} \end{aligned}$$

command

`integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.152 Problem number 322

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(c - d) (3Ac + 5Ad + 5Bc - 13Bd) \cos(fx + e)}{16af (a + a \sin(fx + e))^{\frac{3}{2}}} \\ & - \frac{(A - B) \cos(fx + e) (c + d \sin(fx + e))^2}{4f (a + a \sin(fx + e))^{\frac{5}{2}}} \\ & - \frac{(B(5c^2 + 38cd - 75d^2) + A(3c^2 + 10cd + 19d^2)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\sin(fx+e)}}\right) \sqrt{2}}{32a^{\frac{5}{2}}f} \\ & + \frac{(A - 9B) d^2 \cos(fx + e)}{4a^2f \sqrt{a + a \sin(fx + e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{128 \sqrt{2} B d^2 \sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e)}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} + \frac{\sqrt{2} (3 A \sqrt{a} c^2 + 5 B \sqrt{a} c^2 + 10 A \sqrt{a} c d + 38 B \sqrt{a} c d + 19 A \sqrt{a} d^2 - 75 B \sqrt{a} d^2) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 46.153 Problem number 323

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(A - B)(c - d) \cos(fx + e)}{4f(a + a \sin(fx + e))^{\frac{5}{2}}} - \frac{(3Ac + 5Ad + 5Bc - 13Bd) \cos(fx + e)}{16af(a + a \sin(fx + e))^{\frac{3}{2}}} - \frac{(3Ac + 5Ad + 5Bc + 19Bd) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{32a^{\frac{5}{2}}f}$$

command

```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (3 A \sqrt{a} c + 5 B \sqrt{a} c + 5 A \sqrt{a} d + 19 B \sqrt{a} d) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))} - \frac{\sqrt{2} (3 A \sqrt{a} c + 5 B \sqrt{a} c + 5 A \sqrt{a} d + 19 B \sqrt{a} d) \log(\sin(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4} \pi + \frac{1}{2} f x + \frac{1}{2} e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.154 Problem number 324

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(A - B) \cos(fx + e)}{4f(a + a \sin(fx + e))^{5/2}} - \frac{(3A + 5B) \cos(fx + e)}{16af(a + a \sin(fx + e))^{3/2}} - \frac{(3A + 5B) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{32a^{5/2}f}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (3A\sqrt{a} + 5B\sqrt{a}) \log(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{\sqrt{2} (3A\sqrt{a} + 5B\sqrt{a}) \log(-\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e) + 1)}{a^3 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2(3\sqrt{2}A\sqrt{a} \sin}{$$

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 46.155 Problem number 325

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2}(c + d \sin(e + fx))} dx$$

Optimal antiderivative

$$\frac{(A - B) \cos(fx + e)}{4(c - d)f(a + a \sin(fx + e))^{5/2}} - \frac{(3Ac - 11Ad + 5Bc + 3Bd) \cos(fx + e)}{16a(c - d)^2 f(a + a \sin(fx + e))^{3/2}} - \frac{(B(5c^2 - 34cd - 3d^2) + A(3c^2 - 14cd + 43d^2)) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \sin(fx + e)}}\right) \sqrt{2}}{32a^{5/2}(c - d)^3 f} - \frac{2d^{3/2}(-Ad + Bc) \operatorname{arctanh}\left(\frac{\cos(fx+e)\sqrt{a}\sqrt{d}}{\sqrt{c+d}\sqrt{a + a \sin(fx + e)}}\right)}{a^{5/2}(c - d)^3 f \sqrt{c + d}}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 46.156 Problem number 326

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} (c + d \sin(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^{\frac{3}{2}} (Ad(7c + 5d) - B(5c^2 + 5cd + 2d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{d}}{\sqrt{c+d} \sqrt{a+a \sin(fx+e)}} \right)}{a^{\frac{5}{2}} (c-d)^4 (c+d)^{\frac{3}{2}} f} \\ & - \frac{(A-B) \cos(fx+e)}{4(c-d) f (a+a \sin(fx+e))^{\frac{5}{2}} (c+d \sin(fx+e))} \\ & - \frac{(3Ac - 15Ad + 5Bc + 7Bd) \cos(fx+e)}{16a(c-d)^2 f (a+a \sin(fx+e))^{\frac{3}{2}} (c+d \sin(fx+e))} \\ & - \frac{(B(5c^2 - 58cd - 43d^2) + A(3c^2 - 22cd + 115d^2)) \operatorname{arctanh} \left( \frac{\cos(fx+e) \sqrt{a} \sqrt{2}}{2\sqrt{a+a \sin(fx+e)}} \right) \sqrt{2}}{32a^{\frac{5}{2}} (c-d)^4 f} \\ & - \frac{d(A(3c^2 - 16cd - 35d^2) + B(5c^2 + 32cd + 11d^2)) \cos(fx+e)}{16a^2 (c-d)^3 (c+d) f (c+d \sin(fx+e)) \sqrt{a+a \sin(fx+e)}} \end{aligned}$$

command

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 47 Test file number 78

Test folder name:

test\_cases/4\_Trig\_functions/4.1\_Sine/78\_4.1.4.2-a+b\_sin-^m-c+d\_sin-^n-A+B\_sin+C\_sin^2-

### 47.1 Problem number 7

$$\int \frac{A + C \sin^2(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A + C) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{4af (c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{(A - 3C) \cos(fx + e) \ln(1 - \sin(fx + e))}{4cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{(A + C) \cos(fx + e) \ln(1 + \sin(fx + e))}{4cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

integrate((A+C\*sin(f\*x+e)^2)/(c-c\*sin(f\*x+e))^(3/2)/(a+a\*sin(f\*x+e))^(1/2),x, algorithm="giac")

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(A\sqrt{c} + C\sqrt{c}) \log(-8 \sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)^2 + 8)}{\sqrt{a} c^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} - \frac{2(A\sqrt{a}\sqrt{c} - 3C\sqrt{a}\sqrt{c}) \log(|\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)|)}{ac^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))} + \frac{ac^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{C \sin(fx + e)^2 + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

### 47.2 Problem number 16

$$\int \frac{A + B \sin(e + fx) + C \sin^2(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(A + B + C) \cos(fx + e) \sqrt{a + a \sin(fx + e)}}{4af (c - c \sin(fx + e))^{\frac{3}{2}}} - \frac{(A - B - 3C) \cos(fx + e) \ln(1 - \sin(fx + e))}{4cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}} + \frac{(A - B + C) \cos(fx + e) \ln(1 + \sin(fx + e))}{4cf \sqrt{a + a \sin(fx + e)} \sqrt{c - c \sin(fx + e)}}$$

command

`integrate((A+B*sin(f*x+e)+C*sin(f*x+e)^2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, al`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}+C\sqrt{a}\sqrt{c})\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)}{ac^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2(A\sqrt{a}\sqrt{c}-B\sqrt{a}\sqrt{c}-3C\sqrt{a}\sqrt{c})\log\left(\left|\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{ac^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} \frac{1}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{C \sin^2(fx + e) + B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 48 Test file number 79

Test folder name:

`test_cases/4_Trig_functions/4.1_Sine/79_4.1.7-d_trig-~m-a+b-c_sin-~n-~p`

### 48.1 Problem number 122

$$\int \sin^3(e + fx) \sqrt{a + b \sin^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(a - 3b)(a + b) \arctan\left(\frac{\cos(fx+e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}}\right)}{8b^{\frac{3}{2}}f} - \frac{\cos(fx + e)(a + b - b(\cos^2(fx + e)))^{\frac{3}{2}}}{4bf} + \frac{(a - 3b)\cos(fx + e)\sqrt{a + b - b(\cos^2(fx + e))}}{8bf}$$

command

`integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-b \cos^2(fx + e) + a + b} \left(2 \cos^2(fx + e) - \frac{abf^4 + 5b^2f^4}{b^2f^4}\right) \cos(fx + e)}{8f} + \frac{(a^2 - 2ab - 3b^2) \log\left(\left|\sqrt{-b \cos^2(fx + e) + a + b} + \frac{\sqrt{-bf^2} \cos(fx+e)}{f}\right|\right)}{8\sqrt{-b}b|f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.2 Problem number 123

$$\int \sin(e + fx) \sqrt{a + b \sin^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(a + b) \arctan\left(\frac{\cos(fx + e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}}\right)}{2f\sqrt{b}} - \frac{\cos(fx + e) \sqrt{a + b - b(\cos^2(fx + e))}}{2f}$$

command

```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-b \cos^2(fx + e) + a + b} \cos(fx + e)}{2f} - \frac{(a + b) \log\left(\left|\sqrt{-b \cos^2(fx + e) + a + b} + \frac{\sqrt{-bf^2} \cos(fx + e)}{f}\right|\right)}{2\sqrt{-b}|f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.3 Problem number 132

$$\int \sin^3(e + fx) (a + b \sin^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a - 5b)(a + b)^2 \arctan\left(\frac{\cos(fx + e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}}\right)}{16b^{\frac{3}{2}}f} \\ & + \frac{(a - 5b) \cos(fx + e) (a + b - b(\cos^2(fx + e)))^{\frac{3}{2}}}{24bf} \\ & - \frac{\cos(fx + e) (a + b - b(\cos^2(fx + e)))^{\frac{5}{2}}}{6bf} \\ & + \frac{(a - 5b)(a + b) \cos(fx + e) \sqrt{a + b - b(\cos^2(fx + e))}}{16bf} \end{aligned}$$

command

```
integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-b \cos(fx + e)^2 + a + b} \left( 2 \frac{\left( 4bf^2 \cos(fx+e)^2 - \frac{7ab^4 f^{10} + 13b^5 f^{10}}{b^4 f^8} \right) \cos(fx+e)^2}{f^2} + \frac{3(a^2 b^3 f^8 + 12ab^4 f^8 + 11b^5 f^8)}{b^4 f^8} \right) \cos(fx + e)}{48f} + \frac{(a^3 - 3a^2b - 9ab^2 - 5b^3) \log \left( \left| \sqrt{-b \cos(fx + e)^2 + a + b} + \frac{\sqrt{-bf^2} \cos(fx+e)}{f} \right| \right)}{16 \sqrt{-b} b|f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.4 Problem number 133

$$\int \sin(e + fx) (a + b \sin^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{\cos(fx + e) (a + b - b(\cos^2(fx + e)))^{\frac{3}{2}}}{4f} - \frac{3(a + b)^2 \arctan \left( \frac{\cos(fx+e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}} \right)}{8f\sqrt{b}} - \frac{3(a + b) \cos(fx + e) \sqrt{a + b - b(\cos^2(fx + e))}}{8f}$$

command

```
integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( 2b \cos(fx + e)^2 - \frac{5(ab^2 f^4 + b^3 f^4)}{b^2 f^4} \right) \sqrt{-b \cos(fx + e)^2 + a + b} \cos(fx + e)}{8f} - \frac{3(a^2 + 2ab + b^2) \log \left( \left| \sqrt{-b \cos(fx + e)^2 + a + b} + \frac{\sqrt{-bf^2} \cos(fx+e)}{f} \right| \right)}{8 \sqrt{-b} |f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 48.5 Problem number 144

$$\int \frac{\sin^3(e + fx)}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(a - b) \arctan\left(\frac{\cos(fx+e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}}\right)}{2b^{\frac{3}{2}}f} - \frac{\cos(fx + e) \sqrt{a + b - b(\cos^2(fx + e))}}{2bf}$$

command

```
integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-b \cos^2(fx + e) + a + b} \cos(fx + e)}{2bf} + \frac{(a - b) \log\left(\left|\sqrt{-b \cos^2(fx + e) + a + b} + \frac{\sqrt{-bf^2} \cos(fx + e)}{f}\right|\right)}{2\sqrt{-b} b|f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.6 Problem number 145

$$\int \frac{\sin(e + fx)}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\cos(fx+e)\sqrt{b}}{\sqrt{a + b - b(\cos^2(fx + e))}}\right)}{f\sqrt{b}}$$

command

```
integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-b \cos (fx+e)^2+a+b} \cos (fx+e)}{2 f} - \frac{(a+b) \log \left( \left| \sqrt{-b \cos (fx+e)^2+a+b} + \frac{\sqrt{-bf^2} \cos (fx+e)}{f} \right| \right)}{2 \sqrt{-b} |f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.7 Problem number 153

$$\int \frac{\sin^3(e+fx)}{(a+b \sin^2(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan \left( \frac{\cos (fx+e) \sqrt{b}}{\sqrt{a+b-b \left( \cos^2 (fx+e) \right)}} \right)}{b^{3/2} f} + \frac{a \cos (fx+e)}{b(a+b) f \sqrt{a+b-b \left( \cos^2 (fx+e) \right)}}$$

command

`integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-b \cos (fx+e)^2+a+b} a \cos (fx+e)}{\left( b \cos (fx+e)^2-a-b \right) (ab+b^2) f} - \frac{\log \left( \left| \sqrt{-b \cos (fx+e)^2+a+b} + \frac{\sqrt{-bf^2} \cos (fx+e)}{f} \right| \right)}{\sqrt{-b} b |f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.8 Problem number 324

$$\int \cos^3(e+fx) \sqrt{a+b \sin^2(e+fx)} dx$$

Optimal antiderivative

$$\frac{a(a+4b) \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a+b(\sin^2(fx+e))}}\right)}{8b^{\frac{3}{2}}f} - \frac{\sin(fx+e)(a+b(\sin^2(fx+e)))^{\frac{3}{2}}}{4bf} + \frac{(a+4b)\sin(fx+e)\sqrt{a+b(\sin^2(fx+e))}}{8bf}$$

command

```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{b \sin^2(fx+e) + a} \left(2 \sin^2(fx+e) + \frac{ab-4b^2}{b^2}\right) \sin(fx+e) + \frac{(a^2+4ab) \log\left(-\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a}\right)}{b^{\frac{3}{2}}}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.9 Problem number 325

$$\int \cos(e+fx) \sqrt{a+b \sin^2(e+fx)} dx$$

Optimal antiderivative

$$\frac{a \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a+b(\sin^2(fx+e))}}\right)}{2f\sqrt{b}} + \frac{\sin(fx+e)\sqrt{a+b(\sin^2(fx+e))}}{2f}$$

command

```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(-\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a}\right)}{\sqrt{b}} - \frac{\sqrt{b \sin^2(fx+e) + a} \sin(fx+e)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.10 Problem number 334

$$\int \cos^3(e + fx) (a + b \sin^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a^2(a + 6b) \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a + b(\sin^2(fx + e))}}\right)}{16b^{\frac{3}{2}}f} + \frac{(a + 6b) \sin(fx + e) (a + b(\sin^2(fx + e)))^{\frac{3}{2}}}{24bf} - \frac{\sin(fx + e) (a + b(\sin^2(fx + e)))^{\frac{5}{2}}}{6bf} + \frac{a(a + 6b) \sin(fx + e) \sqrt{a + b(\sin^2(fx + e))}}{16bf}$$

command

```
integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2\left(4b \sin(fx + e)^2 + \frac{7ab^4 - 6b^5}{b^4}\right) \sin(fx + e)^2 + \frac{3(a^2b^3 - 10ab^4)}{b^4}\right) \sqrt{b \sin(fx + e)^2 + a} \sin(fx + e) + \frac{3(a^3 + 6a^2b) \sin(fx + e)}{48f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.11 Problem number 335

$$\int \cos(e + fx) (a + b \sin^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{\sin(fx + e) (a + b(\sin^2(fx + e)))^{\frac{3}{2}}}{4f} + \frac{3a^2 \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a + b(\sin^2(fx + e))}}\right)}{8f\sqrt{b}} + \frac{3a \sin(fx + e) \sqrt{a + b(\sin^2(fx + e))}}{8f}$$

command

```
integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2 \log\left(\left|-\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a}\right|\right)}{\sqrt{b}} - \frac{(2b \sin^2(fx+e) + 5a) \sqrt{b \sin^2(fx+e) + a} \sin(fx+e)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.12 Problem number 345

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{(a+2b) \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a+b(\sin^2(fx+e))}}\right)}{2b^{\frac{3}{2}}f} - \frac{\sin(fx+e) \sqrt{a+b(\sin^2(fx+e))}}{2bf}$$

command

```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a+2b) \log\left(\left|-\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a}\right|\right)}{b^{\frac{3}{2}}} + \frac{\sqrt{b \sin^2(fx+e) + a} \sin(fx+e)}{b}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.13 Problem number 346

$$\int \frac{\cos(e + fx)}{\sqrt{a + b \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a+b(\sin^2(fx+e))}}\right)}{f\sqrt{b}}$$

command

```
integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(\left| -\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a} \right| \right)}{\sqrt{b}} - \frac{\sqrt{b \sin^2(fx+e) + a} \sin(fx+e)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.14 Problem number 354

$$\int \frac{\cos^3(e + fx)}{(a + b \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{b}}{\sqrt{a+b(\sin^2(fx+e))}}\right)}{b^{\frac{3}{2}}f} + \frac{(a+b)\sin(fx+e)}{abf\sqrt{a+b(\sin^2(fx+e))}}$$

command

```
integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\frac{-\sqrt{b} \sin(fx+e) + \sqrt{b \sin^2(fx+e) + a}}{b^{\frac{3}{2}}}\right) + \frac{(a+b) \sin(fx+e)}{\sqrt{b \sin^2(fx+e) + a}}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.15 Problem number 457

$$\int \sqrt{a - a \sin^2(e + fx)} \tan^5(e + fx) dx$$

Optimal antiderivative

$$\frac{a^2}{3f (a (\cos^2(fx + e)))^{\frac{3}{2}}} - \frac{2a}{f \sqrt{a (\cos^2(fx + e))}} - \frac{\sqrt{a (\cos^2(fx + e))}}{f}$$

command

`integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a} \left( \frac{3 \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right)}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1} - \frac{3 \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right) \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 12 \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right) \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 5 \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^3} \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.16 Problem number 458

$$\int \sqrt{a - a \sin^2(e + fx)} \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{a}{f \sqrt{a (\cos^2(fx + e))}} + \frac{\sqrt{a (\cos^2(fx + e))}}{f}$$

command

`integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.17 Problem number 459

$$\int \sqrt{a - a \sin^2(e + fx)} \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{\sqrt{a(\cos^2(fx+e))}}{f}$$

command

`integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-1\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.18 Problem number 461

$$\int \cot^3(e + fx) \sqrt{a - a \sin^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{(a(\cos^2(fx+e)))^{\frac{3}{2}}(\csc^2(fx+e))}{2af} + \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a(\cos^2(fx+e))}}{\sqrt{a}}\right) \sqrt{a}}{2f} - \frac{3\sqrt{a(\cos^2(fx+e))}}{2f}$$



command

```
integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right)\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 6\log\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right) + \frac{3\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right)}{8f}\right)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 48.19 Problem number 462

$$\int \sqrt{a - a \sin^2(e + fx)} \tan^6(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{15 \operatorname{arctanh}(\sin(fx + e)) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{8f} \\ & - \frac{15 \sqrt{a(\cos^2(fx + e))} \tan(fx + e)}{8f} - \frac{5 \sqrt{a(\cos^2(fx + e))} (\tan^3(fx + e))}{8f} \\ & + \frac{\sqrt{a(\cos^2(fx + e))} (\tan^5(fx + e))}{4f} \end{aligned}$$

command

```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 15 \log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2\right|\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1\right) - 15 \log\left(\left|\frac{1}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.20 Problem number 463

$$\int \sqrt{a - a \sin^2(e + fx)} \tan^4(e + fx) dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}(\sin(fx + e)) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{2f} + \frac{3\sqrt{a(\cos^2(fx + e))} \tan(fx + e)}{2f} + \frac{\sqrt{a(\cos^2(fx + e))} (\tan^3(fx + e))}{2f}$$

command

```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( 3 \log \left( \left| \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2 \right| \right) \operatorname{sgn} \left( \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1 \right) - 3 \log \left( \left| \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right| \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.21 Problem number 464

$$\int \sqrt{a - a \sin^2(e + fx)} \tan^2(e + fx) dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e)) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{f} - \frac{\sqrt{a(\cos^2(fx + e))} \tan(fx + e)}{f}$$

command

```
integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \log \left( \left| \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2 \right| \right) \operatorname{sgn} \left( \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1 \right) - \log \left( \left| \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right| \right) \right)}{2f}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.22 Problem number 465

$$\int \cot^2(e + fx) \sqrt{a - a \sin^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\csc(fx + e) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{f} - \frac{\sqrt{a(\cos^2(fx + e))} \tan(fx + e)}{f}$$

command

```
integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right) \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right) + \frac{4 \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)}{\tan(\frac{1}{2}fx + \frac{1}{2}e) + \tan(\frac{1}{2}fx + \frac{1}{2}e)} \right) \sqrt{a}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.23 Problem number 466

$$\int \cot^4(e + fx) \sqrt{a - a \sin^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \csc(fx + e) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{f} - \frac{(\csc^3(fx + e)) \sec(fx + e) \sqrt{a(\cos^2(fx + e))}}{3f} + \frac{\sqrt{a(\cos^2(fx + e))} \tan(fx + e)}{f}$$

command

```
integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right)^3 \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right) - 24 \left( \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right) \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right) \right) \sqrt{a}}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.24 Problem number 467

$$\int \cot^6(e + fx) \sqrt{a - a \sin^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3 \csc(fx + e) \sec(fx + e) \sqrt{a (\cos^2(fx + e))}}{f} \\ & + \frac{(\csc^3(fx + e)) \sec(fx + e) \sqrt{a (\cos^2(fx + e))}}{f} \\ & - \frac{(\csc^5(fx + e)) \sec(fx + e) \sqrt{a (\cos^2(fx + e))}}{5f} - \frac{\sqrt{a (\cos^2(fx + e))} \tan(fx + e)}{f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \left( \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right)^5 \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right) - 20 \left( \frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) \right)^3 \operatorname{sgn}\left(\tan\right.$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.25 Problem number 468

$$\int \frac{\tan^5(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a^2}{5f (a (\cos^2(fx + e)))^{\frac{5}{2}}} - \frac{2a}{3f (a (\cos^2(fx + e)))^{\frac{3}{2}}} + \frac{1}{f \sqrt{a (\cos^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 10 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 1 \right)}{15 \left( \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1 \right)^5 \sqrt{a} f \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.26 Problem number 469

$$\int \frac{\tan^3(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a}{3f(a(\cos^2(fx + e)))^{\frac{3}{2}}} - \frac{1}{f\sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3 \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - 1 \right)}{3 \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - 1 \right)^3 \sqrt{a} f \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^4 - 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.27 Problem number 470

$$\int \frac{\tan(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{1}{f\sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - 1 \right) \sqrt{a} f \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^4 - 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.28 Problem number 472

$$\int \frac{\cot^3(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a(\cos^2(fx + e))}}{\sqrt{a}}\right)}{2f\sqrt{a}} - \frac{(\csc^2(fx + e))\sqrt{a(\cos^2(fx + e))}}{2af}$$

command

`integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\tan(\frac{1}{2}fx + \frac{1}{2}e)^2}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} - \frac{2 \log(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2)}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} + \frac{2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1) \tan(\frac{1}{2}fx + \frac{1}{2}e)^2}}{8\sqrt{a}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.29 Problem number 473

$$\int \frac{\tan^4(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{8f\sqrt{a(\cos^2(fx + e))}} - \frac{3 \tan(fx + e)}{8f\sqrt{a(\cos^2(fx + e))}} + \frac{\tan^3(fx + e)}{4f\sqrt{a(\cos^2(fx + e))}}$$

command

`integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \log\left(\left|\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2\right|\right)}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} - \frac{3 \log\left(\left|\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) - 2\right|\right)}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} - \frac{4 \left(3 \left(\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)^3 - \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)}{\left(\left(\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)^2 - \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)} - \frac{4 \left(3 \left(\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)^3 - \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)}{16\sqrt{a}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.30 Problem number 474

$$\int \frac{\tan^2(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{2f \sqrt{a (\cos^2(fx + e))}} + \frac{\tan(fx + e)}{2f \sqrt{a (\cos^2(fx + e))}}$$

command

`integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\left|\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) + 2\right|\right)}{\operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)} - \frac{\log\left(\left|\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e) - 2\right|\right)}{\operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)} - \frac{4\left(\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)}{\left(\left(\frac{1}{\tan(\frac{1}{2}fx + \frac{1}{2}e)} + \tan(\frac{1}{2}fx + \frac{1}{2}e)\right)^2 - 4\right) \operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)}$$


---


$$4 \sqrt{a} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.31 Problem number 475

$$\int \frac{\cot^2(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cot(fx + e)}{f \sqrt{a (\cos^2(fx + e))}}$$

command

`integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\tan(\frac{1}{2}fx + \frac{1}{2}e)}{\operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right)} + \frac{1}{\operatorname{sgn}\left(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1\right) \tan(\frac{1}{2}fx + \frac{1}{2}e)}$$


---


$$2 \sqrt{a} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.32 Problem number 476

$$\int \frac{\cot^4(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e)}{f \sqrt{a(\cos^2(fx + e))}} - \frac{\cot(fx + e) (\csc^2(fx + e))}{3f \sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 9 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} - \frac{9 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1) \tan(\frac{1}{2}fx + \frac{1}{2}e)^3}$$


---


$$24 \sqrt{a} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.33 Problem number 477

$$\int \frac{\cot^6(e + fx)}{\sqrt{a - a \sin^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cot(fx + e)}{f \sqrt{a(\cos^2(fx + e))}} + \frac{2 \cot(fx + e) (\csc^2(fx + e))}{3f \sqrt{a(\cos^2(fx + e))}} - \frac{\cot(fx + e) (\csc^4(fx + e))}{5f \sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 25 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 150 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} + \frac{150 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 25 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + 3}{\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1) \tan(\frac{1}{2}fx + \frac{1}{2}e)^5}$$


---


$$480 \sqrt{a} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 48.34 Problem number 482

$$\int \frac{\cot^3(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a(\cos^2(fx + e))}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{(\csc^2(fx + e))\sqrt{a(\cos^2(fx + e))}}{2a^2f}$$

command

```
integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\tan(\frac{1}{2}fx + \frac{1}{2}e)^2}{a^{3/2}\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} + \frac{2\log(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2)}{a^{3/2}\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} - \frac{2\sqrt{a}\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + \sqrt{a}}{a^2\operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)\tan(\frac{1}{2}fx + \frac{1}{2}e)^2}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.35 Problem number 483

$$\int \frac{\tan^2(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e))\cos(fx + e)}{8af\sqrt{a(\cos^2(fx + e))}} - \frac{\tan(fx + e)}{8af\sqrt{a(\cos^2(fx + e))}} + \frac{(\sec^2(fx + e))\tan(fx + e)}{4af\sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a\tan(fx + e)^2 + a}\left(\frac{2\tan(fx + e)^2}{a} + \frac{1}{a}\right)\tan(fx + e) + \frac{\log\left(\left|-\sqrt{a}\tan(fx + e) + \sqrt{a\tan(fx + e)^2 + a}\right|\right)}{\sqrt{a}}}{8af}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.36 Problem number 484

$$\int \frac{\cot^2(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{af \sqrt{a(\cos^2(fx + e))}} - \frac{\cot(fx + e)}{af \sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\tan(\frac{1}{2}fx + \frac{1}{2}e)}{a^{\frac{3}{2}} \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)} + \frac{1}{a^{\frac{3}{2}} \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1) \tan(\frac{1}{2}fx + \frac{1}{2}e)}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 48.37 Problem number 485

$$\int \frac{\cot^4(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e) (\csc^2(fx + e))}{3af \sqrt{a(\cos^2(fx + e))}}$$

command

```
integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3\sqrt{a} \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 + \sqrt{a}}{a^2 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1) \tan(\frac{1}{2}fx + \frac{1}{2}e)^3} + \frac{a^{\frac{9}{2}} \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3a^{\frac{9}{2}} \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a^6 \operatorname{sgn}(\tan(\frac{1}{2}fx + \frac{1}{2}e)^4 - 1)}}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.38 Problem number 486

$$\int \frac{\cot^6(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e) (\csc^2(fx + e))}{3af \sqrt{a} (\cos^2(fx + e))} - \frac{\cot(fx + e) (\csc^4(fx + e))}{5af \sqrt{a} (\cos^2(fx + e))}$$

command

`integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{30 \sqrt{a} \tan(\frac{1}{2} fx + \frac{1}{2} e)^4 + 5 \sqrt{a} \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - 3 \sqrt{a}}{a^2 \operatorname{sgn}(\tan(\frac{1}{2} fx + \frac{1}{2} e)^4 - 1) \tan(\frac{1}{2} fx + \frac{1}{2} e)^5} - \frac{3 a^{\frac{17}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)^5 - 5 a^{\frac{17}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)^3 - 30 a^{\frac{17}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)}{a^{10} \operatorname{sgn}(\tan(\frac{1}{2} fx + \frac{1}{2} e)^4 - 1)}$$


---

480 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 48.39 Problem number 487

$$\int \frac{\cot^8(e + fx)}{(a - a \sin^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cot(fx + e) (\csc^2(fx + e))}{3af \sqrt{a} (\cos^2(fx + e))} + \frac{2 \cot(fx + e) (\csc^4(fx + e))}{5af \sqrt{a} (\cos^2(fx + e))} - \frac{\cot(fx + e) (\csc^6(fx + e))}{7af \sqrt{a} (\cos^2(fx + e))}$$

command

`integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{525 \sqrt{a} \tan(\frac{1}{2} fx + \frac{1}{2} e)^6 + 35 \sqrt{a} \tan(\frac{1}{2} fx + \frac{1}{2} e)^4 - 63 \sqrt{a} \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 + 15 \sqrt{a}}{a^2 \operatorname{sgn}(\tan(\frac{1}{2} fx + \frac{1}{2} e)^4 - 1) \tan(\frac{1}{2} fx + \frac{1}{2} e)^7} + \frac{15 a^{\frac{25}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)^7 - 63 a^{\frac{25}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)^5 + 35 a^{\frac{25}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)^3 - 15 a^{\frac{25}{2}} \tan(\frac{1}{2} fx + \frac{1}{2} e)}{a^{14} \operatorname{sgn}(\tan(\frac{1}{2} fx + \frac{1}{2} e)^4 - 1)}$$


---

13440 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 49 Test file number 82

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/82\_4.2.0-a\_cos-^m-b\_trg-^n

### 49.1 Problem number 55

$$\int \frac{1}{(a \cos^4(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cos(x) \sin(x)}{a \sqrt{a (\cos^4(x))}} + \frac{2(\sin^2(x)) \tan(x)}{3a \sqrt{a (\cos^4(x))}} + \frac{(\sin^2(x)) (\tan^3(x))}{5a \sqrt{a (\cos^4(x))}}$$

command

`integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)}{15 a^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 49.2 Problem number 141

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} dx$$

Optimal antiderivative

$$\frac{\sin(dx+c) \sqrt{b \cos(dx+c)}}{d \sqrt{\cos(dx+c)}} - \frac{(\sin^3(dx+c)) \sqrt{b \cos(dx+c)}}{3d \sqrt{\cos(dx+c)}}$$

command

`integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 49.3 Problem number 151

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{b \sin(dx + c) \sqrt{b \cos(dx + c)}}{d \sqrt{\cos(dx + c)}} - \frac{b(\sin^3(dx + c)) \sqrt{b \cos(dx + c)}}{3d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 49.4 Problem number 162

$$\int \sqrt{\cos(c + dx)} (b \cos(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{b^2 \sin(dx + c) \sqrt{b \cos(dx + c)}}{d \sqrt{\cos(dx + c)}} - \frac{b^2(\sin^3(dx + c)) \sqrt{b \cos(dx + c)}}{3d \sqrt{\cos(dx + c)}}$$

command

```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 49.5 Problem number 271

$$\int \sqrt{\csc(a+bx)} \sec(a+bx) dx$$

Optimal antiderivative

$$-\frac{\arctan(\sqrt{\csc}(bx+a))}{b} + \frac{\operatorname{arctanh}(\sqrt{\csc}(bx+a))}{b}$$

command

```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan\left(\sqrt{\sin(bx+a)}\right) + \log\left(\sqrt{\sin(bx+a)} + 1\right) - \log\left(\left|\sqrt{\sin(bx+a)} - 1\right|\right)}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{\csc(bx+a)} \sec(bx+a) dx$$

### 49.6 Problem number 272

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}} dx$$

Optimal antiderivative

$$\frac{\arctan(\sqrt{\csc}(bx+a))}{b} + \frac{\operatorname{arctanh}(\sqrt{\csc}(bx+a))}{b}$$

command

```
integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan\left(\sqrt{\sin(bx+a)}\right) - \log\left(\sqrt{\sin(bx+a)} + 1\right) + \log\left(\left|\sqrt{\sin(bx+a)} - 1\right|\right)}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sec(bx+a)}{\sqrt{\csc(bx+a)}} dx$$

### 49.7 Problem number 275

$$\int \sqrt{\csc(a+bx)} \sec^3(a+bx) dx$$

Optimal antiderivative

$$-\frac{3 \arctan(\sqrt{\csc}(bx+a))}{4b} + \frac{3 \operatorname{arctanh}(\sqrt{\csc}(bx+a))}{4b} + \frac{\sec^2(bx+a)}{2b\sqrt{\csc}(bx+a)}$$

command

```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \sqrt{\sin(bx+a)} - 6 \arctan(\sqrt{\sin(bx+a)}) - 3 \log(\sqrt{\sin(bx+a)} + 1) + 3 \log(|\sqrt{\sin(bx+a)} - 1|)}{8b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{\csc(bx+a)} \sec(bx+a)^3 dx$$

### 49.8 Problem number 276

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}} dx$$

Optimal antiderivative

$$\frac{\arctan(\sqrt{\csc}(bx+a))}{4b} + \frac{\operatorname{arctanh}(\sqrt{\csc}(bx+a))}{4b} + \frac{\sec^2(bx+a)}{2b \csc(bx+a)^{\frac{3}{2}}}$$

command

```
integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \sin(bx+a)^{\frac{3}{2}} + 2 \arctan(\sqrt{\sin(bx+a)}) - \log(\sqrt{\sin(bx+a)} + 1) + \log(|\sqrt{\sin(bx+a)} - 1|)}{8b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sec(bx+a)^3}{\sqrt{\csc(bx+a)}} dx$$

## 50 Test file number 83

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/83\_4.2.10-c+d\_x-^m-a+b\_cos-^n

### 50.1 Problem number 22

$$\int \frac{\cos^3(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{9b^2 \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{8d^3} - \frac{3b^2 \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{8d^3} \\ & - \frac{\cos^3(bx + a)}{2d(dx + c)^2} + \frac{9b^2 \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^3} \\ & + \frac{3b^2 \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^3} + \frac{3b(\cos^2(bx + a)) \sin(bx + a)}{2d^2(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 51 Test file number 85

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/85\_4.2.12-e\_x-^m-a+b\_cos-c+d\_x^n-^p

### 51.1 Problem number 43

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^3} dx$$

Optimal antiderivative

$$-\frac{\sin\left(a + \frac{b}{x^2}\right)}{2b}$$



command

```
integrate(cos(a+b/x^2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^3} dx$$

## 52 Test file number 89

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/89\_4.2.2.1-a+b\_cos-^m-c+d\_cos-^n

### 52.1 Problem number 109

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} + \frac{a^2 \tan(dx+c)}{d\sqrt{a+a\cos(dx+c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 \sqrt{2} a \log \left( \frac{-2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)} \right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 a \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} \right) \sqrt{a}}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 52.2 Problem number 110

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{7a^{3/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4d} + \frac{7a^2 \tan(dx+c)}{4d\sqrt{a+a\cos(dx+c)}} + \frac{a^2 \sec(dx+c) \tan(dx+c)}{2d\sqrt{a+a\cos(dx+c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 7\sqrt{2} a \log\left(\frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left( 14 \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 9 \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left( 2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)^2} \right)}{16d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 52.3 Problem number 111

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{11a^{3/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{8d} + \frac{11a^2 \tan(dx+c)}{8d\sqrt{a+a\cos(dx+c)}} + \frac{11a^2 \sec(dx+c) \tan(dx+c)}{12d\sqrt{a+a\cos(dx+c)}} + \frac{a^2 (\sec^2(dx+c)) \tan(dx+c)}{3d\sqrt{a+a\cos(dx+c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 33\sqrt{2} a \log\left(\frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left( 132 \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 176 \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left( 2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)^2} \right)}{96d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 52.4 Problem number 117

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{5a^{5/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} + \frac{a^3 \sin(dx+c)}{d\sqrt{a+a\cos(dx+c)}} + \frac{a^2 \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{d}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 5 \sqrt{2} a^2 \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) - 8 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) + \dots \right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 52.5 Problem number 118

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{19a^{5/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4d} + \frac{9a^3 \tan(dx+c)}{4d\sqrt{a+a\cos(dx+c)}} + \frac{a^2 \sec(dx+c) \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{2d}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 19 \sqrt{2} a^2 \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + \frac{4(22a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 13a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c))}{(2 \sin(\frac{1}{2} dx + \frac{1}{2} c))^2 - 1} \right)}{16d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 52.6 Problem number 119

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{25a^{5/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{8d} + \frac{25a^3 \tan(dx+c)}{8d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{13a^3 \sec(dx+c) \tan(dx+c)}{12d\sqrt{a+a\cos(dx+c)}} + \frac{a^2 (\sec^2(dx+c)) \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 75 \sqrt{2} a^2 \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + \frac{4(300 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 - 368 a^2}{96 d} \right)$$

96 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 52.7 Problem number 120

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{163a^{5/2} \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{64d} + \frac{163a^3 \tan(dx+c)}{64d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{163a^3 \sec(dx+c) \tan(dx+c)}{96d\sqrt{a+a\cos(dx+c)}} + \frac{17a^3 (\sec^2(dx+c)) \tan(dx+c)}{24d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{a^2 (\sec^3(dx+c)) \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 489 \sqrt{2} a^2 \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left( 3912 a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)^7 - 717 \right)}{f} \right)$$

768

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 52.8 Problem number 223

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx$$

Optimal antiderivative

$$\frac{2 \arcsin \left( \frac{\sin(fx+e)\sqrt{a}}{\sqrt{a - a \cos(fx + e)}} \right) \sqrt{a}}{f}$$

command

```
integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \sqrt{a} \arctan \left( -\frac{1}{2} \sqrt{2} \left( \sqrt{2} + \frac{2 \left( 2\sqrt{2} - \sqrt{-\tan \left( \frac{1}{4} fx + \frac{1}{4} e \right)^4 + 6 \tan \left( \frac{1}{4} fx + \frac{1}{4} e \right)^2 - 1} \right)}{\tan \left( \frac{1}{4} fx + \frac{1}{4} e \right)^2 - 3} \right) \right) \operatorname{sgn} \left( \sin \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 53 Test file number 91

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/91\_4.2.2.3-g\_cos-<sup>p</sup>-a+b\_cos-<sup>m</sup>-c+d\_cos-<sup>n</sup>

### 53.1 Problem number 1

$$\int \frac{(a + a \cos(e + fx))^2 \sec^2(e + fx)}{-c + c \cos(e + fx)} dx$$

Optimal antiderivative

$$-\frac{3a^2 \operatorname{arctanh}(\sin(fx + e))}{cf} + \frac{4a^2 \sin(fx + e)}{cf(1 - \cos(fx + e))} - \frac{a^2 \tan(fx + e)}{cf}$$

command

```
integrate((a+a*cos(f*x+e))^2*sec(f*x+e)^2/(-c+c*cos(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3a^2 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{c} - \frac{3a^2 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{c} - \frac{2(3a^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 2a^2)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - \tan(\frac{1}{2}fx + \frac{1}{2}e))c}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 54 Test file number 92

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/92\_4.2.3.1-a+b\_cos-<sup>m</sup>-c+d\_cos-<sup>n</sup>-A+B\_cos-

### 54.1 Problem number 79

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{(A + 2B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}}\right) \sqrt{a}}{d} + \frac{aA \tan(dx + c)}{d\sqrt{a + a \cos(dx + c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \left( \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 2 B \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) + \frac{4 \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)}{2 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right)}{4 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.2 Problem number 80

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{(3A + 4B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right) \sqrt{a}}{4d} + \frac{a(3A + 4B) \tan(dx + c)}{4d \sqrt{a + a \cos(dx + c)}} + \frac{aA \sec(dx + c) \tan(dx + c)}{2d \sqrt{a + a \cos(dx + c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \left( 3 \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 4 B \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) + \frac{4 \left( 6 \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{2 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right)}{4 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.3 Problem number 81

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{(5A + 6B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}}\right) \sqrt{a}}{8d} + \frac{a(5A + 6B) \tan(dx + c)}{8d \sqrt{a + a \cos(dx + c)}} + \frac{a(5A + 6B) \sec(dx + c) \tan(dx + c)}{12d \sqrt{a + a \cos(dx + c)}} + \frac{aA(\sec^2(dx + c)) \tan(dx + c)}{3d \sqrt{a + a \cos(dx + c)}}$$

command

```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 5 \operatorname{Asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 6 B \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right) \log\left(\frac{\left| -2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}\right) + \frac{4 \left( 60 \operatorname{Asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.4 Problem number 86

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2a^{3/2} A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}}\right)}{d} + \frac{2a^2(3A + 4B) \sin(dx + c)}{3d \sqrt{a + a \cos(dx + c)}} + \frac{2aB \sin(dx + c) \sqrt{a + a \cos(dx + c)}}{3d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\sqrt{2} \left( 8 \operatorname{Asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 + 3 \sqrt{2} A a \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \right)$$

6 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.5 Problem number 87

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{3/2} (3A + 2B) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{d} - \frac{a^2 (A - 2B) \sin(dx+c)}{d \sqrt{a + a \cos(dx+c)}} + \frac{aA \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 8 \operatorname{Asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - \frac{4 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1} - \sqrt{2} (3 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \right)$$

4 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.6 Problem number 88

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{3/2} (7A + 12B) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{4d} + \frac{a^2 (5A + 4B) \tan(dx+c)}{4d \sqrt{a + a \cos(dx+c)}} + \frac{aA \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{2d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \sqrt{2} \left( 7 A \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 12 B \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{-2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right) + \frac{4 \left( 14 A \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 12 B \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{3 \sqrt{2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.7 Problem number 89

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2} (11A + 14B) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{8d} \\ & + \frac{a^2 (11A + 14B) \tan(dx+c)}{8d \sqrt{a + a \cos(dx+c)}} + \frac{a^2 (7A + 6B) \sec(dx+c) \tan(dx+c)}{12d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{aA (\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{3d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 11 A \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 14 B \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{-2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right) + \frac{4 \left( 132 A a \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 120 B a \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{3 \sqrt{2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.8 Problem number 90

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{3}{2}}(75A + 88B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{64d} + \frac{a^2(75A + 88B) \tan(dx+c)}{64d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{a^2(75A + 88B) \sec(dx+c) \tan(dx+c)}{96d\sqrt{a+a\cos(dx+c)}} + \frac{a^2(9A + 8B) (\sec^2(dx+c)) \tan(dx+c)}{24d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{aA(\sec^3(dx+c)) \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3\sqrt{2} (75A \operatorname{asgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) + 88B \operatorname{asgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))) \log\left(\frac{-2\sqrt{2} + 4\sin(\frac{1}{2}dx + \frac{1}{2}c)}{2\sqrt{2} + 4\sin(\frac{1}{2}dx + \frac{1}{2}c)}\right) \right) + \frac{4(1800A + \dots)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.9 Problem number 94

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^{\frac{5}{2}}A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} + \frac{2aB(a + a \cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{5d} \\ & + \frac{2a^3(35A + 32B) \sin(dx+c)}{15d\sqrt{a+a\cos(dx+c)}} + \frac{2a^2(5A + 8B) \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{15d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 48 Ba^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 - 40 Aa^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 160 Ba^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)$$


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Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.10 Problem number 95

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{5/2} (5A + 2B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{d} + \frac{a^3 (3A + 14B) \sin(dx+c)}{3d \sqrt{a + a \cos(dx+c)}} - \frac{a^2 (3A - 2B) \sin(dx+c) \sqrt{a + a \cos(dx+c)}}{3d} + \frac{aA(a + a \cos(dx+c))^{3/2} \tan(dx+c)}{d}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 Ba^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 24 Aa^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - 72 Ba^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.11 Problem number 96

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (19A + 20B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{4d} - \frac{a^3 (9A - 4B) \sin(dx+c)}{4d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{aA(a + a \cos(dx+c))^{3/2} \sec(dx+c) \tan(dx+c)}{2d} \\ & + \frac{a^2 (7A + 4B) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 32 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - \sqrt{2} (19 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 20 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.12 Problem number 97

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (25A + 38B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{8d} \\ & + \frac{aA(a + a \cos(dx+c))^{3/2} (\sec^2(dx+c)) \tan(dx+c)}{3d} + \frac{a^3 (49A + 54B) \tan(dx+c)}{24d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2 (3A + 2B) \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (25 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 38 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4}{300} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 54.13 Problem number 98

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (163A + 200B) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{64d} \\ & + \frac{aA(a + a \cos(dx+c))^{3/2} (\sec^3(dx+c)) \tan(dx+c)}{4d} \\ & + \frac{a^3(163A + 200B) \tan(dx+c)}{64d \sqrt{a + a \cos(dx+c)}} + \frac{a^3(95A + 104B) \sec(dx+c) \tan(dx+c)}{96d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2(11A + 8B) (\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{24d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (163 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 200 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4}{300} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.14 Problem number 99

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (283A + 326B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right)}{128d} \\ & + \frac{aA(a + a \cos(dx + c))^{3/2} (\sec^4(dx + c)) \tan(dx + c)}{5d} \\ & + \frac{a^3(283A + 326B) \tan(dx + c)}{128d\sqrt{a + a \cos(dx + c)}} + \frac{a^3(283A + 326B) \sec(dx + c) \tan(dx + c)}{192d\sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^3(157A + 170B) (\sec^2(dx + c)) \tan(dx + c)}{240d\sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^2(13A + 10B) (\sec^3(dx + c)) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{40d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 \sqrt{2} (283 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 326 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + 4 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 54.15 Problem number 114

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(19A - 12B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right)}{4a^{3/2}d} \\ & - \frac{(13A - 9B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx + c)}} \right) \sqrt{2}}{4a^{3/2}d} - \frac{(A - B) \sec(dx + c) \tan(dx + c)}{2d(a + a \cos(dx + c))^{3/2}} \\ & - \frac{(7A - 6B) \tan(dx + c)}{4ad\sqrt{a + a \cos(dx + c)}} + \frac{(2A - B) \sec(dx + c) \tan(dx + c)}{2ad\sqrt{a + a \cos(dx + c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 13 A \sqrt{a} - 9 B \sqrt{a} \right) \log \left( \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)}{a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{\sqrt{2} \left( 13 A \sqrt{a} - 9 B \sqrt{a} \right) \log \left( -\sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)}{a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{\left( 19 A \sqrt{a} - 12 B \sqrt{a} \right) \log \left( \left| \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right| \right)}{a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 54.16 Problem number 122

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(39A - 20B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{4a^{\frac{5}{2}}d} \\ & - \frac{(219A - 115B) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \cos(dx+c)}} \right) \sqrt{2}}{32a^{\frac{5}{2}}d} \\ & - \frac{(A - B) \sec(dx+c) \tan(dx+c)}{4d(a + a \cos(dx+c))^{\frac{5}{2}}} - \frac{(19A - 11B) \sec(dx+c) \tan(dx+c)}{16ad(a + a \cos(dx+c))^{\frac{3}{2}}} \\ & - \frac{7(9A - 5B) \tan(dx+c)}{16a^2d\sqrt{a + a \cos(dx+c)}} + \frac{(31A - 15B) \sec(dx+c) \tan(dx+c)}{16a^2d\sqrt{a + a \cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 219 A \sqrt{a} - 115 B \sqrt{a} \right) \log \left( \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)}{a^3 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{\sqrt{2} \left( 219 A \sqrt{a} - 115 B \sqrt{a} \right) \log \left( -\sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)}{a^3 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)} - \frac{8 \left( 39 A \sqrt{a} - 20 B \sqrt{a} \right) \log \left( \left| \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right| \right)}{a^3 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 55 Test file number 93

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/93\_4.2.4.1-a+b\_cos-^m-A+B\_cos+C\_cos^2-

### 55.1 Problem number 91

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$$

Optimal antiderivative

$$\frac{(A+C) \sin(dx+c) \sqrt{b \cos(dx+c)}}{d \sqrt{\cos(dx+c)}} - \frac{C(\sin^3(dx+c)) \sqrt{b \cos(dx+c)}}{3d \sqrt{\cos(dx+c)}}$$

command

```
integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56 Test file number 94

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/94\_4.2.4.2-a+b\_cos-^m-c+d\_cos-^n-A+B\_cos+C\_cos^2-

### 56.1 Problem number 79

$$\int \sqrt{a+a \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^2(c+dx) dx$$

Optimal antiderivative

$$\frac{A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a \cos(dx+c)}}\right) \sqrt{a}}{d} - \frac{a(A-2C) \sin(dx+c)}{d \sqrt{a+a \cos(dx+c)}} + \frac{A \sqrt{a+a \cos(dx+c)} \tan(dx+c)}{d}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} A \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) - 8 C \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4 \right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.2 Problem number 80

$$\int \sqrt{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{(3A + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right) \sqrt{a}}{4d} + \frac{aA \tan(dx+c)}{4d \sqrt{a + a \cos(dx+c)}} + \frac{A \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{2d}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} \left( 3 A \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 8 C \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right) \log \left( \frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) + 4 \left( 6 A \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right) \right)}{16d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.3 Problem number 81

$$\int \sqrt{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{(5A + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right) \sqrt{a}}{8d} + \frac{a(5A + 8C) \tan(dx+c)}{8d \sqrt{a + a \cos(dx+c)}} \\ + \frac{aA \sec(dx+c) \tan(dx+c)}{12d \sqrt{a + a \cos(dx+c)}} + \frac{A(\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{3d}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 5 \operatorname{Asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + 8 C \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right) \log\left(\frac{-2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}\right) + \frac{4 \left( 60 \operatorname{Asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.4 Problem number 82

$$\int \sqrt{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\frac{(35A + 48C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right) \sqrt{a}}{64d} + \frac{a(35A + 48C) \tan(dx+c)}{64d \sqrt{a + a \cos(dx+c)}} \\ + \frac{a(35A + 48C) \sec(dx+c) \tan(dx+c)}{96d \sqrt{a + a \cos(dx+c)}} + \frac{aA(\sec^2(dx+c)) \tan(dx+c)}{24d \sqrt{a + a \cos(dx+c)}} \\ + \frac{A(\sec^3(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d}$$

command

`integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 35 \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 48 C \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{\left| -2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|}{\left| 2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right|} \right) + \frac{4 \left( 840 \operatorname{Asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{\dots} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.5 Problem number 86

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2a^{3/2} A \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{d} + \frac{2C(a + a \cos(dx+c))^{3/2} \sin(dx+c)}{5d} + \frac{2a^2(5A + 4C) \sin(dx+c)}{5d \sqrt{a + a \cos(dx+c)}} + \frac{2aC \sin(dx+c) \sqrt{a + a \cos(dx+c)}}{5d}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 C \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)^5 - 40 C \operatorname{asgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)^3 - 5 \sqrt{2} A a \log \left( \dots \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.6 Problem number 87

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{3a^{\frac{3}{2}} A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} - \frac{a^2(3A-8C)\sin(dx+c)}{3d\sqrt{a+a\cos(dx+c)}} - \frac{a(3A-2C)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{3d} + \frac{A(a+a\cos(dx+c))^{\frac{3}{2}}\tan(dx+c)}{d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 16 C \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 \sqrt{2} A a \log\left(\frac{\left| -2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2 \sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{12 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.7 Problem number 88

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{\frac{3}{2}}(7A+8C)\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4d} - \frac{a^2(5A-8C)\sin(dx+c)}{4d\sqrt{a+a\cos(dx+c)}} + \frac{A(a+a\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\tan(dx+c)}{2d} + \frac{3aA\sqrt{a+a\cos(dx+c)}\tan(dx+c)}{4d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 32 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - \sqrt{2} (7 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 8 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \right)$$


---

16 d

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.8 Problem number 89

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{\frac{3}{2}}(11A + 24C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{8d} + \frac{A(a + a \cos(dx+c))^{\frac{3}{2}} (\sec^2(dx+c)) \tan(dx+c)}{3d} + \frac{a^2(19A + 24C) \tan(dx+c)}{24d\sqrt{a + a \cos(dx+c)}} + \frac{aA \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (11 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 24 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4 (132 A a + 128 C a)}{16 d} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.9 Problem number 90

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2}(75A + 112C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{64d} \\ & + \frac{A(a + a \cos(dx+c))^{3/2} (\sec^3(dx+c)) \tan(dx+c)}{4d} \\ & + \frac{a^2(75A + 112C) \tan(dx+c)}{64d\sqrt{a + a \cos(dx+c)}} + \frac{a^2(13A + 16C) \sec(dx+c) \tan(dx+c)}{32d\sqrt{a + a \cos(dx+c)}} \\ & + \frac{aA(\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{8d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \sqrt{2} (75 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 112 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4(600 A a^2 + 112 C a^2)}{64d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.10 Problem number 91

$$\int (a + a \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2}(133A + 176C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{128d} \\ & + \frac{A(a + a \cos(dx+c))^{3/2} (\sec^4(dx+c)) \tan(dx+c)}{5d} + \frac{a^2(133A + 176C) \tan(dx+c)}{128d\sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2(133A + 176C) \sec(dx+c) \tan(dx+c)}{192d\sqrt{a + a \cos(dx+c)}} + \frac{a^2(67A + 80C) (\sec^2(dx+c)) \tan(dx+c)}{240d\sqrt{a + a \cos(dx+c)}} \\ & + \frac{3aA(\sec^3(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{40d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 \sqrt{2} (133 A \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 176 C \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4}{31} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.11 Problem number 95

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^{\frac{5}{2}} A \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}} \right)}{d} + \frac{2aC(a+a\cos(dx+c))^{\frac{3}{2}} \sin(dx+c)}{7d} \\ & + \frac{2C(a+a\cos(dx+c))^{\frac{5}{2}} \sin(dx+c)}{7d} + \frac{2a^3(49A+32C) \sin(dx+c)}{21d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{2a^2(7A+8C) \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{21d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 96 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^7 - 336 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 + 56 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 56.12 Problem number 96

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5a^{5/2} A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} \\ & - \frac{a(5A-2C)(a+a\cos(dx+c))^{3/2} \sin(dx+c)}{5d} + \frac{a^3(15A+64C)\sin(dx+c)}{15d\sqrt{a+a\cos(dx+c)}} \\ & - \frac{a^2(15A-16C)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{15d} + \frac{A(a+a\cos(dx+c))^{5/2} \tan(dx+c)}{d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 96 C a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 320 C a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 75 \sqrt{2} A a^2 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.13 Problem number 97

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2}(19A+8C)\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4d} - \frac{a^3(27A-56C)\sin(dx+c)}{12d\sqrt{a+a\cos(dx+c)}} \\ & - \frac{a^2(21A-8C)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{12d} \\ & + \frac{5aA(a+a\cos(dx+c))^{3/2}\tan(dx+c)}{4d} + \frac{A(a+a\cos(dx+c))^{5/2}\sec(dx+c)\tan(dx+c)}{2d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 64 C a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)^3 - 288 C a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) + 3 \sqrt{2} \left( 19 A \right. \right.$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.14 Problem number 98

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5a^{5/2}(5A + 8C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{8d} - \frac{a^3(49A - 24C) \sin(dx+c)}{24d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{5aA(a + a \cos(dx+c))^{3/2} \sec(dx+c) \tan(dx+c)}{12d} \\ & + \frac{A(a + a \cos(dx+c))^{5/2} (\sec^2(dx+c)) \tan(dx+c)}{3d} \\ & + \frac{a^2(31A + 24C) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{24d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 192 C a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right) - 15 \sqrt{2} \left( 5 A a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 8 C a^2 \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \right. \right. \right.$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.15 Problem number 99

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (163A + 304C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{64d} \\ & + \frac{5aA(a + a \cos(dx+c))^{3/2} (\sec^2(dx+c)) \tan(dx+c)}{24d} \\ & + \frac{A(a + a \cos(dx+c))^{5/2} (\sec^3(dx+c)) \tan(dx+c)}{4d} + \frac{a^3(299A + 432C) \tan(dx+c)}{192d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2(17A + 16C) \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{32d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (163 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 304 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{|-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) + \frac{4}{3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.16 Problem number 100

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (283A + 400C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{128d} \\ & + \frac{aA(a + a \cos(dx+c))^{3/2} (\sec^3(dx+c)) \tan(dx+c)}{8d} \\ & + \frac{A(a + a \cos(dx+c))^{5/2} (\sec^4(dx+c)) \tan(dx+c)}{5d} \\ & + \frac{a^3(283A + 400C) \tan(dx+c)}{128d \sqrt{a + a \cos(dx+c)}} + \frac{a^3(787A + 1040C) \sec(dx+c) \tan(dx+c)}{960d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2(79A + 80C) (\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{240d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 \sqrt{2} (283 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 400 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) + \frac{4}{\dots} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.17 Problem number 101

$$\int (a + a \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^7(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (1015A + 1304C) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right)}{512d} \\ & + \frac{aA(a + a \cos(dx+c))^{3/2} (\sec^4(dx+c)) \tan(dx+c)}{12d} \\ & + \frac{A(a + a \cos(dx+c))^{5/2} (\sec^5(dx+c)) \tan(dx+c)}{6d} \\ & + \frac{a^3(1015A + 1304C) \tan(dx+c)}{512d \sqrt{a + a \cos(dx+c)}} + \frac{a^3(1015A + 1304C) \sec(dx+c) \tan(dx+c)}{768d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^3(109A + 136C) (\sec^2(dx+c)) \tan(dx+c)}{192d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a^2(23A + 24C) (\sec^3(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{96d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (1015 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 1304 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2 \sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) + \frac{4}{\dots} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.18 Problem number 117

$$\int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(19A + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{4a^{\frac{3}{2}}d} \\ & - \frac{(13A + 5C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} - \frac{(A + C) \sec(dx+c) \tan(dx+c)}{2d(a + a \cos(dx+c))^{\frac{3}{2}}} \\ & - \frac{(7A + 2C) \tan(dx+c)}{4ad\sqrt{a + a \cos(dx+c)}} + \frac{(2A + C) \sec(dx+c) \tan(dx+c)}{2ad\sqrt{a + a \cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (13A\sqrt{a} + 5C\sqrt{a}) \log(\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{\sqrt{2} (13A\sqrt{a} + 5C\sqrt{a}) \log(-\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{(19A\sqrt{a} + 8C\sqrt{a}) \log\left(\frac{1}{2}\right)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 56.19 Problem number 118

$$\int \frac{(A + C \cos^2(c + dx)) \sec^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(47A + 24C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{8a^{\frac{3}{2}}d} \\ & + \frac{(17A + 9C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(A + C) (\sec^2(dx+c)) \tan(dx+c)}{2d(a + a \cos(dx+c))^{\frac{3}{2}}} + \frac{3(7A + 4C) \tan(dx+c)}{8ad\sqrt{a + a \cos(dx+c)}} \\ & - \frac{(13A + 6C) \sec(dx+c) \tan(dx+c)}{12ad\sqrt{a + a \cos(dx+c)}} + \frac{(5A + 3C) (\sec^2(dx+c)) \tan(dx+c)}{6ad\sqrt{a + a \cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6\sqrt{2}\left(17A\sqrt{a}+9C\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{6\sqrt{2}\left(17A\sqrt{a}+9C\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{3\left(47A\sqrt{a}+24C\sqrt{a}\right)\log\left(\frac{1}{2}\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 56.20 Problem number 124

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{a^{\frac{5}{2}}d} + \frac{(115A+3C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{32a^{\frac{5}{2}}d} \\ & -\frac{(A+C)\tan(dx+c)}{4d(a+a\cos(dx+c))^{\frac{5}{2}}} - \frac{(15A-C)\tan(dx+c)}{16ad(a+a\cos(dx+c))^{\frac{3}{2}}} + \frac{(35A+3C)\tan(dx+c)}{16a^2d\sqrt{a+a\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{160A \log\left(\frac{\left|-4\sqrt{2}+8\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|4\sqrt{2}+8\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} + \frac{\sqrt{2}\left(115A\sqrt{a}+3C\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{\sqrt{2}\left(115A\sqrt{a}+3C\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 56.21 Problem number 377

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right) \sqrt{a}}{d} + \frac{2a(3B+C) \sin(dx+c)}{3d\sqrt{a+a\cos(dx+c)}} + \frac{2C \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{3d}$$

command

`integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 8 C \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3 \sqrt{2} A \log\left(\frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) - \dots}{6d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.22 Problem number 378

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{(A + 2B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right) \sqrt{a}}{d} - \frac{a(A - 2C) \sin(dx+c)}{d\sqrt{a+a\cos(dx+c)}} + \frac{A \sqrt{a+a\cos(dx+c)} \tan(dx+c)}{d}$$

command

`integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 8 C \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - \sqrt{2} (A \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 2 B \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) \right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.23 Problem number 379

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\frac{(3A + 4B + 8C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right) \sqrt{a}}{4d} + \frac{a(A + 4B) \tan(dx + c)}{4d\sqrt{a + a \cos(dx + c)}} + \frac{A \sec(dx + c) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{2d}$$

command

`integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{2} (3 A \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 4 B \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 8 C \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) \right)}{4d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.24 Problem number 380

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\frac{(5A + 6B + 8C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right) \sqrt{a}}{8d} + \frac{a(5A + 6B + 8C) \tan(dx + c)}{8d\sqrt{a + a \cos(dx + c)}} + \frac{a(A + 6B) \sec(dx + c) \tan(dx + c)}{12d\sqrt{a + a \cos(dx + c)}} + \frac{A(\sec^2(dx + c)) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{3d}$$



command

```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 5 A \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 6 B \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 8 C \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{-2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.25 Problem number 381

$$\int \sqrt{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(35A + 40B + 48C) \operatorname{arctanh} \left( \frac{\sin(dx+c) \sqrt{a}}{\sqrt{a + a \cos(dx+c)}} \right) \sqrt{a}}{64d} \\ & + \frac{a(35A + 40B + 48C) \tan(dx+c)}{64d \sqrt{a + a \cos(dx+c)}} + \frac{a(35A + 40B + 48C) \sec(dx+c) \tan(dx+c)}{96d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a(A + 8B) (\sec^2(dx+c)) \tan(dx+c)}{24d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{A(\sec^3(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} \left( 35 A \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 40 B \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) + 48 C \operatorname{sgn} \left( \cos \left( \frac{1}{2} dx + \frac{1}{2} c \right) \right) \right) \log \left( \frac{-2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)}{2 \sqrt{2} + 4 \sin \left( \frac{1}{2} dx + \frac{1}{2} c \right)} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.26 Problem number 385

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\frac{2a^{3/2} A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} + \frac{2C(a+a\cos(dx+c))^{3/2} \sin(dx+c)}{5d}$$

$$+ \frac{2a^2(15A+20B+12C) \sin(dx+c)}{15d\sqrt{a+a\cos(dx+c)}} + \frac{2a(5B+3C) \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{15d}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 48 C \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 40 B \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 120 C \operatorname{asgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.27 Problem number 386

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\frac{a^{3/2} (3A + 2B) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} - \frac{a^2(3A - 6B - 8C) \sin(dx+c)}{3d\sqrt{a+a\cos(dx+c)}}$$

$$- \frac{a(3A - 2C) \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{3d} + \frac{A(a+a\cos(dx+c))^{3/2} \tan(dx+c)}{d}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 16 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 24 B \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - 48 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.28 Problem number 387

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{3}{2}}(7A + 12B + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}}\right)}{4d} \\ & - \frac{a^2(5A + 4B - 8C) \sin(dx + c)}{4d \sqrt{a + a \cos(dx + c)}} + \frac{A(a + a \cos(dx + c))^{\frac{3}{2}} \sec(dx + c) \tan(dx + c)}{2d} \\ & + \frac{a(3A + 4B) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{4d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 32 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - \sqrt{2} (7 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 12 B \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) - \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.29 Problem number 388

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2}(11A + 14B + 24C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{8d} \\ & + \frac{A(a + a \cos(dx+c))^{3/2} (\sec^2(dx+c)) \tan(dx+c)}{3d} + \frac{a^2(19A + 30B + 24C) \tan(dx+c)}{24d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a(A + 2B) \sec(dx+c) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{4d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (11 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 14 B \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 24 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2 \sqrt{2}}{2 \sqrt{2}} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.30 Problem number 389

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{3/2}(75A + 88B + 112C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{64d} \\ & + \frac{A(a + a \cos(dx+c))^{3/2} (\sec^3(dx+c)) \tan(dx+c)}{4d} \\ & + \frac{a^2(75A + 88B + 112C) \tan(dx+c)}{64d \sqrt{a + a \cos(dx+c)}} + \frac{a^2(39A + 56B + 48C) \sec(dx+c) \tan(dx+c)}{96d \sqrt{a + a \cos(dx+c)}} \\ & + \frac{a(3A + 8B) (\sec^2(dx+c)) \sqrt{a + a \cos(dx+c)} \tan(dx+c)}{24d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 \sqrt{2} (75 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 88 B \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 112 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2 \sqrt{a + a \cos(dx + c)}}{2 \sqrt{a + a \cos(dx + c)}} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.31 Problem number 390

$$\int (a + a \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{3}{2}}(133A + 150B + 176C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right)}{128d} \\ & + \frac{A(a + a \cos(dx + c))^{\frac{3}{2}} (\sec^4(dx + c)) \tan(dx + c)}{5d} \\ & + \frac{a^2(133A + 150B + 176C) \tan(dx + c)}{128d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^2(133A + 150B + 176C) \sec(dx + c) \tan(dx + c)}{192d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^2(67A + 90B + 80C) (\sec^2(dx + c)) \tan(dx + c)}{240d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a(3A + 10B) (\sec^3(dx + c)) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{40d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 \sqrt{2} (133 A \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 150 B \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 176 C \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \log \left( \frac{-2 \sqrt{a + a \cos(dx + c)}}{2 \sqrt{a + a \cos(dx + c)}} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.32 Problem number 394

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^{5/2} A \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} + \frac{2a(7B+5C)(a+a\cos(dx+c))^{3/2} \sin(dx+c)}{35d} \\ & + \frac{2C(a+a\cos(dx+c))^{5/2} \sin(dx+c)}{7d} + \frac{2a^3(245A+224B+160C)\sin(dx+c)}{105d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{2a^2(35A+56B+40C)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{105d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="gia`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 480 Ca^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 336 Ba^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1680 Ca \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.33 Problem number 395

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2}(5A+2B)\operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{d} \\ & - \frac{a(5A-2C)(a+a\cos(dx+c))^{3/2}\sin(dx+c)}{5d} + \frac{a^3(15A+70B+64C)\sin(dx+c)}{15d\sqrt{a+a\cos(dx+c)}} \\ & - \frac{a^2(15A-10B-16C)\sin(dx+c)\sqrt{a+a\cos(dx+c)}}{15d} \\ & + \frac{A(a+a\cos(dx+c))^{5/2}\tan(dx+c)}{d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 96 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 - 80 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 320 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.34 Problem number 396

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (19A + 20B + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4d} \\ & - \frac{a^3 (27A - 12B - 56C) \sin(dx+c)}{12d \sqrt{a+a\cos(dx+c)}} \\ & - \frac{a^2 (21A + 12B - 8C) \sin(dx+c) \sqrt{a+a\cos(dx+c)}}{12d} \\ & + \frac{a(5A + 4B) (a + a \cos(dx+c))^{3/2} \tan(dx+c)}{4d} \\ & + \frac{A(a + a \cos(dx+c))^{5/2} \sec(dx+c) \tan(dx+c)}{2d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 64 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 96 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - 288 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.35 Problem number 397

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (25A + 38B + 40C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx + c)}} \right)}{8d} \\ & - \frac{a^3 (49A + 54B - 24C) \sin(dx + c)}{24d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a(5A + 6B) (a + a \cos(dx + c))^{3/2} \sec(dx + c) \tan(dx + c)}{12d} \\ & + \frac{A(a + a \cos(dx + c))^{5/2} (\sec^2(dx + c)) \tan(dx + c)}{3d} \\ & + \frac{a^2 (31A + 42B + 24C) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{24d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 192 C a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) - 3 \sqrt{2} (25 A a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + 38 B a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.36 Problem number 398

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx$$



Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{5}{2}}(163A + 200B + 304C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{64d} \\ & + \frac{a(5A + 8B)(a + a\cos(dx+c))^{\frac{3}{2}}(\sec^2(dx+c))\tan(dx+c)}{24d} \\ & + \frac{A(a + a\cos(dx+c))^{\frac{5}{2}}(\sec^3(dx+c))\tan(dx+c)}{4d} \\ & + \frac{a^3(299A + 392B + 432C)\tan(dx+c)}{192d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{a^2(17A + 24B + 16C)\sec(dx+c)\sqrt{a+a\cos(dx+c)}\tan(dx+c)}{32d} \end{aligned}$$

command

`integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3\sqrt{2} (163Aa^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) + 200Ba^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) + 304Ca^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))) \log \left( \right. \right.$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.37 Problem number 399

$$\int (a + a\cos(c + dx))^{5/2} (A + B\cos(c + dx) + C\cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{\frac{5}{2}}(283A + 326B + 400C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{128d} \\ & + \frac{a(A + 2B)(a + a\cos(dx+c))^{\frac{3}{2}}(\sec^3(dx+c))\tan(dx+c)}{8d} \\ & + \frac{A(a + a\cos(dx+c))^{\frac{5}{2}}(\sec^4(dx+c))\tan(dx+c)}{5d} \\ & + \frac{a^3(283A + 326B + 400C)\tan(dx+c)}{128d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{a^3(787A + 950B + 1040C)\sec(dx+c)\tan(dx+c)}{960d\sqrt{a+a\cos(dx+c)}} \\ & + \frac{a^2(79A + 110B + 80C)(\sec^2(dx+c))\sqrt{a+a\cos(dx+c)}\tan(dx+c)}{240d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 56.38 Problem number 400

$$\int (a + a \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^7(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^{5/2} (1015A + 1132B + 1304C) \operatorname{arctanh} \left( \frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}} \right)}{512d} \\ & + \frac{a(5A + 12B)(a + a \cos(dx + c))^{3/2} (\sec^4(dx + c)) \tan(dx + c)}{60d} \\ & + \frac{A(a + a \cos(dx + c))^{5/2} (\sec^5(dx + c)) \tan(dx + c)}{6d} \\ & + \frac{a^3(1015A + 1132B + 1304C) \tan(dx + c)}{512d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^3(1015A + 1132B + 1304C) \sec(dx + c) \tan(dx + c)}{768d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^3(545A + 628B + 680C) (\sec^2(dx + c)) \tan(dx + c)}{960d \sqrt{a + a \cos(dx + c)}} \\ & + \frac{a^2(115A + 156B + 120C) (\sec^3(dx + c)) \sqrt{a + a \cos(dx + c)} \tan(dx + c)}{480d} \end{aligned}$$

command

```
integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 56.39 Problem number 416

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(19A - 12B + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a + a \cos(dx+c)}}\right)}{4a^{\frac{3}{2}}d} \\ & - \frac{(13A - 9B + 5C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a + a \cos(dx+c)}}\right) \sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(A - B + C) \sec(dx+c) \tan(dx+c)}{2d(a + a \cos(dx+c))^{\frac{3}{2}}} - \frac{(7A - 6B + 2C) \tan(dx+c)}{4ad\sqrt{a + a \cos(dx+c)}} \\ & + \frac{(2A - B + C) \sec(dx+c) \tan(dx+c)}{2ad\sqrt{a + a \cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (13A\sqrt{a} - 9B\sqrt{a} + 5C\sqrt{a}) \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{\sqrt{2} (13A\sqrt{a} - 9B\sqrt{a} + 5C\sqrt{a}) \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{(19A\sqrt{a}}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 56.40 Problem number 417

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(47A - 38B + 24C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{8a^{\frac{3}{2}}d} \\ & + \frac{(17A - 13B + 9C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{4a^{\frac{3}{2}}d} \\ & - \frac{(A - B + C) (\sec^2(dx+c)) \tan(dx+c)}{2d(a+a\cos(dx+c))^{\frac{3}{2}}} + \frac{(21A - 14B + 12C) \tan(dx+c)}{8ad\sqrt{a+a\cos(dx+c)}} \\ & - \frac{(13A - 12B + 6C) \sec(dx+c) \tan(dx+c)}{12ad\sqrt{a+a\cos(dx+c)}} \\ & + \frac{(5A - 3B + 3C) (\sec^2(dx+c)) \tan(dx+c)}{6ad\sqrt{a+a\cos(dx+c)}} \end{aligned}$$

command

`integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="g`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6\sqrt{2}\left(17A\sqrt{a}-13B\sqrt{a}+9C\sqrt{a}\right)\log\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{6\sqrt{2}\left(17A\sqrt{a}-13B\sqrt{a}+9C\sqrt{a}\right)\log\left(-\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)} - \frac{3\left(47A\right)}{8ad\sqrt{a+a\cos(dx+c)}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 56.41 Problem number 424

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(39A - 20B + 8C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}}{\sqrt{a+a\cos(dx+c)}}\right)}{4a^{\frac{5}{2}}d} \\ & - \frac{(219A - 115B + 43C) \operatorname{arctanh}\left(\frac{\sin(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cos(dx+c)}}\right)\sqrt{2}}{32a^{\frac{5}{2}}d} \\ & - \frac{(A - B + C) \sec(dx+c) \tan(dx+c)}{4d(a+a\cos(dx+c))^{\frac{5}{2}}} - \frac{(19A - 11B + 3C) \sec(dx+c) \tan(dx+c)}{16ad(a+a\cos(dx+c))^{\frac{3}{2}}} \\ & - \frac{(63A - 35B + 11C) \tan(dx+c)}{16a^2d\sqrt{a+a\cos(dx+c)}} + \frac{(31A - 15B + 7C) \sec(dx+c) \tan(dx+c)}{16a^2d\sqrt{a+a\cos(dx+c)}} \end{aligned}$$

command

```
integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (219 A \sqrt{a} - 115 B \sqrt{a} + 43 C \sqrt{a}) \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{\sqrt{2} (219 A \sqrt{a} - 115 B \sqrt{a} + 43 C \sqrt{a}) \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \dots$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 57 Test file number 95

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/95\_4.2.7-d\_trig-<sup>m</sup>-a+b-c\_cos-<sup>n</sup>-<sup>p</sup>

### 57.1 Problem number 51

$$\int \frac{1}{\sqrt{1 - \cos^2(x)}} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arctanh}(\cos(x)) \sin(x)}{\sqrt{2 - 2 \cos(2x)}}$$

command

```
integrate(1/(1-cos(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|\tan(\frac{1}{2} x)|)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2} x\right)^3 + \tan\left(\frac{1}{2} x\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-\cos(x)^2 + 1}} dx$$

## 57.2 Problem number 52

$$\int \frac{1}{\sqrt{-1 + \cos^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(x)) \sin(x)}{\sqrt{-(\sin^2(x))}}$$

command

```
integrate(1/(-1+cos(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i \log(-i \tan(\frac{1}{2}x))}{\operatorname{sgn}\left(-\tan(\frac{1}{2}x)^3 - \tan(\frac{1}{2}x)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{\cos(x)^2 - 1}} dx$$

## 58 Test file number 96

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/96\_4.2.8-a+b\_cos-^m-c+d\_trig-^n

### 58.1 Problem number 9

$$\int \frac{(c + d \sec(e + fx))^4}{a + b \cos(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3(4ac - bd) \operatorname{arctanh}(\sin(fx + e))}{2a^2 f} \\ & + \frac{d(2ac - bd)(2a^2 c^2 - 2abcd + b^2 d^2) \operatorname{arctanh}(\sin(fx + e))}{a^4 f} \\ & + \frac{2(ac - bd)^4 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan(\frac{fx+e}{2})}{\sqrt{a+b}}\right)}{a^4 f \sqrt{a-b} \sqrt{a+b}} \\ & + \frac{d^4 \tan(fx + e)}{af} + \frac{d^2(6a^2 c^2 - 4abcd + b^2 d^2) \tan(fx + e)}{a^3 f} \\ & + \frac{d^3(4ac - bd) \sec(fx + e) \tan(fx + e)}{2a^2 f} + \frac{d^4(\tan^3(fx + e))}{3af} \end{aligned}$$

command

```
integrate((c+d*sec(f*x+e))^4/(a+b*cos(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(8a^3c^3d-12a^2bc^2d^2+4a^3cd^3+8ab^2cd^3-a^2bd^4-2b^3d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^4} - \frac{3(8a^3c^3d-12a^2bc^2d^2+4a^3cd^3+8ab^2cd^3-a^2bd^4-2b^3d^4)}{a^4}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 58.2 Problem number 10

$$\int \frac{(c+d\sec(e+fx))^3}{a+b\cos(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3 \operatorname{arctanh}(\sin(fx+e))}{2af} + \frac{d(3a^2c^2-3abcd+b^2d^2) \operatorname{arctanh}(\sin(fx+e))}{a^3f} \\ & + \frac{2(ac-bd)^3 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx+e}{2}\right)}{\sqrt{a+b}}\right)}{a^3f\sqrt{a-b}\sqrt{a+b}} \\ & + \frac{d^2(3ac-bd)\tan(fx+e)}{a^2f} + \frac{d^3\sec(fx+e)\tan(fx+e)}{2af} \end{aligned}$$

command

```
integrate((c+d*sec(f*x+e))^3/(a+b*cos(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(6a^2c^2d-6abcd^2+a^2d^3+2b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^3} - \frac{(6a^2c^2d-6abcd^2+a^2d^3+2b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^3} - \frac{4(a^3c^3-3a^2bc^2d+3a^2cd^3-3ab^2cd^3-a^2bd^4-2b^3d^4)}{a^3}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 58.3 Problem number 11

$$\int \frac{(c + d \sec(e + fx))^2}{a + b \cos(e + fx)} dx$$

Optimal antiderivative

$$\frac{d(2ac - bd) \operatorname{arctanh}(\sin(fx + e))}{a^2 f} + \frac{2(ac - bd)^2 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a^2 f \sqrt{a-b} \sqrt{a+b}} + \frac{d^2 \tan(fx + e)}{af}$$

command

```
integrate((c+d*sec(f*x+e))^2/(a+b*cos(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)a} - \frac{(2acd - bd^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a^2} + \frac{(2acd - bd^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a^2} + \frac{2(a^2c^2 - 2abcd + b^2d^2)}{f} \left(\pi \left\lfloor \frac{fx+e}{2\pi} \right\rfloor\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 58.4 Problem number 12

$$\int \frac{c + d \sec(e + fx)}{a + b \cos(e + fx)} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctanh}(\sin(fx + e))}{af} + \frac{2(ac - bd) \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{af \sqrt{a-b} \sqrt{a+b}}$$

command

```
integrate((c+d*sec(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{d \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a} - \frac{d \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a} - \frac{2 \left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \operatorname{arctan}\left(-\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{a^2 - b^2}}\right) \right)}{\sqrt{a^2 - b^2} a}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 59 Test file number 97

Test folder name:

test\_cases/4\_Trig\_functions/4.2\_Cosine/97\_4.2.9\_trig^m-a+b\_cos^n+c\_cos^-2\_n^p

### 59.1 Problem number 6

$$\int \frac{\sin^4(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{x}{2c} + \frac{(b^2 - c(a + 2c))x}{c^3} - \frac{b \sin(x)}{c^2} + \frac{\cos(x) \sin(x)}{2c}$$

$$\frac{2 \arctan\left(\frac{\sqrt{b - 2c - \sqrt{-4ac + b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b + 2c - \sqrt{-4ac + b^2}}}\right) \left(b(b^2 - 2c(a + c)) + \frac{-b^4 - 2c^2(a + c)^2 + 2b^2c(2a + c)}{\sqrt{-4ac + b^2}}\right)}{c^3 \sqrt{b - 2c - \sqrt{-4ac + b^2}} \sqrt{b + 2c - \sqrt{-4ac + b^2}}}$$

$$\frac{2 \arctan\left(\frac{\sqrt{b - 2c + \sqrt{-4ac + b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b + 2c + \sqrt{-4ac + b^2}}}\right) \left(b^4 + 2c^2(a + c)^2 - 2b^2c(2a + c) + b^3 \sqrt{-4ac + b^2} - 2bc(a + c)\right) \sqrt{-4ac + b^2}}{c^3 \sqrt{-4ac + b^2} \sqrt{b - 2c + \sqrt{-4ac + b^2}} \sqrt{b + 2c + \sqrt{-4ac + b^2}}}$$

command

```
integrate(sin(x)^4/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 59.2 Problem number 8

$$\int \frac{\csc^2(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{\frac{\sin(x)}{2(a+b+c)(1-\cos(x))} + \frac{\sin(x)}{2(a-b+c)(1+\cos(x))}}{2bc \arctan\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{b^2-2c(a+c)}{b\sqrt{-4ac+b^2}}\right)} - \frac{2bc \arctan\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{-b^2+2c(a+c)}{b\sqrt{-4ac+b^2}}\right)}{(a-b+c)(a+b+c)\sqrt{b-2c-\sqrt{-4ac+b^2}}\sqrt{b+2c-\sqrt{-4ac+b^2}}}$$

command

```
integrate(csc(x)^2/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**59.3 Problem number 13**

$$\int \frac{\cos^4(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{\frac{x}{2c} + \frac{(-ac+b^2)x}{c^3} - \frac{b \sin(x)}{c^2} + \frac{\cos(x) \sin(x)}{2c}}{2 \arctan\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(b^3 - 2abc + \frac{-2a^2c^2+4ab^2c-b^4}{\sqrt{-4ac+b^2}}\right)} - \frac{2 \arctan\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(b^3 - 2abc + \frac{2a^2c^2-4ab^2c+b^4}{\sqrt{-4ac+b^2}}\right)}{c^3 \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}}$$

command

`integrate(cos(x)^4/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 59.4 Problem number 14

$$\int \frac{\cos^3(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{bx}{c^2} + \frac{\sin(x)}{c} \\ & + \frac{2 \arctan\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(b^2 - ac - \frac{b^3}{\sqrt{-4ac+b^2}} + \frac{3abc}{\sqrt{-4ac+b^2}}\right)}{c^2 \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} \\ & + \frac{2 \arctan\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(b^2 - ac + \frac{b^3}{\sqrt{-4ac+b^2}} - \frac{3abc}{\sqrt{-4ac+b^2}}\right)}{c^2 \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}} \end{aligned}$$

command

`integrate(cos(x)^3/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 59.5 Problem number 16

$$\int \frac{\cos(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left(1 - \frac{b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} + \frac{2 \arctan \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left(1 + \frac{b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

command

`integrate(cos(x)/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 59.6 Problem number 18

$$\int \frac{\sec(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(x))}{a} - \frac{2c \arctan \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left(1 + \frac{b}{\sqrt{-4ac+b^2}}\right)}{a \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} - \frac{2c \arctan \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left(1 - \frac{b}{\sqrt{-4ac+b^2}}\right)}{a \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

command

```
integrate(sec(x)/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 59.7 Problem number 19

$$\int \frac{\sec^2(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \operatorname{arctanh}(\sin(x))}{a^2} + \frac{2bc \operatorname{arctan}\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{-2ac+b^2}{b\sqrt{-4ac+b^2}}\right)}{a^2 \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} \\ & + \frac{2bc \operatorname{arctan}\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(1 + \frac{2ac-b^2}{b\sqrt{-4ac+b^2}}\right)}{a^2 \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}} + \frac{\tan(x)}{a} \end{aligned}$$

command

```
integrate(sec(x)^2/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 59.8 Problem number 20

$$\int \frac{\sec^3(x)}{a + b \cos(x) + c \cos^2(x)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}(\sin(x))}{2a} + \frac{(-ac + b^2) \operatorname{arctanh}(\sin(x))}{a^3} \\ & - \frac{2c \operatorname{arctan}\left(\frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}}\right) \left(b^3 - 3abc + (-ac + b^2) \sqrt{-4ac+b^2}\right)}{a^3 \sqrt{-4ac+b^2} \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} \\ & + \frac{2c \operatorname{arctan}\left(\frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}}\right) \left(b^3 - 3abc - (-ac + b^2) \sqrt{-4ac+b^2}\right)}{a^3 \sqrt{-4ac+b^2} \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}} \\ & - \frac{b \tan(x)}{a^2} + \frac{\sec(x) \tan(x)}{2a} \end{aligned}$$

command

```
integrate(sec(x)^3/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 60 Test file number 98

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/98\_4.3.0-a\_trg-^m-b\_tan-^n

## 60.1 Problem number 2

$$\int \tan^2(c + dx) dx$$

Optimal antiderivative

$$-x + \frac{\tan(dx + c)}{d}$$

command

```
integrate(tan(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

---


$$\pi - 4 dx \tan(dx) \tan(c) - \pi \operatorname{sgn}\left(2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 - 2 \tan(dx) - 2 \tan(c)\right) \tan(dx) \tan(c)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 60.2 Problem number 4

$$\int \tan^4(c + dx) dx$$

Optimal antiderivative

$$x - \frac{\tan(dx + c)}{d} + \frac{\tan^3(dx + c)}{3d}$$

command

```
integrate(tan(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 60.3 Problem number 6

$$\int \tan^6(c + dx) dx$$

Optimal antiderivative

$$-x + \frac{\tan(dx + c)}{d} - \frac{\tan^3(dx + c)}{3d} + \frac{\tan^5(dx + c)}{5d}$$

command

```
integrate(tan(d*x+c)^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 60.4 Problem number 8

$$\int \tan^8(c + dx) dx$$

Optimal antiderivative

$$x - \frac{\tan(dx + c)}{d} + \frac{\tan^3(dx + c)}{3d} - \frac{\tan^5(dx + c)}{5d} + \frac{\tan^7(dx + c)}{7d}$$

command

```
integrate(tan(d*x+c)^8,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 60.5 Problem number 17

$$\int (b \tan(c + dx))^{4/3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^{\frac{4}{3}} \arctan\left(\frac{(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{d} - \frac{b^{\frac{4}{3}} \arctan\left(-\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2d} \\ & - \frac{b^{\frac{4}{3}} \arctan\left(\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2d} \\ & + \frac{b^{\frac{4}{3}} \ln\left(b^{\frac{2}{3}} - b^{\frac{1}{3}}\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right) \sqrt{3}}{4d} \\ & - \frac{b^{\frac{4}{3}} \ln\left(b^{\frac{2}{3}} + b^{\frac{1}{3}}\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right) \sqrt{3}}{4d} + \frac{3b(b \tan(dx+c))^{\frac{1}{3}}}{d} \end{aligned}$$

command

`integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4}b \left( \frac{\sqrt{3} |b|^{\frac{1}{3}} \log\left(\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} |b|^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{d} - \frac{\sqrt{3} |b|^{\frac{1}{3}} \log\left(-\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} |b|^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \tan(dx + c))^{\frac{4}{3}} dx$$

## 60.6 Problem number 18

$$\int (b \tan(c + dx))^{2/3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^{\frac{2}{3}} \arctan\left(\frac{(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{d} + \frac{b^{\frac{2}{3}} \arctan\left(-\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2d} \\ & + \frac{b^{\frac{2}{3}} \arctan\left(\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2d} \\ & + \frac{b^{\frac{2}{3}} \ln\left(b^{\frac{2}{3}} - b^{\frac{1}{3}} \sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right) \sqrt{3}}{4d} \\ & - \frac{b^{\frac{2}{3}} \ln\left(b^{\frac{2}{3}} + b^{\frac{1}{3}} \sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right) \sqrt{3}}{4d} \end{aligned}$$

command

`integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{3} |b|^{\frac{5}{3}} \log\left(\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} |b|^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{4bd} \\ & + \frac{\sqrt{3} |b|^{\frac{5}{3}} \log\left(-\sqrt{3} (b \tan(dx+c))^{\frac{1}{3}} |b|^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{4bd} \\ & + \frac{|b|^{\frac{5}{3}} \arctan\left(\frac{\sqrt{3} |b|^{\frac{1}{3}} + 2(b \tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2bd} \\ & + \frac{|b|^{\frac{5}{3}} \arctan\left(-\frac{\sqrt{3} |b|^{\frac{1}{3}} - 2(b \tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2bd} + \frac{|b|^{\frac{5}{3}} \arctan\left(\frac{(b \tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{bd} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \tan(dx+c))^{\frac{2}{3}} dx$$

## 60.7 Problem number 19

$$\int \sqrt[3]{b \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{b^{\frac{1}{3}} \ln\left(b^{\frac{2}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right)}{2d} + \frac{b^{\frac{1}{3}} \ln\left(b^{\frac{4}{3}} - b^{\frac{2}{3}} (b \tan(dx+c))^{\frac{2}{3}} + (b \tan(dx+c))^{\frac{4}{3}}\right)}{4d} \\ & - \frac{b^{\frac{1}{3}} \arctan\left(\frac{(b^{\frac{2}{3}} - 2(b \tan(dx+c))^{\frac{2}{3}}) \sqrt{3}}{3b^{\frac{2}{3}}}\right) \sqrt{3}}{2d} \end{aligned}$$

command

`integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} b \left( \frac{2 \sqrt{3} |b|^{\frac{4}{3}} \arctan \left( \frac{\sqrt{3} (2 (b \tan(dx+c))^{\frac{2}{3}} - |b|^{\frac{2}{3}})}{3 |b|^{\frac{2}{3}}} \right)}{b^2 d} + \frac{|b|^{\frac{4}{3}} \log \left( (b \tan(dx+c))^{\frac{1}{3}} b \tan(dx+c) - (b \tan(dx+c))^{\frac{2}{3}} |b|^{\frac{2}{3}} \right)}{b^2 d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \tan(dx+c))^{\frac{1}{3}} dx$$

## 60.8 Problem number 20

$$\int \frac{1}{\sqrt[3]{b \tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\ln \left( b^{\frac{2}{3}} + (b \tan(dx+c))^{\frac{2}{3}} \right)}{2b^{\frac{1}{3}}d} - \frac{\ln \left( b^{\frac{4}{3}} - b^{\frac{2}{3}}(b \tan(dx+c))^{\frac{2}{3}} + (b \tan(dx+c))^{\frac{4}{3}} \right)}{4b^{\frac{1}{3}}d} - \frac{\arctan \left( \frac{(b^{\frac{2}{3}} - 2(b \tan(dx+c))^{\frac{2}{3}}) \sqrt{3}}{3b^{\frac{2}{3}}} \right) \sqrt{3}}{2b^{\frac{1}{3}}d}$$

command

`integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{3} |b|^{\frac{2}{3}} \arctan \left( \frac{\sqrt{3} (2 (b \tan(dx+c))^{\frac{2}{3}} - |b|^{\frac{2}{3}})}{3 |b|^{\frac{2}{3}}} \right)}{2bd} - \frac{|b|^{\frac{2}{3}} \log \left( (b \tan(dx+c))^{\frac{1}{3}} b \tan(dx+c) - (b \tan(dx+c))^{\frac{2}{3}} |b|^{\frac{2}{3}} + |b|^{\frac{4}{3}} \right)}{4bd} + \frac{|b|^{\frac{2}{3}} \log \left( (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}} \right)}{2bd}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan(dx+c))^{\frac{1}{3}}} dx$$

## 60.9 Problem number 22

$$\int \frac{1}{(b \tan(c + dx))^{4/3}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\arctan\left(\frac{(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}d} - \frac{\arctan\left(-\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2b^{\frac{4}{3}}d} \\ & - \frac{\arctan\left(\sqrt{3} + \frac{2(b \tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{2b^{\frac{4}{3}}d} \\ & - \frac{\ln\left(b^{\frac{2}{3}} - b^{\frac{1}{3}}\sqrt{3}(b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right)\sqrt{3}}{4b^{\frac{4}{3}}d} \\ & + \frac{\ln\left(b^{\frac{2}{3}} + b^{\frac{1}{3}}\sqrt{3}(b \tan(dx+c))^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}}\right)\sqrt{3}}{4b^{\frac{4}{3}}d} - \frac{3}{bd(b \tan(dx+c))^{\frac{1}{3}}} \end{aligned}$$

command

`integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4}b \left( \frac{\sqrt{3}|b|^{\frac{5}{3}} \log\left(\sqrt{3}(b \tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}} + (b \tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{b^4d} - \frac{\sqrt{3}|b|^{\frac{5}{3}} \log\left(-\sqrt{3}(b \tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}\right)}{b^4d} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan(dx+c))^{\frac{4}{3}}} dx$$

## 60.10 Problem number 30

$$\int (b \tan^3(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b^2 \cot(dx+c) \sqrt{b(\tan^3(dx+c))}}{d} \\
& + \frac{b^2 \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{2d \tan(dx+c)^{\frac{3}{2}}} \\
& + \frac{b^2 \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{2d \tan(dx+c)^{\frac{3}{2}}} \\
& - \frac{b^2 \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{4d \tan(dx+c)^{\frac{3}{2}}} \\
& + \frac{b^2 \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{4d \tan(dx+c)^{\frac{3}{2}}} \\
& + \frac{2b^2 \sqrt{b(\tan^3(dx+c))} \tan(dx+c)}{5d} - \frac{2b^2 \sqrt{b(\tan^3(dx+c))} (\tan^3(dx+c))}{9d} \\
& + \frac{2b^2 \sqrt{b(\tan^3(dx+c))} (\tan^5(dx+c))}{13d}
\end{aligned}$$

command

```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2340} \left( \frac{1170 \sqrt{2} b \sqrt{|b|} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan(dx+c)}\right)}{2 \sqrt{|b|}}\right)}{d} + \frac{1170 \sqrt{2} b \sqrt{|b|} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan(dx+c)}\right)}{2 \sqrt{|b|}}\right)}{d} \right) + c)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \left(b \tan(dx+c)^3\right)^{\frac{5}{2}} dx$$

## 60.11 Problem number 31

$$\int (b \tan^3(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b\sqrt{b(\tan^3(dx+c))}}{3d} \\ & + \frac{b \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{2d \tan(dx+c)^{\frac{3}{2}}} \\ & + \frac{b \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{2d \tan(dx+c)^{\frac{3}{2}}} \\ & + \frac{b \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{4d \tan(dx+c)^{\frac{3}{2}}} \\ & - \frac{b \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{b(\tan^3(dx+c))} \sqrt{2}}{4d \tan(dx+c)^{\frac{3}{2}}} \\ & + \frac{2b\sqrt{b(\tan^3(dx+c))} (\tan^2(dx+c))}{7d} \end{aligned}$$

command

`integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{84} b \left( \frac{42 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{|b|} + 2\sqrt{b \tan(dx+c)})}{2\sqrt{|b|}}\right)}{bd} + \frac{42 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{|b|} - 2\sqrt{b \tan(dx+c)})}{2\sqrt{|b|}}\right)}{bd} \right) + c)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \tan(dx+c)^3)^{\frac{3}{2}} dx$$

## 60.12 Problem number 32

$$\int \sqrt{b \tan^3(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cot(dx + c) \sqrt{b (\tan^3(dx + c))}}{d} \\ & - \frac{\arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{b (\tan^3(dx + c))} \sqrt{2}}{2d \tan(dx + c)^{\frac{3}{2}}} \\ & - \frac{\arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{b (\tan^3(dx + c))} \sqrt{2}}{2d \tan(dx + c)^{\frac{3}{2}}} \\ & + \frac{\ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{b (\tan^3(dx + c))} \sqrt{2}}{4d \tan(dx + c)^{\frac{3}{2}}} \\ & - \frac{\ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{b (\tan^3(dx + c))} \sqrt{2}}{4d \tan(dx + c)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( \frac{2\sqrt{2} \sqrt{|b|} \arctan\left(\frac{\sqrt{2}(\sqrt{2}\sqrt{|b|} + 2\sqrt{b \tan(dx+c)})}{2\sqrt{|b|}}\right)}{d} + \frac{2\sqrt{2} \sqrt{|b|} \arctan\left(-\frac{\sqrt{2}(\sqrt{2}\sqrt{|b|} - 2\sqrt{b \tan(dx+c)})}{2\sqrt{|b|}}\right)}{d} \right) + c)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{b \tan(dx + c)^3} dx$$

## 60.13 Problem number 33

$$\int \frac{1}{\sqrt{b \tan^3(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \tan(dx + c)}{d \sqrt{b (\tan^3(dx + c))}} - \frac{\arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{2d \sqrt{b (\tan^3(dx + c))}} \\ & - \frac{\arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{2d \sqrt{b (\tan^3(dx + c))}} \\ & - \frac{\ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{4d \sqrt{b (\tan^3(dx + c))}} \\ & + \frac{\ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{4d \sqrt{b (\tan^3(dx + c))}} \end{aligned}$$

command

```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} b^2 \left( \frac{2 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan(dx + c)}\right)}{2 \sqrt{|b|}}\right)}{b^4 \operatorname{dsgn}(\tan(dx + c))} + \frac{2 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan(dx + c)}\right)}{2 \sqrt{|b|}}\right)}{b^4 \operatorname{dsgn}(\tan(dx + c))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 60.14 Problem number 34

$$\int \frac{1}{(b \tan^3(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2}{3bd\sqrt{b(\tan^3(dx+c))}} - \frac{2(\cot^2(dx+c))}{7bd\sqrt{b(\tan^3(dx+c))}} \\ & + \frac{\arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)\sqrt{2}}{2bd\sqrt{b(\tan^3(dx+c))}} \\ & + \frac{\arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)\sqrt{2}}{2bd\sqrt{b(\tan^3(dx+c))}} \\ & - \frac{\ln\left(1 - \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)\sqrt{2}}{4bd\sqrt{b(\tan^3(dx+c))}} \\ & + \frac{\ln\left(1 + \sqrt{2}\left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right)\left(\tan^{\frac{3}{2}}(dx+c)\right)\sqrt{2}}{4bd\sqrt{b(\tan^3(dx+c))}} \end{aligned}$$

command

`integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{84} b^4 \left( \frac{42\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6 \operatorname{sgn}(\tan(dx+c))} + \frac{42\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6 \operatorname{sgn}(\tan(dx+c))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 60.15 Problem number 35

$$\int \frac{1}{(b \tan^3(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cot(dx + c)}{5b^2 d \sqrt{b(\tan^3(dx + c))}} + \frac{2(\cot^3(dx + c))}{9b^2 d \sqrt{b(\tan^3(dx + c))}} - \frac{2(\cot^5(dx + c))}{13b^2 d \sqrt{b(\tan^3(dx + c))}} \\ & + \frac{2 \tan(dx + c)}{b^2 d \sqrt{b(\tan^3(dx + c))}} + \frac{\arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{2b^2 d \sqrt{b(\tan^3(dx + c))}} \\ & + \frac{\arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{2b^2 d \sqrt{b(\tan^3(dx + c))}} \\ & + \frac{\ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{4b^2 d \sqrt{b(\tan^3(dx + c))}} \\ & - \frac{\ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \left(\tan^{\frac{3}{2}}(dx + c)\right) \sqrt{2}}{4b^2 d \sqrt{b(\tan^3(dx + c))}} \end{aligned}$$

command

```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2340} b^6 \left( \frac{1170 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan(dx + c)}\right)}{2 \sqrt{|b|}}\right)}{b^{10} \operatorname{dsgn}(\tan(dx + c))} + \frac{1170 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan(dx + c)}\right)}{2 \sqrt{|b|}}\right)}{b^{10} \operatorname{dsgn}(\tan(dx + c))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 60.16 Problem number 36

$$\int (b \tan^4(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^2 \cot(dx + c) \sqrt{(\tan^4(dx + c)) b}}{d} - b^2 x (\cot^2(dx + c)) \sqrt{(\tan^4(dx + c)) b} \\ & - \frac{b^2 \sqrt{(\tan^4(dx + c)) b} \tan(dx + c)}{3d} + \frac{b^2 \sqrt{(\tan^4(dx + c)) b} (\tan^3(dx + c))}{5d} \\ & - \frac{b^2 \sqrt{(\tan^4(dx + c)) b} (\tan^5(dx + c))}{7d} + \frac{b^2 \sqrt{(\tan^4(dx + c)) b} (\tan^7(dx + c))}{9d} \end{aligned}$$

command

```
integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 60.17 Problem number 37

$$\int (b \tan^4(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \cot(dx + c) \sqrt{(\tan^4(dx + c)) b}}{d} - bx (\cot^2(dx + c)) \sqrt{(\tan^4(dx + c)) b} \\ & - \frac{b \sqrt{(\tan^4(dx + c)) b} \tan(dx + c)}{3d} + \frac{b \sqrt{(\tan^4(dx + c)) b} (\tan^3(dx + c))}{5d} \end{aligned}$$

command

```
integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**60.18 Problem number 38**

$$\int \sqrt{b \tan^4(c + dx)} dx$$

Optimal antiderivative

$$\frac{\cot(dx + c) \sqrt{(\tan^4(dx + c)) b}}{d} - x(\cot^2(dx + c)) \sqrt{(\tan^4(dx + c)) b}$$

command

```
integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(\pi - 4 dx \tan(dx) \tan(c) - \pi \operatorname{sgn}(2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 - 2 \tan(dx) - 2 \tan(c)) \tan(dx) \tan(c)}{d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**61 Test file number 99**

Test folder name:

```
test_cases/4_Trig_functions/4.3_Tangent/99_4.3.10-c+d_x-^m-a+b_tan-^n
```

**61.1 Problem number 28**

$$\int \frac{1}{(c + dx)^2(a + ia \tan(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{1}{4a^2d(dx+c)} - \frac{if \operatorname{cosineIntegral}\left(\frac{4cf}{d} + 4fx\right) \cos\left(-4e + \frac{4cf}{d}\right)}{a^2d^2} \\
& - \frac{if \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \cos\left(-2e + \frac{2cf}{d}\right)}{a^2d^2} - \frac{\cos(2fx+2e)}{2a^2d(dx+c)} \\
& - \frac{\cos^2(2fx+2e)}{4a^2d(dx+c)} - \frac{f \cos\left(-2e + \frac{2cf}{d}\right) \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right)}{a^2d^2} \\
& - \frac{f \cos\left(-4e + \frac{4cf}{d}\right) \operatorname{sinIntegral}\left(\frac{4cf}{d} + 4fx\right)}{a^2d^2} \\
& + \frac{f \operatorname{cosineIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(-4e + \frac{4cf}{d}\right)}{a^2d^2} \\
& - \frac{if \operatorname{sinIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(-4e + \frac{4cf}{d}\right)}{a^2d^2} \\
& + \frac{f \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{a^2d^2} \\
& - \frac{if \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{a^2d^2} \\
& + \frac{i \sin(2fx+2e)}{2a^2d(dx+c)} + \frac{\sin^2(2fx+2e)}{4a^2d(dx+c)} + \frac{i \sin(4fx+4e)}{4a^2d(dx+c)}
\end{aligned}$$

command

```
integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**61.2 Problem number 33**

$$\int \frac{1}{(c+dx)^2(a+ia \tan(e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{1}{8a^3d(dx+c)} - \frac{3if \operatorname{sinIntegral}\left(\frac{6cf}{d} + 6fx\right) \sin\left(-6e + \frac{6cf}{d}\right)}{4a^3d^2} \\
& - \frac{3if \operatorname{sinIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(-4e + \frac{4cf}{d}\right)}{2a^3d^2} \\
& - \frac{3if \operatorname{cosineIntegral}\left(\frac{4cf}{d} + 4fx\right) \cos\left(-4e + \frac{4cf}{d}\right)}{2a^3d^2} \\
& - \frac{9 \cos(2fx + 2e)}{32a^3d(dx+c)} - \frac{3(\cos^2(2fx + 2e))}{8a^3d(dx+c)} - \frac{\cos^3(2fx + 2e)}{8a^3d(dx+c)} \\
& - \frac{3 \cos(6fx + 6e)}{32a^3d(dx+c)} - \frac{3f \cos\left(-2e + \frac{2cf}{d}\right) \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right)}{4a^3d^2} \\
& - \frac{3f \cos\left(-4e + \frac{4cf}{d}\right) \operatorname{sinIntegral}\left(\frac{4cf}{d} + 4fx\right)}{2a^3d^2} \\
& - \frac{3f \cos\left(-6e + \frac{6cf}{d}\right) \operatorname{sinIntegral}\left(\frac{6cf}{d} + 6fx\right)}{4a^3d^2} \\
& + \frac{3f \operatorname{cosineIntegral}\left(\frac{6cf}{d} + 6fx\right) \sin\left(-6e + \frac{6cf}{d}\right)}{4a^3d^2} \\
& - \frac{3if \operatorname{sinIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{4a^3d^2} \\
& + \frac{3f \operatorname{cosineIntegral}\left(\frac{4cf}{d} + 4fx\right) \sin\left(-4e + \frac{4cf}{d}\right)}{2a^3d^2} \\
& - \frac{3if \operatorname{cosineIntegral}\left(\frac{6cf}{d} + 6fx\right) \cos\left(-6e + \frac{6cf}{d}\right)}{4a^3d^2} \\
& + \frac{3f \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \sin\left(-2e + \frac{2cf}{d}\right)}{4a^3d^2} + \frac{3i \sin(4fx + 4e)}{8a^3d(dx+c)} \\
& + \frac{3i \sin(6fx + 6e)}{32a^3d(dx+c)} + \frac{3(\sin^2(2fx + 2e))}{8a^3d(dx+c)} + \frac{15i \sin(2fx + 2e)}{32a^3d(dx+c)} \\
& - \frac{i(\sin^3(2fx + 2e))}{8a^3d(dx+c)} - \frac{3if \operatorname{cosineIntegral}\left(\frac{2cf}{d} + 2fx\right) \cos\left(-2e + \frac{2cf}{d}\right)}{4a^3d^2}
\end{aligned}$$

command

```
integrate(1/(d*x+c)^2/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 62 Test file number 101

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/101\_4.3.1.2-d\_sec-<sup>m</sup>-a+b\_tan-<sup>n</sup>

### 62.1 Problem number 467

$$\int \sec^2(c + dx)(a + ia \tan(c + dx))^n dx$$

Optimal antiderivative

$$\frac{i(a + ia \tan(dx + c))^{1+n}}{ad(1+n)}$$

command

```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i \left( \frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 2i a \tan(\frac{1}{2} dx + \frac{1}{2} c) - a}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1} \right)^{n+1}}{ad(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (i a \tan(dx + c) + a)^n \sec(dx + c)^2 dx$$

### 62.2 Problem number 515

$$\int \cos^3(c + dx)(a + b \tan(c + dx)) dx$$

Optimal antiderivative

$$-\frac{b(\cos^3(dx + c))}{3d} + \frac{a \sin(dx + c)}{d} - \frac{a(\sin^3(dx + c))}{3d}$$

command

```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 62.3 Problem number 522

$$\int \cos^4(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{(3a^2 + b^2)x}{8} - \frac{(\cos^4(dx + c))(b - a \tan(dx + c))(a + b \tan(dx + c))}{4d} - \frac{(\cos^2(dx + c))(2ab - (3a^2 + b^2)\tan(dx + c))}{8d}$$

command

```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 62.4 Problem number 528

$$\int \cos^3(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$-\frac{ab(\cos^3(dx + c))}{6d} + \frac{(2a^2 + b^2)\sin(dx + c)}{2d} - \frac{(2a^2 + b^2)(\sin^3(dx + c))}{6d} - \frac{b(\cos^3(dx + c))(a + b \tan(dx + c))}{2d}$$

command

```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 62.5 Problem number 529

$$\int \cos^5(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3ab(\cos^5(dx + c))}{20d} + \frac{(4a^2 + b^2) \sin(dx + c)}{4d} - \frac{(4a^2 + b^2) (\sin^3(dx + c))}{6d} \\ & + \frac{(4a^2 + b^2) (\sin^5(dx + c))}{20d} - \frac{b(\cos^5(dx + c)) (a + b \tan(dx + c))}{4d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 62.6 Problem number 530

$$\int \cos^7(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5ab(\cos^7(dx + c))}{42d} + \frac{(6a^2 + b^2) \sin(dx + c)}{6d} \\ & - \frac{(6a^2 + b^2) (\sin^3(dx + c))}{6d} + \frac{(6a^2 + b^2) (\sin^5(dx + c))}{10d} \\ & - \frac{(6a^2 + b^2) (\sin^7(dx + c))}{42d} - \frac{b(\cos^7(dx + c)) (a + b \tan(dx + c))}{6d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 62.7 Problem number 536

$$\int \cos^4(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{3a(a^2 + b^2)x}{8} - \frac{3a(\cos^2(dx + c))(b - a \tan(dx + c))(a + b \tan(dx + c))}{8d} + \frac{(\cos^3(dx + c)) \sin(dx + c)(a + b \tan(dx + c))^3}{4d}$$

command

```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 62.8 Problem number 540

$$\int \cos(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{3ab^2 \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{\cos(dx + c)(b - a \tan(dx + c))(a + b \tan(dx + c))^2}{d} - \frac{b \sec(dx + c)(2a^2 - 2b^2 + ab \tan(dx + c))}{d}$$

command

```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**62.9 Problem number 541**

$$\int \cos^3(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\frac{2(a^2 + b^2) \cos(dx + c) (b - a \tan(dx + c))}{3d} - \frac{(\cos^3(dx + c)) (b - a \tan(dx + c)) (a + b \tan(dx + c))^2}{3d}$$

command

```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**62.10 Problem number 542**

$$\int \cos^5(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{2(4a^2 + b^2) \cos(dx + c) (b - a \tan(dx + c))}{15d} \\ & - \frac{(\cos^3(dx + c)) (b - 4a \tan(dx + c)) (a + b \tan(dx + c))^2}{15d} \\ & + \frac{(\cos^4(dx + c)) \sin(dx + c) (a + b \tan(dx + c))^3}{5d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 62.11 Problem number 543

$$\int \cos^7(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8a(2a^2 + b^2) \sin(dx + c)}{35d} - \frac{3(\cos^5(dx + c))(b - 2a \tan(dx + c))(a + b \tan(dx + c))^2}{35d} \\ & + \frac{(\cos^6(dx + c)) \sin(dx + c)(a + b \tan(dx + c))^3}{7d} \\ & - \frac{2(\cos^3(dx + c))(b(6a^2 + b^2) - a(4a^2 - b^2) \tan(dx + c))}{35d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 62.12 Problem number 673

$$\int (e \cos(c + dx))^{7/2} \sqrt{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{12ia(e \cos(dx + c))^{7/2} (\sec^2(dx + c))}{35d \sqrt{a + ia \tan(dx + c)}} + \frac{32ia(e \cos(dx + c))^{7/2} (\sec^4(dx + c))}{35d \sqrt{a + ia \tan(dx + c)}} \\ & - \frac{2i(e \cos(dx + c))^{7/2} \sqrt{a + ia \tan(dx + c)}}{7d} \\ & - \frac{16i(e \cos(dx + c))^{7/2} (\sec^2(dx + c)) \sqrt{a + ia \tan(dx + c)}}{35d} \end{aligned}$$

command

```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-5i \sqrt{a} e^{\left(\frac{7}{2}i dx + \frac{7}{2}i c\right)} - 35i \sqrt{a} e^{\left(\frac{3}{2}i dx + \frac{3}{2}i c\right)} + 105i \sqrt{a} e^{\left(-\frac{1}{2}i dx - \frac{1}{2}i c\right)} + 7i \sqrt{a} e^{\left(-\frac{5}{2}i dx - \frac{5}{2}i c\right)}\right) e^{\frac{7}{2}}}{140 d}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 62.13 Problem number 674

$$\int (e \cos(c + dx))^{5/2} \sqrt{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{8ia(e \cos(dx + c))^{\frac{5}{2}} (\sec^2(dx + c))}{15d \sqrt{a + ia \tan(dx + c)}} - \frac{2i(e \cos(dx + c))^{\frac{5}{2}} \sqrt{a + ia \tan(dx + c)}}{5d}$$

$$- \frac{16i(e \cos(dx + c))^{\frac{5}{2}} (\sec^2(dx + c)) \sqrt{a + ia \tan(dx + c)}}{15d}$$

command

```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-3i \sqrt{a} e^{\left(\frac{5}{2}i dx + \frac{5}{2}ic\right)} - 30i \sqrt{a} e^{\left(\frac{1}{2}i dx + \frac{1}{2}ic\right)} + 5i \sqrt{a} e^{\left(-\frac{3}{2}i dx - \frac{3}{2}ic\right)}\right) e^{\frac{5}{2}}}{30d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 62.14 Problem number 675

$$\int (e \cos(c + dx))^{3/2} \sqrt{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\frac{4iae \sec(dx + c) \sqrt{e \cos(dx + c)}}{3d \sqrt{a + ia \tan(dx + c)}} - \frac{2i(e \cos(dx + c))^{\frac{3}{2}} \sqrt{a + ia \tan(dx + c)}}{3d}$$

command

```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-i \sqrt{a} e^{\left(\frac{3}{2}i dx + \frac{3}{2}ic\right)} + 3i \sqrt{a} e^{\left(-\frac{1}{2}i dx - \frac{1}{2}ic\right)}\right) e^{\frac{3}{2}}}{3d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 62.15 Problem number 676

$$\int \sqrt{e \cos(c + dx)} \sqrt{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$-\frac{2i \sqrt{e \cos(dx + c)} \sqrt{a + ia \tan(dx + c)}}{d}$$

command

```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2i \sqrt{a} e^{(\frac{1}{2}i dx + \frac{1}{2}i c + \frac{1}{2})}}{d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 63 Test file number 102

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/102\_4.3.1.3-d\_sin-<sup>m</sup>-a+b\_tan-<sup>n</sup>

### 63.1 Problem number 11

$$\int \sin^5(c + dx)(a + b \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{a \cos(dx + c)}{d} + \frac{2a(\cos^3(dx + c))}{3d} - \frac{a(\cos^5(dx + c))}{5d} - \frac{b \sin(dx + c)}{d} - \frac{b(\sin^3(dx + c))}{3d} - \frac{b(\sin^5(dx + c))}{5d}$$

command

```
integrate(sin(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 63.2 Problem number 22

$$\int \sin^4(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(a^2 - 5b^2)x}{8} - \frac{2ab \ln(\cos(dx + c))}{d} + \frac{b^2 \tan(dx + c)}{d} \\ & + \frac{(\cos^2(dx + c))(7b - 5a \tan(dx + c))(a + b \tan(dx + c))}{8d} \\ & + \frac{(\cos^3(dx + c)) \sin(dx + c)(a + b \tan(dx + c))^2}{4d} \end{aligned}$$

command

```
integrate(sin(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 63.3 Problem number 23

$$\int \sin^3(c + dx)(a + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ab \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{a^2 \cos(dx + c)}{d} + \frac{2b^2 \cos(dx + c)}{d} + \frac{a^2(\cos^3(dx + c))}{3d} \\ & - \frac{b^2(\cos^3(dx + c))}{3d} + \frac{b^2 \sec(dx + c)}{d} - \frac{2ab \sin(dx + c)}{d} - \frac{2ab(\sin^3(dx + c))}{3d} \end{aligned}$$

command

```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 63.4 Problem number 32

$$\int \sin^3(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2b \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{5b^3 \operatorname{arctanh}(\sin(dx + c))}{2d} - \frac{a^3 \cos(dx + c)}{d} + \frac{6ab^2 \cos(dx + c)}{d} \\ & + \frac{a^3(\cos^3(dx + c))}{3d} - \frac{ab^2(\cos^3(dx + c))}{d} + \frac{3ab^2 \sec(dx + c)}{d} - \frac{3a^2b \sin(dx + c)}{d} \\ & + \frac{5b^3 \sin(dx + c)}{2d} - \frac{a^2b(\sin^3(dx + c))}{d} + \frac{5b^3(\sin^3(dx + c))}{6d} + \frac{b^3(\sin^3(dx + c))(\tan^2(dx + c))}{2d} \end{aligned}$$

command

```
integrate(sin(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 63.5 Problem number 34

$$\int \sin(c + dx)(a + b \tan(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2b \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{3b^3 \operatorname{arctanh}(\sin(dx + c))}{2d} - \frac{a^3 \cos(dx + c)}{d} + \frac{3ab^2 \cos(dx + c)}{d} \\ & + \frac{3ab^2 \sec(dx + c)}{d} - \frac{3a^2b \sin(dx + c)}{d} + \frac{3b^3 \sin(dx + c)}{2d} + \frac{b^3 \sin(dx + c)(\tan^2(dx + c))}{2d} \end{aligned}$$

command

```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 63.6 Problem number 43

$$\int \sin(c + dx)(a + b \tan(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4a^3b \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{6ab^3 \operatorname{arctanh}(\sin(dx + c))}{d} - \frac{a^4 \cos(dx + c)}{d} \\ & + \frac{6a^2b^2 \cos(dx + c)}{d} - \frac{b^4 \cos(dx + c)}{d} + \frac{6a^2b^2 \sec(dx + c)}{d} - \frac{2b^4 \sec(dx + c)}{d} \\ & + \frac{b^4(\sec^3(dx + c))}{3d} - \frac{4a^3b \sin(dx + c)}{d} + \frac{6ab^3 \sin(dx + c)}{d} + \frac{2ab^3 \sin(dx + c)(\tan^2(dx + c))}{d} \end{aligned}$$

command

```
integrate(sin(d*x+c)*(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 64 Test file number 103

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/103\_4.3.2.1-a+b\_tan<sup>m</sup>-c+d\_tan<sup>n</sup>

### 64.1 Problem number 214

$$\int \frac{1}{\sqrt{\tan(c + dx)} \sqrt{a + ia \tan(c + dx)}} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a} \left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a + ia \tan(dx+c)}}\right)}{d\sqrt{a}} + \frac{\sqrt{\tan(dx+c)}}{d\sqrt{a + ia \tan(dx+c)}}$$

command

```
integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & \left( i a \sqrt{|a|} - |a|^{\frac{3}{2}} \right) \log \left( \frac{-i \left( \frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2} \right) \sqrt{-2(i a \tan(dx+c) + a)a + 2a^2}}{\frac{2 \frac{1}{2} i \left( \frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2} \right) \sqrt{-2(i a \tan(dx+c) + a)a + 2a^2}}}{4 \sqrt{2} \left( a \sqrt{|a|} + i |a|^{\frac{3}{2}} \right)} \right) \\
 & \frac{4 a^2 d}{4 \sqrt{2} \left( a \sqrt{|a|} + i |a|^{\frac{3}{2}} \right)} \\
 & \left( \frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2} \right) \frac{\sqrt{-2(i a \tan(dx+c) + a)a + 2a^2}}{\sqrt{\frac{(i a \tan(dx+c) + a)^2 - 2a^2}{a^2}}}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{i a \tan(dx+c) + a} \sqrt{\tan(dx+c)}} dx$$

## 64.2 Problem number 215

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a} \left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{d\sqrt{a}} + \frac{1}{d\sqrt{\tan(dx+c)} \sqrt{a+ia \tan(dx+c)}} - \frac{3\sqrt{a+ia \tan(dx+c)}}{ad\sqrt{\tan(dx+c)}}$$

command

`integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & \left( a\sqrt{|a|} + i|a|^{\frac{3}{2}} \right) \log \left( \frac{-i \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2}}{\frac{2 \frac{1}{2} i \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2}} \right) \\
 & \frac{4a^2d}{\sqrt{-2(ia \tan(dx+c) + a)a + 2a^2} \sqrt{ia \tan(dx+c) + a} \left( \frac{\tan(dx+c)}{\sqrt{\frac{(ia \tan(dx+c) + a)^2 - 2(ia \tan(dx+c) + a)a - a^2}{a^2}}} \right)} \\
 & \frac{a^3d \tan(dx+c)}{4\sqrt{2} \left(-ia\sqrt{|a|} + |a|^{\frac{3}{2}}\right)} \\
 & \left( \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2} \right) \left( \frac{\tan(dx+c)}{\sqrt{\frac{(ia \tan(dx+c) + a)^2 - 2(ia \tan(dx+c) + a)a - a^2}{a^2}}} \right)
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{ia \tan(dx+c) + a} \tan(dx+c)^{\frac{3}{2}}} dx$$

## 64.3 Problem number 216

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx) \sqrt{a+ia \tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\left(-\frac{1}{2} + \frac{i}{2}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a} \left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{d\sqrt{a}} + \frac{7i\sqrt{a+ia \tan(dx+c)}}{3ad\sqrt{\tan(dx+c)}} \\ + \frac{1}{d\sqrt{a+ia \tan(dx+c)} \tan(dx+c)^{\frac{3}{2}}} - \frac{5\sqrt{a+ia \tan(dx+c)}}{3ad \tan(dx+c)^{\frac{3}{2}}}$$

command

`integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-2(i a \tan(dx+c) + a)a + 2 a^2} \sqrt{i a \tan(dx+c) + a} \left( \frac{\tan(dx+c)}{\sqrt{\frac{(i a \tan(dx+c) + a)^2 - 2(i a \tan(dx+c) + a)a + 2 a^2}{a^2}}} \right)}{3 a^2 \tan(dx+c)^2}$$


---


$$\left( -i a \sqrt{|a|} + |a|^{\frac{3}{2}} \right) \log \left( \frac{-i \frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(i a \tan(dx+c) + a)a + 2 a^2}}{2 \frac{1}{2} i \frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(i a \tan(dx+c) + a)a + 2 a^2}} \right)$$


---


$$+ \frac{4 a^2 d}{4 \sqrt{2} \left( a \sqrt{|a|} + i |a|^{\frac{3}{2}} \right)} \left( \frac{\sqrt{-2(i a \tan(dx+c) + a)a + 2 a^2} \left( \frac{\tan(dx+c)}{\sqrt{\frac{(i a \tan(dx+c) + a)^2 - 2(i a \tan(dx+c) + a)a + 2 a^2}{a^2}}} \right)}{\frac{\sqrt{2} \sqrt{i a \tan(dx+c) + a} \left( -\frac{i|a|}{a} + 1 \right) |a|^{\frac{3}{2}}}{a^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{i a \tan(dx+c) + a} \tan(dx+c)^{\frac{5}{2}}} dx$$

## 64.4 Problem number 217

$$\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{2} - \frac{i}{2}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\sqrt{a}} + \frac{61\sqrt{a+ia\tan(dx+c)}}{15ad\sqrt{\tan(dx+c)}} \\ & + \frac{1}{d\sqrt{a+ia\tan(dx+c)}\tan(dx+c)^{\frac{5}{2}}} - \frac{7\sqrt{a+ia\tan(dx+c)}}{5ad\tan(dx+c)^{\frac{5}{2}}} + \frac{23i\sqrt{a+ia\tan(dx+c)}}{15ad\tan(dx+c)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(1/(a+I*a*tan(d*x+c))^(1/2)/tan(d*x+c)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( a\sqrt{|a|} + i|a|^{\frac{3}{2}} \right) \log \left( \frac{i \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2}}{\sqrt{(ia \tan(dx+c) + a)^2 - 2a^2}} \right) - \frac{2 \left( -\frac{1}{2}i \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2}}{\sqrt{(ia \tan(dx+c) + a)^2 - 2a^2}} \right)}{4a^2d}$$

$$\sqrt{-2(ia \tan(dx+c) + a)a + 2a^2} \left( (ia \tan(dx+c) + a) \left( \frac{23(ia \tan(dx+c) + a)}{a^3d} - \frac{50}{a^2d} \right) + \frac{30}{ad} \right) \sqrt{ia \tan(dx+c) + a}$$

$$\frac{15a^2 \tan(dx+c)^3}{4\sqrt{2} \left( ia\sqrt{|a|} - |a|^{\frac{3}{2}} \right)}$$

$$\left( \frac{\sqrt{2} \sqrt{ia \tan(dx+c) + a} \left(-\frac{i|a|}{a} + 1\right) |a|^{\frac{3}{2}}}{a^2} \sqrt{-2(ia \tan(dx+c) + a)a + 2a^2} \right) \frac{\tan(dx+c)}{\sqrt{(ia \tan(dx+c) + a)^2 - 2a^2}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{ia \tan(dx+c) + a} \tan(dx+c)^{\frac{7}{2}}} dx$$



### 64.5 Problem number 222

$$\int \frac{1}{\sqrt{\tan(c+dx)} (a+ia \tan(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a} \left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{a^{\frac{3}{2}}d} + \frac{7\left(\sqrt{\tan(dx+c)}\right)}{6ad\sqrt{a+ia \tan(dx+c)}} + \frac{\sqrt{\tan(dx+c)}}{3d(a+ia \tan(dx+c))^{\frac{3}{2}}}$$

command

```
integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c) + a)^{\frac{3}{2}} \sqrt{\tan(dx+c)}} dx$$

### 64.6 Problem number 223

$$\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a} \left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{a^{\frac{3}{2}}d} + \frac{11}{6ad\sqrt{\tan(dx+c)} \sqrt{a+ia \tan(dx+c)}} - \frac{25\sqrt{a+ia \tan(dx+c)}}{6a^2d\sqrt{\tan(dx+c)}} + \frac{1}{3d\sqrt{\tan(dx+c)} (a+ia \tan(dx+c))^{\frac{3}{2}}}$$

command

```
integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c) + a)^{\frac{3}{2}} \tan(dx+c)^{\frac{3}{2}}} dx$$

## 64.7 Problem number 224

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\left(-\frac{1}{4} + \frac{i}{4}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{a^{\frac{3}{2}}d} + \frac{13i\sqrt{a+ia \tan(dx+c)}}{2a^2d\sqrt{\tan(dx+c)}} + \frac{5}{2ad\sqrt{a+ia \tan(dx+c)}\tan(dx+c)^{\frac{3}{2}}} - \frac{7\sqrt{a+ia \tan(dx+c)}}{2a^2d\tan(dx+c)^{\frac{3}{2}}} + \frac{1}{3d\tan(dx+c)^{\frac{3}{2}}(a+ia \tan(dx+c))^{\frac{3}{2}}}$$

command

```
integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^{\frac{5}{2}}} dx$$

## 64.8 Problem number 230

$$\int \frac{1}{\sqrt{\tan(c+dx)}(a+ia \tan(c+dx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{a^{\frac{5}{2}}d} + \frac{67\left(\sqrt{\tan(dx+c)}\right)}{60a^2d\sqrt{a+ia \tan(dx+c)}} + \frac{\sqrt{\tan(dx+c)}}{5d(a+ia \tan(dx+c))^{\frac{5}{2}}} + \frac{13\left(\sqrt{\tan(dx+c)}\right)}{30ad(a+ia \tan(dx+c))^{\frac{3}{2}}}$$

command

`integrate(1/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(i a \tan(dx + c) + a)^{\frac{5}{2}} \sqrt{\tan(dx + c)}} dx$$

### 64.9 Problem number 231

$$\int \frac{1}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{1}{8} + \frac{i}{8}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia\tan(dx+c)}}\right)}{a^{\frac{5}{2}}d} + \frac{151}{60a^2d\sqrt{\tan(dx+c)}\sqrt{a+ia\tan(dx+c)}} \\ & - \frac{317\sqrt{a+ia\tan(dx+c)}}{60a^3d\sqrt{\tan(dx+c)}} + \frac{1}{5d\sqrt{\tan(dx+c)}(a+ia\tan(dx+c))^{\frac{5}{2}}} \\ & + \frac{17}{30ad\sqrt{\tan(dx+c)}(a+ia\tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(i a \tan(dx + c) + a)^{\frac{5}{2}} \tan(dx + c)^{\frac{3}{2}}} dx$$

## 64.10 Problem number 232

$$\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))^{\frac{5}{2}}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{8} + \frac{i}{8}\right) \operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\left(\sqrt{\tan(dx+c)}\right)}{\sqrt{a+ia \tan(dx+c)}}\right)}{a^{\frac{5}{2}}d} + \frac{707i\sqrt{a+ia \tan(dx+c)}}{60a^3d\sqrt{\tan(dx+c)}} \\ & + \frac{89}{20a^2d\sqrt{a+ia \tan(dx+c)} \tan(dx+c)^{\frac{3}{2}}} - \frac{361\sqrt{a+ia \tan(dx+c)}}{60a^3d \tan(dx+c)^{\frac{3}{2}}} \\ & + \frac{1}{5d \tan(dx+c)^{\frac{3}{2}}(a+ia \tan(dx+c))^{\frac{5}{2}}} + \frac{7}{10ad \tan(dx+c)^{\frac{3}{2}}(a+ia \tan(dx+c))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c)+a)^{\frac{5}{2}} \tan(dx+c)^{\frac{5}{2}}} dx$$

## 64.11 Problem number 233

$$\int \frac{\tan^{\frac{10}{3}}(c+dx)}{a+ia \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7 \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} - \frac{7 \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\
& - \frac{7 \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} - \frac{5i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{6ad} \\
& + \frac{5i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{12ad} - \frac{5i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{6ad} \\
& + \frac{7 \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\
& - \frac{7 \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\
& + \frac{7\left(\tan^{\frac{1}{3}}(dx+c)\right)}{2ad} - \frac{5i\left(\tan^{\frac{4}{3}}(dx+c)\right)}{4ad} - \frac{\tan^{\frac{7}{3}}(dx+c)}{2d(a+ia \tan(dx+c))}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{17\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24ad} - \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8ad} \\
& + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{8ad} \\
& + \frac{17i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{24ad} \\
& - \frac{17i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{12ad} - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{4ad} \\
& - \frac{i \tan(dx+c)^{\frac{1}{3}}}{2ad(\tan(dx+c) - i)} - \frac{3\left(i a^3 d^3 \tan(dx+c)^{\frac{4}{3}} - 4 a^3 d^3 \tan(dx+c)^{\frac{1}{3}}\right)}{4 a^4 d^4}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{10}{3}}}{i a \tan(dx+c) + a} dx$$

## 64.12 Problem number 234

$$\int \frac{\tan^{\frac{8}{3}}(c+dx)}{a+ia \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5 \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} + \frac{5 \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\ & + \frac{5 \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} + \frac{2i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{3ad} \\ & - \frac{i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{3ad} - \frac{2i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{3ad} \\ & + \frac{5 \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{5 \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{2i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{ad} - \frac{\tan^{\frac{5}{3}}(dx+c)}{2d(a+ia \tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{13\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24ad} - \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8ad} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{8ad} \\ & - \frac{13i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{24ad} + \frac{13i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{12ad} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{4ad} - \frac{3i \tan(dx+c)^{\frac{2}{3}}}{2ad} - \frac{\tan(dx+c)^{\frac{2}{3}}}{2ad(\tan(dx+c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{8}{3}}}{ia \tan(dx+c) + a} dx$$

## 64.13 Problem number 235

$$\int \frac{\tan^{\frac{4}{3}}(c + dx)}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx + c)\right)\right)}{12ad} + \frac{\arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx + c)\right)\right)}{12ad} \\ & + \frac{\arctan\left(\tan^{\frac{1}{3}}(dx + c)\right)}{6ad} + \frac{i \ln\left(1 + \tan^{\frac{2}{3}}(dx + c)\right)}{3ad} \\ & - \frac{i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx + c)\right) + \tan^{\frac{4}{3}}(dx + c)\right)}{6ad} + \frac{i \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx + c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{3ad} \\ & - \frac{\ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx + c)\right) + \tan^{\frac{2}{3}}(dx + c)\right)\sqrt{3}}{24ad} \\ & + \frac{\ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx + c)\right) + \tan^{\frac{2}{3}}(dx + c)\right)\sqrt{3}}{24ad} - \frac{\tan^{\frac{1}{3}}(dx + c)}{2d(a + ia \tan(dx + c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{5\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24ad} + \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8ad} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{8ad} \\ & - \frac{5i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{24ad} + \frac{5i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{12ad} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{4ad} + \frac{i \tan(dx+c)^{\frac{1}{3}}}{2ad(\tan(dx+c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx + c)^{\frac{4}{3}}}{ia \tan(dx + c) + a} dx$$

## 64.14 Problem number 236

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{a+ia \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan\left(-\sqrt{3}+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} + \frac{\arctan\left(\sqrt{3}+2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\ & + \frac{\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} - \frac{i \ln\left(1+\tan^{\frac{2}{3}}(dx+c)\right)}{6ad} \\ & + \frac{i \ln\left(1-\left(\tan^{\frac{2}{3}}(dx+c)\right)+\tan^{\frac{4}{3}}(dx+c)\right)}{12ad} + \frac{i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{6ad} \\ & + \frac{\ln\left(1-\sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{\ln\left(1+\sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right)+\tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} + \frac{i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{2d(a+ia \tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24 ad} + \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8 ad} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}}+i \tan(dx+c)^{\frac{1}{3}}-1\right)}{8 ad} + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}}-i \tan(dx+c)^{\frac{1}{3}}-1\right)}{24 ad} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12 ad} - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4 ad} + \frac{\tan(dx+c)^{\frac{2}{3}}}{2 ad(\tan(dx+c)-i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{2}{3}}}{ia \tan(dx+c)+a} dx$$



## 64.15 Problem number 237

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}(a+ia\tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} - \frac{i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\ & - \frac{i \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} + \frac{\ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{3ad} \\ & - \frac{\ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{6ad} - \frac{\arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{3ad} \\ & - \frac{i \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & + \frac{i \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} + \frac{\tan^{\frac{2}{3}}(dx+c)}{2d(a+ia\tan(dx+c))} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{5i\sqrt{3}\log\left(\frac{-\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}-i}}\right)}{24ad} + \frac{i\sqrt{3}\log\left(\frac{-\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}+i}}\right)}{8ad} \\ & - \frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i\tan(dx+c)^{\frac{1}{3}}-1\right)}{8ad} - \frac{5\log\left(\tan(dx+c)^{\frac{2}{3}}-i\tan(dx+c)^{\frac{1}{3}}-1\right)}{24ad} \\ & + \frac{5\log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{12ad} + \frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{4ad} - \frac{i\tan(dx+c)^{\frac{2}{3}}}{2ad(\tan(dx+c)-i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia\tan(dx+c)+a)\tan(dx+c)^{\frac{1}{3}}} dx$$

## 64.16 Problem number 238

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia \tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{5i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} - \frac{5i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\ & - \frac{5i \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} + \frac{2 \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{3ad} \\ & - \frac{\ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{3ad} + \frac{2 \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{3ad} \\ & + \frac{5i \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{5i \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{2}{ad \tan^{\frac{2}{3}}(dx+c)} + \frac{1}{2d \tan^{\frac{2}{3}}(dx+c)(a+ia \tan(dx+c))} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(5/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{13i\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24ad} - \frac{i\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8ad} \\ & - \frac{\log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{8ad} \\ & - \frac{13 \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{24ad} + \frac{13 \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{12ad} \\ & + \frac{\log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{4ad} - \frac{3}{2ad \tan^{\frac{2}{3}}(dx+c)} - \frac{\tan(dx+c)^{\frac{1}{3}}}{2ad(\tan(dx+c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c) + a) \tan^{\frac{5}{3}}(dx+c)} dx$$

## 64.17 Problem number 239

$$\int \frac{1}{\tan^{\frac{7}{3}}(c+dx)(a+ia \tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} + \frac{7i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{12ad} \\ & + \frac{7i \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{6ad} - \frac{5 \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{6ad} \\ & + \frac{5 \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{12ad} + \frac{5 \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{6ad} \\ & + \frac{7i \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} \\ & - \frac{7i \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{24ad} - \frac{5}{4ad \tan(dx+c)^{\frac{4}{3}}} \\ & + \frac{7i}{2ad \tan(dx+c)^{\frac{1}{3}}} + \frac{1}{2d \tan(dx+c)^{\frac{4}{3}}(a+ia \tan(dx+c))} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{17i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{24 ad} - \frac{i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{8 ad} \\ & + \frac{\log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{8 ad} \\ & + \frac{17 \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{24 ad} - \frac{17 \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{12 ad} \\ & - \frac{\log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{4 ad} - \frac{3(-4i \tan(dx+c) + 1)}{4 ad \tan(dx+c)^{\frac{4}{3}}} + \frac{i \tan(dx+c)^{\frac{2}{3}}}{2 ad(\tan(dx+c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ia \tan(dx+c) + a) \tan(dx+c)^{\frac{7}{3}}} dx$$

## 64.18 Problem number 240

$$\int \frac{\tan^{\frac{14}{3}}(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{121 \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} + \frac{121 \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & + \frac{121 \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} + \frac{14i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & - \frac{7i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{9a^2d} \\ & - \frac{14i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{9a^2d} \\ & + \frac{121 \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & - \frac{121 \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} - \frac{14i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{3a^2d} \\ & - \frac{121\left(\tan^{\frac{5}{3}}(dx+c)\right)}{60a^2d} + \frac{7i\left(\tan^{\frac{8}{3}}(dx+c)\right)}{6a^2d(1+i \tan(dx+c))} - \frac{\tan^{\frac{11}{3}}(dx+c)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(14/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{233 \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144 a^2 d} + \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16 a^2 d} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{16 a^2 d} \\ & - \frac{233 i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{144 a^2 d} \\ & + \frac{233 i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{72 a^2 d} - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{8 a^2 d} \\ & - \frac{23 \tan(dx+c)^{\frac{5}{3}} - 20 i \tan(dx+c)^{\frac{2}{3}}}{12 a^2 d (\tan(dx+c) - i)^2} - \frac{3\left(a^8 d^4 \tan(dx+c)^{\frac{5}{3}} + 5 i a^8 d^4 \tan(dx+c)^{\frac{2}{3}}\right)}{5 a^{10} d^5} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{14}{3}}}{(ia \tan(dx+c)+a)^2} dx$$

### 64.19 Problem number 241

$$\int \frac{\tan^{\frac{10}{3}}(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{49 \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} + \frac{49 \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & + \frac{49 \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} + \frac{5i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & - \frac{5i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{18a^2d} + \frac{5i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{9a^2d} \\ & - \frac{49 \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{49 \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & - \frac{49\left(\tan^{\frac{1}{3}}(dx+c)\right)}{12a^2d} + \frac{5i\left(\tan^{\frac{4}{3}}(dx+c)\right)}{6a^2d(1+i \tan(dx+c))} - \frac{\tan^{\frac{7}{3}}(dx+c)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(10/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{89\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144a^2d} - \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16a^2d} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{16a^2d} \\ & - \frac{89i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{144a^2d} + \frac{89i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{72a^2d} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{8a^2d} - \frac{3 \tan(dx+c)^{\frac{1}{3}}}{a^2d} - \frac{-16i \tan(dx+c)^{\frac{4}{3}} - 13 \tan(dx+c)^{\frac{1}{3}}}{12a^2d(\tan(dx+c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{10}{3}}}{(ia \tan(dx+c)+a)^2} dx$$

### 64.20 Problem number 242

$$\int \frac{\tan^{\frac{8}{3}}(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{25 \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} - \frac{25 \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & - \frac{25 \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} - \frac{2i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & + \frac{i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{9a^2d} + \frac{2i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{9a^2d} \\ & - \frac{25 \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{25 \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{2i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{3a^2d(1+i \tan(dx+c))} - \frac{\tan^{\frac{5}{3}}(dx+c)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(8/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{41 \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144 a^2 d} - \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16 a^2 d} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}}+i \tan(dx+c)^{\frac{1}{3}}-1\right)}{16 a^2 d} \\ & + \frac{41 i \log\left(\tan(dx+c)^{\frac{2}{3}}-i \tan(dx+c)^{\frac{1}{3}}-1\right)}{144 a^2 d} - \frac{41 i \log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72 a^2 d} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8 a^2 d} + \frac{11 \tan(dx+c)^{\frac{5}{3}}-8 i \tan(dx+c)^{\frac{2}{3}}}{12 a^2 d(\tan(dx+c)-i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{8}{3}}}{(ia \tan(dx+c) + a)^2} dx$$

### 64.21 Problem number 243

$$\int \frac{\tan^{\frac{4}{3}}(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} - \frac{\arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & -\frac{\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} + \frac{i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & -\frac{i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{18a^2d} + \frac{i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)}{9a^2d} \sqrt{3} \\ & + \frac{\ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & - \frac{\ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{\tan^{\frac{1}{3}}(dx+c)}{3a^2d(1+i \tan(dx+c))} - \frac{\tan^{\frac{1}{3}}(dx+c)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(4/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{7\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144a^2d} + \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16a^2d} \\ & -\frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{16a^2d} \\ & -\frac{7i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{144a^2d} + \frac{7i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{72a^2d} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{8a^2d} + \frac{-4i \tan(dx+c)^{\frac{4}{3}} - \tan(dx+c)^{\frac{1}{3}}}{12a^2d(\tan(dx+c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{4}{3}}}{(ia \tan(dx+c)+a)^2} dx$$

### 64.22 Problem number 244

$$\int \frac{\tan^{\frac{2}{3}}(c+dx)}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} + \frac{\arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & + \frac{\arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} - \frac{i \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & + \frac{i \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{18a^2d} + \frac{i \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{9a^2d} \\ & + \frac{\ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & - \frac{\ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{i\left(\tan^{\frac{2}{3}}(dx+c)\right)}{3a^2d(1+i \tan(dx+c))} + \frac{\tan^{\frac{5}{3}}(dx+c)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(2/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{7\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144a^2d} + \frac{\sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16a^2d} \\ & + \frac{i \log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{16a^2d} \\ & + \frac{7i \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{144a^2d} - \frac{7i \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{72a^2d} \\ & - \frac{i \log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{8a^2d} + \frac{\tan(dx+c)^{\frac{5}{3}} - 4i \tan(dx+c)^{\frac{2}{3}}}{12a^2d(\tan(dx+c) - i)^2} \end{aligned}$$



Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{2}{3}}}{(ia \tan(dx+c) + a)^2} dx$$

### 64.23 Problem number 245

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)} (a + ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} - \frac{7i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & - \frac{7i \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} + \frac{2 \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & - \frac{\ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{9a^2d} - \frac{2 \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{9a^2d} \\ & - \frac{7i \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{7i \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{144a^2d} \\ & + \frac{7\left(\tan^{\frac{2}{3}}(dx+c)\right)}{12a^2d(1 + i \tan(dx+c))} + \frac{\tan^{\frac{2}{3}}(dx+c)}{4d(a + ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(1/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{23i\sqrt{3} \log\left(\frac{-\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}-i}}\right)}{144a^2d} + \frac{i\sqrt{3} \log\left(\frac{-\sqrt{3}-2\tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2\tan(dx+c)^{\frac{1}{3}+i}}\right)}{16a^2d} \\ & - \frac{\log\left(\tan(dx+c)^{\frac{2}{3}} + i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{16a^2d} \\ & - \frac{23 \log\left(\tan(dx+c)^{\frac{2}{3}} - i \tan(dx+c)^{\frac{1}{3}} - 1\right)}{144a^2d} + \frac{23 \log\left(\tan(dx+c)^{\frac{1}{3}} + i\right)}{72a^2d} \\ & + \frac{\log\left(\tan(dx+c)^{\frac{1}{3}} - i\right)}{8a^2d} + \frac{-7i \tan(dx+c)^{\frac{5}{3}} - 10 \tan(dx+c)^{\frac{2}{3}}}{12a^2d(\tan(dx+c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(i a \tan(dx+c) + a)^2 \tan(dx+c)^{\frac{1}{3}}} dx$$

#### 64.24 Problem number 246

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{55i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} - \frac{55i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{72a^2d} \\ & - \frac{55i \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{36a^2d} + \frac{8 \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{9a^2d} \\ & - \frac{4 \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{9a^2d} + \frac{8 \arctan\left(\frac{\left(1-2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right) \sqrt{3}}{9a^2d} \\ & + \frac{55i \ln\left(1 - \sqrt{3} \left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{144a^2d} \\ & - \frac{55i \ln\left(1 + \sqrt{3} \left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{144a^2d} - \frac{8}{3a^2d \tan(dx+c)^{\frac{2}{3}}} \\ & + \frac{11}{12a^2d(1+i \tan(dx+c)) \tan(dx+c)^{\frac{2}{3}}} + \frac{1}{4d \tan(dx+c)^{\frac{2}{3}}(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(5/3)/(a+i*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{119i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}}+i}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}}-i}\right)}{144 a^2 d} - \frac{i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}}-i}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}}+i}\right)}{16 a^2 d} \\ & - \frac{\log\left(\tan(dx+c)^{\frac{2}{3}}+i \tan(dx+c)^{\frac{1}{3}}-1\right)}{16 a^2 d} \\ & - \frac{119 \log\left(\tan(dx+c)^{\frac{2}{3}}-i \tan(dx+c)^{\frac{1}{3}}-1\right)}{144 a^2 d} + \frac{119 \log\left(\tan(dx+c)^{\frac{1}{3}}+i\right)}{72 a^2 d} \\ & + \frac{\log\left(\tan(dx+c)^{\frac{1}{3}}-i\right)}{8 a^2 d} - \frac{32 \tan(dx+c)^2-53i \tan(dx+c)-18}{12\left(\tan(dx+c)^{\frac{4}{3}}-i \tan(dx+c)^{\frac{1}{3}}\right)^2 a^2 d} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(i a \tan(dx + c) + a)^2 \tan(dx + c)^{\frac{5}{3}}} dx$$

### 64.25 Problem number 247

$$\int \frac{1}{\tan^{\frac{7}{3}}(c + dx)(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{91i \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx + c)\right)\right)}{72a^2d} + \frac{91i \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx + c)\right)\right)}{72a^2d} \\ & + \frac{91i \arctan\left(\tan^{\frac{1}{3}}(dx + c)\right)}{36a^2d} - \frac{25 \ln\left(1 + \tan^{\frac{2}{3}}(dx + c)\right)}{18a^2d} \\ & + \frac{25 \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx + c)\right) + \tan^{\frac{4}{3}}(dx + c)\right)}{36a^2d} + \frac{25 \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx + c)\right)\right)\sqrt{3}}{3}\right)}{18a^2d} \sqrt{3} \\ & + \frac{91i \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx + c)\right) + \tan^{\frac{2}{3}}(dx + c)\right)}{144a^2d} \sqrt{3} \\ & - \frac{91i \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx + c)\right) + \tan^{\frac{2}{3}}(dx + c)\right)}{144a^2d} \sqrt{3} \\ & - \frac{25}{12a^2d \tan(dx + c)^{\frac{4}{3}}} + \frac{13}{12a^2d(1 + i \tan(dx + c)) \tan(dx + c)^{\frac{4}{3}}} \\ & + \frac{91i}{12a^2d \tan(dx + c)^{\frac{1}{3}}} + \frac{1}{4d \tan(dx + c)^{\frac{4}{3}}(a + ia \tan(dx + c))^2} \end{aligned}$$

command

`integrate(1/tan(d*x+c)^(7/3)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{191i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}+i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}-i}}\right)}{144 a^2 d} - \frac{i \sqrt{3} \log\left(-\frac{\sqrt{3}-2 \tan(dx+c)^{\frac{1}{3}-i}}{\sqrt{3}+2 \tan(dx+c)^{\frac{1}{3}+i}}\right)}{16 a^2 d} \\ & + \frac{\log\left(\tan(dx + c)^{\frac{2}{3}} + i \tan(dx + c)^{\frac{1}{3}} - 1\right)}{16 a^2 d} \\ & + \frac{191 \log\left(\tan(dx + c)^{\frac{2}{3}} - i \tan(dx + c)^{\frac{1}{3}} - 1\right)}{144 a^2 d} \\ & - \frac{191 \log\left(\tan(dx + c)^{\frac{1}{3}} + i\right)}{72 a^2 d} - \frac{\log\left(\tan(dx + c)^{\frac{1}{3}} - i\right)}{8 a^2 d} \\ & - \frac{3(-8i \tan(dx + c) + 1)}{4 a^2 d \tan(dx + c)^{\frac{4}{3}}} + \frac{19i \tan(dx + c)^{\frac{5}{3}} + 22 \tan(dx + c)^{\frac{2}{3}}}{12 a^2 d (\tan(dx + c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(i a \tan(dx + c) + a)^2 \tan(dx + c)^{\frac{7}{3}}} dx$$

### 64.26 Problem number 387

$$\int \cot^2(e + fx) \sqrt{1 + \tan(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}\left(\sqrt{1 + \tan(fx + e)}\right)}{f} \\ & + \frac{\operatorname{arctan}\left(\frac{\sqrt{2 + 2\sqrt{2}} - 2\sqrt{1 + \tan(fx + e)}}{\sqrt{-2 + 2\sqrt{2}}}\right) \sqrt{2 + 2\sqrt{2}}}{2f} \\ & - \frac{\operatorname{arctan}\left(\frac{\sqrt{2 + 2\sqrt{2}} + 2\sqrt{1 + \tan(fx + e)}}{\sqrt{-2 + 2\sqrt{2}}}\right) \sqrt{2 + 2\sqrt{2}}}{2f} \\ & - \frac{\ln\left(1 + \sqrt{2} - \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan(fx + e)} + \tan(fx + e)\right)}{2f\sqrt{2 + 2\sqrt{2}}} \\ & + \frac{\ln\left(1 + \sqrt{2} + \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan(fx + e)} + \tan(fx + e)\right)}{2f\sqrt{2 + 2\sqrt{2}}} \\ & - \frac{\cot(fx + e) \sqrt{1 + \tan(fx + e)}}{f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -\frac{\log\left(\sqrt{\tan(fx+e)+1}+1\right)}{2f} + \frac{\log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{2f} \\
& -\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} \\
& -\frac{\left(f^2\sqrt{\sqrt{2}+1}+f\sqrt{\sqrt{2}-1}|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} \\
& -\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} \\
& +\frac{\left(f^2\sqrt{\sqrt{2}-1}-f\sqrt{\sqrt{2}+1}|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} \\
& -\frac{\sqrt{\tan(fx+e)+1}}{f\tan(fx+e)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 64.27 Problem number 388

$$\int \cot^4(e+fx)\sqrt{1+\tan(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{7 \operatorname{arctanh}\left(\sqrt{1+\tan (fx+e)}\right)}{8 f} \\
& - \frac{\operatorname{arctan}\left(\frac{\sqrt{2+2 \sqrt{2}}-2 \sqrt{1+\tan (fx+e)}}{\sqrt{-2+2 \sqrt{2}}}\right) \sqrt{2+2 \sqrt{2}}}{2 f} \\
& + \frac{\operatorname{arctan}\left(\frac{\sqrt{2+2 \sqrt{2}}+2 \sqrt{1+\tan (fx+e)}}{\sqrt{-2+2 \sqrt{2}}}\right) \sqrt{2+2 \sqrt{2}}}{2 f} \\
& + \frac{\ln \left(1+\sqrt{2}-\sqrt{2+2 \sqrt{2}} \sqrt{1+\tan (fx+e)}+\tan (fx+e)\right)}{2 f \sqrt{2+2 \sqrt{2}}} \\
& - \frac{\ln \left(1+\sqrt{2}+\sqrt{2+2 \sqrt{2}} \sqrt{1+\tan (fx+e)}+\tan (fx+e)\right)}{2 f \sqrt{2+2 \sqrt{2}}} \\
& + \frac{9 \cot (fx+e) \sqrt{1+\tan (fx+e)}}{8 f} - \frac{(\cot ^2 (fx+e)) \sqrt{1+\tan (fx+e)}}{12 f} \\
& - \frac{(\cot ^3 (fx+e)) \sqrt{1+\tan (fx+e)}}{3 f}
\end{aligned}$$

command

```
integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{7 \log \left( \sqrt{\tan (fx+e)+1}+1 \right)}{16 f}-\frac{7 \log \left( \left| \sqrt{\tan (fx+e)+1}-1 \right| \right)}{16 f} \\
& +\frac{\left( f^2 \sqrt{\sqrt{2}+1}+f \sqrt{\sqrt{2}-1}|f \right) \arctan \left( \frac{2^{\frac{3}{4}}\left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}+2 \sqrt{\tan (fx+e)+1} \right)}{2 \sqrt{-\sqrt{2}+2}} \right)}{2 f^3} \\
& +\frac{\left( f^2 \sqrt{\sqrt{2}+1}+f \sqrt{\sqrt{2}-1}|f \right) \arctan \left( -\frac{2^{\frac{3}{4}}\left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}-2 \sqrt{\tan (fx+e)+1} \right)}{2 \sqrt{-\sqrt{2}+2}} \right)}{2 f^3} \\
& +\frac{\left( f^2 \sqrt{\sqrt{2}-1}-f \sqrt{\sqrt{2}+1}|f \right) \log \left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (fx+e)+1}+\sqrt{2}+\tan (fx+e)+1 \right)}{4 f^3} \\
& -\frac{\left( f^2 \sqrt{\sqrt{2}-1}-f \sqrt{\sqrt{2}+1}|f \right) \log \left( -2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (fx+e)+1}+\sqrt{2}+\tan (fx+e)+1 \right)}{4 f^3} \\
& +\frac{27(\tan (fx+e)+1)^{\frac{5}{2}}-56(\tan (fx+e)+1)^{\frac{3}{2}}+21 \sqrt{\tan (fx+e)+1}}{24 f \tan (fx+e)^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 64.28 Problem number 398

$$\int \cot^2(e+fx)(1+\tan(e+fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{3 \operatorname{arctanh} \left( \sqrt{1+\tan (fx+e)} \right)}{f} \\
& +\frac{\arctan \left( \frac{3-2 \sqrt{2}+(1-\sqrt{2}) \tan (fx+e)}{\sqrt{-14+10 \sqrt{2}} \sqrt{1+\tan (fx+e)}} \right) \sqrt{\sqrt{2}-1}}{f} \\
& +\frac{\operatorname{arctanh} \left( \frac{3+2 \sqrt{2}+(1+\sqrt{2}) \tan (fx+e)}{\sqrt{14+10 \sqrt{2}} \sqrt{1+\tan (fx+e)}} \right) \sqrt{1+\sqrt{2}}}{f} \\
& -\frac{\cot (fx+e) \sqrt{1+\tan (fx+e)}}{f}
\end{aligned}$$

command

```
integrate(cot(f*x+e)^2*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{3 \log \left( \sqrt{\tan (f x+e)+1}+1 \right)}{2 f}+\frac{3 \log \left( \left| \sqrt{\tan (f x+e)+1}-1 \right| \right)}{2 f} \\
 & \left( f^2 \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}|f| \right) \arctan \left( \frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}+2 \sqrt{\tan (f x+e)+1}\right)}{2 \sqrt{-\sqrt{2}+2}} \right) \\
 & -\frac{\left( f^2 \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}|f| \right) \arctan \left( \frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}-2 \sqrt{\tan (f x+e)+1}\right)}{2 \sqrt{-\sqrt{2}+2}} \right)}{2 f^3} \\
 & +\frac{\left( f^2 \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}|f| \right) \log \left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (f x+e)+1}+\sqrt{2}+\tan (f x+e)+1 \right)}{4 f^3} \\
 & -\frac{\left( f^2 \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}|f| \right) \log \left( -2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (f x+e)+1}+\sqrt{2}+\tan (f x+e)+1 \right)}{4 f^3} \\
 & -\frac{\sqrt{\tan (f x+e)+1}}{f \tan (f x+e)}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 64.29 Problem number 399

$$\int \cot^4(e+fx)(1+\tan(e+fx))^{3/2} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{25 \operatorname{arctanh}\left(\sqrt{1+\tan(fx+e)}\right)}{8f} \\
& - \frac{\operatorname{arctan}\left(\frac{3-2\sqrt{2}+(1-\sqrt{2})\tan(fx+e)}{\sqrt{-14+10\sqrt{2}}\sqrt{1+\tan(fx+e)}}\right)\sqrt{\sqrt{2}-1}}{f} \\
& - \frac{\operatorname{arctanh}\left(\frac{3+2\sqrt{2}+(1+\sqrt{2})\tan(fx+e)}{\sqrt{14+10\sqrt{2}}\sqrt{1+\tan(fx+e)}}\right)\sqrt{1+\sqrt{2}}}{f} \\
& + \frac{7 \cot(fx+e)\sqrt{1+\tan(fx+e)}}{8f} - \frac{7(\cot^2(fx+e))\sqrt{1+\tan(fx+e)}}{12f} \\
& - \frac{(\cot^3(fx+e))\sqrt{1+\tan(fx+e)}}{3f}
\end{aligned}$$

command

```
integrate(cot(f*x+e)^4*(1+tan(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{25 \log\left(\sqrt{\tan(fx+e)+1}+1\right)}{16f} - \frac{25 \log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{16f} \\
& + \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}|f|\right) \operatorname{arctan}\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} \\
& + \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}|f|\right) \operatorname{arctan}\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{2f^3} \\
& - \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}|f|\right) \log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} \\
& + \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}|f|\right) \log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{4f^3} \\
& + \frac{21(\tan(fx+e)+1)^{\frac{5}{2}}-56(\tan(fx+e)+1)^{\frac{3}{2}}+27\sqrt{\tan(fx+e)+1}}{24f \tan(fx+e)^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 64.30 Problem number 409

$$\int \frac{\cot^2(e + fx)}{\sqrt{1 + \tan(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\operatorname{arctanh}\left(\sqrt{1 + \tan(fx + e)}\right)}{f} \\ & + \frac{\ln\left(1 + \sqrt{2} - \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan(fx + e)} + \tan(fx + e)\right)}{4f\sqrt{1 + \sqrt{2}}} \\ & - \frac{\ln\left(1 + \sqrt{2} + \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan(fx + e)} + \tan(fx + e)\right)}{4f\sqrt{1 + \sqrt{2}}} \\ & + \frac{\operatorname{arctan}\left(\frac{\sqrt{2 + 2\sqrt{2}} - 2\sqrt{1 + \tan(fx + e)}}{\sqrt{-2 + 2\sqrt{2}}}\right) \sqrt{1 + \sqrt{2}}}{2f} \\ & - \frac{\operatorname{arctan}\left(\frac{\sqrt{2 + 2\sqrt{2}} + 2\sqrt{1 + \tan(fx + e)}}{\sqrt{-2 + 2\sqrt{2}}}\right) \sqrt{1 + \sqrt{2}}}{2f} \\ & - \frac{\cot(fx + e) \sqrt{1 + \tan(fx + e)}}{f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^2/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{\log\left(\sqrt{\tan(fx+e)+1}+1\right)}{2f} - \frac{\log\left(\left|\sqrt{\tan(fx+e)+1}-1\right|\right)}{2f} \\
& - \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}|f|\right)\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3} \\
& - \frac{\left(f^2\sqrt{2\sqrt{2}-2}+f\sqrt{2\sqrt{2}+2}|f|\right)\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{\tan(fx+e)+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)}{4f^3} \\
& - \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}|f|\right)\log\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3} \\
& + \frac{\left(f^2\sqrt{2\sqrt{2}+2}-f\sqrt{2\sqrt{2}-2}|f|\right)\log\left(-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\sqrt{\tan(fx+e)+1}+\sqrt{2}+\tan(fx+e)+1\right)}{8f^3} \\
& - \frac{\sqrt{\tan(fx+e)+1}}{f\tan(fx+e)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 64.31 Problem number 410

$$\int \frac{\cot^4(e+fx)}{\sqrt{1+\tan(e+fx)}} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{3 \operatorname{arctanh} \left( \sqrt{1 + \tan (fx + e)} \right)}{8f} \\
& - \frac{\ln \left( 1 + \sqrt{2} - \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan (fx + e)} + \tan (fx + e) \right)}{4f\sqrt{1 + \sqrt{2}}} \\
& + \frac{\ln \left( 1 + \sqrt{2} + \sqrt{2 + 2\sqrt{2}} \sqrt{1 + \tan (fx + e)} + \tan (fx + e) \right)}{4f\sqrt{1 + \sqrt{2}}} \\
& - \frac{\operatorname{arctan} \left( \frac{\sqrt{2 + 2\sqrt{2}} - 2\sqrt{1 + \tan (fx + e)}}{\sqrt{-2 + 2\sqrt{2}}} \right) \sqrt{1 + \sqrt{2}}}{2f} \\
& + \frac{\operatorname{arctan} \left( \frac{\sqrt{2 + 2\sqrt{2}} + 2\sqrt{1 + \tan (fx + e)}}{\sqrt{-2 + 2\sqrt{2}}} \right) \sqrt{1 + \sqrt{2}}}{2f} \\
& + \frac{3 \cot (fx + e) \sqrt{1 + \tan (fx + e)}}{8f} + \frac{5(\cot^2 (fx + e)) \sqrt{1 + \tan (fx + e)}}{12f} \\
& - \frac{(\cot^3 (fx + e)) \sqrt{1 + \tan (fx + e)}}{3f}
\end{aligned}$$

command

```
integrate(cot(f*x+e)^4/(1+tan(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -\frac{3 \log \left( \sqrt{\tan (fx+e)+1}+1 \right)}{16 f} + \frac{3 \log \left( \left| \sqrt{\tan (fx+e)+1}-1 \right| \right)}{16 f} \\
& + \frac{\left( f^2 \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}|f| \right) \arctan \left( \frac{2^{\frac{3}{4}} \left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}+2 \sqrt{\tan (fx+e)+1} \right)}{2 \sqrt{-\sqrt{2}+2}} \right)}{4 f^3} \\
& + \frac{\left( f^2 \sqrt{2 \sqrt{2}-2}+f \sqrt{2 \sqrt{2}+2}|f| \right) \arctan \left( -\frac{2^{\frac{3}{4}} \left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2}-2 \sqrt{\tan (fx+e)+1} \right)}{2 \sqrt{-\sqrt{2}+2}} \right)}{4 f^3} \\
& + \frac{\left( f^2 \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}|f| \right) \log \left( 2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (fx+e)+1}+\sqrt{2}+\tan (fx+e)+1 \right)}{8 f^3} \\
& - \frac{\left( f^2 \sqrt{2 \sqrt{2}+2}-f \sqrt{2 \sqrt{2}-2}|f| \right) \log \left( -2^{\frac{1}{4}} \sqrt{\sqrt{2}+2} \sqrt{\tan (fx+e)+1}+\sqrt{2}+\tan (fx+e)+1 \right)}{8 f^3} \\
& + \frac{9(\tan (fx+e)+1)^{\frac{5}{2}}-8(\tan (fx+e)+1)^{\frac{3}{2}}-9 \sqrt{\tan (fx+e)+1}}{24 f \tan (fx+e)^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 64.32 Problem number 655

$$\int \frac{1}{\sqrt{\tan (c+dx)} \sqrt{2+3 \tan (c+dx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \frac{\sqrt{3-2i} \left( \sqrt{\tan (dx+c)} \right)}{\sqrt{2+3 \tan (dx+c)}} \right)}{d \sqrt{3-2i}} + \frac{\operatorname{arctanh} \left( \frac{\sqrt{3+2i} \left( \sqrt{\tan (dx+c)} \right)}{\sqrt{2+3 \tan (dx+c)}} \right)}{d \sqrt{3+2i}}$$

command

`integrate(1/tan(d*x+c)^(1/2)/(2+3*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{3} \left( (3i+2) \sqrt{6 \sqrt{13}-18} \left( -\frac{2i}{\sqrt{13}-3}+1 \right) \log \left( (120i+40) \sqrt{13} \left( \sqrt{3} \sqrt{\tan (dx+c)}-\sqrt{3 \tan (dx+c)} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{3 \tan(dx+c)+2} \sqrt{\tan(dx+c)}} dx$$

### 64.33 Problem number 656

$$\int \frac{1}{\sqrt{\tan(c+dx)} \sqrt{-2+3 \tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{3-2i}(\sqrt{\tan(dx+c)})}{\sqrt{-2+3 \tan(dx+c)}}\right)}{d\sqrt{3-2i}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2i}(\sqrt{\tan(dx+c)})}{\sqrt{-2+3 \tan(dx+c)}}\right)}{d\sqrt{3+2i}}$$

command

`integrate(1/tan(d*x+c)^(1/2)/(-2+3*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{3} \left( -(3i+2) \sqrt{6\sqrt{13}} - 18 \left( -\frac{2i}{\sqrt{13}-3} + 1 \right) \log \left( (120i+40) \sqrt{13} \left( \sqrt{3} \sqrt{\tan(dx+c)} - \sqrt{3 \tan(dx+c)} \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{3 \tan(dx+c)-2} \sqrt{\tan(dx+c)}} dx$$

### 64.34 Problem number 657

$$\int \frac{1}{\sqrt{2-3 \tan(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3-2i}(\sqrt{\tan(dx+c)})}{\sqrt{2-3 \tan(dx+c)}}\right)}{d\sqrt{3-2i}} + \frac{\arctan\left(\frac{\sqrt{3+2i}(\sqrt{\tan(dx+c)})}{\sqrt{2-3 \tan(dx+c)}}\right)}{d\sqrt{3+2i}}$$

command

`integrate(1/(2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

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Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-3 \tan(dx+c)+2} \sqrt{\tan(dx+c)}} dx$$

### 64.35 Problem number 658

$$\int \frac{1}{\sqrt{-2-3 \tan(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{3-2i}(\sqrt{\tan(dx+c)})}{\sqrt{-2-3 \tan(dx+c)}}\right)}{d\sqrt{3-2i}} + \frac{\arctan\left(\frac{\sqrt{3+2i}(\sqrt{\tan(dx+c)})}{\sqrt{-2-3 \tan(dx+c)}}\right)}{d\sqrt{3+2i}}$$

command

`integrate(1/(-2-3*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{3} \arctan\left(\frac{\sqrt{13}\left(\sqrt{3}\sqrt{-\tan(dx+c)} - \sqrt{-3 \tan(dx+c)-2}\right)^2 + 3\left(\sqrt{3}\sqrt{-\tan(dx+c)} - \sqrt{-3 \tan(dx+c)}\right)}{\sqrt{13}\sqrt{6\sqrt{13}+18} + (2i+3)\sqrt{6\sqrt{13}+18}}\right)}{d\sqrt{6\sqrt{13}+18}\left(\frac{2i}{\sqrt{13}+3} + 1\right)} + \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{13}\left(\sqrt{3}\sqrt{-\tan(dx+c)} - \sqrt{-3 \tan(dx+c)-2}\right)^2 + 3\left(\sqrt{3}\sqrt{-\tan(dx+c)} - \sqrt{-3 \tan(dx+c)}\right)}{\sqrt{13}\sqrt{6\sqrt{13}+18} - (2i-3)\sqrt{6\sqrt{13}+18}}\right)}{d\sqrt{6\sqrt{13}+18}\left(-\frac{2i}{\sqrt{13}+3} + 1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-3 \tan(dx+c)-2} \sqrt{\tan(dx+c)}} dx$$

## 64.36 Problem number 659

$$\int \frac{1}{\sqrt{\tan(c+dx)} \sqrt{3+2\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{3+2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{3+2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

`integrate(1/tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \sqrt{\sqrt{13}} - 2 \left( \frac{9i-6}{\sqrt{13}-2} - 2i - 3 \right) \log \left( (915i + 1098) \sqrt{13} \left( \sqrt{2} \sqrt{\tan(dx+c)} - \sqrt{2 \tan(dx+c) + 3} \right)^2 + \right. \right.$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{2 \tan(dx+c) + 3} \sqrt{\tan(dx+c)}} dx$$

## 64.37 Problem number 660

$$\int \frac{1}{\sqrt{3-2\tan(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{3-2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{\arctan\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{3-2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

`integrate(1/(3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-2 \tan(dx+c) + 3} \sqrt{\tan(dx+c)}} dx$$



## 64.38 Problem number 661

$$\int \frac{1}{\sqrt{\tan(c+dx)} \sqrt{-3+2\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3+2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3+2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

`integrate(1/tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{\sqrt{13}} - 2 \left( \frac{9i-6}{\sqrt{13}-2} - 2i - 3 \right) \log \left( (915i + 1098) \sqrt{13} \left( \sqrt{2} \sqrt{\tan(dx+c)} - \sqrt{2 \tan(dx+c) - 3} \right) \right)^2}{-}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{2 \tan(dx+c) - 3} \sqrt{\tan(dx+c)}} dx$$

## 64.39 Problem number 662

$$\int \frac{1}{\sqrt{-3-2\tan(c+dx)} \sqrt{\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3-2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{\arctan\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3-2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

`integrate(1/(-3-2*tan(d*x+c))^(1/2)/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(1904i + 1536) \sqrt{2} \log \left( 3 \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^4 + (24i + 18) \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^2 + (24i - 18) \right)}{23377 d} + \frac{(1904i - 1536) \sqrt{2} \log \left( 3 \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^4 - (24i - 18) \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^2 + (24i + 18) \right)}{23377 d} + \frac{(12720i + 27456) \sqrt{2} \arctan \left( \frac{\sqrt{13} \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^2 + 2 \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)}{2 \left( \sqrt{13} \sqrt{\sqrt{13} + 2} + (3i+2) \sqrt{\sqrt{13} + 2} \right)} \right)}{23377 d \sqrt{\sqrt{13} + 2} \left( \frac{3i}{\sqrt{13} + 2} + 1 \right)} + \frac{(12720i - 27456) \sqrt{2} \arctan \left( \frac{\sqrt{13} \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)^2 + 2 \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2 \tan(dx+c) - 3} \right)}{2 \left( \sqrt{13} \sqrt{\sqrt{13} + 2} - (3i-2) \sqrt{\sqrt{13} + 2} \right)} \right)}{23377 d \sqrt{\sqrt{13} + 2} \left( -\frac{3i}{\sqrt{13} + 2} + 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-2 \tan(dx+c) - 3} \sqrt{\tan(dx+c)}} dx$$

#### 64.40 Problem number 665

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{2-3 \tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{i \arctan \left( \frac{\sqrt{3-2i} \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2-3 \tan(dx+c)}} \right)}{d \sqrt{3-2i}} + \frac{i \arctan \left( \frac{\sqrt{3+2i} \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{2-3 \tan(dx+c)}} \right)}{d \sqrt{3+2i}}$$

command

`integrate(tan(d*x+c)^(1/2)/(2-3*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2028} \sqrt{3} \left( \frac{2 \left( 2 d^2 \sqrt{1014 \sqrt{13}} - 702 - 3 d \sqrt{1014 \sqrt{13}} + 702 |d| \right) \arctan \left( \frac{13 \left( \frac{9}{13} \right)^{\frac{3}{4}} \left( 2 \left( \frac{9}{13} \right)^{\frac{1}{4}} \sqrt{-\frac{3}{26} \sqrt{13}} + \frac{1}{2}} \right)}{d^3} \right)}{d^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 64.41 Problem number 667

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3+2\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{i \operatorname{arctanh} \left( \frac{\sqrt{2-3i} \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{3+2\tan(dx+c)}} \right)}{d\sqrt{2-3i}} - \frac{i \operatorname{arctanh} \left( \frac{\sqrt{2+3i} \left( \sqrt{\tan(dx+c)} \right)}{\sqrt{3+2\tan(dx+c)}} \right)}{d\sqrt{2+3i}}$$

command

`integrate(tan(d*x+c)^(1/2)/(3+2*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{676} \sqrt{2} \left( \frac{3 \left( 2 \sqrt{325 \sqrt{13}} - 1118 \arctan \left( \frac{13 \left( \frac{4}{13} \right)^{\frac{3}{4}} \left( \left( \frac{4}{13} \right)^{\frac{1}{4}} \sqrt{\frac{1}{13} \sqrt{13}} + \frac{1}{2}} + 1 \right)}{4 \sqrt{-\frac{1}{13} \sqrt{13}} + \frac{1}{2}}} \right) + 2 \sqrt{325 \sqrt{13}} - 1118 \arctan \left( \frac{13 \left( \frac{4}{13} \right)^{\frac{3}{4}} \left( \left( \frac{4}{13} \right)^{\frac{1}{4}} \sqrt{\frac{1}{13} \sqrt{13}} + \frac{1}{2}} + 1 \right)}{4 \sqrt{-\frac{1}{13} \sqrt{13}} + \frac{1}{2}}} \right)}{4 \sqrt{-\frac{1}{13} \sqrt{13}} + \frac{1}{2}} \right)}{4 \sqrt{-\frac{1}{13} \sqrt{13}} + \frac{1}{2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 64.42 Problem number 668

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{3-2\tan(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{i \arctan\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{3-2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{i \arctan\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{3-2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

```
integrate(tan(d*x+c)^(1/2)/(3-2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{676} \sqrt{2} \left( \frac{2 \left( 3 d^2 \sqrt{169 \sqrt{13}} - 598 - 2 d \sqrt{169 \sqrt{13}} + 598 |d| \right) \arctan\left( \frac{13 \left(\frac{4}{13}\right)^{\frac{3}{4}} \left( 2 \left(\frac{4}{13}\right)^{\frac{1}{4}} \sqrt{-\frac{1}{13} \sqrt{13}} + \frac{1}{2}} + \frac{\sqrt{13}}{2} \right)}{8 \sqrt{13}} \right)}{d^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 64.43 Problem number 669

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3+2\tan(c+dx)}} dx$$

Optimal antiderivative

$$-\frac{i \operatorname{arctanh}\left(\frac{\sqrt{2-3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3+2\tan(dx+c)}}\right)}{d\sqrt{2-3i}} + \frac{i \operatorname{arctanh}\left(\frac{\sqrt{2+3i}(\sqrt{\tan(dx+c)})}{\sqrt{-3+2\tan(dx+c)}}\right)}{d\sqrt{2+3i}}$$

command

```
integrate(tan(d*x+c)^(1/2)/(-3+2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{676} \sqrt{2} \left( \frac{3 \left( 2 \sqrt{325 \sqrt{13} - 1118} \arctan \left( \frac{13 \left(\frac{4}{13}\right)^{\frac{3}{4}} \left( \left(\frac{4}{13}\right)^{\frac{1}{4}} \sqrt{\frac{1}{13} \sqrt{13} + \frac{1}{2} + 1} \right)}{4 \sqrt{-\frac{1}{13} \sqrt{13} + \frac{1}{2}}} \right) + 2 \sqrt{325 \sqrt{13} - 1118} \arctan \right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

#### 64.44 Problem number 670

$$\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{-3-2\tan(c+dx)}} dx$$

Optimal antiderivative

$$\frac{i \arctan \left( \frac{\sqrt{2-3i} (\sqrt{\tan(dx+c)})}{\sqrt{-3-2\tan(dx+c)}} \right)}{d\sqrt{2-3i}} - \frac{i \arctan \left( \frac{\sqrt{2+3i} (\sqrt{\tan(dx+c)})}{\sqrt{-3-2\tan(dx+c)}} \right)}{d\sqrt{2+3i}}$$

command

```
integrate(tan(d*x+c)^(1/2)/(-3-2*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( -i \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2\tan(dx+c)-3} \right)^4 - 6i \left( \sqrt{2} \sqrt{-\tan(dx+c)} - \sqrt{-2\tan(dx+c)-3} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 64.45 Problem number 671

$$\int \frac{\tan^{\frac{5}{3}}(c+dx)}{a+b\tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} + \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\ & + \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3a^{\frac{5}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2b^{\frac{2}{3}}(a^2+b^2)d} - \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} \\ & + \frac{a^{\frac{5}{3}} \ln(a+b\tan(dx+c))}{2b^{\frac{2}{3}}(a^2+b^2)d} + \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\ & - \frac{a^{\frac{5}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{b^{\frac{2}{3}}(a^2+b^2)d} + \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{2(a^2+b^2)d} \\ & + \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \\ & - \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& - \frac{a^2 \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log \left( \left| -\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \tan(dx+c)^{\frac{1}{3}} \right| \right)}{a^3 d + ab^2 d} \\
& + \frac{(\sqrt{3} a + b) \arctan \left( \sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} \right)}{2(a^2 d + b^2 d)} \\
& - \frac{(\sqrt{3} a - b) \arctan \left( -\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} \right)}{2(a^2 d + b^2 d)} \\
& + \frac{b \arctan \left( \tan(dx+c)^{\frac{1}{3}} \right)}{a^2 d + b^2 d} + \frac{a \log \left( \tan(dx+c)^{\frac{4}{3}} - \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4(a^2 d + b^2 d)} \\
& - \frac{3 b \log \left( \sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4 \left( \sqrt{3} a^2 d + \sqrt{3} b^2 d \right)} \\
& + \frac{3 b \log \left( -\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4 \left( \sqrt{3} a^2 d + \sqrt{3} b^2 d \right)} \\
& - \frac{a \log \left( \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{2(a^2 d + b^2 d)} - \frac{3 \left(-ab^2\right)^{\frac{2}{3}} a \arctan \left( \frac{\sqrt{3} \left( \left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2 \tan(dx+c)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{\left( \sqrt{3} a^2 b^2 + \sqrt{3} b^4 \right) d} \\
& + \frac{\left(-ab^2\right)^{\frac{2}{3}} a \log \left( \left(-\frac{a}{b}\right)^{\frac{2}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} \right)}{2(a^2 b^2 + b^4) d}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{5}{3}}}{b \tan(dx+c) + a} dx$$

**64.46 Problem number 672**

$$\int \frac{\sqrt[3]{\tan(c+dx)}}{a + b \tan(c+dx)} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} + \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& + \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3a^{\frac{1}{3}}b^{\frac{2}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& - \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} + \frac{a^{\frac{1}{3}}b^{\frac{2}{3}} \ln(a + b \tan(dx+c))}{2(a^2+b^2)d} \\
& + \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\
& + \frac{a^{\frac{1}{3}}b^{\frac{2}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right) \sqrt{3}}{(a^2+b^2)d} \\
& - \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right) \sqrt{3}}{2(a^2+b^2)d} \\
& - \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{4(a^2+b^2)d} \\
& + \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right) \sqrt{3}}{4(a^2+b^2)d}
\end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\begin{aligned}
& \frac{ab\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \tan(dx+c)^{\frac{1}{3}}\right|\right)}{a^3d + ab^2d} \\
& - \frac{\left(\sqrt{3}a - b\right) \arctan\left(\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{2(a^2d + b^2d)} \\
& + \frac{\left(\sqrt{3}a + b\right) \arctan\left(-\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{2(a^2d + b^2d)} \\
& + \frac{b \arctan\left(\tan(dx+c)^{\frac{1}{3}}\right)}{a^2d + b^2d} + \frac{a \log\left(\tan(dx+c)^{\frac{4}{3}} - \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4(a^2d + b^2d)} \\
& + \frac{3b \log\left(\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4\left(\sqrt{3}a^2d + \sqrt{3}b^2d\right)} \\
& - \frac{3b \log\left(-\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4\left(\sqrt{3}a^2d + \sqrt{3}b^2d\right)} \\
& - \frac{a \log\left(\tan(dx+c)^{\frac{2}{3}} + 1\right)}{2(a^2d + b^2d)} - \frac{3(-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}a^2 + \sqrt{3}b^2\right)d} \\
& - \frac{\left(-ab^2\right)^{\frac{1}{3}} \log\left(\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}}\right)}{2(a^2 + b^2)d}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\tan(dx+c)^{\frac{1}{3}}}{b \tan(dx+c) + a} dx$$

**64.47 Problem number 673**

$$\int \frac{1}{\sqrt[3]{\tan(c+dx)}(a + b \tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b \arctan\left(-\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} - \frac{b \arctan\left(\sqrt{3} + 2\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2(a^2+b^2)d} \\
& - \frac{b \arctan\left(\tan^{\frac{1}{3}}(dx+c)\right)}{(a^2+b^2)d} - \frac{3b^{\frac{4}{3}} \ln\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)}{2a^{\frac{1}{3}}(a^2+b^2)d} \\
& + \frac{a \ln\left(1 + \tan^{\frac{2}{3}}(dx+c)\right)}{2(a^2+b^2)d} + \frac{b^{\frac{4}{3}} \ln(a + b \tan(dx+c))}{2a^{\frac{1}{3}}(a^2+b^2)d} \\
& - \frac{a \ln\left(1 - \left(\tan^{\frac{2}{3}}(dx+c)\right) + \tan^{\frac{4}{3}}(dx+c)\right)}{4(a^2+b^2)d} \\
& - \frac{b^{\frac{4}{3}} \arctan\left(\frac{\left(a^{\frac{1}{3}} - 2b^{\frac{1}{3}}\left(\tan^{\frac{1}{3}}(dx+c)\right)\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}}{a^{\frac{1}{3}}(a^2+b^2)d} - \frac{a \arctan\left(\frac{\left(1 - 2\left(\tan^{\frac{2}{3}}(dx+c)\right)\right)\sqrt{3}}{3}\right)\sqrt{3}}{2(a^2+b^2)d} \\
& - \frac{b \ln\left(1 - \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d} \\
& + \frac{b \ln\left(1 + \sqrt{3}\left(\tan^{\frac{1}{3}}(dx+c)\right) + \tan^{\frac{2}{3}}(dx+c)\right)\sqrt{3}}{4(a^2+b^2)d}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(1/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& - \frac{b^2 \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log \left( \left| -\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \tan(dx+c)^{\frac{1}{3}} \right| \right)}{a^3 d + ab^2 d} \\
& - \frac{\left(\sqrt{3} a + b\right) \arctan \left(\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2 d + b^2 d\right)} \\
& + \frac{\left(\sqrt{3} a - b\right) \arctan \left(-\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{2\left(a^2 d + b^2 d\right)} \\
& - \frac{b \arctan \left(\tan(dx+c)^{\frac{1}{3}}\right)}{a^2 d + b^2 d} - \frac{a \log \left(\tan(dx+c)^{\frac{4}{3}} - \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4\left(a^2 d + b^2 d\right)} \\
& + \frac{3 b \log \left(\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4\left(\sqrt{3} a^2 d + \sqrt{3} b^2 d\right)} \\
& - \frac{3 b \log \left(-\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1\right)}{4\left(\sqrt{3} a^2 d + \sqrt{3} b^2 d\right)} \\
& + \frac{a \log \left(\tan(dx+c)^{\frac{2}{3}} + 1\right)}{2\left(a^2 d + b^2 d\right)} - \frac{3\left(-ab^2\right)^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2 \tan(dx+c)^{\frac{1}{3}}}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3} a^3 + \sqrt{3} ab^2\right) d} \\
& + \frac{\left(-ab^2\right)^{\frac{2}{3}} \log \left(\left(-\frac{a}{b}\right)^{\frac{2}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}}\right)}{2\left(a^3 + ab^2\right) d}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan(dx+c) + a) \tan(dx+c)^{\frac{1}{3}}} dx$$

**64.48 Problem number 674**

$$\int \frac{1}{\tan^{\frac{5}{3}}(c+dx)(a+b \tan(c+dx))} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{b \arctan \left( -\sqrt{3} + 2 \left( \tan^{\frac{1}{3}}(dx+c) \right) \right)}{2(a^2+b^2)d} - \frac{b \arctan \left( \sqrt{3} + 2 \left( \tan^{\frac{1}{3}}(dx+c) \right) \right)}{2(a^2+b^2)d} \\
& - \frac{b \arctan \left( \tan^{\frac{1}{3}}(dx+c) \right)}{(a^2+b^2)d} - \frac{3b^{\frac{8}{3}} \ln \left( a^{\frac{1}{3}} + b^{\frac{1}{3}} \left( \tan^{\frac{1}{3}}(dx+c) \right) \right)}{2a^{\frac{5}{3}}(a^2+b^2)d} \\
& + \frac{a \ln \left( 1 + \tan^{\frac{2}{3}}(dx+c) \right)}{2(a^2+b^2)d} + \frac{b^{\frac{8}{3}} \ln(a+b \tan(dx+c))}{2a^{\frac{5}{3}}(a^2+b^2)d} \\
& - \frac{a \ln \left( 1 - \left( \tan^{\frac{2}{3}}(dx+c) \right) + \tan^{\frac{4}{3}}(dx+c) \right)}{4(a^2+b^2)d} \\
& + \frac{b^{\frac{8}{3}} \arctan \left( \frac{\left( a^{\frac{1}{3}} - 2b^{\frac{1}{3}} \left( \tan^{\frac{1}{3}}(dx+c) \right) \right) \sqrt{3}}{3a^{\frac{1}{3}}} \right) \sqrt{3}}{a^{\frac{5}{3}}(a^2+b^2)d} + \frac{a \arctan \left( \frac{\left( 1 - 2 \left( \tan^{\frac{2}{3}}(dx+c) \right) \right) \sqrt{3}}{3} \right) \sqrt{3}}{2(a^2+b^2)d} \\
& + \frac{b \ln \left( 1 - \sqrt{3} \left( \tan^{\frac{1}{3}}(dx+c) \right) + \tan^{\frac{2}{3}}(dx+c) \right) \sqrt{3}}{4(a^2+b^2)d} \\
& - \frac{b \ln \left( 1 + \sqrt{3} \left( \tan^{\frac{1}{3}}(dx+c) \right) + \tan^{\frac{2}{3}}(dx+c) \right) \sqrt{3}}{4(a^2+b^2)d} \\
& - \frac{3a}{2(a^2+b^2)d \tan(dx+c)^{\frac{2}{3}}} - \frac{3b^2}{2a(a^2+b^2)d \tan(dx+c)^{\frac{2}{3}}}
\end{aligned}$$

command

```
integrate(1/tan(d*x+c)^(5/3)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{b^3 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left( \left| -\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \tan(dx+c)^{\frac{1}{3}} \right| \right)}{a^4 d + a^2 b^2 d} \\
& - \frac{3 \left(-ab^2\right)^{\frac{1}{3}} b^2 \arctan \left( \frac{\sqrt{3} \left(\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 2 \tan(dx+c)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{\left(\sqrt{3} a^4 + \sqrt{3} a^2 b^2\right) d} \\
& - \frac{\left(-ab^2\right)^{\frac{1}{3}} b^2 \log \left( \left(-\frac{a}{b}\right)^{\frac{2}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} \right)}{2 \left(a^4 + a^2 b^2\right) d} \\
& + \frac{\left(\sqrt{3} a - b\right) \arctan \left( \sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} \right)}{2 \left(a^2 d + b^2 d\right)} \\
& - \frac{\left(\sqrt{3} a + b\right) \arctan \left( -\sqrt{3} + 2 \tan(dx+c)^{\frac{1}{3}} \right)}{2 \left(a^2 d + b^2 d\right)} \\
& - \frac{b \arctan \left( \tan(dx+c)^{\frac{1}{3}} \right)}{a^2 d + b^2 d} - \frac{a \log \left( \tan(dx+c)^{\frac{4}{3}} - \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4 \left(a^2 d + b^2 d\right)} \\
& - \frac{3 b \log \left( \sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4 \left(\sqrt{3} a^2 d + \sqrt{3} b^2 d\right)} \\
& + \frac{3 b \log \left( -\sqrt{3} \tan(dx+c)^{\frac{1}{3}} + \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{4 \left(\sqrt{3} a^2 d + \sqrt{3} b^2 d\right)} \\
& + \frac{a \log \left( \tan(dx+c)^{\frac{2}{3}} + 1 \right)}{2 \left(a^2 d + b^2 d\right)} - \frac{3}{2 a d \tan(dx+c)^{\frac{2}{3}}}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan(dx+c) + a) \tan(dx+c)^{\frac{5}{3}}} dx$$

#### 64.49 Problem number 958

$$\int (a + ia \tan(e + fx)) \sqrt{c - ic \tan(e + fx)} dx$$

Optimal antiderivative

$$\frac{2ia \sqrt{c - ic \tan(fx + e)}}{f}$$

command

`integrate((c-I*c*tan(f*x+e))^(1/2)*(a+I*a*tan(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2i \sqrt{-ic \tan(fx + e) + c} a}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 64.50 Problem number 1046

$$\int (a + ia \tan(e + fx))(c - ic \tan(e + fx))^n dx$$

Optimal antiderivative

$$\frac{ia(c - ic \tan(fx + e))^n}{fn}$$

command

```
integrate((a+I*a*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i(-ic \tan(fx + e) + c)^n a}{fn}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 64.51 Problem number 1054

$$\int (a + ia \tan(e + fx))^m (c - ic \tan(e + fx)) dx$$

Optimal antiderivative

$$\frac{ic(a + ia \tan(fx + e))^m}{fm}$$

command

```
integrate((a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{i(ia \tan(fx + e) + a)^m c}{fm}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 65 Test file number 104

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/104\_4.3.3.1-a+b\_tan<sup>m</sup>-c+d\_tan<sup>n</sup>-A+B\_tan-

### 65.1 Problem number 112

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(-1)^{\frac{1}{4}} a(iA+B) \arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx+c)}\right)\right)}{d} - \frac{2a(iA+B)\left(\sqrt{\tan(dx+c)}\right)}{d} \\ & + \frac{2a(-iB+A)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3d} + \frac{2a(iA+B)\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5d} + \frac{2iaB\left(\tan^{\frac{7}{2}}(dx+c)\right)}{7d} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i-1)\sqrt{2}(Aa-iBa) \arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} \\ & - \frac{2\left(-15iBad^6 \tan(dx+c)^{\frac{7}{2}} - 21iAad^6 \tan(dx+c)^{\frac{5}{2}} - 21Bad^6 \tan(dx+c)^{\frac{5}{2}} - 35Aad^6 \tan(dx+c)^{\frac{3}{2}} + 35iBa\right)}{105d^7} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B \tan(dx+c) + A)(ia \tan(dx+c) + a) \tan(dx+c)^{\frac{5}{2}} dx$$

### 65.2 Problem number 113

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia \tan(c+dx))(A+B \tan(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(-1)^{\frac{1}{4}} a(-iB+A) \arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx+c)}\right)\right)}{d} + \frac{2a(-iB+A)\left(\sqrt{\tan(dx+c)}\right)}{d} \\ & + \frac{2a(iA+B)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3d} + \frac{2iaB\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5d} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i-1)\sqrt{2}(iAa+Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(-3iBad^4\tan(dx+c)^{\frac{5}{2}}-5iAad^4\tan(dx+c)^{\frac{3}{2}}-5Bad^4\tan(dx+c)^{\frac{3}{2}}-15Aad^4\sqrt{\tan(dx+c)}+15iBad^4\right)}{15d^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B\tan(dx+c)+A)(ia\tan(dx+c)+a)\tan(dx+c)^{\frac{3}{2}}dx$$

### 65.3 Problem number 114

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

Optimal antiderivative

$$\frac{2(-1)^{\frac{1}{4}}a(iA+B)\arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx+c)}\right)\right)}{d} + \frac{2a(iA+B)\left(\sqrt{\tan(dx+c)}\right)}{d} + \frac{2iaB\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3d}$$

command

```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i-1)\sqrt{2}(Aa-iBa)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(-iBad^2\tan(dx+c)^{\frac{3}{2}}-3iAad^2\sqrt{\tan(dx+c)}-3Bad^2\sqrt{\tan(dx+c)}\right)}{3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B\tan(dx+c)+A)(ia\tan(dx+c)+a)\sqrt{\tan(dx+c)}dx$$



## 65.4 Problem number 115

$$\int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Optimal antiderivative

$$-\frac{2(-1)^{\frac{1}{4}} a(-iB + A) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} + \frac{2iaB\left(\sqrt{\tan(dx + c)}\right)}{d}$$

command

`integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i - 1) \sqrt{2} (-i Aa - Ba) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} + \frac{2i Ba \sqrt{\tan(dx + c)}}{d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 65.5 Problem number 116

$$\int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$-\frac{2(-1)^{\frac{1}{4}} a(iA + B) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} - \frac{2aA}{d\sqrt{\tan(dx + c)}}$$

command

`integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(i + 1) \sqrt{2} (-i Aa - Ba) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} - \frac{2 Aa}{d\sqrt{\tan(dx + c)}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)}{\tan(dx + c)^{\frac{3}{2}}} dx$$

### 65.6 Problem number 117

$$\int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(-1)^{\frac{1}{4}} a(-iB + A) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} - \frac{2a(iA + B)}{d\sqrt{\tan(dx + c)}} - \frac{2aA}{3d \tan(dx + c)^{\frac{3}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i-1)\sqrt{2}(-iAa - Ba) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx + c)}\right)}{d} - \frac{2(3iAa \tan(dx + c) + 3Ba \tan(dx + c) + Aa)}{3d \tan(dx + c)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)}{\tan(dx + c)^{\frac{5}{2}}} dx$$

### 65.7 Problem number 118

$$\int \frac{(a + ia \tan(c + dx))(A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{2(-1)^{\frac{1}{4}} a(iA + B) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} + \frac{2a(-iB + A)}{d\sqrt{\tan(dx + c)}} - \frac{2aA}{5d \tan(dx + c)^{\frac{5}{2}}} - \frac{2a(iA + B)}{3d \tan(dx + c)^{\frac{3}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i+1)\sqrt{2}(iAa+Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} + \frac{2\left(15Aa\tan(dx+c)^2-15iBa\tan(dx+c)^2-5Aa\tan(dx+c)-5Ba\tan(dx+c)-3Aa\right)}{15d\tan(dx+c)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B\tan(dx+c)+A)(ia\tan(dx+c)+a)}{\tan(dx+c)^{\frac{7}{2}}} dx$$

### 65.8 Problem number 119

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{4(-1)^{\frac{1}{4}}a^2(iA+B)\arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx+c)}\right)\right)}{d} - \frac{4a^2(iA+B)\left(\sqrt{\tan(dx+c)}\right)}{d} \\ & + \frac{4a^2(-iB+A)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{3d} + \frac{4a^2(iA+B)\left(\tan^{\frac{5}{2}}(dx+c)\right)}{5d} \\ & - \frac{2a^2(-11iB+9A)\left(\tan^{\frac{7}{2}}(dx+c)\right)}{63d} + \frac{2iB\left(\tan^{\frac{7}{2}}(dx+c)\right)(a^2+ia^2\tan(dx+c))}{9d} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2i-2)\sqrt{2}(Aa^2-iBa^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{d} - \frac{2\left(35Ba^2d^8\tan(dx+c)^{\frac{9}{2}}+45Aa^2d^8\tan(dx+c)^{\frac{7}{2}}-90iBa^2d^8\tan(dx+c)^{\frac{7}{2}}-126iAa^2d^8\tan(dx+c)^{\frac{5}{2}}-126\right)}{15d^2\tan(dx+c)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B\tan(dx+c)+A)(ia\tan(dx+c)+a)^2\tan(dx+c)^{\frac{5}{2}} dx$$

### 65.9 Problem number 120

$$\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(-1)^{\frac{1}{4}} a^2(-iB + A) \arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx + c)}\right)\right)}{d} \\ & + \frac{4a^2(-iB + A)\left(\sqrt{\tan(dx + c)}\right)}{d} + \frac{4a^2(iA + B)\left(\tan^{\frac{3}{2}}(dx + c)\right)}{3d} \\ & - \frac{2a^2(-9iB + 7A)\left(\tan^{\frac{5}{2}}(dx + c)\right)}{35d} + \frac{2iB\left(\tan^{\frac{5}{2}}(dx + c)\right)(a^2 + ia^2 \tan(dx + c))}{7d} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(2i - 2) \sqrt{2} (i A a^2 + B a^2) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} \\ & - \frac{2 \left(15 B a^2 d^6 \tan(dx + c)^{\frac{7}{2}} + 21 A a^2 d^6 \tan(dx + c)^{\frac{5}{2}} - 42i B a^2 d^6 \tan(dx + c)^{\frac{5}{2}} - 70i A a^2 d^6 \tan(dx + c)^{\frac{3}{2}} - 70 B\right)}{105 d^7} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B \tan(dx + c) + A)(i a \tan(dx + c) + a)^2 \tan(dx + c)^{\frac{3}{2}} dx$$

### 65.10 Problem number 121

$$\int \sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2(A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(-1)^{\frac{1}{4}} a^2(iA + B) \arctan\left((-1)^{\frac{3}{4}}\left(\sqrt{\tan(dx + c)}\right)\right)}{d} + \frac{4a^2(iA + B)\left(\sqrt{\tan(dx + c)}\right)}{d} \\ & - \frac{2a^2(-7iB + 5A)\left(\tan^{\frac{3}{2}}(dx + c)\right)}{15d} + \frac{2iB\left(\tan^{\frac{3}{2}}(dx + c)\right)(a^2 + ia^2 \tan(dx + c))}{5d} \end{aligned}$$

command

`integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2i - 2) \sqrt{2} (Aa^2 - iBa^2) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} - \frac{2 \left(3Ba^2d^4 \tan(dx + c)^{\frac{5}{2}} + 5Aa^2d^4 \tan(dx + c)^{\frac{3}{2}} - 10iBa^2d^4 \tan(dx + c)^{\frac{3}{2}} - 30iAa^2d^4 \sqrt{\tan(dx + c)} - 30Ba^2d^4\right)}{15d^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B \tan(dx + c) + A)(ia \tan(dx + c) + a)^2 \sqrt{\tan(dx + c)} dx$$

### 65.11 Problem number 122

$$\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Optimal antiderivative

$$\frac{4(-1)^{\frac{1}{4}} a^2 (-iB + A) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} - \frac{2a^2(-5iB + 3A) \left(\sqrt{\tan(dx + c)}\right)}{3d} + \frac{2iB \left(\sqrt{\tan(dx + c)}\right) (a^2 + ia^2 \tan(dx + c))}{3d}$$

command

`integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2i - 2) \sqrt{2} (-iAa^2 - Ba^2) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} - \frac{2 \left(Ba^2d^2 \tan(dx + c)^{\frac{3}{2}} + 3Aa^2d^2 \sqrt{\tan(dx + c)} - 6iBa^2d^2 \sqrt{\tan(dx + c)}\right)}{3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 65.12 Problem number 123

$$\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(-1)^{\frac{1}{4}} a^2 (iA + B) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} \\ & + \frac{2a^2 (iA - B) \left( \sqrt{\tan(dx + c)} \right)}{d} - \frac{2A(a^2 + ia^2 \tan(dx + c))}{d \sqrt{\tan(dx + c)}} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{2Ba^2 \sqrt{\tan(dx + c)}}{d} \\ & - \frac{(2i + 2) \sqrt{2} (-iAa^2 - Ba^2) \arctan \left( -\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)} \right)}{d} - \frac{2Aa^2}{d \sqrt{\tan(dx + c)}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)^2}{\tan(dx + c)^{\frac{3}{2}}} dx$$

### 65.13 Problem number 124

$$\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4(-1)^{\frac{1}{4}} a^2 (-iB + A) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} \\ & - \frac{2a^2 (5iA + 3B)}{3d \sqrt{\tan(dx + c)}} - \frac{2A(a^2 + ia^2 \tan(dx + c))}{3d \tan(dx + c)^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2i - 2) \sqrt{2} (-i Aa^2 - Ba^2) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} - \frac{2(6i Aa^2 \tan(dx + c) + 3Ba^2 \tan(dx + c) + Aa^2)}{3d \tan(dx + c)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)^2}{\tan(dx + c)^{\frac{5}{2}}} dx$$

#### 65.14 Problem number 125

$$\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{4(-1)^{\frac{1}{4}} a^2 (iA + B) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} + \frac{4a^2(-iB + A)}{d\sqrt{\tan(dx + c)}} - \frac{2a^2(7iA + 5B)}{15d \tan(dx + c)^{\frac{3}{2}}} - \frac{2A(a^2 + ia^2 \tan(dx + c))}{5d \tan(dx + c)^{\frac{5}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2i + 2) \sqrt{2} (-i Aa^2 - Ba^2) \arctan\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} + \frac{2\left(30 Aa^2 \tan(dx + c)^2 - 30i Ba^2 \tan(dx + c)^2 - 10i Aa^2 \tan(dx + c) - 5 Ba^2 \tan(dx + c) - 3 Aa^2\right)}{15d \tan(dx + c)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)^2}{\tan(dx + c)^{\frac{7}{2}}} dx$$

## 65.15 Problem number 126

$$\int \frac{(a + ia \tan(c + dx))^2 (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4(-1)^{\frac{1}{4}} a^2 (-iB + A) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} + \frac{4a^2 (iA + B)}{d \sqrt{\tan(dx + c)}} \\ & - \frac{2a^2 (9iA + 7B)}{35d \tan(dx + c)^{\frac{5}{2}}} + \frac{4a^2 (-iB + A)}{3d \tan(dx + c)^{\frac{3}{2}}} - \frac{2A(a^2 + ia^2 \tan(dx + c))}{7d \tan(dx + c)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(2i - 2) \sqrt{2} (-iAa^2 - Ba^2) \arctan \left( -\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)} \right)}{d} \\ & - \frac{2 \left( -210iAa^2 \tan(dx + c)^3 - 210Ba^2 \tan(dx + c)^3 - 70Aa^2 \tan(dx + c)^2 + 70iBa^2 \tan(dx + c)^2 + 42iAa^2 \tan(dx + c) \right)}{105d \tan(dx + c)^{\frac{7}{2}}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(ia \tan(dx + c) + a)^2}{\tan(dx + c)^{\frac{9}{2}}} dx$$

## 65.16 Problem number 127

$$\int \tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3 (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(-1)^{\frac{1}{4}} a^3 (-iB + A) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} \\ & + \frac{8a^3 (-iB + A) \left( \sqrt{\tan(dx + c)} \right)}{d} + \frac{8a^3 (iA + B) \left( \tan^{\frac{3}{2}}(dx + c) \right)}{3d} \\ & - \frac{16a^3 (-19iB + 18A) \left( \tan^{\frac{5}{2}}(dx + c) \right)}{315d} + \frac{2iaB \left( \tan^{\frac{5}{2}}(dx + c) \right) (a + ia \tan(dx + c))^2}{9d} \\ & - \frac{2(-13iB + 9A) \left( \tan^{\frac{5}{2}}(dx + c) \right) (a^3 + ia^3 \tan(dx + c))}{63d} \end{aligned}$$



command

`integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4i - 4) \sqrt{2} (i A a^3 + B a^3) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} \\ - \frac{2 \left( 35i B a^3 d^8 \tan(dx + c)^{\frac{9}{2}} + 45i A a^3 d^8 \tan(dx + c)^{\frac{7}{2}} + 135 B a^3 d^8 \tan(dx + c)^{\frac{7}{2}} + 189 A a^3 d^8 \tan(dx + c)^{\frac{5}{2}} - 25 \right)}{105 d^7}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B \tan(dx + c) + A)(i a \tan(dx + c) + a)^3 \tan(dx + c)^{\frac{3}{2}} dx$$

### 65.17 Problem number 128

$$\int \sqrt{\tan(c + dx)} (a + i a \tan(c + dx))^3 (A + B \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{8(-1)^{\frac{1}{4}} a^3 (iA + B) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} + \frac{8a^3 (iA + B) \left(\sqrt{\tan(dx + c)}\right)}{d} \\ - \frac{8a^3 (-23iB + 21A) \left(\tan^{\frac{3}{2}}(dx + c)\right)}{105d} + \frac{2iaB \left(\tan^{\frac{3}{2}}(dx + c)\right) (a + ia \tan(dx + c))^2}{7d} \\ - \frac{2(-11iB + 7A) \left(\tan^{\frac{3}{2}}(dx + c)\right) (a^3 + ia^3 \tan(dx + c))}{35d}$$

command

`integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4i - 4) \sqrt{2} (A a^3 - i B a^3) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} \\ - \frac{2 \left( 15i B a^3 d^6 \tan(dx + c)^{\frac{7}{2}} + 21i A a^3 d^6 \tan(dx + c)^{\frac{5}{2}} + 63 B a^3 d^6 \tan(dx + c)^{\frac{5}{2}} + 105 A a^3 d^6 \tan(dx + c)^{\frac{3}{2}} - 140 \right)}{105 d^7}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (B \tan(dx + c) + A)(i a \tan(dx + c) + a)^3 \sqrt{\tan(dx + c)} dx$$

## 65.18 Problem number 129

$$\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\sqrt{\tan(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(-1)^{\frac{1}{4}} a^3 (-iB + A) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} \\ & - \frac{16a^3 (-6iB + 5A) \left(\sqrt{\tan(dx + c)}\right)}{15d} + \frac{2iaB \left(\sqrt{\tan(dx + c)}\right) (a + ia \tan(dx + c))^2}{5d} \\ & - \frac{2(-9iB + 5A) \left(\sqrt{\tan(dx + c)}\right) (a^3 + ia^3 \tan(dx + c))}{15d} \end{aligned}$$

command

```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(4i - 4) \sqrt{2} (-i A a^3 - B a^3) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} \\ & - \frac{2 \left(3i B a^3 d^4 \tan(dx + c)^{\frac{5}{2}} + 5i A a^3 d^4 \tan(dx + c)^{\frac{3}{2}} + 15 B a^3 d^4 \tan(dx + c)^{\frac{3}{2}} + 45 A a^3 d^4 \sqrt{\tan(dx + c)} - 60i B\right)}{15 d^5} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 65.19 Problem number 130

$$\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8(-1)^{\frac{1}{4}} a^3 (iA + B) \arctan\left((-1)^{\frac{3}{4}} \left(\sqrt{\tan(dx + c)}\right)\right)}{d} - \frac{16a^3 B \left(\sqrt{\tan(dx + c)}\right)}{3d} \\ & - \frac{2aA(a + ia \tan(dx + c))^2}{d \sqrt{\tan(dx + c)}} + \frac{2(3iA - B) \left(\sqrt{\tan(dx + c)}\right) (a^3 + ia^3 \tan(dx + c))}{3d} \end{aligned}$$

command

`integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 A a^3}{d \sqrt{\tan (d x+c)}}-\frac{(4 i+4) \sqrt{2}\left(-i A a^3-B a^3\right) \arctan \left(-\left(\frac{1}{2} i-\frac{1}{2}\right) \sqrt{2} \sqrt{\tan (d x+c)}\right)}{d} \\ -\frac{2\left(i B a^3 d^2 \tan (d x+c)^{\frac{3}{2}}+3 i A a^3 d^2 \sqrt{\tan (d x+c)}+9 B a^3 d^2 \sqrt{\tan (d x+c)}\right)}{3 d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan (d x+c)+A)(i a \tan (d x+c)+a)^3}{\tan (d x+c)^{\frac{3}{2}}} d x$$

## 65.20 Problem number 131

$$\int \frac{(a+i a \tan (c+d x))^3(A+B \tan (c+d x))}{\tan ^{\frac{5}{2}}(c+d x)} d x$$

Optimal antiderivative

$$\frac{8(-1)^{\frac{1}{4}} a^3(-i B+A) \arctan \left((-1)^{\frac{3}{4}}\left(\sqrt{\tan (d x+c)}\right)\right)}{d}-\frac{16 a^3 A\left(\sqrt{\tan (d x+c)}\right)}{3 d} \\ -\frac{2 a A(a+i a \tan (d x+c))^2}{3 d \tan (d x+c)^{\frac{3}{2}}}-\frac{2(7 i A+3 B)\left(a^3+i a^3 \tan (d x+c)\right)}{3 d \sqrt{\tan (d x+c)}}$$

command

`integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 i B a^3 \sqrt{\tan (d x+c)}}{d} \\ -\frac{(4 i-4) \sqrt{2}\left(-i A a^3-B a^3\right) \arctan \left(-\left(\frac{1}{2} i-\frac{1}{2}\right) \sqrt{2} \sqrt{\tan (d x+c)}\right)}{d} \\ +\frac{2\left(-9 i A a^3 \tan (d x+c)-3 B a^3 \tan (d x+c)-A a^3\right)}{3 d \tan (d x+c)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan (d x+c)+A)(i a \tan (d x+c)+a)^3}{\tan (d x+c)^{\frac{5}{2}}} d x$$

### 65.21 Problem number 132

$$\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)} dx$$

Optimal antiderivative

$$\frac{8(-1)^{\frac{1}{4}} a^3 (iA + B) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} + \frac{16a^3 (-5iB + 6A)}{15d \sqrt{\tan(dx + c)}} - \frac{2aA(a + ia \tan(dx + c))^2}{5d \tan(dx + c)^{\frac{5}{2}}} - \frac{2(9iA + 5B)(a^3 + ia^3 \tan(dx + c))}{15d \tan(dx + c)^{\frac{3}{2}}}$$

command

```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4i + 4) \sqrt{2} (i A a^3 + B a^3) \arctan \left( -\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)} \right)}{d} + \frac{2 \left( 60 A a^3 \tan(dx + c)^2 - 45i B a^3 \tan(dx + c)^2 - 15i A a^3 \tan(dx + c) - 5 B a^3 \tan(dx + c) - 3 A a^3 \right)}{15 d \tan(dx + c)^{\frac{5}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(i a \tan(dx + c) + a)^3}{\tan(dx + c)^{\frac{7}{2}}} dx$$

### 65.22 Problem number 133

$$\int \frac{(a + ia \tan(c + dx))^3 (A + B \tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)} dx$$

Optimal antiderivative

$$-\frac{8(-1)^{\frac{1}{4}} a^3 (-iB + A) \arctan \left( (-1)^{\frac{3}{4}} \left( \sqrt{\tan(dx + c)} \right) \right)}{d} + \frac{8a^3 (iA + B)}{d \sqrt{\tan(dx + c)}} + \frac{8a^3 (-21iB + 23A)}{105d \tan(dx + c)^{\frac{3}{2}}} - \frac{2aA(a + ia \tan(dx + c))^2}{7d \tan(dx + c)^{\frac{7}{2}}} - \frac{2(11iA + 7B)(a^3 + ia^3 \tan(dx + c))}{35d \tan(dx + c)^{\frac{5}{2}}}$$

command

`integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4i - 4) \sqrt{2} (-i Aa^3 - Ba^3) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{d} - \frac{2 \left(-420i Aa^3 \tan(dx + c)^3 - 420 Ba^3 \tan(dx + c)^3 - 140 Aa^3 \tan(dx + c)^2 + 105i Ba^3 \tan(dx + c)^2 + 63i Aa^3 \tan(dx + c)\right)}{105 d \tan(dx + c)^{\frac{7}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A)(i a \tan(dx + c) + a)^3}{\tan(dx + c)^{\frac{9}{2}}} dx$$

### 65.23 Problem number 134

$$\int \frac{\tan^{\frac{5}{2}}(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{1}{8} + \frac{i}{8}\right) \left((4 + i) A + (1 + 6i) B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{\left(\frac{1}{8} + \frac{i}{8}\right) \left((4 + i) A + (1 + 6i) B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{\left(-\frac{1}{16} - \frac{i}{16}\right) \left((1 + 4i) A + (-6 - i) B\right) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{ad} \\ & - \frac{\left((3 - 5i) A + (5 + 7i) B\right) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & - \frac{5(iA - B) \left(\sqrt{\tan(dx + c)}\right)}{2ad} - \frac{(7iB + 3A) \left(\tan^{\frac{3}{2}}(dx + c)\right)}{6ad} + \frac{(iA - B) \left(\tan^{\frac{5}{2}}(dx + c)\right)}{2d(a + ia \tan(dx + c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(i-1)\sqrt{2}(-2iA+3B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} + \frac{(i+1)\sqrt{2}(-iA-B)\arctan\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} - \frac{A\sqrt{\tan(dx+c)}+iB\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)} - \frac{2\left(iBa^2d^2\tan(dx+c)^{\frac{3}{2}}+3iAa^2d^2\sqrt{\tan(dx+c)}-3Ba^2d^2\sqrt{\tan(dx+c)}\right)}{3a^3d^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \tan(dx+c)^{\frac{5}{2}}}{i a \tan(dx+c) + a} dx$$

### 65.24 Problem number 135

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)} dx$$

Optimal antiderivative

$$\frac{((1-3i)A+(3+5i)B)\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\sqrt{2}}{8ad} + \frac{\left(-\frac{1}{8}-\frac{i}{8}\right)\left((1+2i)A+(-4-i)B\right)\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\sqrt{2}}{ad} + \frac{\left(-\frac{1}{16}-\frac{i}{16}\right)\left((2+i)A+(1+4i)B\right)\ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right)\sqrt{2}}{ad} + \frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left((2+i)A+(1+4i)B\right)\ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right)\sqrt{2}}{ad} - \frac{(5iB+A)\left(\sqrt{\tan(dx+c)}\right)}{2ad} + \frac{(iA-B)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{2d(a+ia\tan(dx+c))}$$

command

`integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(i+1)\sqrt{2}(A-iB)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{4ad} + \frac{(i-1)\sqrt{2}(A+2iB)\arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{2ad} - \frac{2iB\sqrt{\tan(dx+c)}}{ad} - \frac{-iA\sqrt{\tan(dx+c)}+B\sqrt{\tan(dx+c)}}{2ad(\tan(dx+c)-i)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A) \tan(dx + c)^{\frac{3}{2}}}{i a \tan(dx + c) + a} dx$$

### 65.25 Problem number 136

$$\int \frac{\sqrt{\tan(c + dx)} (A + B \tan(c + dx))}{a + i a \tan(c + dx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A + (2 - i) B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) (A + (2 - i) B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{\left(\frac{1}{16} + \frac{i}{16}\right) (A + (-2 - i) B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{ad} \\ & + \frac{\left(-\frac{1}{16} - \frac{i}{16}\right) (A + (-2 - i) B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{ad} \\ & + \frac{(iA - B) \left(\sqrt{\tan(dx + c)}\right)}{2d(a + i a \tan(dx + c))} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i - 1) \sqrt{2} B \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{2 ad} \\ & - \frac{(i - 1) \sqrt{2} (A - i B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{4 ad} \\ & + \frac{A \sqrt{\tan(dx + c)} + i B \sqrt{\tan(dx + c)}}{2 ad(\tan(dx + c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx + c) + A) \sqrt{\tan(dx + c)}}{i a \tan(dx + c) + a} dx$$

## 65.26 Problem number 137

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{1}{8} - \frac{i}{8}\right) ((2 + i) A + B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{\left(\frac{1}{8} - \frac{i}{8}\right) ((2 + i) A + B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & - \frac{((3 + i) A + (-1 - i) B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & + \frac{((3 + i) A + (-1 - i) B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & + \frac{(iB + A) \left(\sqrt{\tan(dx + c)}\right)}{2d(a + ia \tan(dx + c))} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i - 1) \sqrt{2} A \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{2ad} \\ & - \frac{(i - 1) \sqrt{2} (i A + B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{4ad} \\ & - \frac{i A \sqrt{\tan(dx + c)} - B \sqrt{\tan(dx + c)}}{2ad(\tan(dx + c) - i)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a) \sqrt{\tan(dx + c)}} dx$$



## 65.27 Problem number 138

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((5 + 3i)A + (-3 + i)B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{8ad} \\ & + \frac{((-5 - 3i)A + (3 - i)B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{8ad} \\ & + \frac{\left(-\frac{1}{16} + \frac{i}{16}\right) ((4 + i)A + (1 + 2i)B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{ad} \\ & + \frac{((5 - 3i)A + (3 + i)B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & + \frac{-iB - 5A}{2ad\sqrt{\tan(dx + c)}} + \frac{iB + A}{2d\sqrt{\tan(dx + c)}(a + ia \tan(dx + c))} \end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i + 1) \sqrt{2} (iA + B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{4ad} \\ & + \frac{(i - 1) \sqrt{2} (-2iA + B) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{2ad} \\ & + \frac{-5iA \tan(dx + c) + B \tan(dx + c) - 4A}{2\left(i \tan(dx + c)^{\frac{3}{2}} + \sqrt{\tan(dx + c)}\right)ad} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a) \tan(dx + c)^{\frac{3}{2}}} dx$$

## 65.28 Problem number 139

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{((7 - 5i)A + (5 + 3i)B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{8ad} \\ & + \frac{\left(-\frac{1}{8} + \frac{i}{8}\right) ((6 + i)A + (1 + 4i)B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{ad} \\ & + \frac{((7 + 5i)A + (-5 + 3i)B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & + \frac{((-7 - 5i)A + (5 - 3i)B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{16ad} \\ & + \frac{\frac{5iA}{2} - \frac{5B}{2}}{ad\sqrt{\tan(dx + c)}} + \frac{-3iB - 7A}{6ad \tan(dx + c)^{\frac{3}{2}}} + \frac{iB + A}{2d \tan(dx + c)^{\frac{3}{2}} (a + ia \tan(dx + c))} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i - 1) \sqrt{2} (iA + B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{4ad} \\ & - \frac{(i - 1) \sqrt{2} (3A + 2iB) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{2ad} \\ & - \frac{-iA\sqrt{\tan(dx + c)} + B\sqrt{\tan(dx + c)}}{2ad(\tan(dx + c) - i)} + \frac{2i(3A \tan(dx + c) + 3iB \tan(dx + c) + iA)}{3ad \tan(dx + c)^{\frac{3}{2}}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a) \tan(dx + c)^{\frac{5}{2}}} dx$$

## 65.29 Problem number 140

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{((9+5i)A+(-25+21i)B) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & -\frac{((9+5i)A+(-25+21i)B) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & +\frac{\left(-\frac{1}{64}+\frac{i}{64}\right) \left((7+2i)A+(2+23i)B\right) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{a^2d} \\ & +\frac{\left(\frac{1}{64}-\frac{i}{64}\right) \left((7+2i)A+(2+23i)B\right) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{a^2d} \\ & +\frac{5(iA-5B)\left(\sqrt{\tan(dx+c)}\right)}{8a^2d} +\frac{(7iB+3A)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{8a^2d(1+i \tan(dx+c))} +\frac{(iA-B)\left(\tan^{\frac{5}{2}}(dx+c)\right)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & -\frac{(i+1) \sqrt{2}(-iA-B) \arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{8a^2d} \\ & +\frac{(i-1) \sqrt{2}(-7iA+23B) \arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^2d} -\frac{2B \sqrt{\tan(dx+c)}}{a^2d} \\ & +\frac{7A \tan(dx+c)^{\frac{3}{2}}+11iB \tan(dx+c)^{\frac{3}{2}}-5iA \sqrt{\tan(dx+c)}+9B \sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c)+A) \tan(dx+c)^{\frac{5}{2}}}{(ia \tan(dx+c)+a)^2} dx$$

## 65.30 Problem number 141

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((1+3i)A+(9+5i)B) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & - \frac{((1+3i)A+(9+5i)B) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & + \frac{((1-3i)A+(-9+5i)B) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{64a^2d} \\ & - \frac{((1-3i)A+(-9+5i)B) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{64a^2d} \\ & + \frac{(5iB+A)\left(\sqrt{\tan(dx+c)}\right)}{8a^2d(1+i \tan(dx+c))} + \frac{(iA-B)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i+1) \sqrt{2} (A-iB) \arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{8a^2d} \\ & + \frac{(i-1) \sqrt{2} (A-7iB) \arctan\left(-\left(\frac{1}{2}i+\frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^2d} \\ & - \frac{3iA \tan(dx+c)^{\frac{3}{2}} - 7B \tan(dx+c)^{\frac{3}{2}} + A\sqrt{\tan(dx+c)} + 5iB\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \tan(dx+c)^{\frac{3}{2}}}{(ia \tan(dx+c) + a)^2} dx$$

## 65.31 Problem number 142

$$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((-1+3i)A+(1+3i)B) \arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & - \frac{((-1+3i)A+(1+3i)B) \arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^2d} \\ & + \frac{((1+3i)A+(1-3i)B) \ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{64a^2d} \\ & - \frac{((1+3i)A+(1-3i)B) \ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right) \sqrt{2}}{64a^2d} \\ & + \frac{(iA+3B)\left(\sqrt{\tan(dx+c)}\right)}{8a^2d(1+i \tan(dx+c))} + \frac{(iA-B)\left(\sqrt{\tan(dx+c)}\right)}{4d(a+ia \tan(dx+c))^2} \end{aligned}$$

command

`integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i-1)\sqrt{2}(iA-B) \arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^2d} \\ & - \frac{(i+1)\sqrt{2}(-iA-B) \arctan\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{8a^2d} \\ & + \frac{A \tan(dx+c)^{\frac{3}{2}} - 3iB \tan(dx+c)^{\frac{3}{2}} - 3iA\sqrt{\tan(dx+c)} - B\sqrt{\tan(dx+c)}}{8a^2d(\tan(dx+c)-i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \sqrt{\tan(dx+c)}}{(ia \tan(dx+c) + a)^2} dx$$

## 65.32 Problem number 143

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{32} - \frac{i}{32}\right) \left((-2 + 7i)A + (1 + 2i)B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^2 d} \\ & + \frac{\left((9 - 5i)A + (1 - 3i)B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{32a^2 d} \\ & + \frac{\left(\frac{1}{64} + \frac{i}{64}\right) \left((-7 + 2i)A + (2 + i)B\right) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^2 d} \\ & + \frac{\left((9 + 5i)A + (-1 - 3i)B\right) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{64a^2 d} \\ & + \frac{(iB + 5A) \left(\sqrt{\tan(dx + c)}\right)}{8a^2 d (1 + i \tan(dx + c))} + \frac{(iB + A) \left(\sqrt{\tan(dx + c)}\right)}{4d (a + ia \tan(dx + c))^2} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i + 1) \sqrt{2} (A - iB) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{8a^2 d} \\ & + \frac{(i - 1) \sqrt{2} (7A - iB) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^2 d} \\ & - \frac{5iA \tan(dx + c)^{\frac{3}{2}} - B \tan(dx + c)^{\frac{3}{2}} + 7A \sqrt{\tan(dx + c)} + 3iB \sqrt{\tan(dx + c)}}{8a^2 d (\tan(dx + c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a)^2 \sqrt{\tan(dx + c)}} dx$$

## 65.33 Problem number 144

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((25 + 21i)A + (-9 + 5i)B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{32a^2d} \\ & + \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) ((2 + 23i)A + (-7 - 2i)B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^2d} \\ & + \frac{\left(-\frac{1}{64} + \frac{i}{64}\right) ((23 + 2i)A + (2 + 7i)B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^2d} \\ & + \frac{\left(\frac{1}{64} - \frac{i}{64}\right) ((23 + 2i)A + (2 + 7i)B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^2d} \\ & - \frac{5(iB + 5A)}{8a^2d\sqrt{\tan(dx + c)}} + \frac{3iB + 7A}{8a^2d\sqrt{\tan(dx + c)}(1 + i \tan(dx + c))} \\ & + \frac{iB + A}{4d\sqrt{\tan(dx + c)}(a + ia \tan(dx + c))^2} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i - 1) \sqrt{2} (A - i B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{8a^2d} \\ & + \frac{(i - 1) \sqrt{2} (-23iA + 7B) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^2d} - \frac{2A}{a^2d\sqrt{\tan(dx + c)}} \\ & - \frac{9A \tan(dx + c)^{\frac{3}{2}} + 5iB \tan(dx + c)^{\frac{3}{2}} - 11iA \sqrt{\tan(dx + c)} + 7B \sqrt{\tan(dx + c)}}{8a^2d(\tan(dx + c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a)^2 \tan(dx + c)^{\frac{3}{2}}} dx$$

## 65.34 Problem number 145

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) \left((47 + 2i)A + (2 + 23i)B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^2 d} \\ & + \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) \left((47 + 2i)A + (2 + 23i)B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^2 d} \\ & + \frac{\left((49 + 45i)A + (-25 + 21i)B\right) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{64 a^2 d} \\ & + \frac{\left(-\frac{1}{64} + \frac{i}{64}\right) \left((2 + 47i)A + (-23 - 2i)B\right) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^2 d} \\ & + \frac{\frac{45iA}{8} - \frac{25B}{8}}{a^2 d \sqrt{\tan(dx + c)}} - \frac{7(3iB + 7A)}{24 a^2 d \tan(dx + c)^{\frac{3}{2}}} \\ & + \frac{5iB + 9A}{8 a^2 d (1 + i \tan(dx + c)) \tan(dx + c)^{\frac{3}{2}}} + \frac{iB + A}{4d \tan(dx + c)^{\frac{3}{2}} (a + ia \tan(dx + c))^2} \end{aligned}$$

command

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i - 1) \sqrt{2} (47 A + 23i B) \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16 a^2 d} \\ & + \frac{(i + 1) \sqrt{2} (A - i B) \arctan\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{8 a^2 d} \\ & - \frac{2(-6i A \tan(dx + c) + 3 B \tan(dx + c) + A)}{3 a^2 d \tan(dx + c)^{\frac{3}{2}}} \\ & - \frac{-13i A \tan(dx + c)^{\frac{3}{2}} + 9 B \tan(dx + c)^{\frac{3}{2}} - 15 A \sqrt{\tan(dx + c)} - 11i B \sqrt{\tan(dx + c)}}{8 a^2 d (\tan(dx + c) - i)^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a)^2 \tan(dx + c)^{\frac{5}{2}}} dx$$



## 65.35 Problem number 146

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{32} - \frac{i}{32}\right) \left((29+i)A + (1+76i)B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{a^3 d} \\ & + \frac{\left(-\frac{1}{32} - \frac{i}{32}\right) \left((29+i)A + (1+76i)B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{a^3 d} \\ & - \frac{\left((28-30i)A + (75+77i)B\right) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{64a^3 d} \\ & + \frac{\left(-\frac{1}{64} - \frac{i}{64}\right) \left((1+29i)A + (-76-i)B\right) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{a^3 d} \\ & + \frac{15(2iA-5B) \left(\sqrt{\tan(dx+c)}\right)}{8a^3 d} + \frac{7(11iB+4A) \left(\tan^{\frac{3}{2}}(dx+c)\right)}{24a^3 d} \\ & + \frac{(iA-B) \left(\tan^{\frac{9}{2}}(dx+c)\right)}{6d(a+ia \tan(dx+c))^3} + \frac{(2iB+A) \left(\tan^{\frac{7}{2}}(dx+c)\right)}{4ad(a+ia \tan(dx+c))^2} - \frac{3(2iA-5B) \left(\tan^{\frac{5}{2}}(dx+c)\right)}{8d(a^3+ia^3 \tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i+1) \sqrt{2} (-iA-B) \arctan\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^3 d} \\ & + \frac{(i-1) \sqrt{2} (-29iA+76B) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^3 d} \\ & + \frac{60A \tan(dx+c)^{\frac{5}{2}} + 105iB \tan(dx+c)^{\frac{5}{2}} - 98iA \tan(dx+c)^{\frac{3}{2}} + 182B \tan(dx+c)^{\frac{3}{2}} - 42A \sqrt{\tan(dx+c)} - 8}{24a^3 d (\tan(dx+c) - i)^3} \\ & - \frac{2 \left(-iBa^6 d^2 \tan(dx+c)^{\frac{3}{2}} - 3iAa^6 d^2 \sqrt{\tan(dx+c)} + 9Ba^6 d^2 \sqrt{\tan(dx+c)}\right)}{3a^9 d^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \tan(dx+c)^{\frac{9}{2}}}{(ia \tan(dx+c) + a)^3} dx$$

## 65.36 Problem number 147

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(\frac{1}{32} + \frac{i}{32}\right) \left((1+6i)A + (-29-i)B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{a^3 d} \\ & - \frac{\left((5-7i)A + (28+30i)B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{32a^3 d} \\ & + \frac{\left(\frac{1}{64} + \frac{i}{64}\right) \left((6+i)A + (1+29i)B\right) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{a^3 d} \\ & + \frac{\left(-\frac{1}{64} - \frac{i}{64}\right) \left((6+i)A + (1+29i)B\right) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{a^3 d} \\ & + \frac{5(6iB+A) \left(\sqrt{\tan(dx+c)}\right)}{8a^3 d} + \frac{(iA-B) \left(\tan^{\frac{7}{2}}(dx+c)\right)}{6d(a+ia \tan(dx+c))^3} \\ & + \frac{(5iB+2A) \left(\tan^{\frac{5}{2}}(dx+c)\right)}{12ad(a+ia \tan(dx+c))^2} - \frac{7(iA-4B) \left(\tan^{\frac{3}{2}}(dx+c)\right)}{24d(a^3+ia^3 \tan(dx+c))} \end{aligned}$$

command

```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i-1) \sqrt{2} (6A+29iB) \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^3 d} \\ & + \frac{(i+1) \sqrt{2} (A-iB) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16a^3 d} + \frac{2iB \sqrt{\tan(dx+c)}}{a^3 d} \\ & + \frac{-27iA \tan(dx+c)^{\frac{5}{2}} + 60B \tan(dx+c)^{\frac{5}{2}} - 38A \tan(dx+c)^{\frac{3}{2}} - 98iB \tan(dx+c)^{\frac{3}{2}} + 15iA \sqrt{\tan(dx+c)} - 4}{24a^3 d (\tan(dx+c) - i)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \tan(dx+c)^{\frac{7}{2}}}{(ia \tan(dx+c) + a)^3} dx$$

## 65.37 Problem number 148

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(2A + (5 - 7i) B) \arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{32a^3d} \\ & - \frac{(2A + (5 - 7i) B) \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) \right) \sqrt{2}}{32a^3d} \\ & - \frac{(2A + (-5 - 7i) B) \ln \left( 1 - \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{64a^3d} \\ & + \frac{(2A + (-5 - 7i) B) \ln \left( 1 + \sqrt{2} \left( \sqrt{\tan(dx+c)} \right) + \tan(dx+c) \right) \sqrt{2}}{64a^3d} \\ & + \frac{(iA - B) \left( \tan^{\frac{5}{2}}(dx+c) \right)}{6d(a+ia \tan(dx+c))^3} + \frac{(4iB + A) \left( \tan^{\frac{3}{2}}(dx+c) \right)}{12ad(a+ia \tan(dx+c))^2} + \frac{5B \left( \sqrt{\tan(dx+c)} \right)}{8d(a^3+ia^3 \tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i+1) \sqrt{2} (A - 6i B) \arctan \left( \left( \frac{1}{2}i + \frac{1}{2} \right) \sqrt{2} \sqrt{\tan(dx+c)} \right)}{16a^3d} \\ & + \frac{(i-1) \sqrt{2} (A - i B) \arctan \left( -\left( \frac{1}{2}i - \frac{1}{2} \right) \sqrt{2} \sqrt{\tan(dx+c)} \right)}{16a^3d} \\ & - \frac{6A \tan(dx+c)^{\frac{5}{2}} + 27iB \tan(dx+c)^{\frac{5}{2}} - 2iA \tan(dx+c)^{\frac{3}{2}} + 38B \tan(dx+c)^{\frac{3}{2}} - 15iB \sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c) - i)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \tan(dx+c)^{\frac{5}{2}}}{(ia \tan(dx+c) + a)^3} dx$$

## 65.38 Problem number 149

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((1+i)A+2B)\arctan\left(-1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\sqrt{2}}{32a^3d} \\ & - \frac{((1+i)A+2B)\arctan\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)\right)\sqrt{2}}{32a^3d} \\ & - \frac{((-1+i)A+2B)\ln\left(1-\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right)\sqrt{2}}{64a^3d} \\ & + \frac{((-1+i)A+2B)\ln\left(1+\sqrt{2}\left(\sqrt{\tan(dx+c)}\right)+\tan(dx+c)\right)\sqrt{2}}{64a^3d} \\ & + \frac{(iA-B)\left(\tan^{\frac{3}{2}}(dx+c)\right)}{6d(a+ia\tan(dx+c))^3} + \frac{iB\left(\sqrt{\tan(dx+c)}\right)}{4ad(a+ia\tan(dx+c))^2} + \frac{(-2iB+A)\left(\sqrt{\tan(dx+c)}\right)}{8d(a^3+ia^3\tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i+1)\sqrt{2}B\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} \\ & + \frac{(i-1)\sqrt{2}(iA+B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)}{16a^3d} \\ & - \frac{3iA\tan(dx+c)^{\frac{5}{2}}+6B\tan(dx+c)^{\frac{5}{2}}+10A\tan(dx+c)^{\frac{3}{2}}-2iB\tan(dx+c)^{\frac{3}{2}}-3iA\sqrt{\tan(dx+c)}}{24a^3d(\tan(dx+c)-i)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{3}{2}}}{(ia\tan(dx+c)+a)^3} dx$$

## 65.39 Problem number 150

$$\int \frac{\sqrt{\tan(c+dx)} (A+B \tan(c+dx))}{(a+ia \tan(c+dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{32} - \frac{i}{32}\right) \left((1+i)A+B\right) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{a^3 d} \\ & + \frac{\left(-\frac{1}{32} - \frac{i}{32}\right) \left((1+i)A+B\right) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right)\right) \sqrt{2}}{a^3 d} \\ & + \frac{(2iA+(1-i)B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{64a^3 d} \\ & - \frac{(2iA+(1-i)B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx+c)}\right) + \tan(dx+c)\right) \sqrt{2}}{64a^3 d} \\ & + \frac{(iA-B) \left(\sqrt{\tan(dx+c)}\right)}{6d(a+ia \tan(dx+c))^3} + \frac{(iA+2B) \left(\sqrt{\tan(dx+c)}\right)}{12ad(a+ia \tan(dx+c))^2} + \frac{B \left(\sqrt{\tan(dx+c)}\right)}{8d(a^3+ia^3 \tan(dx+c))} \end{aligned}$$

command

`integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i+1) \sqrt{2} A \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16 a^3 d} \\ & - \frac{(i-1) \sqrt{2} (A-iB) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx+c)}\right)}{16 a^3 d} \\ & - \frac{3i B \tan(dx+c)^{\frac{5}{2}} + 2i A \tan(dx+c)^{\frac{3}{2}} + 10 B \tan(dx+c)^{\frac{3}{2}} + 6 A \sqrt{\tan(dx+c)} - 3i B \sqrt{\tan(dx+c)}}{24 a^3 d (\tan(dx+c) - i)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(B \tan(dx+c) + A) \sqrt{\tan(dx+c)}}{(ia \tan(dx+c) + a)^3} dx$$

## 65.40 Problem number 151

$$\int \frac{A + B \tan(c + dx)}{\sqrt{\tan(c + dx)} (a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{((7 - 5i)A - 2iB) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{32a^3d} \\ & + \frac{((7 - 5i)A - 2iB) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{32a^3d} \\ & - \frac{((7 + 5i)A - 2iB) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{64a^3d} \\ & + \frac{((7 + 5i)A - 2iB) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{64a^3d} \\ & + \frac{(iB + A) \left(\sqrt{\tan(dx + c)}\right)}{6d(a + ia \tan(dx + c))^3} + \frac{(iB + 4A) \left(\sqrt{\tan(dx + c)}\right)}{12ad(a + ia \tan(dx + c))^2} + \frac{5A \left(\sqrt{\tan(dx + c)}\right)}{8d(a^3 + ia^3 \tan(dx + c))} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i + 1) \sqrt{2} (6iA + B) \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^3d} \\ & + \frac{(i - 1) \sqrt{2} (-iA - B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^3d} \\ & - \frac{15iA \tan(dx + c)^{\frac{5}{2}} + 38A \tan(dx + c)^{\frac{3}{2}} + 2iB \tan(dx + c)^{\frac{3}{2}} - 27iA \sqrt{\tan(dx + c)} + 6B \sqrt{\tan(dx + c)}}{24a^3d(\tan(dx + c) - i)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a)^3 \sqrt{\tan(dx + c)}} dx$$

## 65.41 Problem number 152

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{((30 + 28i)A + (-7 + 5i)B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{32a^3d} \\ & + \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) ((1 + 29i)A + (-6 - i)B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^3d} \\ & + \frac{\left(-\frac{1}{64} + \frac{i}{64}\right) ((29 + i)A + (1 + 6i)B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^3d} \\ & + \frac{\left(\frac{1}{64} - \frac{i}{64}\right) ((29 + i)A + (1 + 6i)B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^3d} \\ & - \frac{5(iB + 6A)}{8a^3d\sqrt{\tan(dx + c)}} + \frac{iB + A}{6d\sqrt{\tan(dx + c)}(a + ia \tan(dx + c))^3} \\ & + \frac{2iB + 5A}{12ad\sqrt{\tan(dx + c)}(a + ia \tan(dx + c))^2} + \frac{\frac{7iB}{24} + \frac{7A}{6}}{d\sqrt{\tan(dx + c)}(a^3 + ia^3 \tan(dx + c))} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{(i + 1) \sqrt{2} (29A + 6iB) \arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^3d} \\ & - \frac{(i + 1) \sqrt{2} (iA + B) \arctan\left(\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16a^3d} - \frac{2A}{a^3d\sqrt{\tan(dx + c)}} \\ & - \frac{42iA \tan(dx + c)^{\frac{5}{2}} - 15B \tan(dx + c)^{\frac{5}{2}} + 98A \tan(dx + c)^{\frac{3}{2}} + 38iB \tan(dx + c)^{\frac{3}{2}} - 60iA \sqrt{\tan(dx + c)} + 27}{24a^3d(-i \tan(dx + c) - 1)^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(ia \tan(dx + c) + a)^3 \tan(dx + c)^{\frac{3}{2}}} dx$$

## 65.42 Problem number 153

$$\int \frac{A + B \tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia \tan(c + dx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) ((76 + i) A + (1 + 29i) B) \arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^3 d} \\ & + \frac{\left(-\frac{1}{32} + \frac{i}{32}\right) ((76 + i) A + (1 + 29i) B) \arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right)\right) \sqrt{2}}{a^3 d} \\ & + \frac{((77 + 75i) A + (-30 + 28i) B) \ln\left(1 - \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{64 a^3 d} \\ & + \frac{\left(-\frac{1}{64} + \frac{i}{64}\right) ((1 + 76i) A + (-29 - i) B) \ln\left(1 + \sqrt{2} \left(\sqrt{\tan(dx + c)}\right) + \tan(dx + c)\right) \sqrt{2}}{a^3 d} \\ & + \frac{\frac{75iA}{8} - \frac{15B}{4}}{a^3 d \sqrt{\tan(dx + c)}} - \frac{7(4iB + 11A)}{24 a^3 d \tan(dx + c)^{\frac{3}{2}}} + \frac{iB + A}{6 d \tan(dx + c)^{\frac{3}{2}} (a + ia \tan(dx + c))^3} \\ & + \frac{iB + 2A}{4 a d \tan(dx + c)^{\frac{3}{2}} (a + ia \tan(dx + c))^2} + \frac{\frac{3iB}{4} + \frac{15A}{8}}{d \tan(dx + c)^{\frac{3}{2}} (a^3 + ia^3 \tan(dx + c))} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(i - 1) \sqrt{2} (i A + B) \arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16 a^3 d} \\ & - \frac{(i - 1) \sqrt{2} (76 A + 29i B) \arctan\left(-\left(\frac{1}{2}i + \frac{1}{2}\right) \sqrt{2} \sqrt{\tan(dx + c)}\right)}{16 a^3 d} \\ & - \frac{225 A \tan(dx + c)^4 + 90i B \tan(dx + c)^4 - 598i A \tan(dx + c)^3 + 242 B \tan(dx + c)^3 - 489 A \tan(dx + c)^2 - 200 A \tan(dx + c) + 200}{24 \left(-i \tan(dx + c)^{\frac{3}{2}} - \sqrt{\tan(dx + c)}\right)^3 a^3 d} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(i a \tan(dx + c) + a)^3 \tan(dx + c)^{\frac{5}{2}}} dx$$



## 65.43 Problem number 373

$$\int \frac{1 + i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{-ib + a}}\right)}{d\sqrt{-ib + a}}$$

command

```
integrate((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \operatorname{arctan}\left(\frac{2\left(i\sqrt{b \tan(dx + c) + a} a + i\sqrt{a^2 + b^2} \sqrt{b \tan(dx + c) + a}\right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} a - i\sqrt{2a + 2\sqrt{a^2 + b^2}} b + \sqrt{a^2 + b^2} \sqrt{2a + 2\sqrt{a^2 + b^2}}}\right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} d\left(-\frac{ib}{a + \sqrt{a^2 + b^2}} + 1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 65.44 Problem number 374

$$\int \frac{1 - i \tan(c + dx)}{\sqrt{a + b \tan(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2i \operatorname{arctanh}\left(\frac{\sqrt{a + b \tan(dx + c)}}{\sqrt{ib + a}}\right)}{d\sqrt{ib + a}}$$

command

```
integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \operatorname{arctan}\left(\frac{2\left(-i\sqrt{b \tan(dx + c) + a} a - i\sqrt{a^2 + b^2} \sqrt{b \tan(dx + c) + a}\right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} a + i\sqrt{2a + 2\sqrt{a^2 + b^2}} b + \sqrt{a^2 + b^2} \sqrt{2a + 2\sqrt{a^2 + b^2}}}\right)}{\sqrt{2a + 2\sqrt{a^2 + b^2}} d\left(\frac{ib}{a + \sqrt{a^2 + b^2}} + 1\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 65.45 Problem number 475

$$\int \frac{A + B \tan(c + dx)}{\sqrt[3]{a + b \tan(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(-iB + A)x}{4(-ib + a)^{\frac{1}{3}}} - \frac{(iB + A)x}{4(ib + a)^{\frac{1}{3}}} - \frac{(iA - B) \ln(\cos(dx + c))}{4(ib + a)^{\frac{1}{3}} d} \\ & + \frac{(iA + B) \ln(\cos(dx + c))}{4(-ib + a)^{\frac{1}{3}} d} + \frac{3(iA + B) \ln\left(\left(-ib + a\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4(-ib + a)^{\frac{1}{3}} d} \\ & - \frac{3(iA - B) \ln\left(\left(ib + a\right)^{\frac{1}{3}} - (a + b \tan(dx + c))^{\frac{1}{3}}\right)}{4(ib + a)^{\frac{1}{3}} d} \\ & + \frac{(iA + B) \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(-ib + a)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2(-ib + a)^{\frac{1}{3}} d} \\ & - \frac{(iA - B) \arctan\left(\frac{\left(1 + \frac{2(a + b \tan(dx + c))^{\frac{1}{3}}}{(ib + a)^{\frac{1}{3}}}\right) \sqrt{3}}{3}\right) \sqrt{3}}{2(ib + a)^{\frac{1}{3}} d} \end{aligned}$$

command

`integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{A^3 - 3iA^2B - 3AB^2 + iB^3}{8ia + 8b}\right)^{\frac{1}{3}} \log\left(-a + ib - (-a^2 + 2iab + b^2)^{\frac{1}{3}}(b \tan(dx + c) + a)^{\frac{1}{3}}\right) + \left(-\frac{A^3 + 3iA^2B - 3AB^2 - iB^3}{8ia - 8b}\right)^{\frac{1}{3}} \log\left(-a - ib - (-a^2 - 2iab + b^2)^{\frac{1}{3}}(b \tan(dx + c) + a)^{\frac{1}{3}}\right)}{d}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{B \tan(dx + c) + A}{(b \tan(dx + c) + a)^{\frac{1}{3}}} dx$$

## 66 Test file number 105

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/105\_4.3.4.2-a+b\_tan<sup>m</sup>-c+d\_tan<sup>n</sup>-A+B\_tan+C\_tan<sup>2</sup>-

### 66.1 Problem number 50

$$\int (a + b \tan(e + fx))^3 (c + d \tan(e + fx)) (A + B \tan(e + fx) + C \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & (a^3(Ac - Bd - cC) - 3ab^2(Ac - Bd - cC) - 3a^2b(Bc + (A - C)d) + b^3(Bc + (A - C)d)) x \\ & - \frac{(3a^2b(Ac - Bd - cC) - b^3(Ac - Bd - cC) + a^3(Bc + (A - C)d) - 3ab^2(Bc + (A - C)d)) \ln(\cos(fx + e))}{f} \\ & + \frac{b(2ab(Ac - Bd - cC) + a^2(Bc + (A - C)d) - b^2(Bc + (A - C)d)) \tan(fx + e)}{f} \\ & + \frac{(aAd + Abc + aBc - bBd - aCd - bcC)(a + b \tan(fx + e))^2}{2f} \\ & + \frac{(Bc + (A - C)d)(a + b \tan(fx + e))^3}{3f} \\ & - \frac{(aCd - 5b(Bd + cC))(a + b \tan(fx + e))^4}{20b^2f} + \frac{Cd \tan(fx + e)(a + b \tan(fx + e))^4}{5bf} \end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 66.2 Problem number 57

$$\int (a + b \tan(e + fx))^3 (c + d \tan(e + fx))^2 (A + B \tan(e + fx) + C \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - (a^3(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) - 3ab^2(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2))) \\
& + 3a^2b(2c(A - C)d + B(c^2 - d^2)) - b^3(2c(A - C)d + B(c^2 - d^2)) x \\
& + \frac{(3a^2b(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) - b^3(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) - a^3(2c(A - C)d + B(c^2 - d^2)))}{f} \\
& + \frac{d(3a^2b(Ac - Bd - cC) - b^3(Ac - Bd - cC) + a^3(Bc + (A - C)d) - 3ab^2(Bc + (A - C)d)) \tan(fx + e)}{f} \\
& + \frac{(a^3B - 3ab^2B + 3a^2b(A - C) - b^3(A - C))(c + d \tan(fx + e))^2}{2f} \\
& + \frac{(4a^3Cd^3 - 3a^2bd^2(-16Bd + 3cC) + 3ab^2d(2c^2C - 5Bcd + 20(A - C)d^2) - b^3(c^3C - 2Bc^2d + 5c(A - C)d^2 + 60d^4f))}{60d^4f} \\
& + \frac{b(5b(Ab + aB - bC)d^2 + (-ad + bc)(-2bBd - aCd + bcC)) \tan(fx + e)(c + d \tan(fx + e))^3}{20d^3f} \\
& - \frac{(-2bBd - aCd + bcC)(a + b \tan(fx + e))^2(c + d \tan(fx + e))^3}{10d^2f} \\
& + \frac{C(a + b \tan(fx + e))^3(c + d \tan(fx + e))^3}{6df}
\end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^3*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**66.3 Problem number 58**

$$\int (a + b \tan(e + fx))^2 (c + d \tan(e + fx))^2 (A + B \tan(e + fx) + C \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& - (a^2(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) \\
& - b^2(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) + 2ab(2c(A - C)d + B(c^2 - d^2))) x \\
& + \frac{(2ab(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) - a^2(2c(A - C)d + B(c^2 - d^2)) + b^2(2c(A - C)d + B(c^2 - d^2))) \ln(c)}{f} \\
& + \frac{d(2ab(Ac - Bd - cC) + a^2(Bc + (A - C)d) - b^2(Bc + (A - C)d)) \tan(fx + e)}{f} \\
& + \frac{(a^2B - b^2B + 2ab(A - C))(c + d \tan(fx + e))^2}{2f} \\
& + \frac{(8a^2C d^2 - 10abd(-4Bd + cC) + b^2(2c^2C - 5Bcd + 20(A - C)d^2))(c + d \tan(fx + e))^3}{60d^3 f} \\
& - \frac{b(-5bBd - 2aCd + 2bcC) \tan(fx + e)(c + d \tan(fx + e))^3}{20d^2 f} \\
& + \frac{C(a + b \tan(fx + e))^2(c + d \tan(fx + e))^3}{5df}
\end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^2*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**66.4 Problem number 64**

$$\int (a + b \tan(e + fx))^2 (c + d \tan(e + fx))^3 (A + B \tan(e + fx) + C \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& (a^2(Ac^3 - 3Ac^2d - 3Bc^2d + Bd^3 - c^3C + 3cCd^2) \\
& + b^2(c^3C + 3Bc^2d - 3cCd^2 - Bd^3 - A(c^3 - 3cd^2)) - 2ab((A - C)d(3c^2 - d^2) + B(c^3 - 3cd^2))) x \\
& + \frac{(2ab(c^3C + 3Bc^2d - 3cCd^2 - Bd^3 - A(c^3 - 3cd^2)) - a^2((A - C)d(3c^2 - d^2) + B(c^3 - 3cd^2)) + b^2((A - C)d(3c^2 - d^2) + B(c^3 - 3cd^2)))}{f} \\
& - \frac{d(2ab(c^2C + 2Bcd - Cd^2 - A(c^2 - d^2)) - a^2(2c(A - C)d + B(c^2 - d^2)) + b^2(2c(A - C)d + B(c^2 - d^2)))}{f} \tan(fx + e) \\
& + \frac{(2ab(Ac - Bd - cC) + a^2(Bc + (A - C)d) - b^2(Bc + (A - C)d))(c + d \tan(fx + e))^2}{2f} \\
& + \frac{(a^2B - b^2B + 2ab(A - C))(c + d \tan(fx + e))^3}{3f} \\
& + \frac{(5a^2Cd^2 - 6abd(-5Bd + cC) + b^2(c^2C - 3Bcd + 15(A - C)d^2))(c + d \tan(fx + e))^4}{60d^3f} \\
& - \frac{b(-3bBd - aCd + bcC) \tan(fx + e)(c + d \tan(fx + e))^4}{15d^2f} \\
& + \frac{C(a + b \tan(fx + e))^2(c + d \tan(fx + e))^4}{6df}
\end{aligned}$$

command

`integrate((a+b*tan(f*x+e))^2*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm=`

`Giac 1.9.0-11 via sagemath 9.6 output`

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 66.5 Problem number 65

$$\int (a + b \tan(e + fx))(c + d \tan(e + fx))^3 (A + B \tan(e + fx) + C \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned}
& (a(Ac^3 - 3Ac^2d - 3Bc^2d + Bd^3 - c^3C + 3cCd^2) - b((A - C)d(3c^2 - d^2) + B(c^3 - 3cd^2))) x \\
& - \frac{(A(3ac^2d - ad^3 + bc^3 - 3bcd^2) - b(3Bc^2d - Bd^3 + c^3C - 3cCd^2) + a(Bc^3 - 3Bcd^2 - 3c^2Cd + Cd^3))}{f} \ln(cc + d \tan(fx + e)) \\
& + \frac{d(a(Bc^2 - Bd^2 - 2cCd) - b(2Bcd + c^2C - Cd^2) + A(2acd + b(c^2 - d^2)))}{f} \tan(fx + e) \\
& + \frac{(aAd + Abc + aBc - bBd - aCd - bcC)(c + d \tan(fx + e))^2}{2f} \\
& + \frac{(Ab + aB - bC)(c + d \tan(fx + e))^3}{3f} \\
& - \frac{(-5bBd - 5aCd + bcC)(c + d \tan(fx + e))^4}{20d^2f} + \frac{bC \tan(fx + e)(c + d \tan(fx + e))^4}{5df}
\end{aligned}$$

command

```
integrate((a+b*tan(f*x+e))*(c+d*tan(f*x+e))^3*(A+B*tan(f*x+e)+C*tan(f*x+e)^2),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67 Test file number 106

Test folder name:

test\_cases/4\_Trig\_functions/4.3\_Tangent/106\_4.3.7-d\_trig-<sup>m</sup>-a+b-c\_tan-<sup>n</sup>-<sup>p</sup>

### 67.1 Problem number 1

$$\int (b \tan^2(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{b^2 \cot(fx + e) \ln(\cos(fx + e)) \sqrt{b(\tan^2(fx + e))}}{f} \\ & - \frac{b^2 \sqrt{b(\tan^2(fx + e))} \tan(fx + e)}{2f} + \frac{b^2 \sqrt{b(\tan^2(fx + e))} (\tan^3(fx + e))}{4f} \end{aligned}$$

command

```
integrate((b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.2 Problem number 2

$$\int (b \tan^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{b \cot (fx + e) \ln (\cos (fx + e)) \sqrt{b (\tan^2 (fx + e))}}{f} + \frac{b \sqrt{b (\tan^2 (fx + e))} \tan (fx + e)}{2f}$$

command

```
integrate((b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \log \left( \frac{4 (\tan (fx)^4 \tan (e)^2 - 2 \tan (fx)^3 \tan (e) + \tan (fx)^2 \tan (e)^2 + \tan (fx)^2 - 2 \tan (fx) \tan (e) + 1)}{\tan (e)^2 + 1} \right) \right) \tan (fx)^2 \tan (e)^2 + \tan (fx)^2 \tan$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.3 Problem number 3

$$\int \sqrt{b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\cot (fx + e) \ln (\cos (fx + e)) \sqrt{b (\tan^2 (fx + e))}}{f}$$

command

```
integrate((b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{b} \log (|\cos (fx + e)|) \operatorname{sgn}(\tan (fx + e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 67.4 Problem number 4

$$\int \frac{1}{\sqrt{b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\ln(\sin(fx + e)) \tan(fx + e)}{f \sqrt{b (\tan^2(fx + e))}}$$

command

```
integrate(1/(b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{\sqrt{b} \operatorname{sgn}(\tan(fx+e))} - \frac{2 \log\left(\frac{|-\cos(fx+e)-1|}{|\cos(fx+e)+1|} + 1\right)}{\sqrt{b} \operatorname{sgn}(\tan(fx+e))}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.5 Problem number 5

$$\int \frac{1}{(b \tan^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cot(fx + e)}{2bf \sqrt{b (\tan^2(fx + e))}} - \frac{\ln(\sin(fx + e)) \tan(fx + e)}{bf \sqrt{b (\tan^2(fx + e))}}$$

command

```
integrate(1/(b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{\sqrt{b} \operatorname{sgn}(\tan(fx+e))} - \frac{8 \log\left(\frac{|-\cos(fx+e)-1|}{|\cos(fx+e)+1|} + 1\right)}{\sqrt{b} \operatorname{sgn}(\tan(fx+e))} - \frac{\left(\sqrt{b} + \frac{4\sqrt{b}(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{b(\cos(fx+e)-1)\operatorname{sgn}(\tan(fx+e))} - \frac{\cos(fx+e)-1}{\sqrt{b}(\cos(fx+e)+1)\operatorname{sgn}(\tan(fx+e))}$$


---


$$8bf$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.6 Problem number 6

$$\int \frac{1}{(b \tan^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e)}{2b^2 f \sqrt{b(\tan^2(fx + e))}} - \frac{\cot^3(fx + e)}{4b^2 f \sqrt{b(\tan^2(fx + e))}} + \frac{\ln(\sin(fx + e)) \tan(fx + e)}{b^2 f \sqrt{b(\tan^2(fx + e))}}$$

command

```
integrate(1/(b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{b^{\frac{5}{2}} \operatorname{sgn}(\tan(fx+e))} - \frac{64 \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{b^{\frac{5}{2}} \operatorname{sgn}(\tan(fx+e))} - \frac{\left(\sqrt{b} + \frac{12\sqrt{b}(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{48\sqrt{b}(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{b^3(\cos(fx+e)-1)^2 \operatorname{sgn}(\tan(fx+e))} - \frac{12 b^{\frac{7}{2}} (\cos(fx+e)+1)^2}{64 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.7 Problem number 7

$$\int (b \tan^3(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& - \frac{2b^2 \cot (fx + e) \sqrt{b (\tan^3 (fx + e))}}{f} \\
& + \frac{b^2 \arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{2f \tan (fx + e)^{\frac{3}{2}}} \\
& + \frac{b^2 \arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{2f \tan (fx + e)^{\frac{3}{2}}} \\
& - \frac{b^2 \ln \left( 1 - \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{4f \tan (fx + e)^{\frac{3}{2}}} \\
& + \frac{b^2 \ln \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{4f \tan (fx + e)^{\frac{3}{2}}} \\
& + \frac{2b^2 \sqrt{b (\tan^3 (fx + e))} \tan (fx + e)}{5f} - \frac{2b^2 \sqrt{b (\tan^3 (fx + e))} (\tan^3 (fx + e))}{9f} \\
& + \frac{2b^2 \sqrt{b (\tan^3 (fx + e))} (\tan^5 (fx + e))}{13f}
\end{aligned}$$

command

```
integrate((b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2340} \left( \frac{1170 \sqrt{2} b \sqrt{|b|} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{f} + \frac{1170 \sqrt{2} b \sqrt{|b|} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} \right)}{f} \right)}{f} \right)$$

+ e))

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.8 Problem number 8

$$\int (b \tan^3(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b\sqrt{b(\tan^3(fx+e))}}{3f} \\ & + \frac{b \arctan\left(-1 + \sqrt{2}\left(\sqrt{\tan(fx+e)}\right)\right) \sqrt{b(\tan^3(fx+e))} \sqrt{2}}{2f \tan(fx+e)^{\frac{3}{2}}} \\ & + \frac{b \arctan\left(1 + \sqrt{2}\left(\sqrt{\tan(fx+e)}\right)\right) \sqrt{b(\tan^3(fx+e))} \sqrt{2}}{2f \tan(fx+e)^{\frac{3}{2}}} \\ & + \frac{b \ln\left(1 - \sqrt{2}\left(\sqrt{\tan(fx+e)}\right) + \tan(fx+e)\right) \sqrt{b(\tan^3(fx+e))} \sqrt{2}}{4f \tan(fx+e)^{\frac{3}{2}}} \\ & - \frac{b \ln\left(1 + \sqrt{2}\left(\sqrt{\tan(fx+e)}\right) + \tan(fx+e)\right) \sqrt{b(\tan^3(fx+e))} \sqrt{2}}{4f \tan(fx+e)^{\frac{3}{2}}} \\ & + \frac{2b\sqrt{b(\tan^3(fx+e))} (\tan^2(fx+e))}{7f} \end{aligned}$$

command

```
integrate((b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{84} b \left( \frac{42 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|} + 2\sqrt{b \tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{bf} + \frac{42 \sqrt{2} |b|^{\frac{3}{2}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|} - 2\sqrt{b \tan(fx+e)}\right)}{2\sqrt{|b|}}\right)}{bf} \right) + e))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.9 Problem number 9

$$\int \sqrt{b \tan^3(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \cot (fx + e) \sqrt{b (\tan^3 (fx + e))}}{f} \\ & - \frac{\arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{2 f \tan (fx + e)^{\frac{3}{2}}} \\ & - \frac{\arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{2 f \tan (fx + e)^{\frac{3}{2}}} \\ & + \frac{\ln \left( 1 - \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{4 f \tan (fx + e)^{\frac{3}{2}}} \\ & - \frac{\ln \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \sqrt{b (\tan^3 (fx + e))} \sqrt{2}}{4 f \tan (fx + e)^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate((b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( \frac{2 \sqrt{2} \sqrt{|b|} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{f} + \frac{2 \sqrt{2} \sqrt{|b|} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{f} \right) + e))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.10 Problem number 10

$$\int \frac{1}{\sqrt{b \tan^3(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2 \tan (fx + e)}{f \sqrt{b \left(\tan^3 (fx + e)\right)}} - \frac{\arctan \left(-1 + \sqrt{2} \left(\sqrt{\tan (fx + e)}\right)\right) \left(\tan^{\frac{3}{2}} (fx + e)\right) \sqrt{2}}{2 f \sqrt{b \left(\tan^3 (fx + e)\right)}} \\ & - \frac{\arctan \left(1 + \sqrt{2} \left(\sqrt{\tan (fx + e)}\right)\right) \left(\tan^{\frac{3}{2}} (fx + e)\right) \sqrt{2}}{2 f \sqrt{b \left(\tan^3 (fx + e)\right)}} \\ & - \frac{\ln \left(1 - \sqrt{2} \left(\sqrt{\tan (fx + e)}\right) + \tan (fx + e)\right) \left(\tan^{\frac{3}{2}} (fx + e)\right) \sqrt{2}}{4 f \sqrt{b \left(\tan^3 (fx + e)\right)}} \\ & + \frac{\ln \left(1 + \sqrt{2} \left(\sqrt{\tan (fx + e)}\right) + \tan (fx + e)\right) \left(\tan^{\frac{3}{2}} (fx + e)\right) \sqrt{2}}{4 f \sqrt{b \left(\tan^3 (fx + e)\right)}} \end{aligned}$$

command

```
integrate(1/(b*tan(f*x+e)^3)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} b^2 \left( \frac{2 \sqrt{2} |b|^{\frac{3}{2}} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{b^4 f \operatorname{sgn}(\tan (fx + e))} + \frac{2 \sqrt{2} |b|^{\frac{3}{2}} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{b^4 f \operatorname{sgn}(\tan (fx + e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.11 Problem number 11

$$\int \frac{1}{(b \tan^3(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2}{3bf \sqrt{b(\tan^3(fx + e))}} - \frac{2(\cot^2(fx + e))}{7bf \sqrt{b(\tan^3(fx + e))}} \\ & + \frac{\arctan\left(-1 + \sqrt{2} \left(\sqrt{\tan(fx + e)}\right)\right) \left(\tan^{\frac{3}{2}}(fx + e)\right) \sqrt{2}}{2bf \sqrt{b(\tan^3(fx + e))}} \\ & + \frac{\arctan\left(1 + \sqrt{2} \left(\sqrt{\tan(fx + e)}\right)\right) \left(\tan^{\frac{3}{2}}(fx + e)\right) \sqrt{2}}{2bf \sqrt{b(\tan^3(fx + e))}} \\ & - \frac{\ln\left(1 - \sqrt{2} \left(\sqrt{\tan(fx + e)}\right) + \tan(fx + e)\right) \left(\tan^{\frac{3}{2}}(fx + e)\right) \sqrt{2}}{4bf \sqrt{b(\tan^3(fx + e))}} \\ & + \frac{\ln\left(1 + \sqrt{2} \left(\sqrt{\tan(fx + e)}\right) + \tan(fx + e)\right) \left(\tan^{\frac{3}{2}}(fx + e)\right) \sqrt{2}}{4bf \sqrt{b(\tan^3(fx + e))}} \end{aligned}$$

command

`integrate(1/(b*tan(f*x+e)^3)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{84} b^4 \left( \frac{42 \sqrt{2} \sqrt{|b|} \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan(fx + e)}\right)}{2 \sqrt{|b|}}\right)}{b^6 \operatorname{sgn}(\tan(fx + e))} + \frac{42 \sqrt{2} \sqrt{|b|} \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|b|} - 2 \sqrt{b \tan(fx + e)}\right)}{2 \sqrt{|b|}}\right)}{b^6 \operatorname{sgn}(\tan(fx + e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan(fx + e)^3)^{\frac{3}{2}}} dx$$

## 67.12 Problem number 12

$$\int \frac{1}{(b \tan^3(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \cot (fx + e)}{5b^2 f \sqrt{b (\tan^3 (fx + e))}} + \frac{2(\cot^3 (fx + e))}{9b^2 f \sqrt{b (\tan^3 (fx + e))}} - \frac{2(\cot^5 (fx + e))}{13b^2 f \sqrt{b (\tan^3 (fx + e))}} \\ & + \frac{2 \tan (fx + e)}{b^2 f \sqrt{b (\tan^3 (fx + e))}} + \frac{\arctan \left( -1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \left( \tan^{\frac{3}{2}} (fx + e) \right) \sqrt{2}}{2b^2 f \sqrt{b (\tan^3 (fx + e))}} \\ & + \frac{\arctan \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) \right) \left( \tan^{\frac{3}{2}} (fx + e) \right) \sqrt{2}}{2b^2 f \sqrt{b (\tan^3 (fx + e))}} \\ & + \frac{\ln \left( 1 - \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \left( \tan^{\frac{3}{2}} (fx + e) \right) \sqrt{2}}{4b^2 f \sqrt{b (\tan^3 (fx + e))}} \\ & - \frac{\ln \left( 1 + \sqrt{2} \left( \sqrt{\tan (fx + e)} \right) + \tan (fx + e) \right) \left( \tan^{\frac{3}{2}} (fx + e) \right) \sqrt{2}}{4b^2 f \sqrt{b (\tan^3 (fx + e))}} \end{aligned}$$

command

```
integrate(1/(b*tan(f*x+e)^3)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2340} b^6 \left( \frac{1170 \sqrt{2} |b|^{\frac{3}{2}} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} + 2 \sqrt{b \tan (fx + e)} \right)}{2 \sqrt{|b|}} \right)}{b^{10} f \operatorname{sgn} (\tan (fx + e))} + \frac{1170 \sqrt{2} |b|^{\frac{3}{2}} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} - \dots \right)}{2} \right)}{b^{10} f \operatorname{sgn} (\tan (fx + e))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(b \tan (fx + e)^3)^{\frac{5}{2}}} dx$$



## 67.13 Problem number 13

$$\int (b \tan^4(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^2 \cot(fx + e) \sqrt{b(\tan^4(fx + e))}}{f} - b^2 x (\cot^2(fx + e)) \sqrt{b(\tan^4(fx + e))} \\ & - \frac{b^2 \sqrt{b(\tan^4(fx + e))} \tan(fx + e)}{3f} + \frac{b^2 \sqrt{b(\tan^4(fx + e))} (\tan^3(fx + e))}{5f} \\ & - \frac{b^2 \sqrt{b(\tan^4(fx + e))} (\tan^5(fx + e))}{7f} + \frac{b^2 \sqrt{b(\tan^4(fx + e))} (\tan^7(fx + e))}{9f} \end{aligned}$$

command

```
integrate((b*tan(f*x+e)^4)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.14 Problem number 14

$$\int (b \tan^4(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \cot(fx + e) \sqrt{b(\tan^4(fx + e))}}{f} - bx (\cot^2(fx + e)) \sqrt{b(\tan^4(fx + e))} \\ & - \frac{b \sqrt{b(\tan^4(fx + e))} \tan(fx + e)}{3f} + \frac{b \sqrt{b(\tan^4(fx + e))} (\tan^3(fx + e))}{5f} \end{aligned}$$

command

```
integrate((b*tan(f*x+e)^4)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.15 Problem number 15

$$\int \sqrt{b \tan^4(e + fx)} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e) \sqrt{b(\tan^4(fx + e))}}{f} - x(\cot^2(fx + e)) \sqrt{b(\tan^4(fx + e))}$$

command

```
integrate((b*tan(f*x+e)^4)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(\pi - 4fx \tan(fx) \tan(e) - \pi \operatorname{sgn}(2 \tan(fx)^2 \tan(e) + 2 \tan(fx) \tan(e)^2 - 2 \tan(fx) - 2 \tan(e)) \tan(fx) \tan(e))}{1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.16 Problem number 30

$$\int \sin^5(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{(a - 3b) \cos(fx + e)}{f} + \frac{(2a - 3b) (\cos^3(fx + e))}{3f} - \frac{(a - b) (\cos^5(fx + e))}{5f} + \frac{b \sec(fx + e)}{f}$$

command

```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.17 Problem number 33

$$\int \csc(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{a \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{b \sec(fx + e)}{f}$$

command

`integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) + \frac{4b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.18 Problem number 34

$$\int \csc^3(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{(a + 2b) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{a \cot(fx + e) \csc(fx + e)}{2f} + \frac{b \sec(fx + e)}{f}$$

command

`integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(a + 2b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a + \frac{14b(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{2b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + \frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.19 Problem number 35

$$\int \csc^5(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a + 4b) \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{(5a + 4b) \cot(fx + e) \csc(fx + e)}{8f} \\ & - \frac{a(\cot^3(fx + e)) \csc(fx + e)}{4f} + \frac{b \sec(fx + e)}{f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$12(a + 4b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{\left(a - \frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{18a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{72b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}$$


---

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.20 Problem number 36

$$\int \sin^6(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(a - 7b)x}{16} - \frac{(11a - 29b) \cos(fx + e) \sin(fx + e)}{16f} + \frac{(13a - 19b) (\cos^3(fx + e)) \sin(fx + e)}{24f} \\ & - \frac{(a - b) (\cos^5(fx + e)) \sin(fx + e)}{6f} + \frac{b \tan(fx + e)}{f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^6*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.21 Problem number 37**

$$\int \sin^4(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{3(a - 5b)x}{8} - \frac{(5a - 9b) \cos(fx + e) \sin(fx + e)}{8f} + \frac{(a - b) (\cos^3(fx + e)) \sin(fx + e)}{4f} + \frac{b \tan(fx + e)}{f}$$

command

```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.22 Problem number 39**

$$\int (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$ax - bx + \frac{b \tan(fx + e)}{f}$$

command

```
integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$ax + \frac{\left(\pi - 4fx \tan(fx) \tan(e) - \pi \operatorname{sgn}\left(2 \tan(fx)^2 \tan(e) + 2 \tan(fx) \tan(e)^2 - 2 \tan(fx) - 2 \tan(e)\right) \tan(fx) \tan(e)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.23 Problem number 43

$$\int \sin^5(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{(a^2 - 6ab + 6b^2) \cos(fx + e)}{f} + \frac{2(a - 2b)(a - b) \cos^3(fx + e)}{3f} \\ - \frac{(a - b)^2 \cos^5(fx + e)}{5f} + \frac{2(a - 2b)b \sec(fx + e)}{f} + \frac{b^2 \sec^3(fx + e)}{3f}$$

command

```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 67.24 Problem number 46

$$\int \csc(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{a^2 \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{(2a - b)b \sec(fx + e)}{f} + \frac{b^2 \sec^3(fx + e)}{3f}$$

command

```
integrate(csc(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2 \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) + \frac{8\left(3ab-b^2 + \frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{3ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.25 Problem number 47

$$\int \csc^3(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{a(a + 4b) \operatorname{arctanh}(\cos(fx + e))}{2f} + \frac{a(a + 4b) \sec(fx + e)}{2f} - \frac{a^2(\csc^2(fx + e)) \sec(fx + e)}{2f} + \frac{b^2(\sec^3(fx + e))}{3f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - 6(a^2 + 4ab) \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right) - \frac{3\left(a^2 - \frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8ab(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1} - \frac{16(6ab)}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.26 Problem number 48

$$\int \csc^5(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{(3a^2 + 24ab + 8b^2) \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{a(a + 8b) \cot(fx + e) \csc(fx + e)}{8f} + \frac{(a^2 + 8ab + 4b^2) \sec(fx + e)}{4f} - \frac{a^2(\csc^4(fx + e)) \sec(fx + e)}{4f} + \frac{b^2(\sec^3(fx + e))}{3f}$$

command

```
integrate(csc(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{24a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{48ab(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{3a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - 12(3a^2 + 24ab + 8b^2) \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right) + \frac{3(a^2 - 8ab)}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.27 Problem number 49

$$\int \sin^4(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{(3a^2 - 30ab + 35b^2)x}{8} - \frac{(a - 9b)(a - b) \cos(fx + e) \sin(fx + e)}{8f} - \frac{(a^2 - 10ab + 13b^2) \tan(fx + e)}{4f} + \frac{(a - b)^2 (\sin^4(fx + e)) \tan(fx + e)}{4f} + \frac{b^2 (\tan^3(fx + e))}{3f}$$

command

```
integrate(sin(f*x+e)^4*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.28 Problem number 58

$$\int \frac{\csc(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(fx + e))}{af} - \frac{\operatorname{arctan}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a-b}}\right)\sqrt{b}}{af\sqrt{a-b}}$$

command

```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2b \operatorname{arctan}\left(-\frac{a \cos(fx+e) - b \cos(fx+e) - b}{\sqrt{ab-b^2} \cos(fx+e) + \sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2} a} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 67.29 Problem number 59

$$\int \frac{\csc^3(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(a - 2b) \operatorname{arctanh}(\cos(fx + e))}{2a^2 f} - \frac{\cot(fx + e) \csc(fx + e)}{2af} - \frac{\operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{a - b} \sqrt{b}}{a^2 f}$$

command

```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(a-2b) \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right) - \frac{8\sqrt{ab-b^2} \operatorname{arctan}\left(-\frac{a \cos(fx+e) - b \cos(fx+e) - b}{\sqrt{ab-b^2} \cos(fx+e) + \sqrt{ab-b^2}}\right)}{a^2} + \frac{\left(a - \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{a^2(\cos(fx+e)-1)}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.30 Problem number 60

$$\int \frac{\csc^5(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(3a^2 - 12ab + 8b^2) \operatorname{arctanh}(\cos(fx + e))}{8a^3 f} - \frac{(5a - 4b) \cot(fx + e) \csc(fx + e)}{8a^2 f} - \frac{(\cot^3(fx + e)) \csc(fx + e)}{4af} - \frac{(a - b)^{\frac{3}{2}} \operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{b}}{a^3 f}$$

command

```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8b(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2} - \frac{4(3a^2 - 12ab + 8b^2) \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right)}{a^3} + \frac{64(a^2b - 2ab^2 + b^3) \operatorname{arctan}\left(-\frac{a \cos(fx+e) - b \cos(fx+e) - b}{\sqrt{ab-b^2} \cos(fx+e) + \sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2} a^3}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.31 Problem number 71

$$\int \frac{\csc(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}(\cos(fx + e))}{a^2 f} - \frac{b \sec(fx + e)}{2a(a - b)f(a - b + b(\sec^2(fx + e)))} \\ & - \frac{(3a - 2b) \operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{b}}{2a^2(a - b)^{\frac{3}{2}} f} \end{aligned}$$

command

```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(3ab - 2b^2) \operatorname{arctan}\left(\frac{a \cos(fx + e) - b \cos(fx + e) - b}{\sqrt{ab - b^2} \cos(fx + e) + \sqrt{ab - b^2}}\right)}{(a^3 - a^2b) \sqrt{ab - b^2}} + \frac{2 \left( ab + \frac{ab(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{2b^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1} \right)}{(a^3 - a^2b) \left( a + \frac{2a(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{4b(\cos(fx + e) - 1)}{\cos(fx + e) + 1} + \frac{a(\cos(fx + e) - 1)^2}{(\cos(fx + e) + 1)^2} \right)} - \frac{\log(\dots)}{2f} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.32 Problem number 72

$$\int \frac{\csc^3(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a - 4b) \operatorname{arctanh}(\cos(fx + e))}{2a^3 f} - \frac{\cot(fx + e) \csc(fx + e)}{2af(a - b + b(\sec^2(fx + e)))} \\ & - \frac{b \sec(fx + e)}{a^2 f(a - b + b(\sec^2(fx + e)))} - \frac{(3a - 4b) \operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{b}}{2a^3 f \sqrt{a - b}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(a-4b)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3} - \frac{12(3ab-4b^2)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}a^3} - \frac{3(\cos(fx+e)-1)}{a^2(\cos(fx+e)+1)} + \frac{3a^2+\frac{4a^2(\cos(fx+e))}{\cos(fx+e)+1}}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.33 Problem number 73

$$\int \frac{\csc^5(e+fx)}{(a+b\tan^2(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a^2-8ab+8b^2)\operatorname{arctanh}(\cos(fx+e))}{8a^4f} - \frac{(5a-6b)\cot(fx+e)\csc(fx+e)}{8a^2f(a-b+b(\sec^2(fx+e)))} \\ & - \frac{(\cot^3(fx+e))\csc(fx+e)}{4af(a-b+b(\sec^2(fx+e)))} - \frac{3(3a-4b)b\sec(fx+e)}{8a^3f(a-b+b(\sec^2(fx+e)))} \\ & - \frac{3(a-2b)\arctan\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a-b}}\right)\sqrt{a-b}\sqrt{b}}{2a^4f} \end{aligned}$$

command

`integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12(a^2-8ab+8b^2)\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^4} - \frac{96(a^2b-3ab^2+2b^3)\arctan\left(-\frac{a\cos(fx+e)-b\cos(fx+e)-b}{\sqrt{ab-b^2}\cos(fx+e)+\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}a^4} - \frac{8a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{16ab(\cos(fx+e))}{\cos(fx+e)+1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.34 Problem number 83

$$\int \frac{\csc(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cos(fx + e))}{a^3 f} - \frac{b \sec(fx + e)}{4a(a - b) f (a - b + b(\sec^2(fx + e)))^2} \\ & - \frac{(7a - 4b) b \sec(fx + e)}{8a^2(a - b)^2 f (a - b + b(\sec^2(fx + e)))} - \frac{(15a^2 - 20ab + 8b^2) \operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{b}}{8a^3(a - b)^{\frac{5}{2}} f} \end{aligned}$$

command

```
integrate(csc(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(15a^2b - 20ab^2 + 8b^3) \operatorname{arctan}\left(-\frac{a \cos(fx + e) - b \cos(fx + e) - b}{\sqrt{ab - b^2} \cos(fx + e) + \sqrt{ab - b^2}}\right)}{(a^5 - 2a^4b + a^3b^2) \sqrt{ab - b^2}} + 2 \left( \frac{9a^3b - 6a^2b^2 + \frac{27a^3b(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{68a^2b^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1} + \frac{32ab^3}{\cos(fx + e) + 1}}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.35 Problem number 84

$$\int \frac{\csc^3(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a - 6b) \operatorname{arctanh}(\cos(fx + e))}{2a^4 f} - \frac{\cot(fx + e) \csc(fx + e)}{2af(a - b + b(\sec^2(fx + e)))^2} \\ & - \frac{3b \sec(fx + e)}{4a^2 f (a - b + b(\sec^2(fx + e)))^2} - \frac{(11a - 12b) b \sec(fx + e)}{8a^3(a - b) f (a - b + b(\sec^2(fx + e)))} \\ & - \frac{(15a^2 - 40ab + 24b^2) \operatorname{arctan}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a - b}}\right) \sqrt{b}}{8a^4(a - b)^{\frac{3}{2}} f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(15a^2b - 40ab^2 + 24b^3) \arctan\left(-\frac{a \cos(fx+e) - b \cos(fx+e) - b}{\sqrt{ab - b^2} \cos(fx+e) + \sqrt{ab - b^2}}\right)}{(a^5 - a^4b) \sqrt{ab - b^2}} + \frac{2 \left(9a^3b - 10a^2b^2 + \frac{27a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{80a^2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{56a^2b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{(a^5 - a^4b) \sqrt{ab - b^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.36 Problem number 85

$$\int \frac{\csc^5(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a^2 - 12ab + 16b^2) \operatorname{arctanh}(\cos(fx + e))}{8a^5 f} - \frac{(5a - 8b) \cot(fx + e) \csc(fx + e)}{8a^2 f (a - b + b(\sec^2(fx + e)))^2} \\ & - \frac{(\cot^3(fx + e)) \csc(fx + e)}{4af(a - b + b(\sec^2(fx + e)))^2} - \frac{(7a - 12b)b \sec(fx + e)}{8a^3 f (a - b + b(\sec^2(fx + e)))^2} \\ & - \frac{3(a - 2b)b \sec(fx + e)}{2a^4 f (a - b + b(\sec^2(fx + e)))} - \frac{3(5a^2 - 20ab + 16b^2) \arctan\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a-b}}\right) \sqrt{b}}{8a^5 f \sqrt{a-b}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.37 Problem number 92**

$$\int \sin^5(e + fx) \sqrt{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a - 4b) (\cos^3(fx + e)) (a - b + b(\sec^2(fx + e)))^{\frac{3}{2}}}{15(a - b)^2 f} \\ & - \frac{(\cos^5(fx + e)) (a - b + b(\sec^2(fx + e)))^{\frac{3}{2}}}{5(a - b) f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a - b + b(\sec^2(fx + e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.38 Problem number 93**

$$\int \sin^3(e + fx) \sqrt{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(\cos^3(fx + e)) (a - b + b(\sec^2(fx + e)))^{\frac{3}{2}}}{3(a - b) f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a - b + b(\sec^2(fx + e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} \left( \frac{\left( 3ab \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) - 3b^2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 3a\sqrt{-b}\sqrt{b} - 4\sqrt{-b}b^{\frac{3}{2}} \right) \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx+e))}{a\sqrt{-b}f^2 - \sqrt{-b}bf^2} - \frac{3b \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.39 Problem number 94

$$\int \sin(e+fx) \sqrt{a+b \tan^2(e+fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a-b+b(\sec^2(fx+e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx+e) \sqrt{a-b+b(\sec^2(fx+e))}}{f}$$

command

`integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$- \left( \frac{b \arctan\left(\frac{\sqrt{a \cos(fx+e)^2 - b \cos(fx+e)^2 + b}}{\sqrt{-b}}\right) \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx+e))}{\sqrt{-b}f^2} + \frac{\sqrt{a \cos(fx+e)^2 - b \cos(fx+e)^2 + b}}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.40 Problem number 105

$$\int \sin^3(e + fx) (a + b \tan^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a - 5b) \cos(fx + e) (a - b + b(\sec^2(fx + e)))^{3/2}}{3(a - b)f} \\ & + \frac{(\cos^3(fx + e)) (a - b + b(\sec^2(fx + e)))^{5/2}}{3(a - b)f} \\ & + \frac{(3a - 5b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a - b + b(\sec^2(fx + e))}}\right) \sqrt{b}}{2f} \\ & + \frac{(3a - 5b)b \sec(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{2(a - b)f} \end{aligned}$$

command

`integrate(sin(f*x+e)^3*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6} \left( \frac{3(3ab \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) - 5b^2 \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))) \operatorname{arctan}\left(\frac{\sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b} f^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.41 Problem number 106

$$\int \sin(e + fx) (a + b \tan^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e) (a - b + b(\sec^2(fx + e)))^{3/2}}{f} \\ & + \frac{3(a - b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a - b + b(\sec^2(fx + e))}}\right) \sqrt{b}}{2f} \\ & + \frac{3b \sec(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{2f} \end{aligned}$$



command

```
integrate(sin(f*x+e)*(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} \left( \frac{3 (ab \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) - b^2 \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))) \arctan \left( \frac{\sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b}}{\sqrt{-b}} \right)}{\sqrt{-b} f^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.42 Problem number 116

$$\int \frac{\sin^5(e + fx)}{\sqrt{a + b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(15a^2 - 10ab + 3b^2) \cos(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{15(a - b)^3 f} \\ & + \frac{2(5a - 3b) (\cos^3(fx + e)) \sqrt{a - b + b(\sec^2(fx + e))}}{15(a - b)^2 f} \\ & - \frac{(\cos^5(fx + e)) \sqrt{a - b + b(\sec^2(fx + e))}}{5(a - b) f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.43 Problem number 117

$$\int \frac{\sin^3(e + fx)}{\sqrt{a + b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{(3a - b) \cos(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{3(a - b)^2 f} + \frac{(\cos^3(fx + e)) \sqrt{a - b + b(\sec^2(fx + e))}}{3(a - b) f}$$

command

```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(3a\sqrt{b} - b^{\frac{3}{2}}) \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))}{3(a^2|f| - 2ab|f| + b^2|f|)} + \frac{(a \cos(fx + e)^2 - b \cos(fx + e)^2 + b)^{\frac{3}{2}} f^2}{3(a|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) - b|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)))(af^2 - bf^2)}$$

$$-\frac{\sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b} a}{a^2|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) - 2ab|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) + b^2|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.44 Problem number 118

$$\int \frac{\sin(e + fx)}{\sqrt{a + b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) \sqrt{a - b + b(\sec^2(fx + e))}}{(a - b) f}$$

command

```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{b} \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))}{a|f| - b|f|} - \frac{\sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b}}{a|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e)) - b|f| \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.45 Problem number 121

$$\int \frac{\csc^5(e + fx)}{\sqrt{a + b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3(a-b)^2 \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a}}{\sqrt{a-b+b(\sec^2(fx+e))}}\right)}{8a^{\frac{5}{2}}f} \\ & - \frac{(5a-3b) \cot(fx+e) \csc(fx+e) \sqrt{a-b+b(\sec^2(fx+e))}}{8a^2f} \\ & - \frac{(\cot^3(fx+e)) \csc(fx+e) \sqrt{a-b+b(\sec^2(fx+e))}}{4af} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.46 Problem number 128

$$\int \frac{\sin^5(e + fx)}{(a + b \tan^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(15a^2 + 10ab - b^2) \cos(fx + e)}{15(a-b)^3 f \sqrt{a-b+b(\sec^2(fx+e))}} + \frac{2(5a-2b) (\cos^3(fx+e))}{15(a-b)^2 f \sqrt{a-b+b(\sec^2(fx+e))}} \\ & - \frac{\cos^5(fx+e)}{5(a-b) f \sqrt{a-b+b(\sec^2(fx+e))}} - \frac{2b(15a^2 + 10ab - b^2) \sec(fx+e)}{15(a-b)^4 f \sqrt{a-b+b(\sec^2(fx+e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.47 Problem number 129

$$\int \frac{\sin^3(e + fx)}{(a + b \tan^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(3a + b) \cos(fx + e)}{3(a - b)^2 f \sqrt{a - b + b(\sec^2(fx + e))}} + \frac{\cos^3(fx + e)}{3(a - b) f \sqrt{a - b + b(\sec^2(fx + e))}} - \frac{2b(3a + b) \sec(fx + e)}{3(a - b)^3 f \sqrt{a - b + b(\sec^2(fx + e))}}$$

command

```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sin(fx + e)^3}{(b \tan(fx + e)^2 + a)^{3/2}} dx$$

## 67.48 Problem number 130

$$\int \frac{\sin(e + fx)}{(a + b \tan^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)}{(a - b) f \sqrt{a - b + b(\sec^2(fx + e))}} - \frac{2b \sec(fx + e)}{(a - b)^2 f \sqrt{a - b + b(\sec^2(fx + e))}}$$

command

```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{f^2 \left( \frac{\sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b}}{a|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx + e)) - b|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx + e))} + \frac{b}{(a|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx + e)) - b|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx + e)))} \sqrt{a \cos(fx + e)^2 - b \cos(fx + e)^2 + b} \right)}{af^2 - bf^2} + \frac{2\sqrt{b} \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx + e))}{a^2|f| - 2ab|f| + b^2|f|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.49 Problem number 133

$$\int \frac{\csc^5(e + fx)}{(a + b \tan^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3(a-5b)(a-b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a}}{\sqrt{a-b+b(\sec^2(fx+e))}}\right)}{8a^{\frac{7}{2}}f} \\ & - \frac{5(a-b) \cot(fx+e) \csc(fx+e)}{8a^2 f \sqrt{a-b+b(\sec^2(fx+e))}} \\ & - \frac{(\cot^3(fx+e)) \csc(fx+e)}{4af \sqrt{a-b+b(\sec^2(fx+e))}} - \frac{(13a-15b)b \sec(fx+e)}{8a^3 f \sqrt{a-b+b(\sec^2(fx+e))}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.50 Problem number 140

$$\int \frac{\sin^5(e + fx)}{(a + b \tan^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(5a^2 + 10ab + b^2) \cos(fx + e)}{5(a-b)^3 f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} + \frac{2(5a-b) (\cos^3(fx+e))}{15(a-b)^2 f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} \\ & - \frac{\cos^5(fx+e)}{5(a-b) f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} - \frac{4b(5a^2 + 10ab + b^2) \sec(fx+e)}{15(a-b)^4 f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} \\ & - \frac{8b(5a^2 + 10ab + b^2) \sec(fx+e)}{15(a-b)^5 f \sqrt{a-b+b(\sec^2(fx+e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 67.51 Problem number 141

$$\int \frac{\sin^3(e + fx)}{(a + b \tan^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a + b) \cos(fx + e)}{(a - b)^2 f (a - b + b(\sec^2(fx + e)))^{3/2}} + \frac{\cos^3(fx + e)}{3(a - b) f (a - b + b(\sec^2(fx + e)))^{3/2}} \\ & - \frac{4b(a + b) \sec(fx + e)}{3(a - b)^3 f (a - b + b(\sec^2(fx + e)))^{3/2}} - \frac{8b(a + b) \sec(fx + e)}{3(a - b)^4 f \sqrt{a - b + b(\sec^2(fx + e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^3/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sin^3(fx + e)}{(b \tan^2(fx + e) + a)^{5/2}} dx$$

### 67.52 Problem number 142

$$\int \frac{\sin(e + fx)}{(a + b \tan^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\cos(fx + e)}{(a - b) f (a - b + b(\sec^2(fx + e)))^{3/2}} - \frac{4b \sec(fx + e)}{3(a - b)^2 f (a - b + b(\sec^2(fx + e)))^{3/2}} \\ & - \frac{8b \sec(fx + e)}{3(a - b)^3 f \sqrt{a - b + b(\sec^2(fx + e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$f^4 \left( \frac{3 \sqrt{a \cos(fx+e)^2 - b \cos(fx+e)^2 + b}}{a|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx+e)) - b|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx+e))} + \frac{6(a \cos(fx+e)^2 - b \cos(fx+e)^2 + b)b - b^2}{(a|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx+e)) - b|f|\operatorname{sgn}(f)\operatorname{sgn}(\cos(fx+e)))(a \cos(fx+e)^2 - b \cos(fx+e)^2 + b)} \right) \\ - \frac{8 \sqrt{b} \operatorname{sgn}(f) \operatorname{sgn}(\cos(fx+e))}{3(a^3|f| - 3a^2b|f| + 3ab^2|f| - b^3|f|)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 67.53 Problem number 145

$$\int \frac{\csc^5(e+fx)}{(a+b \tan^2(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(3a^2 - 30ab + 35b^2) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a}}{\sqrt{a-b+b(\sec^2(fx+e))}}\right)}{8a^{\frac{9}{2}}f} \\ - \frac{(5a-7b) \cot(fx+e) \csc(fx+e)}{8a^2 f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} - \frac{(\cot^3(fx+e)) \csc(fx+e)}{4af (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} \\ - \frac{(23a-35b) b \sec(fx+e)}{24a^3 f (a-b+b(\sec^2(fx+e)))^{\frac{3}{2}}} - \frac{5(11a-21b) b \sec(fx+e)}{24a^4 f \sqrt{a-b+b(\sec^2(fx+e))}}$$

command

```
integrate(csc(f*x+e)^5/(a+b*tan(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.54 Problem number 188

$$\int \cot(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{b \ln(\cos(fx + e))}{f} + \frac{a \ln(\sin(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log(\sin(fx + e)^2) - b \log(|\sin(fx + e)^2 - 1|)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.55 Problem number 189

$$\int \cot^3(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{a(\cot^2(fx + e))}{2f} - \frac{(a - b) \ln(\sin(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4(a - b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 8(a - b) \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right) - \frac{\left(a + \frac{4a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



**67.56 Problem number 190**

$$\int \cot^5(e + fx) (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{(a - b) (\cot^2(fx + e))}{2f} - \frac{a(\cot^4(fx + e))}{4f} + \frac{(a - b) \ln(\sin(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$32(a - b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 64(a - b) \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right) - \frac{\left(a + \frac{12a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{48a(\cos(fx+e)-1)}{(\cos(fx+e)+1)^2}\right)}{64f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.57 Problem number 194**

$$\int (a + b \tan^2(e + fx)) dx$$

Optimal antiderivative

$$ax - bx + \frac{b \tan(fx + e)}{f}$$

command

```
integrate(a+b*tan(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$ax + \frac{\left(\pi - 4fx \tan(fx) \tan(e) - \pi \operatorname{sgn}\left(2 \tan(fx)^2 \tan(e) + 2 \tan(fx) \tan(e)^2 - 2 \tan(fx) - 2 \tan(e)\right) \tan(fx) \tan(e)\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.58 Problem number 198

$$\int \tan^5(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a-b)^2 \ln(\cos(fx+e))}{f} - \frac{(a-b)^2 (\tan^2(fx+e))}{2f} \\ & + \frac{(a-b)^2 (\tan^4(fx+e))}{4f} + \frac{(2a-b)b(\tan^6(fx+e))}{6f} + \frac{b^2(\tan^8(fx+e))}{8f} \end{aligned}$$

command

```
integrate(tan(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.59 Problem number 201

$$\int \cot(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{(a-b)^2 \ln(\cos(fx+e))}{f} + \frac{a^2 \ln(\tan(fx+e))}{f} + \frac{b^2(\tan^2(fx+e))}{2f}$$

command

```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \log(\sin(fx+e)^2) - (2ab - b^2) \log(|\sin(fx+e)^2 - 1|) + \frac{2ab \sin(fx+e)^2 - b^2 \sin(fx+e)^2 - 2ab}{\sin(fx+e)^2 - 1}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.60 Problem number 202**

$$\int \cot^3(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{a^2(\cot^2(fx + e))}{2f} - \frac{(a - b)^2 \ln(\cos(fx + e))}{f} - \frac{a(a - 2b) \ln(\tan(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)^3*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right) - 4b^2 \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right| \right) + 4(a^2 - 2ab + b^2) \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} \right| \right)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.61 Problem number 203**

$$\int \cot^5(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{a(a - 2b) (\cot^2(fx + e))}{2f} - \frac{a^2(\cot^4(fx + e))}{4f} + \frac{(a - b)^2 \ln(\cos(fx + e))}{f} + \frac{(a - b)^2 \ln(\tan(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)^5*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{12a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{16ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - 32(a^2 - 2ab + b^2) \log \left( \frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|} \right) + 64(a^2 - 2ab - b^2) \log \left( \frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|} \right)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.62 Problem number 204

$$\int \tan^6(e + fx) (a + b \tan^2(e + fx))^2 dx$$

Optimal antiderivative

$$-(a-b)^2 x + \frac{(a-b)^2 \tan(fx+e)}{f} - \frac{(a-b)^2 (\tan^3(fx+e))}{3f} \\ + \frac{(a-b)^2 (\tan^5(fx+e))}{5f} + \frac{(2a-b)b(\tan^7(fx+e))}{7f} + \frac{b^2(\tan^9(fx+e))}{9f}$$

command

```
integrate(tan(f*x+e)^6*(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.63 Problem number 211

$$\int \frac{\tan^5(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\ln(\cos(fx+e))}{(a-b)f} - \frac{a^2 \ln(a + b(\tan^2(fx+e)))}{2(a-b)b^2 f} + \frac{\tan^2(fx+e)}{2bf}$$

command

```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^3 \log\left(-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a + 4b\right)}{a^2 b^2 - ab^3} - \frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right)}{a-b} - \frac{(a+b) \log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right)}{b^2}$$

---

$2f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.64 Problem number 212

$$\int \frac{\tan^3(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\ln(\cos(fx + e))}{(a - b)f} + \frac{a \ln(a + b \tan^2(fx + e))}{2(a - b)bf}$$

command

```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \log\left(\left| -a \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right) - 2a + 4b \right|\right)}{a^2b - ab^2} - \frac{\log\left(\left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2 \right|\right)}{a-b} - \frac{\log\left(\left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right|\right)}{b}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.65 Problem number 213

$$\int \frac{\tan(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\ln(a(\cos^2(fx + e)) + b(\sin^2(fx + e)))}{2(a - b)f}$$

command

```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a-b} - \frac{2 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a-b}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.66 Problem number 214

$$\int \frac{\cot(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\ln(\cos(fx + e))}{(a - b)f} + \frac{\ln(\tan(fx + e))}{af} + \frac{b \ln(a + b(\tan^2(fx + e)))}{2a(a - b)f}$$

command

```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \log\left(\left|-a \sin(fx+e)^2 + b \sin(fx+e)^2 + a\right|\right)}{a^2 - ab} + \frac{\log(\sin(fx+e)^2)}{a}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.67 Problem number 215

$$\int \frac{\cot^3(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\cot^2(fx + e)}{2af} - \frac{\ln(\cos(fx + e))}{(a - b)f} - \frac{(a + b) \ln(\tan(fx + e))}{a^2 f} - \frac{b^2 \ln(a + b(\tan^2(fx + e)))}{2a^2(a - b)f}$$

command

```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4b^2 \log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3 - a^2 b} + \frac{4(a+b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2} - \frac{8 \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right)}{a-b} - \left(a + \frac{4a}{\cos}\right)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.68 Problem number 216

$$\int \frac{\cot^5(e + fx)}{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + b) (\cot^2(fx + e))}{2a^2 f} - \frac{\cot^4(fx + e)}{4af} + \frac{\ln(\cos(fx + e))}{(a - b) f} \\ & + \frac{(a^2 + ab + b^2) \ln(\tan(fx + e))}{a^3 f} + \frac{b^3 \ln(a + b(\tan^2(fx + e)))}{2a^3 (a - b) f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 b^3 \log\left(a + \frac{2 a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4 b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4 - a^3 b} - \frac{64 \log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right)}{a - b} - \frac{\frac{12 a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{8 b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.69 Problem number 224

$$\int \frac{\tan^5(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{\ln(\cos(fx + e))}{(a - b)^2 f} + \frac{a(a - 2b) \ln(a + b(\tan^2(fx + e)))}{2(a - b)^2 b^2 f} + \frac{a^2}{2(a - b) b^2 f (a + b(\tan^2(fx + e)))}$$

command

```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a^3 - 2a^2b) \log\left(-a \left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a + 4b\right)}{a^3 b^2 - 2a^2 b^3 + ab^4} + \frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right)}{a^2 - 2ab + b^2} - \frac{a^3 \left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a^2 b}{(a^2 b^2 - 2ab^3 + b^4) \left(a \left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a + 4b\right)}$$

2 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.70 Problem number 225

$$\int \frac{\tan^3(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\ln(a(\cos^2(fx + e)) + b(\sin^2(fx + e)))}{2(a - b)^2 f} - \frac{a}{2(a - b)bf(a + b(\tan^2(fx + e)))}$$

command

```
integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2 - 2ab + b^2} - \frac{2 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^2 - 2ab + b^2} - \frac{a + \frac{6a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a^2}{\cos(fx+e)+1}}{(a^2 - 2ab + b^2)\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.71 Problem number 226

$$\int \frac{\tan(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{\ln(a(\cos^2(fx + e)) + b(\sin^2(fx + e)))}{2(a - b)^2 f} + \frac{1}{2(a - b)bf(a + b(\tan^2(fx + e)))}$$

command

```
integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2 - 2ab + b^2} - \frac{2 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^2 - 2ab + b^2} - \frac{a^2 + \frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}}{(a^3 - 2a^2b + ab^2)\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 67.72 Problem number 227

$$\int \frac{\cot(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\ln(\cos(fx + e))}{(a - b)^2 f} + \frac{\ln(\tan(fx + e))}{a^2 f} + \frac{(2a - b)b \ln(a + b(\tan^2(fx + e)))}{2a^2(a - b)^2 f} - \frac{b}{2a(a - b)f(a + b(\tan^2(fx + e)))}$$

command

```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2ab - b^2) \log\left(\left| -a \sin(fx + e)^2 + b \sin(fx + e)^2 + a \right|\right)}{a^4 - 2a^3b + a^2b^2} - \frac{2ab \sin(fx + e)^2 - b^2 \sin(fx + e)^2 - 2ab}{(a^3 - a^2b)(a \sin(fx + e)^2 - b \sin(fx + e)^2 - a)} + \frac{\log(\sin(fx + e)^2)}{a^2}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.73 Problem number 228

$$\int \frac{\cot^3(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\cot^2(fx + e)}{2a^2 f} - \frac{\ln(\cos(fx + e))}{(a - b)^2 f} - \frac{(a + 2b) \ln(\tan(fx + e))}{a^3 f} - \frac{(3a - 2b)b^2 \ln(a + b(\tan^2(fx + e)))}{2a^3(a - b)^2 f} + \frac{b^2}{2a^2(a - b)f(a + b(\tan^2(fx + e)))}$$

command

```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12(3ab^2 - 2b^3) \log\left(a + \frac{2a(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{4b(\cos(fx + e) - 1)}{\cos(fx + e) + 1} + \frac{a(\cos(fx + e) - 1)^2}{(\cos(fx + e) + 1)^2}\right)}{a^5 - 2a^4b + a^3b^2} - \frac{24 \log\left(\left| -\frac{\cos(fx + e) - 1}{\cos(fx + e) + 1} + 1 \right|\right)}{a^2 - 2ab + b^2} - \frac{3a^4 - 6a^3b + 3a^2b^2 + \frac{10a^4 \cos(fx + e)}{\cos(fx + e)}}{a^2 - 2ab + b^2}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.74 Problem number 229

$$\int \frac{\cot^5(e + fx)}{(a + b \tan^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 2b) (\cot^2(fx + e))}{2a^3 f} - \frac{\cot^4(fx + e)}{4a^2 f} \\ & + \frac{\ln(\cos(fx + e))}{(a - b)^2 f} + \frac{(a^2 + 2ab + 3b^2) \ln(\tan(fx + e))}{a^4 f} \\ & + \frac{(4a - 3b)b^3 \ln(a + b(\tan^2(fx + e)))}{2a^4(a - b)^2 f} - \frac{b^3}{2a^3(a - b)f(a + b(\tan^2(fx + e)))} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32(4ab^3 - 3b^4) \log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^6 - 2a^5b + a^4b^2} - \frac{64 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^2 - 2ab + b^2} - \frac{32\left(4a^2b^3 - 3ab^4 + \frac{8a^2b^3(\cos(fx+e))}{\cos(fx+e)}\right)}{(a^6 - 2a^5b + a^4b^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.75 Problem number 237

$$\int \frac{\tan^5(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\ln(a(\cos^2(fx + e)) + b(\sin^2(fx + e)))}{2(a - b)^3 f} + \frac{a^2}{4(a - b)b^2 f(a + b(\tan^2(fx + e)))^2} \\ & - \frac{a(a - 2b)}{2(a - b)^2 b^2 f(a + b(\tan^2(fx + e)))} \end{aligned}$$

command

```
integrate(tan(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{4 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{3a^2 + \frac{20a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{32ab(\cos(fx+e)-1)}{\cos(fx+e)+1}}{a^3 - 3a^2b + 3ab^2 - b^3}$$

4 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.76 Problem number 238**

$$\int \frac{\tan^3(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{\ln(a(\cos^2(fx + e)) + b(\sin^2(fx + e)))}{2(a - b)^3 f} - \frac{a}{4(a - b)bf(a + b(\tan^2(fx + e)))^2}$$

$$- \frac{1}{2(a - b)^2 f(a + b(\tan^2(fx + e)))}$$

command

`integrate(tan(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{4 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{3a^3 + \frac{20a^3(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{32a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{a^3 - 3a^2b + 3ab^2 - b^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.77 Problem number 239

$$\int \frac{\tan(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{\ln(a \cos^2(fx + e) + b \sin^2(fx + e))}{2(a - b)^3 f} + \frac{1}{4(a - b) f (a + b (\tan^2(fx + e)))^2} + \frac{1}{2(a - b)^2 f (a + b (\tan^2(fx + e)))}$$

command

`integrate(tan(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(a + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{4 \log\left(\left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right|\right)}{a^3 - 3a^2b + 3ab^2 - b^3} - \frac{3a^4 + \frac{12a^4(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{a^3 - 3a^2b + 3ab^2 - b^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.78 Problem number 240

$$\int \frac{\cot(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{\ln(\cos(fx + e))}{(a - b)^3 f} + \frac{\ln(\tan(fx + e))}{a^3 f} + \frac{b(3a^2 - 3ab + b^2) \ln(a + b(\tan^2(fx + e)))}{2a^3(a - b)^3 f} - \frac{b}{4a(a - b) f (a + b(\tan^2(fx + e)))^2} - \frac{(2a - b)b}{2a^2(a - b)^2 f (a + b(\tan^2(fx + e)))}$$

command

`integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(3a^2b - 3ab^2 + b^3) \log\left(\left| -a \sin(fx+e)^2 + b \sin(fx+e)^2 + a \right|\right)}{a^6 - 3a^5b + 3a^4b^2 - a^3b^3} - \frac{9a^3b \sin(fx+e)^4 - 18a^2b^2 \sin(fx+e)^4 + 12ab^3 \sin(fx+e)^4 - 3b^4 \sin(fx+e)^4 - 18a^3b}{(a^5 - 2a^4b + a^3b^2)(a \sin(fx+e)^2 - t)} - \frac{4f}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.79 Problem number 241

$$\int \frac{\cot^3(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cot^2(fx + e)}{2a^3 f} - \frac{\ln(\cos(fx + e))}{(a - b)^3 f} - \frac{(a + 3b) \ln(\tan(fx + e))}{a^4 f} \\ & - \frac{b^2(6a^2 - 8ab + 3b^2) \ln(a + b(\tan^2(fx + e)))}{2a^4(a - b)^3 f} \\ & + \frac{b^2}{4a^2(a - b) f (a + b(\tan^2(fx + e)))^2} + \frac{(3a - 2b)b^2}{2a^3(a - b)^2 f (a + b(\tan^2(fx + e)))} \end{aligned}$$

command

```
integrate(cot(f*x+e)^3/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 67.80 Problem number 242

$$\int \frac{\cot^5(e + fx)}{(a + b \tan^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a + 3b) \cot^2(fx + e)}{2a^4 f} - \frac{\cot^4(fx + e)}{4a^3 f} + \frac{\ln(\cos(fx + e))}{(a - b)^3 f} \\ & + \frac{(a^2 + 3ab + 6b^2) \ln(\tan(fx + e))}{a^5 f} + \frac{b^3(10a^2 - 15ab + 6b^2) \ln(a + b(\tan^2(fx + e)))}{2a^5(a - b)^3 f} \\ & - \frac{b^3}{4a^3(a - b) f (a + b(\tan^2(fx + e)))^2} - \frac{(4a - 3b)b^3}{2a^4(a - b)^2 f (a + b(\tan^2(fx + e)))} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5/(a+b*tan(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.81 Problem number 253**

$$\int (a + b \tan^2(c + dx)) dx$$

Optimal antiderivative

$$ax - bx + \frac{b \tan(dx + c)}{d}$$

command

```
integrate(a+b*tan(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$+ \frac{ax \left( \pi - 4 dx \tan(dx) \tan(c) - \pi \operatorname{sgn} \left( 2 \tan(dx)^2 \tan(c) + 2 \tan(dx) \tan(c)^2 - 2 \tan(dx) - 2 \tan(c) \right) \tan(dx) \tan(c)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.82 Problem number 271**

$$\int (a + a \tan^2(c + dx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a^{3/2} \operatorname{arctanh} \left( \frac{\sqrt{a} \tan(dx+c)}{\sqrt{a} (\sec^2(dx+c))} \right)}{2d} + \frac{a \sqrt{a} (\sec^2(dx+c)) \tan(dx+c)}{2d}$$

command

```
integrate((a+a*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.83 Problem number 272**

$$\int (a + a \tan^2(c + dx))^{5/2} dx$$

Optimal antiderivative

$$\frac{3a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \tan(dx+c)}{\sqrt{a(\sec^2(dx+c))}}\right)}{8d} + \frac{a(a(\sec^2(dx+c)))^{3/2} \tan(dx+c)}{4d} + \frac{3a^2 \sqrt{a(\sec^2(dx+c))} \tan(dx+c)}{8d}$$

command

```
integrate((a+a*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.84 Problem number 283**

$$\int \frac{1}{\sqrt{a + a \tan^2(c + dx)}} dx$$

Optimal antiderivative

$$\frac{\tan(dx+c)}{d\sqrt{a(\sec^2(dx+c))}}$$

command

```
integrate(1/(a+a*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\sqrt{a} d \left( \frac{1}{\tan(\frac{1}{2} dx + \frac{1}{2} c)} + \tan(\frac{1}{2} dx + \frac{1}{2} c) \right) \operatorname{sgn} \left( \tan(\frac{1}{2} dx + \frac{1}{2} c)^4 - 1 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a \tan^2(dx+c) + a}} dx$$

**67.85 Problem number 296**

$$\int \cot(e + fx) \sqrt{a + b \tan^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tan^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tan^2(fx + e))}}{\sqrt{a - b}}\right) \sqrt{a - b}}{f}$$

command

```
integrate(cot(f*x+e)*(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{2a \operatorname{arctan}\left(\frac{\sqrt{a - b} \sin(fx + e)^2 - \sqrt{a \sin^4(fx + e) - b \sin^4(fx + e) - 2a \sin^2(fx + e) + b \sin^2(fx + e) + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} \right) + \sqrt{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.86 Problem number 323**

$$\int \frac{\cot(e + fx)}{\sqrt{a + b \tan^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tan^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tan^2(fx + e))}}{\sqrt{a - b}}\right)}{f\sqrt{a - b}}$$

command

```
integrate(cot(f*x+e)/(a+b*tan(f*x+e)^2)^(1/2),x, algorithm="giac")
```



Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan\left(\frac{\sqrt{a-b} \sin(fx+e)^2 - \sqrt{a \sin(fx+e)^4 - b \sin(fx+e)^4 - 2a \sin(fx+e)^2 + b \sin(fx+e)^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\log\left(\frac{\dots}{\dots}\right)}{2 f \operatorname{sgn}(s)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 67.87 Problem number 374

$$\int (a + b \tan^3(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & (a^4 - 6a^2b^2 + b^4)x + \frac{4ab(a^2 - b^2) \ln(\cos(dx + c))}{d} + \frac{b^2(6a^2 - b^2) \tan(dx + c)}{d} \\ & + \frac{2ab(a^2 - b^2) (\tan^2(dx + c))}{d} - \frac{b^2(6a^2 - b^2) (\tan^3(dx + c))}{3d} + \frac{ab^3 (\tan^4(dx + c))}{d} \\ & + \frac{b^2(6a^2 - b^2) (\tan^5(dx + c))}{5d} - \frac{2ab^3 (\tan^6(dx + c))}{3d} + \frac{b^4 (\tan^7(dx + c))}{7d} \\ & + \frac{ab^3 (\tan^8(dx + c))}{2d} - \frac{b^4 (\tan^9(dx + c))}{9d} + \frac{b^4 (\tan^{11}(dx + c))}{11d} \end{aligned}$$

command

```
integrate((a+b*tan(d*x+c)^3)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.88 Problem number 375

$$\int (a + b \tan^3(c + dx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & a(a^2 - 3b^2)x + \frac{b(3a^2 - b^2) \ln(\cos(dx + c))}{d} + \frac{3ab^2 \tan(dx + c)}{d} \\ & + \frac{b(3a^2 - b^2) (\tan^2(dx + c))}{2d} - \frac{ab^2 (\tan^3(dx + c))}{d} + \frac{b^3 (\tan^4(dx + c))}{4d} \\ & + \frac{3ab^2 (\tan^5(dx + c))}{5d} - \frac{b^3 (\tan^6(dx + c))}{6d} + \frac{b^3 (\tan^8(dx + c))}{8d} \end{aligned}$$

command

```
integrate((a+b*tan(d*x+c)^3)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 67.89 Problem number 381

$$\int (a + b \tan^4(c + dx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & (a + b)^4 x - \frac{b(2a + b) (2a^2 + 2ab + b^2) \tan(dx + c)}{d} \\ & + \frac{b(2a + b) (2a^2 + 2ab + b^2) (\tan^3(dx + c))}{3d} - \frac{b^2 (6a^2 + 4ab + b^2) (\tan^5(dx + c))}{5d} \\ & + \frac{b^2 (6a^2 + 4ab + b^2) (\tan^7(dx + c))}{7d} - \frac{b^3 (4a + b) (\tan^9(dx + c))}{9d} \\ & + \frac{b^3 (4a + b) (\tan^{11}(dx + c))}{11d} - \frac{b^4 (\tan^{13}(dx + c))}{13d} + \frac{b^4 (\tan^{15}(dx + c))}{15d} \end{aligned}$$

command

```
integrate((a+tan(d*x+c)^4*b)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.90 Problem number 382**

$$\int (a + b \tan^4(c + dx))^3 dx$$

Optimal antiderivative

$$(a + b)^3 x - \frac{b(3a^2 + 3ab + b^2) \tan(dx + c)}{d} + \frac{b(3a^2 + 3ab + b^2) (\tan^3(dx + c))}{3d} \\ - \frac{b^2(3a + b) (\tan^5(dx + c))}{5d} + \frac{b^2(3a + b) (\tan^7(dx + c))}{7d} \\ - \frac{b^3 (\tan^9(dx + c))}{9d} + \frac{b^3 (\tan^{11}(dx + c))}{11d}$$

command

```
integrate((a+tan(d*x+c)^4*b)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.91 Problem number 384**

$$\int (a + b \tan^4(c + dx)) dx$$

Optimal antiderivative

$$ax + bx - \frac{b \tan(dx + c)}{d} + \frac{b(\tan^3(dx + c))}{3d}$$

command

```
integrate(a+tan(d*x+c)^4*b,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.92 Problem number 429**

$$\int \cos^5(c + dx) (a + b \tan^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{a \sin(dx + c)}{d} - \frac{(2a - b) (\sin^3(dx + c))}{3d} + \frac{(a - b) (\sin^5(dx + c))}{5d}$$

command

```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.93 Problem number 430**

$$\int \cos^7(c + dx) (a + b \tan^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{a \sin(dx + c)}{d} - \frac{(3a - b) (\sin^3(dx + c))}{3d} + \frac{(3a - 2b) (\sin^5(dx + c))}{5d} - \frac{(a - b) (\sin^7(dx + c))}{7d}$$

command

```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.94 Problem number 435**

$$\int \cos^4(c + dx) (a + b \tan^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{(3a + b)x}{8} + \frac{(3a + b) \cos(dx + c) \sin(dx + c)}{8d} + \frac{(a - b) (\cos^3(dx + c)) \sin(dx + c)}{4d}$$

command

```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.95 Problem number 436**

$$\int \cos^6(c + dx) (a + b \tan^2(c + dx)) dx$$

Optimal antiderivative

$$\frac{(5a + b)x}{16} + \frac{(5a + b) \cos(dx + c) \sin(dx + c)}{16d} + \frac{(5a + b) (\cos^3(dx + c)) \sin(dx + c)}{24d} + \frac{(a - b) (\cos^5(dx + c)) \sin(dx + c)}{6d}$$

command

```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**67.96 Problem number 441**

$$\int \cos^5(c + dx) (a + b \tan^2(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{a^2 \sin(dx + c)}{d} - \frac{2a(a - b) (\sin^3(dx + c))}{3d} + \frac{(a - b)^2 (\sin^5(dx + c))}{5d}$$

command

```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.97 Problem number 448**

$$\int \cos^4(c + dx) (a + b \tan^2(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{(3a^2 + 2ab + 3b^2) x}{8} + \frac{3(a^2 - b^2) \cos(dx + c) \sin(dx + c)}{8d} + \frac{(a - b) (\cos^3(dx + c)) \sin(dx + c) (a + b(\tan^2(dx + c)))}{4d}$$

command

```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**67.98 Problem number 449**

$$\int \cos^6(c + dx) (a + b \tan^2(c + dx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(5a^2 + 2ab + b^2)x}{16} + \frac{(5a^2 + 2ab + b^2) \cos(dx + c) \sin(dx + c)}{16d} \\ & + \frac{(a - b)(5a + 3b) (\cos^3(dx + c)) \sin(dx + c)}{24d} \\ & + \frac{(a - b) (\cos^5(dx + c)) \sin(dx + c) (a + b(\tan^2(dx + c)))}{6d} \end{aligned}$$

command

```
integrate(cos(d*x+c)^6*(a+b*tan(d*x+c)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**68 Test file number 108**

Test folder name:

test\_cases/4\_Trig\_functions/4.4\_Cotangent/108\_4.4.0-a\_trg-^m-b\_cot-^n

**68.1 Problem number 45**

$$\int (d \cot(e + fx))^n \csc^2(e + fx) dx$$

Optimal antiderivative

$$\frac{(d \cot(fx + e))^{1+n}}{df(1+n)}$$

command

```
integrate((d*cot(f*x+e))^n*csc(f*x+e)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-\frac{d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - d}{2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}\right)^{n+1}}{df(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (d \cot (fx + e))^n \csc (fx + e)^2 dx$$

## 69 Test file number 110

Test folder name:

test\_cases/4\_Trig\_functions/4.4\_Cotangent/110\_4.4.1.2-d\_csc-^m-a+b\_cot-^n

### 69.1 Problem number 23

$$\int (a + b \cot(x))^n \csc^2(x) dx$$

Optimal antiderivative

$$\frac{(a + b \cot(x))^{1+n}}{b(1+n)}$$

command

`integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(-\frac{b \tan\left(\frac{1}{2}x\right)^2 - 2a \tan\left(\frac{1}{2}x\right) - b}{2 \tan\left(\frac{1}{2}x\right)}\right)^{n+1}}{b(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \cot(x) + a)^n \csc(x)^2 dx$$



## 70 Test file number 113

Test folder name:

test\_cases/4\_Trig\_functions/4.4\_Cotangent/113\_4.4.7-d\_trig-<sup>m</sup>-a+b-c\_cot-<sup>n</sup>-<sup>p</sup>

### 70.1 Problem number 1

$$\int \frac{A + C \cot^2(c + dx)}{\sqrt{b \tan(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(A - C) \arctan\left(1 - \frac{\sqrt{2} \sqrt{b \tan(dx + c)}}{\sqrt{b}}\right) \sqrt{2}}{2d\sqrt{b}} \\ & + \frac{(A - C) \arctan\left(1 + \frac{\sqrt{2} \sqrt{b \tan(dx + c)}}{\sqrt{b}}\right) \sqrt{2}}{2d\sqrt{b}} \\ & - \frac{(A - C) \ln\left(\sqrt{b} - \sqrt{2} \sqrt{b \tan(dx + c)} + \sqrt{b} \tan(dx + c)\right) \sqrt{2}}{4d\sqrt{b}} \\ & + \frac{(A - C) \ln\left(\sqrt{b} + \sqrt{2} \sqrt{b \tan(dx + c)} + \sqrt{b} \tan(dx + c)\right) \sqrt{2}}{4d\sqrt{b}} - \frac{2bC}{3d(b \tan(dx + c))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate((A+C*cot(d*x+c)^2)/(b*tan(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{2} \left( A\sqrt{|b|} - C\sqrt{|b|} \right) \arctan\left( \frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} + 2\sqrt{b \tan(dx + c)} \right)}{2\sqrt{|b|}} \right)}{2bd} \\ & + \frac{\sqrt{2} \left( A\sqrt{|b|} - C\sqrt{|b|} \right) \arctan\left( -\frac{\sqrt{2} \left( \sqrt{2} \sqrt{|b|} - 2\sqrt{b \tan(dx + c)} \right)}{2\sqrt{|b|}} \right)}{2bd} \\ & + \frac{\sqrt{2} \left( A\sqrt{|b|} - C\sqrt{|b|} \right) \log\left( b \tan(dx + c) + \sqrt{2} \sqrt{b \tan(dx + c)} \sqrt{|b|} + |b| \right)}{4bd} \\ & - \frac{\sqrt{2} \left( A\sqrt{|b|} - C\sqrt{|b|} \right) \log\left( b \tan(dx + c) - \sqrt{2} \sqrt{b \tan(dx + c)} \sqrt{|b|} + |b| \right)}{4bd} \\ & - \frac{2C}{3\sqrt{b \tan(dx + c)} d \tan(dx + c)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{C \cot(dx + c)^2 + A}{\sqrt{b \tan(dx + c)}} dx$$

## 70.2 Problem number 9

$$\int \sqrt{1 + \cot^2(x)} dx$$

Optimal antiderivative

$$- \operatorname{arcsinh}(\cot(x))$$

command

`integrate((1+cot(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 70.3 Problem number 10

$$\int \frac{1}{\sqrt{1 + \cot^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\cot(x)}{\sqrt{\csc^2(x)}}$$

command

`integrate(1/(1+cot(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\left(\frac{\cos(x)-1}{\cos(x)+1} - 1\right) \operatorname{sgn}(\sin(x))} + 2 \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 70.4 Problem number 11

$$\int (-1 - \cot^2(x))^{3/2} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\cot(x)}{\sqrt{-(\csc^2(x))}}\right)}{2} + \frac{\cot(x) \sqrt{-(\csc^2(x))}}{2}$$

command

```
integrate((-1-cot(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} \left( \frac{2i \cos(x)}{\cos(x)^2 - 1} - i \log(\cos(x) + 1) + i \log(-\cos(x) + 1) \right) \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 70.5 Problem number 12

$$\int \sqrt{-1 - \cot^2(x)} dx$$

Optimal antiderivative

$$\arctan\left(\frac{\cot(x)}{\sqrt{-(\csc^2(x))}}\right)$$

command

```
integrate((-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$i \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right) \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 70.6 Problem number 13

$$\int \frac{1}{\sqrt{-1 - \cot^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\cot(x)}{\sqrt{-(\csc^2(x))}}$$

command

```
integrate(1/(-1-cot(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2i}{\left(\frac{\cos(x)-1}{\cos(x)+1} - 1\right) \operatorname{sgn}(\sin(x))} - 2i \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 70.7 Problem number 15

$$\int \frac{\cot^2(x)}{\sqrt{a + a \cot^2(x)}} dx$$

Optimal antiderivative

$$\frac{\cot(x)}{\sqrt{a(\csc^2(x))}} - \frac{\operatorname{arctanh}(\cos(x)) \csc(x)}{\sqrt{a(\csc^2(x))}}$$

command

```
integrate(cot(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \sqrt{a} \left( \frac{2 \cos(x)}{a \operatorname{sgn}(\sin(x))} - \frac{\log(\cos(x) + 1)}{a \operatorname{sgn}(\sin(x))} + \frac{\log(-\cos(x) + 1)}{a \operatorname{sgn}(\sin(x))} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 70.8 Problem number 18

$$\int \frac{\tan^2(x)}{\sqrt{a + a \cot^2(x)}} dx$$

Optimal antiderivative

$$\frac{\cot(x)}{\sqrt{a(\csc^2(x))}} + \frac{\csc(x) \sec(x)}{\sqrt{a(\csc^2(x))}}$$

command

`integrate(tan(x)^2/(a+a*cot(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2 \operatorname{sgn}(\sin(x))}{\sqrt{a}} + \frac{\frac{1}{\cos(x)} + \cos(x)}{\sqrt{a} \operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 70.9 Problem number 45

$$\int \frac{\cot^2(x)}{\sqrt{a + b \cot^2(x)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{a-b}}{\sqrt{a+b(\cot^2(x))}}\right)}{\sqrt{a-b}} - \frac{\operatorname{arctanh}\left(\frac{\cot(x)\sqrt{b}}{\sqrt{a+b(\cot^2(x))}}\right)}{\sqrt{b}}$$

command

`integrate(cot(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2a \arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right) - 2b \arctan\left(\frac{\sqrt{-a+b}\sqrt{b}}{\sqrt{ab-b^2}}\right) + \sqrt{ab-b^2} \log\left(-a - 2\sqrt{-a+b}\sqrt{b} + 2b\right)\right) \operatorname{sgn}(\sin(x))}{2\sqrt{ab-b^2}\sqrt{-a+b}} + \frac{2\sqrt{-a+b} \arctan\left(\frac{\left(\sqrt{-a+b}\cos(x) - \sqrt{-a\cos^2(x) + b\cos(x)^2 + a}\right)^2}{2\sqrt{ab-b^2}}\right)}{\sqrt{ab-b^2}} + \frac{\log\left(\left(\sqrt{-a+b}\cos(x) - \sqrt{-a\cos^2(x) + b\cos(x)^2 + a}\right)\right)}{\sqrt{-a+b}}$$

$$-\frac{\phantom{0}}{2 \operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 70.10 Problem number 48

$$\int \frac{\tan^2(x)}{\sqrt{a + b \cot^2(x)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\cot(x)\sqrt{a-b}}{\sqrt{a+b(\cot^2(x))}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a+b(\cot^2(x))} \tan(x)}{a}$$

command

```
integrate(tan(x)^2/(a+b*cot(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a \log\left(-a - 2\sqrt{-a+b}\sqrt{b} + 2b\right) + \sqrt{-a+b}\sqrt{b} \log\left(-a - 2\sqrt{-a+b}\sqrt{b} + 2b\right) - b \log\left(-a - 2\sqrt{-a+b}\sqrt{b} + 2b\right)\right)}{2\left(a\sqrt{-a+b} - a\sqrt{b} - \sqrt{-a+b}b + b^{\frac{3}{2}}\right)}$$

$$\frac{\log\left(\left(\sqrt{-a+b}\cos(x) - \sqrt{-a\cos^2(x) + b\cos^2(x) + a}\right)^2\right)}{\sqrt{-a+b}} + \frac{4\sqrt{-a+b}}{\left(\sqrt{-a+b}\cos(x) - \sqrt{-a\cos^2(x) + b\cos^2(x) + a}\right)^2}$$


---


$$- \frac{1}{2 \operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 70.11 Problem number 49

$$\int \frac{\cot^3(x)}{(a + b \cot^2(x))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\cot^2(x))}}{\sqrt{a-b}}\right)}{(a-b)^{\frac{3}{2}}} + \frac{a}{(a-b)b\sqrt{a+b(\cot^2(x))}}$$

command

```
integrate(cot(x)^3/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{-\frac{\log(|b|) \operatorname{sgn}(\sin(x))}{2(\sqrt{a-b}a - \sqrt{a-b}b)} + \frac{\frac{a \sin(x)}{\sqrt{a \sin(x)^2 - b \sin(x)^2 + b}}{(ab-b^2)} + \frac{\log\left(\left|-\sqrt{a-b} \sin(x) + \sqrt{a \sin(x)^2 - b \sin(x)^2 + b}\right|\right)}{(a-b)^{\frac{3}{2}}}}{\operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 70.12 Problem number 51

$$\int \frac{\cot(x)}{(a + b \cot^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a - b}}\right)}{(a - b)^{\frac{3}{2}}} - \frac{1}{(a - b) \sqrt{a + b(\cot^2(x))}}$$

command

```
integrate(cot(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\log(|b|) \operatorname{sgn}(\sin(x))}{2(\sqrt{a-b}a - \sqrt{a-b}b)} + \frac{\log\left(\left|-\sqrt{a-b} \sin(x) + \sqrt{a \sin(x)^2 - b \sin(x)^2 + b}\right|\right)}{(a-b)^{\frac{3}{2}}} + \frac{\frac{\sin(x)}{\sqrt{a \sin(x)^2 - b \sin(x)^2 + b}}{(a-b)}}{\operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 70.13 Problem number 52

$$\int \frac{\tan(x)}{(a + b \cot^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a - b}}\right)}{(a - b)^{3/2}} + \frac{b}{a(a - b)\sqrt{a + b(\cot^2(x))}}$$

command

```
integrate(tan(x)/(a+b*cot(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2\sqrt{a-b}a^2\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right) - 4\sqrt{a-b}ab\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right) + 2\sqrt{a-b}b^2\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)\right)}{2\left(\sqrt{-a^2+ab}a^3 - 2\sqrt{-a^2+ab}a^2b + \sqrt{-a^2+ab}ab^2\right)} + \frac{\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x) - \sqrt{a\sin^2(x) - b\sin^2(x) + b}\right)^2\right)}{a^2 - 2ab + b^2} + \frac{2b\sin(x)}{\sqrt{a\sin^2(x) - b\sin^2(x) + b}(a^2 - ab)} + \frac{2\sqrt{a-b}\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)}{2\operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 70.14 Problem number 53

$$\int \frac{\tan^2(x)}{(a + b \cot^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctan}\left(\frac{\cot(x)\sqrt{a-b}}{\sqrt{a+b(\cot^2(x))}}\right)}{(a-b)^{3/2}} + \frac{b\tan(x)}{a(a-b)\sqrt{a+b(\cot^2(x))}} + \frac{(a-2b)\sqrt{a+b(\cot^2(x))}\tan(x)}{a^2(a-b)}$$





## 70.16 Problem number 56

$$\int \frac{\cot(x)}{(a + b \cot^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a - b}}\right)}{(a - b)^{\frac{5}{2}}} - \frac{1}{3(a - b)(a + b(\cot^2(x)))^{\frac{3}{2}}} - \frac{1}{(a - b)^2 \sqrt{a + b(\cot^2(x))}}$$

command

```
integrate(cot(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(|b|) \operatorname{sgn}(\sin(x))}{2\left(\sqrt{a - b} a^2 - 2\sqrt{a - b} ab + \sqrt{a - b} b^2\right)} + \frac{\left(\frac{4(a^2 b - 2ab^2 + b^3) \sin(x)^2}{a^3 b - 3a^2 b^2 + 3ab^3 - b^4} + \frac{3(ab^2 - b^3)}{a^3 b - 3a^2 b^2 + 3ab^3 - b^4}\right) \sin(x)}{(a \sin(x)^2 - b \sin(x)^2 + b)^{\frac{3}{2}}} + \frac{3 \log\left(\left|-\sqrt{a - b} \sin(x) + \sqrt{a \sin(x)^2 - b \sin(x)^2 + b}\right|\right)}{(a^2 - 2ab + b^2) \sqrt{a - b}}$$


---


$$3 \operatorname{sgn}(\sin(x))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 70.17 Problem number 57

$$\int \frac{\tan(x)}{(a + b \cot^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\cot^2(x))}}{\sqrt{a - b}}\right)}{(a - b)^{\frac{5}{2}}} + \frac{b}{3a(a - b)(a + b(\cot^2(x)))^{\frac{3}{2}}} + \frac{(2a - b)b}{a^2(a - b)^2 \sqrt{a + b(\cot^2(x))}}$$

command

```
integrate(tan(x)/(a+b*cot(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(2\sqrt{a-b}a^3\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)-6\sqrt{a-b}a^2b\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)+6\sqrt{a-b}ab^2\arctan\left(-\frac{a-b}{\sqrt{-a^2+ab}}\right)\right)}{2\left(\sqrt{-a^2+ab}a^5-3\sqrt{-a^2+ab}a^4b+3\sqrt{-a^2+ab}a^3b^2-3\sqrt{-a^2+ab}a^2b^3+b^5\right)} + \frac{3\sqrt{a-b}\log\left(\left(\sqrt{a-b}\sin(x)-\sqrt{a\sin^2(x)-b\sin(x)^2+b}\right)^2\right)}{a^3-3a^2b+3ab^2-b^3} + \frac{2\left(\frac{(7a^5b^2-17a^4b^3+13a^3b^4-3a^2b^5)\sin(x)^2}{a^7b-3a^6b^2+3a^5b^3-a^4b^4}+\frac{3(2a^4b^3-3a^3b^4+a^2b^5)}{a^7b-3a^6b^2+3a^5b^3-a^4b^4}\right)}{(a\sin(x)^2-b\sin(x)^2+b)^{\frac{3}{2}}}$$

$$+ \frac{1}{6\operatorname{sgn}(\sin(x))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 71 Test file number 115

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/115\_4.5.0-a\_sec~m-b\_trg~n

### 71.1 Problem number 228

$$\int (d \sec(a + bx))^{5/2} \sin^3(a + bx) dx$$

Optimal antiderivative

$$\frac{2d(d \sec(bx + a))^{\frac{3}{2}}}{3b} + \frac{2d^3}{b\sqrt{d \sec(bx + a)}}$$

command

```
integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(3\sqrt{d\cos(bx+a)}d+\frac{d^2}{\sqrt{d\cos(bx+a)}\cos(bx+a)}\right)d\operatorname{sgn}(\cos(bx+a))}{3b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (d \sec(bx + a))^{\frac{5}{2}} \sin(bx + a)^3 dx$$

## 72 Test file number 118

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/118\_4.5.1.2-d\_sec-<sup>n</sup>-a+b\_sec-<sup>m</sup>

### 72.1 Problem number 123

$$\int \frac{1}{\sqrt{a + a \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{\sqrt{a} \tan(dx+c)}{\sqrt{a + a \sec(dx+c)}}\right)}{d\sqrt{a}} - \frac{\arctan\left(\frac{\sqrt{a} \tan(dx+c)\sqrt{2}}{2\sqrt{a + a \sec(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}}$$

command

`integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{-a + \frac{a}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2 \arctan\left(\frac{\sqrt{2} \sqrt{-a + \frac{a}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}}}{2\sqrt{a}}\right)}{\sqrt{a}}$$

$d$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a \sec(dx + c) + a}} dx$$

### 72.2 Problem number 142

$$\int \frac{1}{\sqrt{a - a \sec(c + dx)}} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{\sqrt{a} \tan(dx+c)}{\sqrt{a - a \sec(dx+c)}}\right)}{d\sqrt{a}} - \frac{\arctan\left(\frac{\sqrt{a} \tan(dx+c)\sqrt{2}}{2\sqrt{a - a \sec(dx+c)}}\right)\sqrt{2}}{d\sqrt{a}}$$

command

```
integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2 \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a}}{2 \sqrt{a}}\right)}{\sqrt{a}}$$


---

$d$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-a \sec(dx + c) + a}} dx$$

### 72.3 Problem number 244

$$\int \sqrt{-\sec(e + fx)} \sqrt{a - a \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arcsinh}\left(\frac{\sqrt{a} \tan(fx + e)}{\sqrt{a - a \sec(fx + e)}}\right) \sqrt{a}}{f}$$

command

```
integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{\sqrt{2} a^2 \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a}}{2 \sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{2} a^2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}} \right) |a| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} e\right)\right)}{a^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 73 Test file number 120

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/120\_4.5.1.4-d\_tan<sup>n</sup>-a+b\_sec<sup>m</sup>

### 73.1 Problem number 207

$$\int \frac{\tan^2(e + fx)}{(a + a \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{a^{\frac{9}{2}} f} + \frac{91 \arctan\left(\frac{\sqrt{a} \tan(fx+e)\sqrt{2}}{2\sqrt{a + a \sec(fx+e)}}\right) \sqrt{2}}{64a^{\frac{9}{2}} f} \\ & + \frac{\tan(fx+e)}{3af(a + a \sec(fx+e))^{\frac{7}{2}}} + \frac{11 \tan(fx+e)}{24a^2 f(a + a \sec(fx+e))^{\frac{5}{2}}} + \frac{27 \tan(fx+e)}{32a^3 f(a + a \sec(fx+e))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(tan(f*x+e)^2/(a+a*sec(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \left(2 \left(\frac{4\sqrt{2} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}{a^5 \operatorname{sgn}(\cos(fx+e))} - \frac{19\sqrt{2}}{a^5 \operatorname{sgn}(\cos(fx+e))}\right) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + \frac{111\sqrt{2}}{a^5 \operatorname{sgn}(\cos(fx+e))}\right) \tan}{192 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 73.2 Problem number 300

$$\int \frac{\tan^7(c + dx)}{(a + b \sec(c + dx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(dx+c))}{a^2 d} + \frac{(a^2 - b^2)^2 (5a^2 + b^2) \ln(a + b \sec(dx+c))}{a^2 b^6 d} - \frac{2a(2a^2 - 3b^2) \sec(dx+c)}{b^5 d} \\ & + \frac{3(a^2 - b^2) (\sec^2(dx+c))}{2b^4 d} - \frac{2a(\sec^3(dx+c))}{3b^3 d} + \frac{\sec^4(dx+c)}{4b^2 d} + \frac{(a^2 - b^2)^3}{a b^6 d (a + b \sec(dx+c))} \end{aligned}$$

command

```
integrate(tan(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 73.3 Problem number 330

$$\int \frac{\cot^3(c + dx)}{\sqrt{a + b \sec(c + dx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a + b \sec(dx + c)}}{\sqrt{a - b}}\right)}{4(a - b)^{\frac{3}{2}} d} + \frac{b \operatorname{arctanh}\left(\frac{\sqrt{a + b \sec(dx + c)}}{\sqrt{a + b}}\right)}{4(a + b)^{\frac{3}{2}} d} \\ & -\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a + b \sec(dx + c)}}{\sqrt{a}}\right)}{d\sqrt{a}} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sec(dx + c)}}{\sqrt{a - b}}\right)}{d\sqrt{a - b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \sec(dx + c)}}{\sqrt{a + b}}\right)}{d\sqrt{a + b}} \\ & + \frac{\sqrt{a + b \sec(dx + c)}}{4(a + b)d(1 - \sec(dx + c))} + \frac{\sqrt{a + b \sec(dx + c)}}{4(a - b)d(1 + \sec(dx + c))} \end{aligned}$$

command

```
integrate(cot(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4a-5b)\sqrt{a-b} \log\left(\left(\sqrt{a-b} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - \sqrt{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 2a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2}\right)\right)}{a^2 - 2ab + b^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74 Test file number 121

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/121\_4.5.2.1-a+b\_sec-^m-c+d\_sec-^n

### 74.1 Problem number 1

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^5 dx$$

Optimal antiderivative

$$\begin{aligned} & a^2 c^5 x - \frac{19a^2 c^5 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{a^2 c^5 \tan(fx + e)}{f} \\ & + \frac{17a^2 c^5 \sec(fx + e) \tan(fx + e)}{16f} + \frac{a^2 c^5 (\sec^3(fx + e)) \tan(fx + e)}{8f} \\ & + \frac{a^2 c^5 (\tan^3(fx + e))}{3f} - \frac{3a^2 c^5 \sec(fx + e) (\tan^3(fx + e))}{4f} \\ & - \frac{a^2 c^5 (\sec^3(fx + e)) (\tan^3(fx + e))}{6f} + \frac{3a^2 c^5 (\tan^5(fx + e))}{5f} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$240 (fx + e)a^2 c^5 - 285 a^2 c^5 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) + 285 a^2 c^5 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right) + \frac{2 \left( 525 a^2 c^5 \tan \left( \frac{1}{2} fx \right) \right)}{240}$$

240

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.2 Problem number 2

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & a^2 c^4 x - \frac{3a^2 c^4 \operatorname{arctanh}(\sin(fx + e))}{4f} - \frac{a^2 c^4 \tan(fx + e)}{f} + \frac{3a^2 c^4 \sec(fx + e) \tan(fx + e)}{4f} \\ & + \frac{a^2 c^4 (\tan^3(fx + e))}{3f} - \frac{a^2 c^4 \sec(fx + e) (\tan^3(fx + e))}{2f} + \frac{a^2 c^4 (\tan^5(fx + e))}{5f} \end{aligned}$$



command

```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$60 (fx + e)a^2c^4 - 45 a^2c^4 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) + 45 a^2c^4 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right) + \frac{2 \left( 105 a^2c^4 \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{60 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.3 Problem number 3

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$a^2c^3x - \frac{3a^2c^3 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{a^2(8c^3 - 3c^3 \sec(fx + e)) \tan(fx + e)}{8f} + \frac{a^2(4c^3 - 3c^3 \sec(fx + e)) (\tan^3(fx + e))}{12f}$$

command

```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$24 (fx + e)a^2c^3 - 9 a^2c^3 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) + 9 a^2c^3 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right) + \frac{2 \left( 33 a^2c^3 \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)^7}{24 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.4 Problem number 5

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$a^2 cx + \frac{a^2 c \operatorname{arctanh}(\sin(fx + e))}{2f} - \frac{c(2a^2 + a^2 \sec(fx + e)) \tan(fx + e)}{2f}$$

command

```
integrate((a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(fx + e)a^2c + a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) + \frac{2\left(a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^2}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.5 Problem number 6

$$\int \frac{(a + a \sec(e + fx))^2}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{a^2 x}{c} - \frac{a^2 \operatorname{arctanh}(\sin(fx + e))}{cf} - \frac{4a^2 \tan(fx + e)}{cf(1 - \sec(fx + e))}$$

command

```
integrate((a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(fx+e)a^2}{c} - \frac{a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{c} + \frac{a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{c} + \frac{4a^2}{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.6 Problem number 11

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^5 dx$$

Optimal antiderivative

$$\begin{aligned} & a^3 c^5 x - \frac{5a^3 c^5 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{a^3 c^5 \tan(fx + e)}{f} + \frac{5a^3 c^5 \sec(fx + e) \tan(fx + e)}{8f} \\ & + \frac{a^3 c^5 (\tan^3(fx + e))}{3f} - \frac{5a^3 c^5 \sec(fx + e) (\tan^3(fx + e))}{12f} \\ & - \frac{a^3 c^5 (\tan^5(fx + e))}{5f} + \frac{a^3 c^5 \sec(fx + e) (\tan^5(fx + e))}{3f} - \frac{a^3 c^5 (\tan^7(fx + e))}{7f} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$840 (fx + e) a^3 c^5 - 525 a^3 c^5 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) + 525 a^3 c^5 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right) + \frac{2 \left( 1365 a^3 c^5 \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.7 Problem number 12

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & a^3 c^4 x - \frac{5a^3 c^4 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{a^3 (16c^4 - 5c^4 \sec(fx + e)) \tan(fx + e)}{16f} \\ & + \frac{a^3 (8c^4 - 5c^4 \sec(fx + e)) (\tan^3(fx + e))}{24f} - \frac{a^3 (6c^4 - 5c^4 \sec(fx + e)) (\tan^5(fx + e))}{30f} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$240 (fx + e) a^3 c^4 - 75 a^3 c^4 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) + 75 a^3 c^4 \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right) + \frac{2 \left( 315 a^3 c^4 \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.8 Problem number 14

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$a^3 c^2 x + \frac{3a^3 c^2 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{c^2(8a^3 + 3a^3 \sec(fx + e)) \tan(fx + e)}{8f} + \frac{c^2(4a^3 + 3a^3 \sec(fx + e)) (\tan^3(fx + e))}{12f}$$

command

`integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$24(fx + e)a^3c^2 + 9a^3c^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 9a^3c^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) + \frac{2\left(15a^3c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 - \dots\right)}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.9 Problem number 15

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$a^3 cx + \frac{a^3 c \operatorname{arctanh}(\sin(fx + e))}{f} - \frac{a^3 c \tan(fx + e)}{f} - \frac{a^3 c \sec(fx + e) \tan(fx + e)}{f} - \frac{a^3 c (\tan^3(fx + e))}{3f}$$

command

`integrate((a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$3(fx + e)a^3c + 3a^3c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3a^3c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{4\left(a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^2}$$

3f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.10 Problem number 16

$$\int \frac{(a + a \sec(e + fx))^3}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{a^3 x}{c} - \frac{4a^3 \operatorname{arctanh}(\sin(fx + e))}{cf} + \frac{8a^3 \cot(fx + e)}{cf} + \frac{8a^3 \csc(fx + e)}{cf} - \frac{a^3 \tan(fx + e)}{cf}$$

command

```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(fx+e)a^3}{c} - \frac{4a^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{c} + \frac{4a^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{c} + \frac{2(5a^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 4a^3)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - \tan(\frac{1}{2}fx + \frac{1}{2}e))c}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.11 Problem number 17

$$\int \frac{(a + a \sec(e + fx))^3}{(c - c \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^3 x}{c^2} + \frac{a^3 \operatorname{arctanh}(\sin(fx + e))}{c^2 f} - \frac{8a^3 \tan(fx + e)}{3c^2 f (1 - \sec(fx + e))^2} + \frac{4a^3 \tan(fx + e)}{3c^2 f (1 - \sec(fx + e))}$$

command

```
integrate((a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3(fx+e)a^3}{c^2} + \frac{3a^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{c^2} - \frac{3a^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{c^2} - \frac{4a^3}{c^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3}}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.12 Problem number 21

$$\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^5 x}{a^2} - \frac{47c^5 \operatorname{arctanh}(\sin(fx + e))}{2a^2 f} + \frac{13c^5 \tan(fx + e)}{2a^2 f} + \frac{112c^5 \tan(fx + e)}{3a^2 f (1 + \sec(fx + e))} \\ & - \frac{32c^5 \tan(fx + e)}{3f (a + a \sec(fx + e))^2} + \frac{(c^5 - c^5 \sec(fx + e)) \tan(fx + e)}{2a^2 f} \end{aligned}$$

command

```
integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(fx+e)c^5}{a^2} - \frac{141c^5 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} + \frac{141c^5 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2} - \frac{6(15c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 13c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)^2 a^2} + \frac{32(a^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 3a^4 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.13 Problem number 22

$$\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^4 x}{a^2} - \frac{6c^4 \operatorname{arctanh}(\sin(fx + e))}{a^2 f} - \frac{16c^4 \cot(fx + e)}{a^2 f} \\ & - \frac{32c^4 (\cot^3(fx + e))}{3a^2 f} + \frac{32c^4 (\csc^3(fx + e))}{3a^2 f} + \frac{c^4 \tan(fx + e)}{a^2 f} \end{aligned}$$

command

```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(fx+e)c^4}{a^2} - \frac{18c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} + \frac{18c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2} - \frac{6c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)a^2} + \frac{8(a^4 c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3a^4 c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.14 Problem number 23

$$\int \frac{(c - c \sec(e + fx))^3}{(a + a \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{c^3 x}{a^2} - \frac{c^3 \operatorname{arctanh}(\sin(fx + e))}{a^2 f} - \frac{8c^3 \tan(fx + e)}{3a^2 f (1 + \sec(fx + e))^2} + \frac{4c^3 \tan(fx + e)}{3a^2 f (1 + \sec(fx + e))}$$

command

`integrate((c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{4c^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3}{a^2} + \frac{3(fx+e)c^3}{a^2} - \frac{3c^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} + \frac{3c^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2}}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.15 Problem number 31

$$\int \frac{(c - c \sec(e + fx))^5}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^5 x}{a^3} + \frac{8c^5 \operatorname{arctanh}(\sin(fx + e))}{a^3 f} + \frac{32c^5 \cot(fx + e)}{a^3 f} \\ & + \frac{128c^5 (\cot^3(fx + e))}{3a^3 f} + \frac{128c^5 (\cot^5(fx + e))}{5a^3 f} - \frac{16c^5 \csc(fx + e)}{a^3 f} \\ & + \frac{64c^5 (\csc^3(fx + e))}{3a^3 f} - \frac{128c^5 (\csc^5(fx + e))}{5a^3 f} - \frac{c^5 \tan(fx + e)}{a^3 f} \end{aligned}$$

command

`integrate((c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{15(fx+e)c^5}{a^3} + \frac{120c^5 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^3} - \frac{120c^5 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^3} + \frac{30c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)a^3} - \frac{8(3a^{12}c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 + 5a^{12}c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 3a^{12}c^5 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 5a^{12}c^5)}{15f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.16 Problem number 32

$$\int \frac{(c - c \sec(e + fx))^4}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^4 x}{a^3} + \frac{c^4 \operatorname{arctanh}(\sin(fx + e))}{a^3 f} - \frac{3c^4 \tan(fx + e)}{a^3 f (1 + \sec(fx + e))^3} \\ & - \frac{c^4 (\sec^2(fx + e)) \tan(fx + e)}{5a^3 f (1 + \sec(fx + e))^3} + \frac{14c^4 \tan(fx + e)}{5a^3 f (1 + \sec(fx + e))^2} - \frac{23c^4 \tan(fx + e)}{5a^3 f (1 + \sec(fx + e))} \end{aligned}$$

command

```
integrate((c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5 \frac{(fx+e)c^4}{a^3} + \frac{5c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^3} - \frac{5c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^3} - \frac{4(a^{12}c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 + 5a^{12}c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{a^{15}}}{5f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.17 Problem number 42

$$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2c^4 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{f} - \frac{2a c^4 \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2a^2 c^4 (\tan^3(fx + e))}{3f (a + a \sec(fx + e))^{\frac{3}{2}}} - \frac{2a^3 c^4 (\tan^5(fx + e))}{5f (a + a \sec(fx + e))^{\frac{5}{2}}} + \frac{2a^4 c^4 (\tan^7(fx + e))}{7f (a + a \sec(fx + e))^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((c-c*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$105 \sqrt{-a} a c^4 \log \left( \frac{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a \right|}{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a \right|} \right) \operatorname{sgn}(\cos(fx+e))}{|a|} - \frac{2 \left( 105 \sqrt{2} a^4 c^4 \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.18 Problem number 43

$$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{2c^3 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right) \sqrt{a}}{f} - \frac{2a c^3 \tan(fx+e)}{f \sqrt{a + a \sec(fx+e)}} + \frac{2a^2 c^3 (\tan^3(fx+e))}{3f (a + a \sec(fx+e))^{\frac{3}{2}}} - \frac{2a^3 c^3 (\tan^5(fx+e))}{5f (a + a \sec(fx+e))^{\frac{5}{2}}}$$

command

`integrate((c-c*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 \sqrt{-a} a c^3 \log \left( \frac{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a \right|}{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a \right|} \right) \operatorname{sgn}(\cos(fx+e))}{|a|} + \frac{2 \left( 15 \sqrt{2} a^3 c^3 \operatorname{sgn}(\cos(fx+e)) \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.19 Problem number 44

$$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{2c^2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right) \sqrt{a}}{f} - \frac{2ac^2 \tan(fx+e)}{f \sqrt{a + a \sec(fx+e)}} + \frac{2a^2c^2 (\tan^3(fx+e))}{3f (a + a \sec(fx+e))^{\frac{3}{2}}}$$

command

`integrate((c-c*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \sqrt{-a} ac^2 \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right|^{-4 \sqrt{2} |a| - 6a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right|^{+4 \sqrt{2} |a| - 6a}} \right) \operatorname{sgn}(\cos(fx+e))}{|a|} + \frac{2 \left( 5 \sqrt{2} a^2 c^2 \operatorname{sgn}(\cos(fx+e)) \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.20 Problem number 45

$$\int \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2c \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right) \sqrt{a}}{f} - \frac{2ac \tan(fx+e)}{f \sqrt{a + a \sec(fx+e)}}$$

command

`integrate((c-c*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2} \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \operatorname{acsgn}(\cos(fx+e)) \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a} - \frac{\sqrt{-a} \operatorname{aclog} \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2 + 4\sqrt{2}|a| - 6a \right|} \right)}{f|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**74.21 Problem number 46**

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{cf} + \frac{2 \cot(fx + e) \sqrt{a + a \sec(fx + e)}}{cf}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{-a} \operatorname{alog} \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2 + 4\sqrt{2}|a| - 6a \right|} \right)}{c|a|} + \left( \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2 \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.22 Problem number 47

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(\cot^3(fx + e))(a + a \sec(fx + e))^{\frac{3}{2}}}{3ac^2f} + \frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{c^2f} \\ & + \frac{2 \cot(fx + e) \sqrt{a + a \sec(fx + e)}}{c^2f} \end{aligned}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{3\sqrt{2} \sqrt{-a} a \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right|^{-4} \sqrt{2}^{|a|-6a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right|^{+4} \sqrt{2}^{|a|-6a}} \right)}{c^2|a|} \right) + \frac{2 \left( 9 \left( \sqrt{-a} \tan\left(\frac{1}{2}fx\right) \right) \right)}{c^2|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.23 Problem number 48

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(\cot^3(fx + e))(a + a \sec(fx + e))^{\frac{3}{2}}}{3ac^3f} + \frac{2(\cot^5(fx + e))(a + a \sec(fx + e))^{\frac{5}{2}}}{5a^2c^3f} \\ & + \frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{c^3f} + \frac{2 \cot(fx + e) \sqrt{a + a \sec(fx + e)}}{c^3f} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{15 \sqrt{2} \sqrt{-a} a \log \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a| - 6a}{2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a| - 6a} \right)}{c^3 |a|} \right) + \frac{105 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right)}{c^3 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.24 Problem number 49

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2(\cot^3(fx + e))(a + a \sec(fx + e))^{\frac{3}{2}}}{3a^3 c^4 f} + \frac{2(\cot^5(fx + e))(a + a \sec(fx + e))^{\frac{5}{2}}}{5a^2 c^4 f} \\ & - \frac{2(\cot^7(fx + e))(a + a \sec(fx + e))^{\frac{7}{2}}}{7a^3 c^4 f} \\ & + \frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx + e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{c^4 f} + \frac{2 \cot(fx + e) \sqrt{a + a \sec(fx + e)}}{c^4 f} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{210 \sqrt{2} \sqrt{-a} a \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^{-4 \sqrt{2} |a| - 6 a}}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^{-4 \sqrt{2} |a| - 6 a}} \right)}{c^4 |a|} \right) + \frac{1575 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right)}{c^4 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.25 Problem number 50

$$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{2a^{3/2} c^3 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{f} - \frac{2a^2 c^3 \tan(fx+e)}{f \sqrt{a + a \sec(fx+e)}} + \frac{2a^3 c^3 (\tan^3(fx+e))}{3f (a + a \sec(fx+e))^{3/2}} - \frac{2a^4 c^3 (\tan^5(fx+e))}{5f (a + a \sec(fx+e))^{5/2}} - \frac{2a^5 c^3 (\tan^7(fx+e))}{7f (a + a \sec(fx+e))^{7/2}}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105 \sqrt{-a} a^2 c^3 \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^{-4 \sqrt{2} |a| - 6 a}}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^{-4 \sqrt{2} |a| - 6 a}} \right)}{|a|} \operatorname{sgn}(\cos(fx+e))}{c^4 |a|} - \frac{2 \left( 105 \sqrt{2} a^5 c^3 \right)}{c^4 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.26 Problem number 51

$$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{2a^{3/2}c^2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a\sec(fx+e)}}\right)}{f} - \frac{2a^2c^2 \tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}} + \frac{2a^3c^2(\tan^3(fx+e))}{3f(a+a\sec(fx+e))^{3/2}} + \frac{2a^4c^2(\tan^5(fx+e))}{5f(a+a\sec(fx+e))^{5/2}}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15\sqrt{-a}a^2c^2 \log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right) \operatorname{sgn}(\cos(fx+e))}{|a|} + \frac{2\left(15\sqrt{2}a^4c^2\operatorname{sgn}(\cos(fx+e))\right)}{15f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.27 Problem number 52

$$\int (a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2a^{3/2}c \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a\sec(fx+e)}}\right)}{f} - \frac{2a^2c \tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}} - \frac{2a^3c(\tan^3(fx+e))}{3f(a+a\sec(fx+e))^{3/2}}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3 \sqrt{-a} a^2 c \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} f x + \frac{1}{2} e\right) + a} \right|^{-4 \sqrt{2} |a| - 6 a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} f x + \frac{1}{2} e\right) + a} \right|^{+4 \sqrt{2} |a| - 6 a}} \right) \operatorname{sgn}(\cos(f x + e)) + \frac{2 \left( \sqrt{2} a^3 \operatorname{csgn}(\cos(f x + e)) \right)}{|a|} + \frac{\left( a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**74.28 Problem number 53**

$$\int \frac{(a + a \sec(e + f x))^{3/2}}{c - c \sec(e + f x)} dx$$

Optimal antiderivative

$$\frac{2 a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{a} \tan(f x + e)}{\sqrt{a + a \sec(f x + e)}}\right)}{c f} + \frac{4 a \cot(f x + e) \sqrt{a + a \sec(f x + e)}}{c f}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{-a} a^3 \left( \frac{\sqrt{2} \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} f x + \frac{1}{2} e\right) + a} \right|^{-4 \sqrt{2} |a| - 6 a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} f x + \frac{1}{2} e\right) + a} \right|^{+4 \sqrt{2} |a| - 6 a}} \right)}{a c |a|} + \left( \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) \right) \right) \right)$$



Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.29 Problem number 54

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{c^2 f} - \frac{4(\cot^3(fx+e))(a + a \sec(fx+e))^{\frac{3}{2}}}{3c^2 f} + \frac{2a \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{c^2 f}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{-a} a^2 \log\left(\frac{\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a}\right)^2 - 4\sqrt{2}|a| - 6a}{\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a}\right)^2 + 4\sqrt{2}|a| - 6a}\right) \operatorname{sgn}(\cos(fx+e))}{c^2|a|} + \frac{4\sqrt{2}}{3} \left(\sqrt{-a}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.30 Problem number 55

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{2a^{3/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{c^3 f} - \frac{2(\cot^3(fx+e))(a + a \sec(fx+e))^{3/2}}{3c^3 f} + \frac{4(\cot^5(fx+e))(a + a \sec(fx+e))^{5/2}}{5a c^3 f} + \frac{2a \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{c^3 f}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 \sqrt{-a} a^2 \log \left( \frac{\left| \frac{2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right|}{c^3 |a|} \right)^{\operatorname{sgn}(\cos(fx+e))} + \frac{2 \sqrt{2} \left( 30 \left( \sqrt{-a} \right) \right)}{c^3 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.31 Problem number 56

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^4} dx$$

Optimal antiderivative

$$\frac{2a^{3/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{c^4 f} - \frac{2(\cot^3(fx+e))(a + a \sec(fx+e))^{3/2}}{3c^4 f} + \frac{2(\cot^5(fx+e))(a + a \sec(fx+e))^{5/2}}{5a c^4 f} - \frac{4(\cot^7(fx+e))(a + a \sec(fx+e))^{7/2}}{7a^2 c^4 f} + \frac{2a \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{c^4 f}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$105 \sqrt{-a} a^2 \log \left( \frac{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 - 4\sqrt{2}|a|-6a \right|}{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 + 4\sqrt{2}|a|-6a \right|} \right)^{\operatorname{sgn}(\cos(fx+e))}}{c^4|a|} + 420 \sqrt{2} \left( \sqrt{-a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.32 Problem number 57

$$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{2a^{\frac{5}{2}}c^3 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a \sec(fx+e)}}\right)}{f} - \frac{2a^3c^3 \tan(fx+e)}{f \sqrt{a+a \sec(fx+e)}} + \frac{2a^4c^3(\tan^3(fx+e))}{3f(a+a \sec(fx+e))^{\frac{3}{2}}} - \frac{2a^5c^3(\tan^5(fx+e))}{5f(a+a \sec(fx+e))^{\frac{5}{2}}} - \frac{6a^6c^3(\tan^7(fx+e))}{7f(a+a \sec(fx+e))^{\frac{7}{2}}} - \frac{2a^7c^3(\tan^9(fx+e))}{9f(a+a \sec(fx+e))^{\frac{9}{2}}}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$315 \sqrt{-a} a^3 c^3 \log \left( \frac{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 - 4\sqrt{2}|a|-6a \right|}{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 + 4\sqrt{2}|a|-6a \right|} \right)^{\operatorname{sgn}(\cos(fx+e))}}{|a|} + 2 \left( 315 \sqrt{2} a^7 c^3 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.33 Problem number 58

$$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{2a^{5/2}c^2 \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a \sec(fx+e)}}\right)}{f} - \frac{2a^3c^2 \tan(fx+e)}{f\sqrt{a+a \sec(fx+e)}} + \frac{2a^4c^2(\tan^3(fx+e))}{3f(a+a \sec(fx+e))^{3/2}} + \frac{6a^5c^2(\tan^5(fx+e))}{5f(a+a \sec(fx+e))^{5/2}} + \frac{2a^6c^2(\tan^7(fx+e))}{7f(a+a \sec(fx+e))^{7/2}}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105 \sqrt{-a} a^3 c^2 \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right|^2 - 4\sqrt{2} |a|^{-6} a}{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right|^2 + 4\sqrt{2} |a|^{-6} a} \right)}{|a|} \operatorname{sgn}(\cos(fx+e))}{2} - \frac{2 \left( 105 \sqrt{2} a^6 c^2 \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.34 Problem number 59

$$\int (a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2a^{5/2} c \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{f} - \frac{2a^3 c \tan(fx+e)}{f \sqrt{a + a \sec(fx+e)}} - \frac{2a^4 c (\tan^3(fx+e))}{f (a + a \sec(fx+e))^{3/2}} - \frac{2a^5 c (\tan^5(fx+e))}{5f (a + a \sec(fx+e))^{5/2}}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$5 \sqrt{-a} a^3 c \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right|^{-4} \sqrt{2}^{|a|-6a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right|^{+4} \sqrt{2}^{|a|-6a}} \right)^{\operatorname{sgn}(\cos(fx+e))} - \frac{2 \left( \sqrt{2} a^5 \operatorname{csgn}(\cos(fx+e)) \right)}{5f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.35 Problem number 60

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{cf} + \frac{8a^2 \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{cf} - \frac{2a^3 \tan(fx+e)}{cf \sqrt{a + a \sec(fx+e)}}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}a^3\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)c} - \frac{\sqrt{-a}a^3\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)+4\sqrt{2}|a|-6a}\right)}{c|a|}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**74.36 Problem number 61**

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right)}{c^2 f} - \frac{8a(\cot^3(fx + e))(a + a \sec(fx + e))^{3/2}}{3c^2 f}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}\sqrt{-a}a^5 \left( \frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)\right|^{-4}\sqrt{2}|a|-6a}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)+4\sqrt{2}|a|-6a}\right)}{a^2c^2|a|}\right)}{6f} + \frac{8\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.37 Problem number 62

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right)}{c^3 f} + \frac{8(\cot^5(fx + e))(a + a \sec(fx + e))^{5/2}}{5c^3 f} + \frac{2a^2 \cot(fx + e) \sqrt{a + a \sec(fx + e)}}{c^3 f}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$5 \sqrt{-a} a^3 \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right)^{\operatorname{sgn}(\cos(fx+e))}}{c^3 |a|} + \frac{4 \sqrt{2}}{5} \left( \sqrt{-a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.38 Problem number 63

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^4} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{c^4 f} - \frac{2a(\cot^3(fx+e))(a + a \sec(fx+e))^{3/2}}{3c^4 f} - \frac{8(\cot^7(fx+e))(a + a \sec(fx+e))^{7/2}}{7a c^4 f} + \frac{2a^2 \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{c^4 f}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$21 \sqrt{-a} a^3 \log \left( \frac{\left| \frac{2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right|}{c^4 |a|} \right)^{\text{sgn}(\cos(fx+e))} + \frac{4 \left( 21 \sqrt{2} \left( \sqrt{-a} \right) \right)}{c^4 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.39 Problem number 64

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^5} dx$$

Optimal antiderivative

$$\frac{2a^{5/2} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx+e)}}\right)}{c^5 f} - \frac{2a(\cot^3(fx+e))(a + a \sec(fx+e))^{3/2}}{3c^5 f} + \frac{2(\cot^5(fx+e))(a + a \sec(fx+e))^{5/2}}{5c^5 f} + \frac{8(\cot^9(fx+e))(a + a \sec(fx+e))^{9/2}}{9a^2 c^5 f} + \frac{2a^2 \cot(fx+e) \sqrt{a + a \sec(fx+e)}}{c^5 f}$$



command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$45 \sqrt{-a} a^3 \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 - 4\sqrt{2}|a|-6a}{\left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 + 4\sqrt{2}|a|-6a} \right)^{\operatorname{sgn}(\cos(fx+e))} + \frac{4 \left( 45 \sqrt{2} \left( \sqrt{-a} \right) \right)}{c^5 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.40 Problem number 90

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{\sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{a \ln(1 - \cos(fx + e)) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\sqrt{-ac} a \log\left(|a| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{c|a|} - \frac{\sqrt{-ac} a \log\left(\left|-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a\right|\right)}{c|a|}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.41 Problem number 91

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{a \tan(fx + e)}{f(c - c \sec(fx + e))^{3/2} \sqrt{a + a \sec(fx + e)}} + \frac{a \ln(1 - \cos(fx + e)) \tan(fx + e)}{cf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2 \sqrt{2} \sqrt{-ac} a \log\left(2 \left| a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 \right| \right)}{c^2 |a|} - \frac{2 \sqrt{2} \sqrt{-ac} a \log\left(\left| -a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a \right| \right)}{c^2 |a|} - \frac{\sqrt{2} \left( 2 \left( a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a \right) \sqrt{-ac}}{ac^2 |a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.42 Problem number 92

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a \tan(fx + e)}{2f(c - c \sec(fx + e))^{5/2} \sqrt{a + a \sec(fx + e)}} - \frac{a \tan(fx + e)}{cf(c - c \sec(fx + e))^{3/2} \sqrt{a + a \sec(fx + e)}} + \frac{a \ln(1 - \cos(fx + e)) \tan(fx + e)}{c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{8 \sqrt{2} \sqrt{-ac} a \log\left(2 \left| a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 \right| \right)}{c^3 |a|} - \frac{8 \sqrt{2} \sqrt{-ac} a \log\left(\left| -a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a \right| \right)}{c^3 |a|} - \frac{\sqrt{2} \left( 12 \left( a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a \right) \sqrt{-ac}}{c^3 |a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)} \right)}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.43 Problem number 93

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c - c \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a \tan (fx + e)}{3f (c - c \sec (fx + e))^{\frac{7}{2}} \sqrt{a + a \sec (fx + e)}} \\ & - \frac{a \tan (fx + e)}{2cf (c - c \sec (fx + e))^{\frac{5}{2}} \sqrt{a + a \sec (fx + e)}} \\ & - \frac{a \tan (fx + e)}{c^2 f (c - c \sec (fx + e))^{\frac{3}{2}} \sqrt{a + a \sec (fx + e)}} \\ & + \frac{a \ln (1 - \cos (fx + e)) \tan (fx + e)}{c^3 f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}} \end{aligned}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{24 \sqrt{2} \sqrt{-ac} a \log \left( 2 \left| a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 \right| \right)}{c^4 |a|} - \frac{24 \sqrt{2} \sqrt{-ac} a \log \left( \left| -a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right| \right)}{c^4 |a|} - \frac{\sqrt{2} \left( 44 \left( a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right) \right)}{48 f} \right)$$

48 f

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.44 Problem number 98

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2a^2 \tan (fx + e)}{f (c - c \sec (fx + e))^{\frac{3}{2}} \sqrt{a + a \sec (fx + e)}} + \frac{a^2 \ln (1 - \cos (fx + e)) \tan (fx + e)}{cf \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\sqrt{-ac} a^2 \log\left(|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c^2|a|} - \frac{\sqrt{-ac} a^2 \log\left(|-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a|\right)}{c^2|a|} - \frac{\left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right) \sqrt{-ac} a}{c^2|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 74.45 Problem number 99

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a^2 \tan(fx + e)}{f(c - c \sec(fx + e))^{\frac{5}{2}} \sqrt{a + a \sec(fx + e)}} - \frac{a^2 \tan(fx + e)}{cf(c - c \sec(fx + e))^{\frac{3}{2}} \sqrt{a + a \sec(fx + e)}} + \frac{a^2 \ln(1 - \cos(fx + e)) \tan(fx + e)}{c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{4 \sqrt{-ac} a^2 \log\left(|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c^3|a|} - \frac{4 \sqrt{-ac} a^2 \log\left(|-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a|\right)}{c^3|a|} - \frac{6 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^2 \sqrt{-ac} a^2 + 8 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right) \sqrt{-ac} a}{a^2 c^3 |a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 74.46 Problem number 100

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{2a^2 \tan (fx + e)}{3f (c - c \sec (fx + e))^{\frac{7}{2}} \sqrt{a + a \sec (fx + e)}} - \frac{a^2 \tan (fx + e)}{2cf (c - c \sec (fx + e))^{\frac{5}{2}} \sqrt{a + a \sec (fx + e)}} - \frac{c^2 f (c - c \sec (fx + e))^{\frac{3}{2}} \sqrt{a + a \sec (fx + e)}}{a^2 \ln (1 - \cos (fx + e)) \tan (fx + e)} + \frac{a^2 \ln (1 - \cos (fx + e)) \tan (fx + e)}{c^3 f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{24 \sqrt{-ac} a^2 \log \left( \left| a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 \right. \right)}{c^4 |a|} - \frac{24 \sqrt{-ac} a^2 \log \left( \left| -a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right. \right)}{c^4 |a|} - \frac{44 \left( a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right)^3 \sqrt{-ac} a^2 + 108 \left( a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right)}{24 f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.47 Problem number 106

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2a^3 \tan (fx + e)}{f (c - c \sec (fx + e))^{\frac{5}{2}} \sqrt{a + a \sec (fx + e)}} + \frac{a^3 \ln (1 - \cos (fx + e)) \tan (fx + e)}{c^2 f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{-ac} a^3 \log \left( \left| a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 \right. \right)}{c^3 |a|} - \frac{2 \sqrt{-ac} a^3 \log \left( \left| -a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right. \right)}{c^3 |a|} - \frac{3 \left( a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right)^2 \sqrt{-ac} a^3 + 4 \left( a \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - a \right)}{2 f a^2 c^3 |a| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.48 Problem number 107

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3 \tan(fx + e)}{3f(c - c \sec(fx + e))^{\frac{7}{2}} \sqrt{a + a \sec(fx + e)}} \\ & - \frac{a^3 \tan(fx + e)}{c^2 f (c - c \sec(fx + e))^{\frac{3}{2}} \sqrt{a + a \sec(fx + e)}} \\ & + \frac{a^3 \ln(1 - \cos(fx + e)) \tan(fx + e)}{c^3 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6 \sqrt{-ac} a^3 \log\left(|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c^4 |a|} - \frac{6 \sqrt{-ac} a^3 \log\left(|-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right|)}{c^4 |a|} - \frac{11 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^3 \sqrt{-ac} a^3 + 27 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right) \sqrt{-ac} a^3}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.49 Problem number 108

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^3 \tan(fx + e)}{f(c - c \sec(fx + e))^{\frac{9}{2}} \sqrt{a + a \sec(fx + e)}} \\ & - \frac{a^3 \tan(fx + e)}{2c^2 f (c - c \sec(fx + e))^{\frac{5}{2}} \sqrt{a + a \sec(fx + e)}} \\ & - \frac{a^3 \tan(fx + e)}{c^3 f (c - c \sec(fx + e))^{\frac{3}{2}} \sqrt{a + a \sec(fx + e)}} \\ & + \frac{a^3 \ln(1 - \cos(fx + e)) \tan(fx + e)}{c^4 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{48 \sqrt{-ac} a^3 \log\left(|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c^5 |a|} - \frac{48 \sqrt{-ac} a^3 \log\left(|-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a|\right)}{c^5 |a|} - \frac{100 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^4 \sqrt{-ac} a^3 + 352 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^2 \sqrt{-ac} a^3 + 128 \sqrt{-ac} a^3}{c^5 |a|}$$

48 f

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.50 Problem number 109

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{11/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4a^3 \tan(fx + e)}{5f (c - c \sec(fx + e))^{\frac{11}{2}} \sqrt{a + a \sec(fx + e)}} \\ & - \frac{a^3 \tan(fx + e)}{3c^2 f (c - c \sec(fx + e))^{\frac{7}{2}} \sqrt{a + a \sec(fx + e)}} \\ & - \frac{2c^3 f (c - c \sec(fx + e))^{\frac{5}{2}} \sqrt{a + a \sec(fx + e)}}{a^3 \tan(fx + e)} \\ & - \frac{c^4 f (c - c \sec(fx + e))^{\frac{3}{2}} \sqrt{a + a \sec(fx + e)}}{a^3 \ln(1 - \cos(fx + e)) \tan(fx + e)} \\ & + \frac{c^5 f \sqrt{a + a \sec(fx + e)}}{c^5 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{120 \sqrt{-ac} a^3 \log\left(|a| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c^6 |a|} - \frac{120 \sqrt{-ac} a^3 \log\left(|-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a|\right)}{c^6 |a|} - \frac{274 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^5 \sqrt{-ac} a^3 + 1250 \left(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a\right)^3 \sqrt{-ac} a^3 + 125 \sqrt{-ac} a^3}{c^6 |a|}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 74.51 Problem number 113

$$\int \frac{\sqrt{c - c \sec(e + fx)}}{\sqrt{a + a \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{c \ln(1 + \cos(fx + e)) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-ac} c \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c\right|\right)}{af|c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.52 Problem number 114

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\ln(\sin(fx + e)) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(1/(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\sqrt{-ac} \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right|\right)}{a|c|} - \frac{2 \sqrt{-ac} \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c\right|\right)}{a|c|}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 74.53 Problem number 115

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\tan(fx + e)}{2cf(1 - \cos(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{3 \ln(1 - \cos(fx + e)) \tan(fx + e)}{4cf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\ln(1 + \cos(fx + e)) \tan(fx + e)}{4cf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

`integrate(1/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3 \log(|c| \tan(\frac{1}{2} fx + \frac{1}{2} e)^2)}{\sqrt{-ac} |c|} + \frac{4 \sqrt{-ac} \log(|c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 + c|)}{ac|c|} - \frac{3c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c}{\sqrt{-ac} |c| \tan(\frac{1}{2} fx + \frac{1}{2} e)^2}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.54 Problem number 116

$$\int \frac{1}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(fx + e)) \tan(fx + e)}{c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{7 \ln(1 - \sec(fx + e)) \tan(fx + e)}{8c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\ln(1 + \sec(fx + e)) \tan(fx + e)}{8c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{\tan(fx + e)}{4c^2 f (1 - \sec(fx + e))^2 \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{3 \tan(fx + e)}{4c^2 f (1 - \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate(1/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{14 \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{\sqrt{-ac} |c|} + \frac{16 \sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c\right)}{ac^2 |c|} - \frac{21 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 + 34 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)c + 14c^2}{\sqrt{-ac} c^3 |c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4}}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 74.55 Problem number 119

$$\int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{2c^2 \tan(fx + e)}{f(a + a \sec(fx + e))^{3/2} \sqrt{c - c \sec(fx + e)}} + \frac{c^2 \ln(1 + \cos(fx + e)) \tan(fx + e)}{af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{\sqrt{-ac} c^2 \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c\right)}{a^2 |c|} - \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac} c}{a^2 |c|}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.56 Problem number 120

$$\int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{c \tan(fx + e)}{f(a + a \sec(fx + e))^{3/2} \sqrt{c - c \sec(fx + e)}} + \frac{c \ln(1 + \cos(fx + e)) \tan(fx + e)}{af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{2\sqrt{2}\sqrt{-ac}c \log\left(\left|-2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 2c\right|\right)}{a^2|c|} - \frac{\sqrt{2}\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)\sqrt{-ac}}{a^2|c|} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.57 Problem number 121

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(fx + e)) \tan(fx + e)}{af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\ln(1 - \sec(fx + e)) \tan(fx + e)}{4af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{3 \ln(1 + \sec(fx + e)) \tan(fx + e)}{4af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{\tan(fx + e)}{2af(1 + \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\sqrt{-ac} \log\left(\left|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right|\right)}{a^2|c|} - \frac{4\sqrt{-ac} \log\left(\left|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c\right|\right)}{a^2|c|} + \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)\sqrt{-ac}}{a^2c|c|}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.58 Problem number 122

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e)}{2acf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} + \frac{\ln(\sin(fx + e)) \tan(fx + e)}{acf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{4 \log\left(|c| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{\sqrt{-ac} a|c|} + \frac{8 \sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c|\right)}{a^2 c|c|} - \frac{\left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right) \sqrt{-ac}}{a^2 c^2 |c|} - \frac{4 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c}{\sqrt{-ac} ac|c| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.59 Problem number 123

$$\int \frac{1}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(fx + e)) \tan(fx + e)}{a c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{11 \ln(1 - \sec(fx + e)) \tan(fx + e)}{16 a c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{5 \ln(1 + \sec(fx + e)) \tan(fx + e)}{16 a c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{\tan(fx + e)}{8 a c^2 f (1 - \sec(fx + e))^2 \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{2 a c^2 f (1 - \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}{\tan(fx + e)} \\ & - \frac{8 a c^2 f (1 + \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}{\tan(fx + e)} \end{aligned}$$

command

```
integrate(1/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{22 \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{\sqrt{-ac} ac|c|} + \frac{32 \sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c|\right)}{a^2 c^2 |c|} - \frac{2 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac}}{a^2 c^3 |c|} - \frac{33 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2}{\sqrt{-ac} ac^3 |c|}}{32 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.60 Problem number 124

$$\int \frac{(c - c \sec(e + fx))^{7/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c^4 \ln(\cos(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{2c^4 \ln(1 + \sec(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{4c^4 \tan(fx + e)}{a^2 f (1 + \sec(fx + e))^2 \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{4c^4 \tan(fx + e)}{a^2 f (1 + \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate((c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 \sqrt{-ac} a^2 |c| + 2 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac} a^2 |c|\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^5 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.61 Problem number 125

$$\int \frac{(c - c \sec(e + fx))^{5/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{2c^3 \tan(fx + e)}{f(a + a \sec(fx + e))^{5/2} \sqrt{c - c \sec(fx + e)}} + \frac{c^3 \ln(1 + \cos(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{2\sqrt{-ac} c^3 \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c\right|\right)}{a^3 |c|} + \frac{\left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right)^2 \sqrt{-ac} |c|}{a^3 c}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.62 Problem number 126

$$\int \frac{(c - c \sec(e + fx))^{3/2}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{c^2 \tan(fx + e)}{f(a + a \sec(fx + e))^{5/2} \sqrt{c - c \sec(fx + e)}} - \frac{c^2 \tan(fx + e)}{af(a + a \sec(fx + e))^{3/2} \sqrt{c - c \sec(fx + e)}} + \frac{c^2 \ln(1 + \cos(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate((c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{4\sqrt{-ac} c^2 \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c\right|\right)}{a^3 |c|} + \frac{\left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right)^2 \sqrt{-ac} a^3 |c| - 2\left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right) \sqrt{-ac} a^3 c |c|}{a^6 c^2}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.63 Problem number 127

$$\int \frac{\sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{c \tan(fx + e)}{2f(a + a \sec(fx + e))^{5/2} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{c \tan(fx + e)}{af(a + a \sec(fx + e))^{3/2} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{c \ln(1 + \cos(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

`integrate((c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \frac{8 \sqrt{2} \sqrt{-ac} c \log\left(\left|c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + c\right|\right)}{a^3 |c|} + \frac{\sqrt{2} \left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right)^2 \sqrt{-ac} a^3 c |c| - 4 \sqrt{2} \left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right) \sqrt{-ac} a^3}{a^6 c^4} \right)}{16 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.64 Problem number 128

$$\int \frac{1}{(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(fx + e)) \tan(fx + e)}{a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\ln(1 - \sec(fx + e)) \tan(fx + e)}{8a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{7 \ln(1 + \sec(fx + e)) \tan(fx + e)}{8a^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{\tan(fx + e)}{4a^2 f (1 + \sec(fx + e))^2 \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{3 \tan(fx + e)}{4a^2 f (1 + \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{-ac} \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{a^3|c|} - \frac{16\sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c|\right)}{a^3|c|} - \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 \sqrt{-ac} a^3 c^2 |c| - 6 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^6 c^6}$$

16 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.65 Problem number 129

$$\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\ln(\cos(fx + e)) \tan(fx + e)}{a^2 c f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{5 \ln(1 - \sec(fx + e)) \tan(fx + e)}{16 a^2 c f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & + \frac{11 \ln(1 + \sec(fx + e)) \tan(fx + e)}{16 a^2 c f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{\tan(fx + e)}{8 a^2 c f (1 - \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} \\ & - \frac{8 a^2 c f (1 + \sec(fx + e))^2 \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}{\tan(fx + e)} \\ & - \frac{2 a^2 c f (1 + \sec(fx + e)) \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}{\tan(fx + e)} \end{aligned}$$

command

```
integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10 \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{\sqrt{-ac} a^2 |c|} + \frac{32 \sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c|\right)}{a^3 |c|} - \frac{2 \left(5 c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)}{\sqrt{-ac} a^2 |c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2} + \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 \sqrt{-ac}}{32 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 74.66 Problem number 130

$$\int \frac{1}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cot(fx + e)}{2a^2c^2f\sqrt{a + a \sec(fx + e)}\sqrt{c - c \sec(fx + e)}} - \frac{\cot^3(fx + e)}{4a^2c^2f\sqrt{a + a \sec(fx + e)}\sqrt{c - c \sec(fx + e)}} + \frac{\ln(\sin(fx + e))\tan(fx + e)}{a^2c^2f\sqrt{a + a \sec(fx + e)}\sqrt{c - c \sec(fx + e)}}$$

command

`integrate(1/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{\sqrt{-ac} a^2c|c|} + \frac{64 \sqrt{-ac} \log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c|\right)}{a^3c^2|c|} - \frac{48 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 + 84 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)c + 37c^2}{\sqrt{-ac} a^2c^3|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4} + \frac{64f}{64f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 74.67 Problem number 143

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{c + c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \arctan\left(\frac{\sqrt{a} \tan(fx + e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{cf} - \frac{\arctan\left(\frac{\sqrt{a} \tan(fx + e) \sqrt{2}}{2\sqrt{a + a \sec(fx + e)}}\right) \sqrt{2} \sqrt{a}}{cf}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{\sqrt{2} \sqrt{-a} a \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2}^{|a|-6a}} \right)}{c|a|} + \frac{\sqrt{-a} \log \left( \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2}^{|a|-6a}} \right)}{2f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.68 Problem number 147

$$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ad(2c + d)(2c^2 + 2cd + d^2) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}} \\ & - \frac{2d^2(6c^2 + 8cd + 3d^2)(a - a \sec(fx + e)) \tan(fx + e)}{3f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2d^3(4c + 3d)(a - a \sec(fx + e))^2 \tan(fx + e)}{5af \sqrt{a + a \sec(fx + e)}} - \frac{2d^4(a - a \sec(fx + e))^3 \tan(fx + e)}{7a^2 f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2a^{\frac{3}{2}} c^4 \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}} \end{aligned}$$

command

`integrate((c+d*sec(f*x+e))^4*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$105 \sqrt{-a} a c^4 \log \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right|}{|a|} \right)^{\operatorname{sgn}(\cos(fx+e))} + \frac{2 \left( 420 \sqrt{2} a^4 c^3 \right)}{|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.69 Problem number 148

$$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{2ad(3c^2 + 3cd + d^2) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}} - \frac{2d^2(3c + 2d)(a - a \sec(fx + e)) \tan(fx + e)}{3f \sqrt{a + a \sec(fx + e)}} + \frac{2d^3(a - a \sec(fx + e))^2 \tan(fx + e)}{5af \sqrt{a + a \sec(fx + e)}} + \frac{2a^{\frac{3}{2}} c^3 \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}}$$

command

`integrate((c+d*sec(f*x+e))^3*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 \sqrt{-a} a c^3 \log \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a|^{-6} a}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a|^{-6} a} \right|}{|a|} \right)^{\operatorname{sgn}(\cos(fx+e))} - \frac{2 \left( \left( \sqrt{2} (45 a^3 c^3) \right) \right)}{|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.70 Problem number 149

$$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{2ad(2c + d) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}} - \frac{2d^2(a - a \sec(fx + e)) \tan(fx + e)}{3f \sqrt{a + a \sec(fx + e)}} + \frac{2a^{\frac{3}{2}} c^2 \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}}$$

command

`integrate((c+d*sec(f*x+e))^2*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3 \sqrt{-a} a c^2 \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right|^{-4 \sqrt{2} |a| - 6 a}}{\left| \sqrt{-a} \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + a} \right|^{+4 \sqrt{2} |a| - 6 a}} \right) \operatorname{sgn}(\cos(fx+e))}{|a|} + \frac{2 \left( 6 \sqrt{2} a^2 c d \operatorname{sgn}(\cos(fx+e)) \right)}{3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.71 Problem number 150

$$\int \sqrt{a + a \sec(e + fx)} (c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2c \operatorname{arctan}\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a + a \sec(fx + e)}}\right) \sqrt{a}}{f} + \frac{2ad \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}}$$

command

```
integrate((c+d*sec(f*x+e))*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\operatorname{adsgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a} + \frac{\sqrt{-a}\operatorname{aclog}\left(\frac{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)}{2\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**74.72 Problem number 151**

$$\int \frac{\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{2\arctan\left(\frac{\sqrt{a}\tan(fx+e)}{\sqrt{a+a\sec(fx+e)}}\right)\sqrt{a}}{cf} - \frac{2\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(fx+e)}{\sqrt{c+d}\sqrt{a+a\sec(fx+e)}}\right)\sqrt{a}\sqrt{d}}{cf\sqrt{c+d}}$$

command

```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{2\sqrt{2}\sqrt{-a}\operatorname{darctan}\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)^2+a\right)^2}{4\sqrt{-cd-d^2}a}\right)}{\sqrt{-cd-d^2}c} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.73 Problem number 152

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{ad \tan(fx + e)}{c(c + d) f (c + d \sec(fx + e)) \sqrt{a + a \sec(fx + e)}} + \frac{2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{c^2 f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}} + \frac{a^{\frac{3}{2}} (3c + 2d) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a - a \sec(fx + e)}}{\sqrt{a} \sqrt{c + d}}\right) \sqrt{d} \tan(fx + e)}{c^2 (c + d)^{\frac{3}{2}} f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}}$$

command

`integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{\sqrt{2} (3 \sqrt{-a} acd + 2 \sqrt{-a} ad^2) \operatorname{arctan} \left( \frac{\sqrt{2} \left( \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2}{4 \sqrt{-cd - d^2} a} \right)}{(c^3 + c^2 d) \sqrt{-cd - d^2} a} \right)}{(c^3 + c^2 d) \sqrt{-cd - d^2} a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.74 Problem number 153

$$\int \frac{\sqrt{a + a \sec(e + fx)}}{(c + d \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{ad \tan (fx + e)}{2c(c + d) f (c + d \sec (fx + e))^2 \sqrt{a + a \sec (fx + e)}} \\ & - \frac{ad(7c + 4d) \tan (fx + e)}{4c^2 (c + d)^2 f (c + d \sec (fx + e)) \sqrt{a + a \sec (fx + e)}} \\ & + \frac{2a^{\frac{3}{2}} \operatorname{arctanh} \left( \frac{\sqrt{a - a \sec (fx + e)}}{\sqrt{a}} \right) \tan (fx + e)}{c^3 f \sqrt{a - a \sec (fx + e)} \sqrt{a + a \sec (fx + e)}} \\ & - \frac{a^{\frac{3}{2}} (15c^2 + 20cd + 8d^2) \operatorname{arctanh} \left( \frac{\sqrt{d} \sqrt{a - a \sec (fx + e)}}{\sqrt{a} \sqrt{c + d}} \right) \sqrt{d} \tan (fx + e)}{4c^3 (c + d)^{\frac{5}{2}} f \sqrt{a - a \sec (fx + e)} \sqrt{a + a \sec (fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.75 Problem number 154

$$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(6c + 13d) (c + d \sec (fx + e))^2 \tan (fx + e)}{35f \sqrt{a + a \sec (fx + e)}} + \frac{2a^2(c + d \sec (fx + e))^3 \tan (fx + e)}{7f \sqrt{a + a \sec (fx + e)}} \\ & + \frac{2a^2(72c^3 + 486c^2d + 378cd^2 + 104d^3 + d(24c^2 + 111cd + 52d^2) \sec (fx + e)) \tan (fx + e)}{105f \sqrt{a + a \sec (fx + e)}} \\ & + \frac{2a^{\frac{5}{2}}c^3 \operatorname{arctanh} \left( \frac{\sqrt{a - a \sec (fx + e)}}{\sqrt{a}} \right) \tan (fx + e)}{f \sqrt{a - a \sec (fx + e)} \sqrt{a + a \sec (fx + e)}} \end{aligned}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$105 \sqrt{-a} a^2 c^3 \log \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a| - 6 a}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a| - 6 a} \right|}{|a|} \right) \operatorname{sgn}(\cos(fx+e)) + \frac{2 \left( 105 \sqrt{2} a^5 c^3 \right)}{|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.76 Problem number 155

$$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^2(c + d \sec(fx + e))^2 \tan(fx + e)}{5f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2a^2(12c^2 + 50cd + 18d^2 + d(4c + 9d) \sec(fx + e)) \tan(fx + e)}{15f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2a^{\frac{5}{2}} c^2 \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}} \end{aligned}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 \sqrt{-a} a^2 c^2 \log \left( \frac{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 - 4 \sqrt{2} |a| - 6 a}{\left| 2 \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a} \right)^2 + 4 \sqrt{2} |a| - 6 a} \right|}{|a|} \right) \operatorname{sgn}(\cos(fx+e)) + \frac{2 \left( \left( \sqrt{2} (15 a^4 c^2) \right) \right)}{|a|}$$



Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.77 Problem number 156

$$\int (a + a \sec(e + fx))^{3/2} (c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2a^{3/2}c \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a \sec(fx+e)}}\right)}{f} + \frac{2a^2(3c+4d) \tan(fx+e)}{3f\sqrt{a+a \sec(fx+e)}} + \frac{2ad\sqrt{a+a \sec(fx+e)} \tan(fx+e)}{3f}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\sqrt{-a} a^2 c \log\left(\frac{\left|2\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}\right)^2 - 4\sqrt{2}|a|-6a\right)}{\left|2\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}\right)^2 + 4\sqrt{2}|a|-6a\right)}\right)}{|a|} \operatorname{sgn}(\cos(fx+e)) + \frac{2\left(3\sqrt{2} a^3 \operatorname{csgn}(\cos(fx+e))\right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 74.78 Problem number 157

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a \sec(fx+e)}}\right)}{cf} + \frac{2a^{\frac{3}{2}}(c-d) \arctan\left(\frac{\sqrt{a} \sqrt{d} \tan(fx+e)}{\sqrt{c+d} \sqrt{a+a \sec(fx+e)}}\right)}{cf\sqrt{d} \sqrt{c+d}}$$

command

`integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{-a} a^3 \left( \frac{2 \sqrt{2} (c-d) \arctan\left(\frac{\sqrt{2} \left(\left(\sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a}\right)^2}{4 \sqrt{-cd - d^2} a}\right)}{c - \left(\sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a}\right)}\right)}{\sqrt{-cd - d^2} a^2 c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.79 Problem number 158

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(c-d) \tan(fx+e)}{c(c+d) f (c+d \sec(fx+e)) \sqrt{a+a \sec(fx+e)}} \\ & + \frac{2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sec(fx+e)}}{\sqrt{a}}\right) \tan(fx+e)}{c^2 f \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \\ & + \frac{a^{\frac{5}{2}}(c^2-3cd-2d^2) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(fx+e)}{c^2(c+d)^{\frac{3}{2}} f \sqrt{d} \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{-a} a^2 \log \left( \frac{\left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 - 4\sqrt{2}|a|^{-6a}}{\left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan^2\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a} \right)^2 + 4\sqrt{2}|a|^{-6a}} \right)}{c^2|a|} \operatorname{sgn}(\cos(fx+e)) \sqrt{2} \left( \sqrt{-a} a^2 c^2 \operatorname{sgn}(\cos(fx+e)) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.80 Problem number 159

$$\int \frac{(a + a \sec(e + fx))^{3/2}}{(c + d \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(c-d) \tan(fx+e)}{2c(c+d) f (c+d \sec(fx+e))^2 \sqrt{a+a \sec(fx+e)}} \\ & + \frac{a^2(3c^2-7cd-4d^2) \tan(fx+e)}{4c^2(c+d)^2 f (c+d \sec(fx+e)) \sqrt{a+a \sec(fx+e)}} \\ & + \frac{2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sec(fx+e)}}{\sqrt{a}}\right) \tan(fx+e)}{c^3 f \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \\ & + \frac{a^{\frac{5}{2}}(3c^3-15c^2d-20cd^2-8d^3) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(fx+e)}{4c^3(c+d)^{\frac{5}{2}} f \sqrt{d} \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.81 Problem number 160

$$\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3(3c^3 + 12c^2d + 12cd^2 + 4d^3) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2ad(3c^2 + 15cd + 13d^2) (a - a \sec(fx + e))^2 \tan(fx + e)}{5f \sqrt{a + a \sec(fx + e)}} \\ & - \frac{6d^2(c + 2d) (a - a \sec(fx + e))^3 \tan(fx + e)}{7f \sqrt{a + a \sec(fx + e)}} + \frac{2d^3(a - a \sec(fx + e))^4 \tan(fx + e)}{9af \sqrt{a + a \sec(fx + e)}} \\ & - \frac{2(c^3 + 12c^2d + 24cd^2 + 12d^3) (a^3 - a^3 \sec(fx + e)) \tan(fx + e)}{3f \sqrt{a + a \sec(fx + e)}} \\ & + \frac{2a^{7/2} c^3 \operatorname{arctanh}\left(\frac{\sqrt{a - a \sec(fx + e)}}{\sqrt{a}}\right) \tan(fx + e)}{f \sqrt{a - a \sec(fx + e)} \sqrt{a + a \sec(fx + e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{315 \sqrt{-a} a^3 c^3 \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right|^2 - 4 \sqrt{2} |a|^{-6} a}{\left| \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right|^2 + 4 \sqrt{2} |a|^{-6} a} \right)}{|a|} \operatorname{sgn}(\cos(fx + e))}{2 \left( 945 \sqrt{2} a^7 c^3 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.82 Problem number 161

$$\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2a^3(c+2d)(3c+2d)\tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}} + \frac{2ad(2c+5d)(a-a\sec(fx+e))^2\tan(fx+e)}{5f\sqrt{a+a\sec(fx+e)}} \\ & - \frac{2d^2(a-a\sec(fx+e))^3\tan(fx+e)}{7f\sqrt{a+a\sec(fx+e)}} \\ & - \frac{2(c^2+8cd+8d^2)(a^3-a^3\sec(fx+e))\tan(fx+e)}{3f\sqrt{a+a\sec(fx+e)}} \\ & + \frac{2a^{7/2}c^2 \operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(fx+e)}}{\sqrt{a}}\right)\tan(fx+e)}{f\sqrt{a-a\sec(fx+e)}\sqrt{a+a\sec(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105\sqrt{-a}a^3c^2 \log \left( \frac{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right|^2}{\left| \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right|^2} \right)^{-4\sqrt{2}|a|-6a}}{|a|} \operatorname{sgn}(\cos(fx+e)) + \frac{2(315\sqrt{2}a^6c^2}{|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.83 Problem number 162

$$\int (a + a \sec(e + fx))^{5/2} (c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{2a^{\frac{5}{2}} c \arctan\left(\frac{\sqrt{a} \tan(fx+e)}{\sqrt{a+a \sec(fx+e)}}\right)}{f} + \frac{2ad(a+a \sec(fx+e))^{\frac{3}{2}} \tan(fx+e)}{5f}$$

$$+ \frac{2a^3(35c+32d) \tan(fx+e)}{15f \sqrt{a+a \sec(fx+e)}} + \frac{2a^2(5c+8d) \sqrt{a+a \sec(fx+e)} \tan(fx+e)}{15f}$$

command

`integrate((a+a*sec(f*x+e))^(5/2)*(c+d*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 \sqrt{-a} a^3 c \log \left( \frac{\left| \left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right)^2 - 4 \sqrt{2}^{|a|-6} a \right|}{\left( \sqrt{-a} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{-a \tan^2\left(\frac{1}{2} fx + \frac{1}{2} e\right) + a} \right)^2 + 4 \sqrt{2}^{|a|-6} a} \right) \operatorname{sgn}(\cos(fx+e))}{|a|} - \frac{2 \left( 45 \sqrt{2} a^5 \operatorname{csgn}(\cos(fx+e)) \right)}{|a|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 74.84 Problem number 163

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2a^3 \tan(fx+e)}{df \sqrt{a+a \sec(fx+e)}} + \frac{2a^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sec(fx+e)}}{\sqrt{a}}\right) \tan(fx+e)}{cf \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}}$$

$$- \frac{2a^{\frac{7}{2}} (c-d)^2 \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(fx+e)}{cd^{\frac{3}{2}} f \sqrt{c+d} \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a^3\operatorname{sgn}(\cos(fx+e))\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a)d} + \frac{\sqrt{-a}a^2\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right)\right)}{c}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.85 Problem number 164

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^3(c-d)^2 \tan(fx+e)}{cd(c+d)f(c+d\sec(fx+e))\sqrt{a+a\sec(fx+e)}} \\ & + \frac{2a^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{a-a\sec(fx+e)}}{\sqrt{a}}\right) \tan(fx+e)}{c^2 f \sqrt{a-a\sec(fx+e)} \sqrt{a+a\sec(fx+e)}} \\ & - \frac{a^{7/2}(c-d)^2 \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(fx+e)}}{\sqrt{a}\sqrt{c+d}}\right) \tan(fx+e)}{cd^{3/2}(c+d)^{3/2} f \sqrt{a-a\sec(fx+e)} \sqrt{a+a\sec(fx+e)}} \\ & + \frac{2a^{7/2}(c-d) \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-a\sec(fx+e)}}{\sqrt{a}\sqrt{c+d}}\right) \sqrt{c+d} \tan(fx+e)}{c^2 d^{3/2} f \sqrt{a-a\sec(fx+e)} \sqrt{a+a\sec(fx+e)}} \end{aligned}$$

command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \sqrt{-a} a^5 \left( \frac{\sqrt{2} (c^3 + 4c^2d - 3cd^2 - 2d^3) \arctan \left( \frac{\sqrt{2} \left( \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2}{c - \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} \right)}{4 \sqrt{-cd - d^2} a} \right)}{(a^2c^3d + a^2c^2d^2) \sqrt{-cd - d^2} a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 74.86 Problem number 165

$$\int \frac{(a + a \sec(e + fx))^{5/2}}{(c + d \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(c-d)^2 \tan(fx+e)}{2cd(c+d)f(c+d \sec(fx+e))^2 \sqrt{a+a \sec(fx+e)}} \\ & + \frac{a^3(c-d) \tan(fx+e)}{c^2df(c+d \sec(fx+e)) \sqrt{a+a \sec(fx+e)}} \\ & - \frac{3a^3(c-d)^2 \tan(fx+e)}{4cd(c+d)^2 f(c+d \sec(fx+e)) \sqrt{a+a \sec(fx+e)}} \\ & + \frac{2a^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sec(fx+e)}}{\sqrt{a}}\right) \tan(fx+e)}{c^3f \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \\ & - \frac{3a^{7/2}(c-d)^2 \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(fx+e)}{4cd^{3/2}(c+d)^{5/2} f \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \\ & + \frac{a^{7/2}(c-d) \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \tan(fx+e)}{c^2d^{3/2}f \sqrt{c+d} \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \\ & - \frac{2a^{7/2} \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a-a \sec(fx+e)}}{\sqrt{a} \sqrt{c+d}}\right) \sqrt{d} \tan(fx+e)}{c^3f \sqrt{c+d} \sqrt{a-a \sec(fx+e)} \sqrt{a+a \sec(fx+e)}} \end{aligned}$$



command

```
integrate((a+a*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75 Test file number 122

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/122\_4.5.2.3-g\_sec-^p-a+b\_sec-^m-c+d\_sec-^n

### 75.1 Problem number 1

$$\int \sec(e + fx)(a + a \sec(e + fx))(c - c \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\frac{7a c^4 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{a c^4 \sec(fx + e) \tan(fx + e)}{8f} - \frac{3a c^4 (\sec^3(fx + e)) \tan(fx + e)}{4f} + \frac{4a c^4 (\tan^3(fx + e))}{3f} + \frac{a c^4 (\tan^5(fx + e))}{5f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$105 a c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right) - 105 a c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right) - \frac{2\left(105 a c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 + 790 a c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 210 a c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 35 a c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 5 a c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{120 f}$$

---

120 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.2 Problem number 2

$$\int \sec(e + fx)(a + a \sec(e + fx))(c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{5a c^3 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{3a c^3 \sec(fx + e) \tan(fx + e)}{8f} - \frac{a c^3 (\sec^3(fx + e)) \tan(fx + e)}{4f} + \frac{2a c^3 (\tan^3(fx + e))}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15ac^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15ac^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2\left(15ac^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 + 73ac^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^5}{24f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.3 Problem number 3

$$\int \sec(e + fx)(a + a \sec(e + fx))(c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{a c^2 \operatorname{arctanh}(\sin(fx + e))}{2f} - \frac{a c^2 \sec(fx + e) \tan(fx + e)}{2f} + \frac{a c^2 (\tan^3(fx + e))}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3ac^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3ac^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2\left(3ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 8ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 3ac^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^2 - 1\right)^3}{6f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.4 Problem number 4

$$\int \sec(e + fx)(a + a \sec(e + fx))(c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{ac \operatorname{arctanh}(\sin(fx + e))}{2f} - \frac{ac \sec(fx + e) \tan(fx + e)}{2f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{ac \log(|\sin(fx + e) + 1|) - ac \log(|\sin(fx + e) - 1|) + \frac{2ac \sin(fx + e)}{\sin(fx + e)^2 - 1}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.5 Problem number 5

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$-\frac{a \operatorname{arctanh}(\sin(fx + e))}{cf} - \frac{2a \tan(fx + e)}{f(c - c \sec(fx + e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{a \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{c} - \frac{a \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{c} - \frac{2a}{c \tan(\frac{1}{2}fx + \frac{1}{2}e)}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.6 Problem number 10

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c - c \sec(e + fx))^5 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9a^2c^5 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{3a^2c^5 \sec(fx + e) \tan(fx + e)}{16f} \\ & - \frac{3a^2c^5 (\sec^3(fx + e)) \tan(fx + e)}{8f} + \frac{a^2c^5 \sec(fx + e) (\tan^3(fx + e))}{4f} \\ & + \frac{a^2c^5 (\sec^3(fx + e)) (\tan^3(fx + e))}{2f} - \frac{4a^2c^5 (\tan^5(fx + e))}{5f} - \frac{a^2c^5 (\tan^7(fx + e))}{7f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$315 a^2 c^5 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 315 a^2 c^5 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 315 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{13} - 2100 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{11} + 10500 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^9 - 21000 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^7 + 15750 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^5 - 5250 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + 735 a^2 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{13}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.7 Problem number 11

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c - c \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{7a^2c^4 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{5a^2c^4 \sec(fx + e) \tan(fx + e)}{16f} \\ & - \frac{a^2c^4 (\sec^3(fx + e)) \tan(fx + e)}{8f} + \frac{a^2c^4 \sec(fx + e) (\tan^3(fx + e))}{4f} \\ & + \frac{a^2c^4 (\sec^3(fx + e)) (\tan^3(fx + e))}{6f} - \frac{2a^2c^4 (\tan^5(fx + e))}{5f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$105 a^2 c^4 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 105 a^2 c^4 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 105 a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{11} - 595 a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^9 + 105 a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^7 - 35 a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^5 + 7 a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 - a^2 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{240 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.8 Problem number 12

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c - c \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\frac{3a^2 c^3 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{3a^2 c^3 \sec(fx + e) \tan(fx + e)}{8f} + \frac{a^2 c^3 \sec(fx + e) (\tan^3(fx + e))}{4f} - \frac{a^2 c^3 (\tan^5(fx + e))}{5f}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$15 a^2 c^3 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 15 a^2 c^3 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 15 a^2 c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^9 - 70 a^2 c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^7 + 15 a^2 c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^5 - 5 a^2 c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + a^2 c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{40 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.9 Problem number 13

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{3a^2 c^2 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{3a^2 c^2 \sec(fx + e) \tan(fx + e)}{8f} + \frac{a^2 c^2 \sec(fx + e) (\tan^3(fx + e))}{4f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2c^2 \log(|\sin(fx+e)+1|) - 3a^2c^2 \log(|\sin(fx+e)-1|) + \frac{2(5a^2c^2 \sin(fx+e)^3 - 3a^2c^2 \sin(fx+e))}{(\sin(fx+e)^2 - 1)^2}}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.10 Problem number 14

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx)) dx$$

Optimal antiderivative

$$\frac{a^2c \operatorname{arctanh}(\sin(fx+e))}{2f} - \frac{a^2c \sec(fx+e) \tan(fx+e)}{2f} - \frac{a^2c(\tan^3(fx+e))}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|) - 3a^2c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|) - \frac{2(3a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 8a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 3a^2c)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)^3}}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.11 Problem number 15

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{3a^2 \operatorname{arctanh}(\sin(fx+e))}{cf} - \frac{3a^2 \tan(fx+e)}{cf} - \frac{2(a^2 + a^2 \sec(fx+e)) \tan(fx+e)}{f(c-c\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{c} - \frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{c} - \frac{2\left(3a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 2a^2\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^3 - \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}c}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.12 Problem number 16

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^2}{(c - c \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^2 \operatorname{arctanh}(\sin(fx + e))}{c^2 f} - \frac{2(a^2 + a^2 \sec(fx + e)) \tan(fx + e)}{3f(c - c \sec(fx + e))^2} + \frac{2a^2 \tan(fx + e)}{f(c^2 - c^2 \sec(fx + e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{c^2} - \frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{c^2} - \frac{2\left(3a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a^2\right)}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3}}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.13 Problem number 21

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c - c \sec(e + fx))^6 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{55a^3c^6 \operatorname{arctanh}(\sin(fx+e))}{128f} - \frac{25a^3c^6 \sec(fx+e) \tan(fx+e)}{128f} \\ & - \frac{15a^3c^6 (\sec^3(fx+e)) \tan(fx+e)}{64f} + \frac{5a^3c^6 \sec(fx+e) (\tan^3(fx+e))}{24f} \\ & + \frac{5a^3c^6 (\sec^3(fx+e)) (\tan^3(fx+e))}{16f} - \frac{a^3c^6 \sec(fx+e) (\tan^5(fx+e))}{6f} \\ & - \frac{3a^3c^6 (\sec^3(fx+e)) (\tan^5(fx+e))}{8f} + \frac{4a^3c^6 (\tan^7(fx+e))}{7f} + \frac{a^3c^6 (\tan^9(fx+e))}{9f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3465 a^3 c^6 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 3465 a^3 c^6 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 3465 a^3 c^6 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{17} - 30030 a^3 c^6 \right)}{17}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.14 Problem number 22

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{45a^3c^5 \operatorname{arctanh}(\sin(fx+e))}{128f} - \frac{35a^3c^5 \sec(fx+e) \tan(fx+e)}{128f} \\ & - \frac{5a^3c^5 (\sec^3(fx+e)) \tan(fx+e)}{64f} + \frac{5a^3c^5 \sec(fx+e) (\tan^3(fx+e))}{24f} \\ & + \frac{5a^3c^5 (\sec^3(fx+e)) (\tan^3(fx+e))}{48f} - \frac{a^3c^5 \sec(fx+e) (\tan^5(fx+e))}{6f} \\ & - \frac{a^3c^5 (\sec^3(fx+e)) (\tan^5(fx+e))}{8f} + \frac{2a^3c^5 (\tan^7(fx+e))}{7f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$315 a^3 c^5 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 315 a^3 c^5 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 315 a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{15} - 2415 a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{13} + 105 a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{11} - 21 a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^9 + 2 a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^7 - a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^5 + a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 - a^3 c^5 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{15}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.15 Problem number 23

$$\int \sec(e + f x) (a + a \sec(e + f x))^3 (c - c \sec(e + f x))^4 dx$$

Optimal antiderivative

$$\frac{5a^3 c^4 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{5a^3 c^4 \sec(fx + e) \tan(fx + e)}{16f} + \frac{5a^3 c^4 \sec(fx + e) (\tan^3(fx + e))}{24f} - \frac{a^3 c^4 \sec(fx + e) (\tan^5(fx + e))}{6f} + \frac{a^3 c^4 (\tan^7(fx + e))}{7f}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$105 a^3 c^4 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 105 a^3 c^4 \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 105 a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{13} - 700 a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^{11} + 105 a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^9 - 21 a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^7 + 2 a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^5 - a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + a^3 c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{13}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.16 Problem number 24

$$\int \sec(e + f x) (a + a \sec(e + f x))^3 (c - c \sec(e + f x))^3 dx$$

Optimal antiderivative

$$\frac{5a^3 c^3 \operatorname{arctanh}(\sin(fx + e))}{16f} - \frac{5a^3 c^3 \sec(fx + e) \tan(fx + e)}{16f} + \frac{5a^3 c^3 \sec(fx + e) (\tan^3(fx + e))}{24f} - \frac{a^3 c^3 \sec(fx + e) (\tan^5(fx + e))}{6f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 a^3 c^3 \log(|\sin(fx + e) + 1|) - 15 a^3 c^3 \log(|\sin(fx + e) - 1|) + \frac{2(33 a^3 c^3 \sin(fx+e)^5 - 40 a^3 c^3 \sin(fx+e)^3 + 15 a^3 c^3 \sin(fx+e))}{(\sin(fx+e)^2 - 1)^3}}{96 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.17 Problem number 25

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c - c \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{3a^3c^2 \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{3a^3c^2 \sec(fx + e) \tan(fx + e)}{8f} + \frac{a^3c^2 \sec(fx + e) (\tan^3(fx + e))}{4f} + \frac{a^3c^2 (\tan^5(fx + e))}{5f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 a^3 c^2 \log(|\tan(\frac{1}{2} fx + \frac{1}{2} e) + 1|) - 15 a^3 c^2 \log(|\tan(\frac{1}{2} fx + \frac{1}{2} e) - 1|) - \frac{2(15 a^3 c^2 \tan(\frac{1}{2} fx + \frac{1}{2} e)^9 - 70 a^3 c^2 \tan(\frac{1}{2} fx + \frac{1}{2} e))}{40 f}}{40 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.18 Problem number 26

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{5a^3c \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{3a^3c \sec(fx + e) \tan(fx + e)}{8f} - \frac{a^3c(\sec^3(fx + e)) \tan(fx + e)}{4f} - \frac{2a^3c(\tan^3(fx + e))}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15a^3c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15a^3c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2\left(15a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 - 55a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + \dots\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^2} \frac{1}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.19 Problem number 27

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^3}{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$-\frac{15a^3 \operatorname{arctanh}(\sin(fx + e))}{2cf} - \frac{10a^3 \tan(fx + e)}{cf} - \frac{5a^3 \sec(fx + e) \tan(fx + e)}{2cf} - \frac{2a(a + a \sec(fx + e))^2 \tan(fx + e)}{f(c - c \sec(fx + e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15a^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{c} - \frac{15a^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{c} - \frac{16a^3}{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)} - \frac{2\left(7a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 9a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^2 - 1} \frac{1}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.20 Problem number 28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\frac{5a^3 \operatorname{arctanh}(\sin(fx+e))}{c^2 f} + \frac{5a^3 \tan(fx+e)}{c^2 f} - \frac{2a(a+a\sec(fx+e))^2 \tan(fx+e)}{3f(c-c\sec(fx+e))^2} + \frac{10(a^3+a^3\sec(fx+e)) \tan(fx+e)}{3f(c^2-c^2\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15a^3 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{c^2} - \frac{15a^3 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{c^2} - \frac{6a^3 \tan(\frac{1}{2}fx+\frac{1}{2}e)}{(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-1)c^2} - \frac{4(6a^3 \tan(\frac{1}{2}fx+\frac{1}{2}e)^2+a^3)}{c^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)^3}$$


---


$$3f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.21 Problem number 29

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx$$

Optimal antiderivative

$$-\frac{a^3 \operatorname{arctanh}(\sin(fx+e))}{c^3 f} - \frac{2a(a+a\sec(fx+e))^2 \tan(fx+e)}{5f(c-c\sec(fx+e))^3} + \frac{2(a^3+a^3\sec(fx+e)) \tan(fx+e)}{3cf(c-c\sec(fx+e))^2} - \frac{2a^3 \tan(fx+e)}{f(c^3-c^3\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15a^3 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{c^3} - \frac{15a^3 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{c^3} - \frac{2(15a^3 \tan(\frac{1}{2}fx+\frac{1}{2}e)^4+5a^3 \tan(\frac{1}{2}fx+\frac{1}{2}e)^2+3a^3)}{c^3 \tan(\frac{1}{2}fx+\frac{1}{2}e)^5}$$


---


$$15f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.22 Problem number 34

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{35c^4 \operatorname{arctanh}(\sin(fx+e))}{2af} + \frac{28c^4 \tan(fx+e)}{af} - \frac{21c^4 \sec(fx+e) \tan(fx+e)}{2af} \\ & + \frac{2c(c-c\sec(fx+e))^3 \tan(fx+e)}{f(a+a\sec(fx+e))} + \frac{7c^4 (\tan^3(fx+e))}{3af} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a} - \frac{105c^4 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a} - \frac{96c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a} + \frac{2(87c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^5 - 136c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)^3} c$$


---


$$6f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.23 Problem number 35

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{15c^3 \operatorname{arctanh}(\sin(fx+e))}{2af} + \frac{10c^3 \tan(fx+e)}{af} \\ & - \frac{5c^3 \sec(fx+e) \tan(fx+e)}{2af} + \frac{2c(c-c\sec(fx+e))^2 \tan(fx+e)}{f(a+a\sec(fx+e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15c^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a} - \frac{15c^3 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a} - \frac{16c^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a} + \frac{2(9c^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 7c^3 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)^2} a$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.24 Problem number 36

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$-\frac{3c^2 \operatorname{arctanh}(\sin(fx+e))}{af} + \frac{3c^2 \tan(fx+e)}{af} + \frac{2(c^2 - c^2 \sec(fx+e)) \tan(fx+e)}{f(a+a\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3c^2 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a} - \frac{3c^2 \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a} - \frac{4c^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a} + \frac{2c^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)}{(\tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - 1)a}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.25 Problem number 37

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$-\frac{c \operatorname{arctanh}(\sin(fx+e))}{af} + \frac{2c \tan(fx+e)}{f(a+a\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\frac{c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a} - \frac{c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a} - \frac{2c \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.26 Problem number 42

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{105c^5 \operatorname{arctanh}(\sin(fx+e))}{2a^2f} - \frac{84c^5 \tan(fx+e)}{a^2f} \\ & + \frac{63c^5 \sec(fx+e) \tan(fx+e)}{2a^2f} - \frac{6c^2(c-c\sec(fx+e))^3 \tan(fx+e)}{f(a^2+a^2\sec(fx+e))} \\ & + \frac{2c(c-c\sec(fx+e))^4 \tan(fx+e)}{3f(a+a\sec(fx+e))^2} - \frac{7c^5(\tan^3(fx+e))}{a^2f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{315c^5 \log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{315c^5 \log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} + \frac{2\left(165c^5 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 - 280c^5 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 123c^5 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3 a^2}$$


---


$$6f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.27 Problem number 43

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{35c^4 \operatorname{arctanh}(\sin(fx+e))}{2a^2f} - \frac{70c^4 \tan(fx+e)}{3a^2f} + \frac{35c^4 \sec(fx+e) \tan(fx+e)}{6a^2f} \\ & + \frac{2c(c-c\sec(fx+e))^3 \tan(fx+e)}{3f(a+a\sec(fx+e))^2} - \frac{14(c^2-c^2\sec(fx+e))^2 \tan(fx+e)}{3f(a^2+a^2\sec(fx+e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105 c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^2} - \frac{105 c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^2} + \frac{6 \left(13 c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 11 c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1\right)^2 a^2} - \frac{16 \left(a^4 c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{6 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.28 Problem number 44

$$\int \frac{\sec(e + f x)(c - c \sec(e + f x))^3}{(a + a \sec(e + f x))^2} dx$$

Optimal antiderivative

$$\frac{5c^3 \operatorname{arctanh}(\sin(fx + e))}{a^2 f} - \frac{5c^3 \tan(fx + e)}{a^2 f} + \frac{2c(c - c \sec(fx + e))^2 \tan(fx + e)}{3f(a + a \sec(fx + e))^2} - \frac{10(c^3 - c^3 \sec(fx + e)) \tan(fx + e)}{3f(a^2 + a^2 \sec(fx + e))}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 c^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^2} - \frac{15 c^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^2} + \frac{6 c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1\right) a^2} - \frac{4 \left(a^4 c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 6 a^4 c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{a^6}$$


---


$$3 f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.29 Problem number 45

$$\int \frac{\sec(e + f x)(c - c \sec(e + f x))^2}{(a + a \sec(e + f x))^2} dx$$

Optimal antiderivative

$$\frac{c^2 \operatorname{arctanh}(\sin(fx + e))}{a^2 f} - \frac{2c^2 \tan(fx + e)}{f(a^2 + a^2 \sec(fx + e))} + \frac{2(c^2 - c^2 \sec(fx + e)) \tan(fx + e)}{3f(a + a \sec(fx + e))^2}$$



command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3c^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a^2} - \frac{3c^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a^2} - \frac{2\left(a^4c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^3 + 3a^4c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a^6}}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.30 Problem number 52

$$\int \frac{\sec(e + fx)(c - c \sec(e + fx))^6}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{231c^6 \operatorname{arctanh}(\sin(fx + e))}{2a^3f} + \frac{924c^6 \tan(fx + e)}{5a^3f} - \frac{693c^6 \sec(fx + e) \tan(fx + e)}{10a^3f} \\ & - \frac{22c^2(c - c \sec(fx + e))^4 \tan(fx + e)}{15af(a + a \sec(fx + e))^2} + \frac{2c(c - c \sec(fx + e))^5 \tan(fx + e)}{5f(a + a \sec(fx + e))^3} \\ & + \frac{66(c^2 - c^2 \sec(fx + e))^3 \tan(fx + e)}{5f(a^3 + a^3 \sec(fx + e))} + \frac{77c^6(\tan^3(fx + e))}{5a^3f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{3465c^6 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a^3} - \frac{3465c^6 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a^3} + \frac{10\left(267c^6 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 472c^6 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 213c^6 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)^3 a^3}}{30f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.31 Problem number 53

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{63c^5 \operatorname{arctanh}(\sin(fx+e))}{2a^3 f} + \frac{42c^5 \tan(fx+e)}{a^3 f} \\ & -\frac{21c^5 \sec(fx+e) \tan(fx+e)}{2a^3 f} - \frac{6c^2(c-c\sec(fx+e))^3 \tan(fx+e)}{5af(a+a\sec(fx+e))^2} \\ & + \frac{2c(c-c\sec(fx+e))^4 \tan(fx+e)}{5f(a+a\sec(fx+e))^3} + \frac{42c(c^2-c^2\sec(fx+e))^2 \tan(fx+e)}{5f(a^3+a^3\sec(fx+e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{315c^5 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{a^3} - \frac{315c^5 \log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{a^3} + \frac{10(17c^5 \tan(\frac{1}{2}fx+\frac{1}{2}e)^3 - 15c^5 \tan(\frac{1}{2}fx+\frac{1}{2}e))}{(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2 - 1)^2 a^3} - \frac{16(a^{12}c^5 \tan(\frac{1}{2}fx+\frac{1}{2}e))}{10f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.32 Problem number 54

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7c^4 \operatorname{arctanh}(\sin(fx+e))}{a^3 f} + \frac{7c^4 \tan(fx+e)}{a^3 f} + \frac{2c(c-c\sec(fx+e))^3 \tan(fx+e)}{5f(a+a\sec(fx+e))^3} \\ & -\frac{14(c^2-c^2\sec(fx+e))^2 \tan(fx+e)}{15af(a+a\sec(fx+e))^2} + \frac{14(c^4-c^4\sec(fx+e)) \tan(fx+e)}{3f(a^3+a^3\sec(fx+e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{105 c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^3} - \frac{105 c^4 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^3} + \frac{30 c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1\right) a^3} - \frac{4 \left(3 a^{12} c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 10 a^{12} c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{a^{15}}$$


---

15 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.33 Problem number 55

$$\int \frac{\sec(e + f x)(c - c \sec(e + f x))^3}{(a + a \sec(e + f x))^3} dx$$

Optimal antiderivative

$$-\frac{c^3 \operatorname{arctanh}(\sin(fx + e))}{a^3 f} + \frac{2c^3 \tan(fx + e)}{f(a^3 + a^3 \sec(fx + e))} + \frac{2c(c - c \sec(fx + e))^2 \tan(fx + e)}{5f(a + a \sec(fx + e))^3} - \frac{2(c^3 - c^3 \sec(fx + e)) \tan(fx + e)}{3af(a + a \sec(fx + e))^2}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 c^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^3} - \frac{15 c^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^3} - \frac{2 \left(3 a^{12} c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 5 a^{12} c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 15 a^{12} c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}{a^{15}}$$


---

15 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.34 Problem number 68

$$\int \frac{\sec(e + f x)(a + a \sec(e + f x))}{\sqrt{c - c \sec(e + f x)}} dx$$

Optimal antiderivative

$$-\frac{2a \operatorname{arctan}\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx + e)}}\right) \sqrt{2}}{f \sqrt{c}} + \frac{2a \tan(fx + e)}{f \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2a \frac{\left( \sqrt{2} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) \right)}{\sqrt{c}} + \frac{\sqrt{2}}{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}$$


---

$f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.35 Problem number 69

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))}{(c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \arctan \left( \frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx + e)}} \right) \sqrt{2}}{2c^{\frac{3}{2}}f} - \frac{a \tan(fx + e)}{f(c - c \sec(fx + e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( \sqrt{c} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) + \frac{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}{\tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2} \right) a}{2c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.36 Problem number 70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{a \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{16c^{\frac{5}{2}}f} - \frac{a \tan(fx+e)}{2f(c-c\sec(fx+e))^{\frac{5}{2}}} + \frac{a \tan(fx+e)}{8cf(c-c\sec(fx+e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( a\sqrt{c} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right) + \frac{(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c)^{\frac{3}{2}} ac - \sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c} ac^2}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4} \right)}{16c^3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.37 Problem number 75

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}} dx$$

Optimal antiderivative

$$\frac{4a^2 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{f\sqrt{c}} + \frac{16a^2 \tan(fx+e)}{3f\sqrt{c-c\sec(fx+e)}} - \frac{2a^2 \sqrt{c-c\sec(fx+e)} \tan(fx+e)}{3cf}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4a^2 \left( \frac{3\sqrt{2} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{\sqrt{c}} + \frac{\sqrt{2} (3c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - 4c)}{(c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c)^{\frac{3}{2}}} \right)$$


---


$$3f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.38 Problem number 76

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^2}{(c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3a^2 \arctan \left( \frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx + e)}} \right) \sqrt{2}}{c^{\frac{3}{2}} f} - \frac{2a^2 \tan(fx + e)}{f(c - c \sec(fx + e))^{\frac{3}{2}}} - \frac{2a^2 \tan(fx + e)}{cf \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a^2 \left( \frac{3\sqrt{2} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{c^{\frac{3}{2}}} + \frac{\sqrt{2} (3c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c)}{\left( (c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^{\frac{3}{2}} + \sqrt{c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c} \right) c} \right)$$


---


$$f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.39 Problem number 77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3a^2 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{8c^{5/2}f} - \frac{a^2 \tan(fx+e)}{f(c-c\sec(fx+e))^{5/2}} + \frac{5a^2 \tan(fx+e)}{4cf(c-c\sec(fx+e))^{3/2}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3\sqrt{c} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right) + \frac{3(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c)^{3/2} c^{5/2} \sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c^2}}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4} \right) a^2}{8c^3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.40 Problem number 78

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{a^2 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{32c^{7/2}f} - \frac{(a^2 + a^2 \sec(fx+e)) \tan(fx+e)}{3f(c-c\sec(fx+e))^{7/2}} + \frac{a^2 \tan(fx+e)}{4cf(c-c\sec(fx+e))^{5/2}} - \frac{a^2 \tan(fx+e)}{16c^2 f(c-c\sec(fx+e))^{3/2}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 3 a^2 \sqrt{c} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) + \frac{3 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{5}{2}} a^2 c + 8 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{3}{2}} a^2 c^2 - 3 \sqrt{c} \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^6}{c^3 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^6} \right)$$


---

$96 c^4 f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.41 Problem number 83

$$\int \frac{\sec(e + f x)(a + a \sec(e + f x))^3}{\sqrt{c - c \sec(e + f x)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{8 a^3 \arctan \left( \frac{\sqrt{c} \tan(f x + e) \sqrt{2}}{2 \sqrt{c - c \sec(f x + e)}} \right) \sqrt{2}}{f \sqrt{c}} + \frac{8 a^3 \tan(f x + e)}{f \sqrt{c - c \sec(f x + e)}} \\ & + \frac{2 a(a + a \sec(f x + e))^2 \tan(f x + e)}{5 f \sqrt{c - c \sec(f x + e)}} + \frac{4(a^3 + a^3 \sec(f x + e)) \tan(f x + e)}{3 f \sqrt{c - c \sec(f x + e)}} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$8 a^3 \left( \frac{15 \sqrt{2} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{\sqrt{c}} + \frac{\sqrt{2} \left( 15 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^2 - 5 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right) c + 3 c^2 \right)}{\left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{5}{2}}} \right)$$


---

$15 f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 75.42 Problem number 84

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{10a^3 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{c^{\frac{3}{2}} f} - \frac{a(a+a\sec(fx+e))^2 \tan(fx+e)}{f(c-c\sec(fx+e))^{\frac{3}{2}}} - \frac{10a^3 \tan(fx+e)}{cf\sqrt{c-c\sec(fx+e)}} - \frac{5(a^3+a^3\sec(fx+e)) \tan(fx+e)}{3cf\sqrt{c-c\sec(fx+e)}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^3 \left( \frac{15\sqrt{2} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2} (6c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 7c)}{(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c)^{\frac{3}{2}} c} + \frac{3\sqrt{2} \sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2} \right)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.43 Problem number 85

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{15a^3 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{4c^{\frac{5}{2}} f} - \frac{a(a+a\sec(fx+e))^2 \tan(fx+e)}{2f(c-c\sec(fx+e))^{\frac{5}{2}}} + \frac{5(a^3+a^3\sec(fx+e)) \tan(fx+e)}{4cf(c-c\sec(fx+e))^{\frac{3}{2}}} + \frac{15a^3 \tan(fx+e)}{4c^2 f \sqrt{c-c\sec(fx+e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$a^3 \left( \frac{15 \sqrt{2} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{c^{\frac{5}{2}}} + \frac{8 \sqrt{2}}{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c c^2}} + \frac{7 \sqrt{2} \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{3}{2}} + 9 \sqrt{2}}{c^4 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)} \right) \frac{1}{4 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

#### 75.44 Problem number 89

$$\int \frac{\sec(e + f x) \sqrt{c - c \sec(e + f x)}}{a + a \sec(e + f x)} dx$$

Optimal antiderivative

$$\frac{2c \tan(fx + e)}{f(a + a \sec(fx + e)) \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c} \operatorname{sgn} \left( \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn}(\cos(fx + e))}{af}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.45 Problem number 90

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx)) \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx + e)}}\right) \sqrt{2}}{2af\sqrt{c}} + \frac{\tan(fx + e)}{f(a + a \sec(fx + e)) \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{\arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{c} \right)$$

$2af$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.46 Problem number 91

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))(c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx + e)}}\right) \sqrt{2}}{8ac^{\frac{3}{2}}f} - \frac{3 \tan(fx + e)}{4af(c - c \sec(fx + e))^{\frac{3}{2}}} + \frac{\tan(fx + e)}{f(a + a \sec(fx + e))(c - c \sec(fx + e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 3 \sqrt{c} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) - 2 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c} - \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2} \right)}{8 a c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.47 Problem number 92

$$\int \frac{\sec(e + f x)}{(a + a \sec(e + f x))(c - c \sec(e + f x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{15 \arctan \left( \frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx+e)}} \right) \sqrt{2}}{64 a c^{\frac{5}{2}} f} - \frac{5 \tan(fx+e)}{8 a f (c - c \sec(fx+e))^{\frac{5}{2}}} \\ & + \frac{\tan(fx+e)}{f (a + a \sec(fx+e))(c - c \sec(fx+e))^{\frac{5}{2}}} - \frac{15 \tan(fx+e)}{32 a c f (c - c \sec(fx+e))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( 15 \sqrt{c} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) - 8 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c} - \frac{9 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{3}{2}} c + 7 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{c^2 \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2} \right)}{64 a c^3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.48 Problem number 96

$$\int \frac{\sec(e+fx) \sqrt{c - c \sec(e+fx)}}{(a + a \sec(e+fx))^2} dx$$

Optimal antiderivative

$$\frac{2c \tan(fx + e)}{3f (a + a \sec(fx + e))^2 \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^{\frac{3}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}(\cos(fx + e))}{6a^2cf}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.49 Problem number 97

$$\int \frac{\sec(e+fx)}{(a + a \sec(e+fx))^2 \sqrt{c - c \sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx+e)}}\right) \sqrt{2}}{4a^2f\sqrt{c}} + \frac{\tan(fx+e)}{3f(a + a \sec(fx+e))^2 \sqrt{c - c \sec(fx+e)}} + \frac{\tan(fx+e)}{2f(a^2 + a^2 \sec(fx+e)) \sqrt{c - c \sec(fx+e)}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{3 \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{\sqrt{c}} + \frac{(c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c)^{\frac{3}{2}} c^4 - 3 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c} c^5}{c^6} \right)$$


---


$$12 a^2 f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.50 Problem number 98

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^2 (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5 \arctan \left( \frac{\sqrt{c} \tan(fx+e) \sqrt{2}}{2 \sqrt{c - c \sec(fx+e)}} \right) \sqrt{2}}{16 a^2 c^{\frac{3}{2}} f} - \frac{5 \tan(fx+e)}{8 a^2 f (c - c \sec(fx+e))^{\frac{3}{2}}} \\ & + \frac{\tan(fx+e)}{3 f (a + a \sec(fx+e))^2 (c - c \sec(fx+e))^{\frac{3}{2}}} \\ & + \frac{5 \tan(fx+e)}{6 f (a^2 + a^2 \sec(fx+e)) (c - c \sec(fx+e))^{\frac{3}{2}}} \end{aligned}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 15 \sqrt{c} \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right) - \frac{3 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2} + \frac{2 \left( (c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c)^{\frac{3}{2}} c^2 - 6 \right)}{c^6} \right)$$


---


$$48 a^2 c^2 f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.51 Problem number 99

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^2 (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{35 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx+e)}}\right) \sqrt{2}}{128a^2c^{\frac{5}{2}}f} - \frac{35 \tan(fx+e)}{48a^2f(c - c \sec(fx+e))^{\frac{5}{2}}} \\ & + \frac{\tan(fx+e)}{3f(a + a \sec(fx+e))^2 (c - c \sec(fx+e))^{\frac{5}{2}}} \\ & + \frac{7 \tan(fx+e)}{6f(a^2 + a^2 \sec(fx+e))(c - c \sec(fx+e))^{\frac{5}{2}}} - \frac{35 \tan(fx+e)}{64a^2cf(c - c \sec(fx+e))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 105 \sqrt{c} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right) + \frac{8 \left( (c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c)^{\frac{3}{2}} c^2 - 9 \sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c} c^3 \right)}{c^3} \right)$$


---


$$384 a^2 c^3 f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.52 Problem number 103

$$\int \frac{\sec(e + fx) \sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{2c \tan(fx + e)}{5f(a + a \sec(fx + e))^3 \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{5}{2}} \operatorname{sgn} \left( \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right) \operatorname{sgn}(\cos(fx + e))}{20 a^3 c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.53 Problem number 104

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^3 \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\arctan \left( \frac{\sqrt{c} \tan(fx+e) \sqrt{2}}{2 \sqrt{c - c \sec(fx + e)}} \right) \sqrt{2}}{8 a^3 f \sqrt{c}} + \frac{\tan(fx + e)}{5 f (a + a \sec(fx + e))^3 \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\tan(fx + e)}{6 a f (a + a \sec(fx + e))^2 \sqrt{c - c \sec(fx + e)}} \\ & + \frac{\tan(fx + e)}{4 f (a^3 + a^3 \sec(fx + e)) \sqrt{c - c \sec(fx + e)}} \end{aligned}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \frac{15 \arctan \left( \frac{\sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c}}{\sqrt{c}} \right)}{\sqrt{c}} - \frac{3 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{5}{2}} c^{12} - 5 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^{\frac{3}{2}} c^{13} + 15 \sqrt{c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c} c^{15}}{120 a^3 f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 75.54 Problem number 105

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^3 (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{7 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c - c \sec(fx+e)}}\right) \sqrt{2}}{32a^3 c^{\frac{3}{2}} f} - \frac{7 \tan(fx+e)}{16a^3 f (c - c \sec(fx+e))^{\frac{3}{2}}} \\ & + \frac{\tan(fx+e)}{5f (a + a \sec(fx+e))^3 (c - c \sec(fx+e))^{\frac{3}{2}}} \\ & + \frac{7 \tan(fx+e)}{30af (a + a \sec(fx+e))^2 (c - c \sec(fx+e))^{\frac{3}{2}}} \\ & + \frac{7 \tan(fx+e)}{12f (a^3 + a^3 \sec(fx+e)) (c - c \sec(fx+e))^{\frac{3}{2}}} \end{aligned}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 105 \sqrt{c} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c}}{\sqrt{c}}\right) - \frac{15 \sqrt{c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c}}{\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2} - \frac{2 \left(3 \left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right)^{\frac{5}{2}}\right)}{480 a^3 c^2 f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.55 Problem number 106

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^3 (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{63 \arctan\left(\frac{\sqrt{c} \tan(fx+e)\sqrt{2}}{2\sqrt{c-c\sec(fx+e)}}\right) \sqrt{2}}{256a^3c^{\frac{5}{2}}f} - \frac{21 \tan(fx+e)}{32a^3f(c-c\sec(fx+e))^{\frac{5}{2}}} \\
 & + \frac{\tan(fx+e)}{5f(a+a\sec(fx+e))^3(c-c\sec(fx+e))^{\frac{5}{2}}} \\
 & + \frac{3 \tan(fx+e)}{10af(a+a\sec(fx+e))^2(c-c\sec(fx+e))^{\frac{5}{2}}} \\
 & + \frac{21 \tan(fx+e)}{20f(a^3+a^3\sec(fx+e))(c-c\sec(fx+e))^{\frac{5}{2}}} - \frac{63 \tan(fx+e)}{128a^3cf(c-c\sec(fx+e))^{\frac{3}{2}}}
 \end{aligned}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( 315 \sqrt{c} \arctan\left(\frac{\sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}}{\sqrt{c}}\right) - \frac{5 \left(17 \left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^{\frac{3}{2}} c + 15 \sqrt{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c}\right)}{c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4} \right)$$


---

1280 a<sup>3</sup>c<sup>3</sup>f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.56 Problem number 107

$$\int \sec(e+fx) \sqrt{a+a\sec(e+fx)} (c-c\sec(e+fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a(c-c\sec(fx+e))^{\frac{5}{2}} \tan(fx+e)}{3f\sqrt{a+a\sec(fx+e)}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 3 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^2 c^2 + 3 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right) c^3 + c^4 \right) \sqrt{-ac} |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{3 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.57 Problem number 108

$$\int \sec(e + fx) \sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a(c - c \sec(fx + e))^{\frac{3}{2}} \tan(fx + e)}{2f \sqrt{a + a \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 2 \left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right) c^3 + c^4 \right) \sqrt{-ac} |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\left( c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right)^2 - c \right)^2 c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.58 Problem number 109

$$\int \sec(e + fx) \sqrt{a + a \sec(e + fx)} \sqrt{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{c \sqrt{a + a \sec(fx + e)} \tan(fx + e)}{f \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{-ac}|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(c\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.59 Problem number 111

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\sqrt{a+a\sec(fx+e)}\tan(fx+e)}{2f(c-c\sec(fx+e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2\left(\frac{1}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2} - 1\right)}{2\sqrt{-ac}cf|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.60 Problem number 112

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{a\tan(fx+e)}{2f(c-c\sec(fx+e))^{\frac{5}{2}}\sqrt{a+a\sec(fx+e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \left( \frac{2 \left( a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - a \right) a + a^2}{a^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4} - 1 \right)}{8 \sqrt{-ac} c^2 f |a| \operatorname{sgn} \left( \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.61 Problem number 113

$$\int \sec(e + fx)(a + a \sec(e + fx))^{3/2}(c - c \sec(e + fx))^{7/2} dx$$

Optimal antiderivative

$$\frac{a^2(c - c \sec(fx + e))^{\frac{7}{2}} \tan(fx + e)}{10f \sqrt{a + a \sec(fx + e)}} + \frac{a(c - c \sec(fx + e))^{\frac{7}{2}} \sqrt{a + a \sec(fx + e)} \tan(fx + e)}{5f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( 10 \left( c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c \right)^3 c^3 + 20 \left( c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c \right)^2 c^4 + 15 \left( c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c \right) c^5 + 4 c^6 \right) \sqrt{a + a \sec(fx + e)}}{5 \left( c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c \right)^5 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.62 Problem number 114**

$$\int \sec(e + fx)(a + a \sec(e + fx))^{3/2}(c - c \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\frac{a^2(c - c \sec(fx + e))^{\frac{5}{2}} \tan(fx + e)}{6f \sqrt{a + a \sec(fx + e)}} + \frac{a(c - c \sec(fx + e))^{\frac{5}{2}} \sqrt{a + a \sec(fx + e)} \tan(fx + e)}{4f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 6 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^2 c^3 + 8 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right) c^4 + 3 c^5 \right) \sqrt{-ac} a |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{3 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^4 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.63 Problem number 115**

$$\int \sec(e + fx)(a + a \sec(e + fx))^{3/2}(c - c \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{c^2(a + a \sec(fx + e))^{\frac{3}{2}} \tan(fx + e)}{3f \sqrt{c - c \sec(fx + e)}} - \frac{c(a + a \sec(fx + e))^{\frac{3}{2}} \sqrt{c - c \sec(fx + e)} \tan(fx + e)}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 3 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right) c^4 + 2 c^5 \right) \sqrt{-ac} a |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{3 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^3 c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.64 Problem number 116

$$\int \sec(e + fx)(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{c(a + a \sec(fx + e))^{\frac{3}{2}} \tan(fx + e)}{2f \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \sqrt{-ac} ac|c|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.65 Problem number 119

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^{3/2}}{(c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(a + a \sec(fx + e))^{\frac{3}{2}} \tan(fx + e)}{4f (c - c \sec(fx + e))^{\frac{5}{2}}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a - \frac{a}{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4}\right) a^2}{4 \sqrt{-ac} c^2 f |a| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.66 Problem number 120

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{(a+a\sec(fx+e))^{3/2} \tan(fx+e)}{6f(c-c\sec(fx+e))^{7/2}} - \frac{(a+a\sec(fx+e))^{3/2} \tan(fx+e)}{24cf(c-c\sec(fx+e))^{5/2}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a - \frac{3\left(a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a\right)a^3 + a^4}{a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6}\right)a^2}{24\sqrt{-ac}c^3f|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.67 Problem number 121

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{a^2 \tan(fx+e)}{12cf(c-c\sec(fx+e))^{7/2} \sqrt{a+a\sec(fx+e)}} - \frac{a\sqrt{a+a\sec(fx+e)} \tan(fx+e)}{4f(c-c\sec(fx+e))^{9/2}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a - \frac{6\left(a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a\right)^2 a^3 + 4\left(a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a\right)a^4 + a^5}{a^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8}\right)a^2}{96\sqrt{-ac}c^4f|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 75.68 Problem number 122

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{a^2 \tan(fx+e)}{20cf(c-c\sec(fx+e))^{9/2} \sqrt{a+a\sec(fx+e)}} - \frac{a\sqrt{a+a\sec(fx+e)} \tan(fx+e)}{5f(c-c\sec(fx+e))^{11/2}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( a - \frac{10 \left( a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a \right)^3 a^3 + 10 \left( a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a \right)^2 a^4 + 5 \left( a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - a \right) a^5 + a^6}{a^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{10}} \right) a^2}{320 \sqrt{-ac} c^5 f |a| \operatorname{sgn} \left( \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.69 Problem number 123

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(a+a\sec(fx+e))^{3/2}(c-c\sec(fx+e))^{7/2} \tan(fx+e)}{6f} \\ & + \frac{a^3(c-c\sec(fx+e))^{7/2} \tan(fx+e)}{15f\sqrt{a+a\sec(fx+e)}} \\ & + \frac{2a^2(c-c\sec(fx+e))^{7/2} \sqrt{a+a\sec(fx+e)} \tan(fx+e)}{15f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 20 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^3 c^4 + 45 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^2 c^5 + 36 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right) c^6 + 10 c^7 \right) \sqrt{}}{15 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^6 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.70 Problem number 124

$$\int \sec(e + fx)(a + a \sec(e + fx))^{5/2}(c - c \sec(e + fx))^{5/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c(a + a \sec(fx + e))^{\frac{5}{2}}(c - c \sec(fx + e))^{\frac{3}{2}} \tan(fx + e)}{5f} \\ & - \frac{2c^3(a + a \sec(fx + e))^{\frac{5}{2}} \tan(fx + e)}{15f \sqrt{c - c \sec(fx + e)}} \\ & - \frac{c^2(a + a \sec(fx + e))^{\frac{5}{2}} \sqrt{c - c \sec(fx + e)} \tan(fx + e)}{5f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( 10 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^2 c^4 + 15 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right) c^5 + 6 c^6 \right) \sqrt{-ac} a^2 |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right) + 15 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^5 f}{15 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^5 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.71 Problem number 125

$$\int \sec(e + fx)(a + a \sec(e + fx))^{5/2}(c - c \sec(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{c^2(a + a \sec(fx + e))^{\frac{5}{2}} \tan(fx + e)}{6f \sqrt{c - c \sec(fx + e)}} - \frac{c(a + a \sec(fx + e))^{\frac{5}{2}} \sqrt{c - c \sec(fx + e)} \tan(fx + e)}{4f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \left( 4 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right) c^5 + 3 c^6 \right) \sqrt{-ac} a^2 |c| \operatorname{sgn} \left( \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^3 + \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) \right)}{3 \left( c \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right)^2 - c \right)^4 c^2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.72 Problem number 126

$$\int \sec(e + fx)(a + a \sec(e + fx))^{5/2} \sqrt{c - c \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{c(a + a \sec(fx + e))^{5/2} \tan(fx + e)}{3f \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \sqrt{-ac} a^2 c^2 |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}{3 \left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right)^3 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.73 Problem number 130

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^{5/2}}{(c - c \sec(e + fx))^{7/2}} dx$$

Optimal antiderivative

$$\frac{(a + a \sec(fx + e))^{5/2} \tan(fx + e)}{6f (c - c \sec(fx + e))^{7/2}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a^2 - \frac{a^2}{\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6}\right) a^2}{6 \sqrt{-ac} c^3 f |a| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.74 Problem number 131

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}} dx$$

Optimal antiderivative

$$\frac{(a+a\sec(fx+e))^{5/2} \tan(fx+e)}{8f(c-c\sec(fx+e))^{9/2}} - \frac{(a+a\sec(fx+e))^{5/2} \tan(fx+e)}{48cf(c-c\sec(fx+e))^{7/2}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a^2 - \frac{4(a \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - a)a^5 + a^6}{a^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^8}\right)a^2}{48\sqrt{-ac}c^4f|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.75 Problem number 132

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}} dx$$

Optimal antiderivative

$$\frac{(a+a\sec(fx+e))^{5/2} \tan(fx+e)}{10f(c-c\sec(fx+e))^{11/2}} - \frac{(a+a\sec(fx+e))^{5/2} \tan(fx+e)}{40cf(c-c\sec(fx+e))^{9/2}} - \frac{(a+a\sec(fx+e))^{5/2} \tan(fx+e)}{240c^2f(c-c\sec(fx+e))^{7/2}}$$

command

`integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(a^2 - \frac{10(a \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - a)^2 a^5 + 5(a \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - a)a^6 + a^7}{a^5 \tan(\frac{1}{2}fx + \frac{1}{2}e)^{10}}\right)a^2}{240\sqrt{-ac}c^5f|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.76 Problem number 133

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{c(c-c\sec(fx+e))^{\frac{3}{2}}\tan(fx+e)}{2f\sqrt{a+a\sec(fx+e)}} - \frac{4c^3\ln(1+\sec(fx+e))\tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}\sqrt{c-c\sec(fx+e)}} \\ & - \frac{2c^2\sqrt{c-c\sec(fx+e)}\tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}} \end{aligned}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2c^2 \left( \frac{2\sqrt{-ac} \operatorname{clog}\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)}{a|c|} - \frac{3\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 \sqrt{-ac} c + 4\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac} c^2 + \sqrt{-ac} c^3}{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2 a|c|} \right) \operatorname{sgn}\left(\frac{t}{f}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.77 Problem number 134

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{2c^2\ln(1+\sec(fx+e))\tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}\sqrt{c-c\sec(fx+e)}} - \frac{c\sqrt{c-c\sec(fx+e)}\tan(fx+e)}{f\sqrt{a+a\sec(fx+e)}}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\sqrt{-ac} c^2 \log\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)}{a|c|} - \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac} c^2 + \sqrt{-ac} c^3}{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) a|c|} \right) \operatorname{sgn}\left(\frac{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{f}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.78 Problem number 135

$$\int \frac{\sec(e+fx) \sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{c \ln(1 + \sec(fx + e)) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^2 \log\left(\left|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right|\right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{-ac} f|c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.79 Problem number 136

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)} \sqrt{c-c\sec(e+fx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^2 \left( \frac{\log\left(|c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right)}{c} - \frac{\log(|c|)}{c} \right)}{2 \sqrt{-ac} f|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.80 Problem number 137

$$\int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\tan(fx + e)}{2f(c - c \sec(fx + e))^{3/2} \sqrt{a + a \sec(fx + e)}} - \frac{\operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{2cf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c}{c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2} - \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right) + \log(|c|)$$

$$\frac{4\sqrt{-ac} f|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{4\sqrt{-ac} f|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.81 Problem number 138

$$\int \frac{\sec(e + fx)}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\tan(fx + e)}{4f(c - c \sec(fx + e))^{5/2} \sqrt{a + a \sec(fx + e)}} - \frac{\tan(fx + e)}{4cf(c - c \sec(fx + e))^{3/2} \sqrt{a + a \sec(fx + e)}} - \frac{\operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{4c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

`integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3\left(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c\right)^2 + 2\left(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c\right)c}{c^2 \tan(\frac{1}{2}fx + \frac{1}{2}e)^4} - 2 \log\left(|c| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right) + 2 \log(|c|)$$

$$\frac{16\sqrt{-ac} cf|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{16\sqrt{-ac} cf|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.82 Problem number 139

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{c(c-c\sec(fx+e))^{\frac{3}{2}} \tan(fx+e)}{f(a+a\sec(fx+e))^{\frac{3}{2}}} + \frac{4c^3 \ln(1+\sec(fx+e)) \tan(fx+e)}{af\sqrt{a+a\sec(fx+e)}\sqrt{c-c\sec(fx+e)}} \\ + \frac{2c^2\sqrt{c-c\sec(fx+e)}\tan(fx+e)}{af\sqrt{a+a\sec(fx+e)}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2c^2 \left( \frac{2\sqrt{-ac} c \log\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)}{a^2|c|} + \frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac}}{a^2|c|} - \frac{2\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \sqrt{-ac} c + \sqrt{-ac} c^2}{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) a^2|c|} \right) \operatorname{sgn}(t)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.83 Problem number 140

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{c^2 \ln(1+\sec(fx+e)) \tan(fx+e)}{af\sqrt{a+a\sec(fx+e)}\sqrt{c-c\sec(fx+e)}} + \frac{c\sqrt{c-c\sec(fx+e)}\tan(fx+e)}{f(a+a\sec(fx+e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + c \log\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) - c\right) c^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{\sqrt{-ac} af|c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 75.84 Problem number 141

$$\int \frac{\sec(e + fx) \sqrt{c - c \sec(e + fx)}}{(a + a \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\sqrt{c - c \sec(fx + e)} \tan(fx + e)}{2f (a + a \sec(fx + e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c\right) c}{2 \sqrt{-ac} af|c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.85 Problem number 142

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^{3/2} \sqrt{c - c \sec(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\tan(fx + e)}{2f (a + a \sec(fx + e))^{\frac{3}{2}} \sqrt{c - c \sec(fx + e)}} - \frac{\operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{2af \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^2 \left( \frac{\log\left(|c| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right)}{c} - \frac{\log(|c|)}{c} - \frac{c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - c}{c^2} \right)}{4 \sqrt{-ac} af|c| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.86 Problem number 143

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\csc(fx + e)}{2acf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} - \frac{\operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{2acf \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c}{c} + \frac{2 c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c}{c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2} - 2 \log\left(|c| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right) + 2 \log(|c|) - 1}{8 \sqrt{-ac} a f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.87 Problem number 144

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^{3/2} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3 \csc(fx + e)}{8a c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}} - \frac{\tan(fx + e)}{4f (a + a \sec(fx + e))^{\frac{3}{2}} (c - c \sec(fx + e))^{\frac{5}{2}}} - \frac{3 \operatorname{arctanh}(\cos(fx + e)) \tan(fx + e)}{8a c^2 f \sqrt{a + a \sec(fx + e)} \sqrt{c - c \sec(fx + e)}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2(c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c)}{c} + \frac{9(c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c)^2 + 12(c \tan(\frac{1}{2} fx + \frac{1}{2} e)^2 - c)c + 4c^2}{c^2 \tan(\frac{1}{2} fx + \frac{1}{2} e)^4} - 6 \log\left(|c| \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2\right) + 6 \log(|c|)}{32 \sqrt{-ac} a f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.88 Problem number 145

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{c(c-c\sec(fx+e))^{\frac{3}{2}} \tan(fx+e)}{2f(a+a\sec(fx+e))^{\frac{5}{2}}} - \frac{c^3 \ln(1+\sec(fx+e)) \tan(fx+e)}{a^2 f \sqrt{a+a\sec(fx+e)} \sqrt{c-c\sec(fx+e)}} - \frac{c^2 \sqrt{c-c\sec(fx+e)} \tan(fx+e)}{af(a+a\sec(fx+e))^{\frac{3}{2}}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{c^4 \left( \frac{(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c)^2 c^2 + 4(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c)c^3}{c^4} + 2 \log\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right) \right) \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{2\sqrt{-ac} a^2 f |c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.89 Problem number 146

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(c-c\sec(fx+e))^{\frac{3}{2}} \tan(fx+e)}{4f(a+a\sec(fx+e))^{\frac{5}{2}}}$$

command

```
integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^2 + 2 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right) c \right) c}{4\sqrt{-ac} a^2 f |c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.90 Problem number 147**

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{c \tan(fx+e)}{2f(a+a\sec(fx+e))^{5/2} \sqrt{c-c\sec(fx+e)}}$$

command

`integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c\right)^2}{8\sqrt{-ac} a^2 f |c|}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.91 Problem number 148**

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2} \sqrt{c-c\sec(e+fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\tan(fx+e)}{4f(a+a\sec(fx+e))^{5/2} \sqrt{c-c\sec(fx+e)}} \\ & + \frac{\tan(fx+e)}{4af(a+a\sec(fx+e))^{3/2} \sqrt{c-c\sec(fx+e)}} \\ & - \frac{\operatorname{arctanh}(\cos(fx+e)) \tan(fx+e)}{4a^2 f \sqrt{a+a\sec(fx+e)} \sqrt{c-c\sec(fx+e)}} \end{aligned}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{c^2 \left( \frac{2 \log(|c| \tan(\frac{1}{2}fx + \frac{1}{2}e)^2)}{c} - \frac{2 \log(|c|)}{c} + \frac{(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c)^2 c^3 - 2(c \tan(\frac{1}{2}fx + \frac{1}{2}e)^2 - c)c^4}{c^6} \right)}{16\sqrt{-ac} a^2 f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.92 Problem number 149

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3 \csc (fx + e)}{8a^2cf \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}} + \frac{\tan (fx + e)}{4f (a + a \sec (fx + e))^{\frac{5}{2}} (c - c \sec (fx + e))^{\frac{3}{2}}} - \frac{3 \operatorname{arctanh} (\cos (fx + e)) \tan (fx + e)}{8a^2cf \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}}$$

command

```
integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( 3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)}{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2} - \frac{\left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right)^2 c^2 - 4 \left( c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - c \right) c^3}{c^4} - 6 \log\left(\left|c\right| \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2\right) + 6 \log\left(\left|c\right|\right) - \frac{32 \sqrt{-ac} a^2 f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{32 \sqrt{-ac} a^2 f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.93 Problem number 150

$$\int \frac{\sec(e + fx)}{(a + a \sec(e + fx))^{5/2} (c - c \sec(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{3 \csc (fx + e)}{8a^2c^2f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}} - \frac{(\cot^2 (fx + e)) \csc (fx + e)}{4a^2c^2f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}} - \frac{3 \operatorname{arctanh} (\cos (fx + e)) \tan (fx + e)}{8a^2c^2f \sqrt{a + a \sec (fx + e)} \sqrt{c - c \sec (fx + e)}}$$

command

`integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right)^2 c^2 - 6 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right) c^3}{c^4} - \frac{18 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right)^2 + 28 \left(c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - c\right) c + 11 c^2}{c^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4} + 12 \log\left(\left|c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right|\right)}{64 \sqrt{-ac} a^2 c f |c| \operatorname{sgn}\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 75.94 Problem number 168

$$\int \sec^2(e + fx)(a + a \sec(e + fx))^3(c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^3 c \operatorname{arctanh}(\sin(fx + e))}{4f} + \frac{a^3 c \sec(fx + e) \tan(fx + e)}{4f} - \frac{a^3 c (\sec^3(fx + e)) \tan(fx + e)}{2f} - \frac{2a^3 c (\tan^3(fx + e))}{3f} - \frac{a^3 c (\tan^5(fx + e))}{5f}$$

command

`integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15 a^3 c \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right) - 15 a^3 c \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right) - \frac{2 \left(15 a^3 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 - 70 a^3 c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + \dots\right)}{60 f}}{60 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.95 Problem number 169

$$\int \sec^2(e + fx)(a + a \sec(e + fx))^2(c - c \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2 c \operatorname{arctanh}(\sin(fx + e))}{8f} + \frac{a^2 c \sec(fx + e) \tan(fx + e)}{8f} - \frac{a^2 c (\sec^3(fx + e)) \tan(fx + e)}{4f} - \frac{a^2 c (\tan^3(fx + e))}{3f}$$

command

```
integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3a^2c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2\left(3a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 - 11a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 53a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 11a^2c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + a^2c\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)^2}}{24f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.96 Problem number 171

$$\int \frac{\sec^2(e + fx)(c - c \sec(e + fx))}{a + a \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{2c \operatorname{arctanh}(\sin(fx + e))}{af} - \frac{c \tan(fx + e)}{af} - \frac{2c \tan(fx + e)}{f(a + a \sec(fx + e))}$$

command

```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\frac{c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a} - \frac{c \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a} - \frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{a} + \frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)a}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.97 Problem number 172

$$\int \frac{\sec^2(e + fx)(c - c \sec(e + fx))}{(a + a \sec(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{c \operatorname{arctanh}(\sin(fx + e))}{a^2 f} + \frac{7c \tan(fx + e)}{3a^2 f (1 + \sec(fx + e))} - \frac{2c \tan(fx + e)}{3f (a + a \sec(fx + e))^2}$$

command

```
integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} - \frac{3c \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2} - \frac{a^4 c \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 6a^4 c \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a^6}$$


---


$$3f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.98 Problem number 179

$$\int \frac{\sec^2(e + fx)}{\sqrt{a + a \sec(e + fx)} (c - c \sec(e + fx))} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctan}\left(\frac{\sqrt{a} \tan(fx+e)\sqrt{2}}{2\sqrt{a+a\sec(fx+e)}}\right)\sqrt{2}}{2cf\sqrt{a}} + \frac{\cot(fx+e)\sqrt{a+a\sec(fx+e)}}{acf}$$

command

```
integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \log\left(\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}\right)^2\right)}{\sqrt{-a} \operatorname{csgn}(\cos(fx+e))} - \frac{4\sqrt{2}\sqrt{-a}}{\left(\left(\sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a}\right)^2\right)} 4f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 75.99 Problem number 184

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(fx+e)}{\sqrt{c-d}\sqrt{a+a\sec(fx+e)}}\right)\sqrt{a}}{f\sqrt{c-d}\sqrt{d}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2\sqrt{-a}\operatorname{arctan}\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c+\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\right)}{4\sqrt{cd-d^2}a}\right)}{\sqrt{cd-d^2}f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.100 Problem number 185

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(fx+e))}{8f} \\ & + \frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(fx+e)}{30f} \\ & + \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(fx+e)\tan(fx+e)}{120f} \\ & + \frac{a(12c^2+35cd+16d^2)(c+d\sec(fx+e))^2\tan(fx+e)}{60f} \\ & + \frac{a(4c+5d)(c+d\sec(fx+e))^3\tan(fx+e)}{20f} + \frac{a(c+d\sec(fx+e))^4\tan(fx+e)}{5f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15 (8ac^4 + 16ac^3d + 24ac^2d^2 + 12acd^3 + 3ad^4) \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) - 15 (8ac^4 + 16ac^3d + 24ac^2d^2 + 12acd^3 + 3ad^4) \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.101 Problem number 186

$$\int \sec(e + fx)(a + a \sec(e + fx))(c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a(8c^3 + 12c^2d + 12cd^2 + 3d^3) \operatorname{arctanh}(\sin(fx + e))}{8f} \\ & + \frac{a(3c^3 + 16c^2d + 12cd^2 + 4d^3) \tan(fx + e)}{6f} \\ & + \frac{ad(6c^2 + 20cd + 9d^2) \sec(fx + e) \tan(fx + e)}{24f} \\ & + \frac{a(3c + 4d)(c + d \sec(fx + e))^2 \tan(fx + e)}{12f} + \frac{a(c + d \sec(fx + e))^3 \tan(fx + e)}{4f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3 (8ac^3 + 12ac^2d + 12acd^2 + 3ad^3) \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right| \right) - 3 (8ac^3 + 12ac^2d + 12acd^2 + 3ad^3) \log \left( \left| \tan \left( \frac{1}{2} fx + \frac{1}{2} e \right) - 1 \right| \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.102 Problem number 187

$$\int \sec(e + fx)(a + a \sec(e + fx))(c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{a(2c^2 + 2cd + d^2) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{2a(c^2 + 3cd + d^2) \tan(fx + e)}{3f} \\ + \frac{ad(2c + 3d) \sec(fx + e) \tan(fx + e)}{6f} + \frac{a(c + d \sec(fx + e))^2 \tan(fx + e)}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(2ac^2 + 2acd + ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3(2ac^2 + 2acd + ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2(6ac^2 + \dots)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.103 Problem number 188

$$\int \sec(e + fx)(a + a \sec(e + fx))(c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{a(2c + d) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{a(c + d) \tan(fx + e)}{f} + \frac{ad \sec(fx + e) \tan(fx + e)}{2f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$(2ac + ad) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - (2ac + ad) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2(2ac \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + ad \tan(\frac{1}{2}fx + \frac{1}{2}e))}{2f \tan(\frac{1}{2}fx + \frac{1}{2}e)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.104 Problem number 189

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{a \operatorname{arctanh}(\sin(fx + e))}{df} - \frac{2a \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right) \sqrt{c-d}}{df \sqrt{c+d}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1|\right)}{d} - \frac{a \log\left(|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1|\right)}{d} + \frac{2 \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2c-2d) + \operatorname{arctan}\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2 + d^2}}\right) \right) (ac-ad)}{\sqrt{-c^2 + d^2} d} \cdot f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.105 Problem number 193

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c + d \sec(e + fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(24c^4 + 64c^3d + 84c^2d^2 + 48cd^3 + 11d^4) \operatorname{arctanh}(\sin(fx + e))}{16f} \\ & - \frac{a^2(4c^5 - 48c^4d - 311c^3d^2 - 448c^2d^3 - 288cd^4 - 64d^5) \tan(fx + e)}{60df} \\ & - \frac{a^2(8c^4 - 96c^3d - 438c^2d^2 - 464cd^3 - 165d^4) \sec(fx + e) \tan(fx + e)}{240f} \\ & - \frac{a^2(4c^3 - 48c^2d - 123cd^2 - 64d^3) (c + d \sec(fx + e))^2 \tan(fx + e)}{120df} \\ & - \frac{a^2(4c^2 - 48cd - 55d^2) (c + d \sec(fx + e))^3 \tan(fx + e)}{120df} \\ & - \frac{a^2(c - 12d) (c + d \sec(fx + e))^4 \tan(fx + e)}{30df} + \frac{a^2(c + d \sec(fx + e))^5 \tan(fx + e)}{6df} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.106 Problem number 194

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2(2c + d)(2c^2 + 3cd + 2d^2) \operatorname{arctanh}(\sin(fx + e))}{8f} \\ & - \frac{a^2(c^4 - 10c^3d - 44c^2d^2 - 40cd^3 - 12d^4) \tan(fx + e)}{10df} \\ & - \frac{a^2(2c^3 - 20c^2d - 57cd^2 - 30d^3) \sec(fx + e) \tan(fx + e)}{40f} \\ & - \frac{a^2(c^2 - 10cd - 12d^2)(c + d \sec(fx + e))^2 \tan(fx + e)}{20df} \\ & - \frac{a^2(c - 10d)(c + d \sec(fx + e))^3 \tan(fx + e)}{20df} + \frac{a^2(c + d \sec(fx + e))^4 \tan(fx + e)}{5df} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15(4a^2c^3 + 8a^2c^2d + 7a^2cd^2 + 2a^2d^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15(4a^2c^3 + 8a^2c^2d + 7a^2cd^2 + 2a^2d^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.107 Problem number 195**

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{a^2(12c^2 + 16cd + 7d^2) \operatorname{arctanh}(\sin(fx + e))}{8f} - \frac{a^2(c^3 - 8c^2d - 20cd^2 - 8d^3) \tan(fx + e)}{6df}$$

$$- \frac{a^2(2c(c - 8d) - 21d^2) \sec(fx + e) \tan(fx + e)}{24f}$$

$$- \frac{a^2(c - 8d)(c + d \sec(fx + e))^2 \tan(fx + e)}{12df} + \frac{a^2(c + d \sec(fx + e))^3 \tan(fx + e)}{4df}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(12a^2c^2 + 16a^2cd + 7a^2d^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3(12a^2c^2 + 16a^2cd + 7a^2d^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.108 Problem number 196**

$$\int \sec(e + fx)(a + a \sec(e + fx))^2(c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{a^2(3c + 2d) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{2a^2(3c + 2d) \tan(fx + e)}{3f}$$

$$+ \frac{a^2(3c + 2d) \sec(fx + e) \tan(fx + e)}{6f} + \frac{d(a + a \sec(fx + e))^2 \tan(fx + e)}{3f}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(3a^2c + 2a^2d) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3(3a^2c + 2a^2d) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2(9a^2c \tan(\frac{1}{2}fx + \frac{1}{2}e))^5}{6f}$$


---

6f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.109 Problem number 197

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^2}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$-\frac{a^2(c-2d) \operatorname{arctanh}(\sin(fx+e))}{d^2 f} + \frac{2a^2(c-d)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right)}{d^2 f \sqrt{c+d}} + \frac{a^2 \tan(fx+e)}{df}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)d} + \frac{(a^2c - 2a^2d) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{d^2} - \frac{(a^2c - 2a^2d) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{d^2} + \frac{2(a^2c^2 - 2a^2cd + a^2d^2)}{f} \left(\pi \left\lfloor \frac{fx+e}{2\pi} \right\rfloor\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.110 Problem number 198

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^2}{(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a^2 \operatorname{arctanh}(\sin(fx+e))}{d^2 f} - \frac{2a^2(c+2d) \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right) \sqrt{c-d}}{d^2 (c+d)^{\frac{3}{2}} f} - \frac{a^2(c-d) \tan(fx+e)}{d(c+d) f (c+d \sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{d^2} - \frac{a^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{d^2} + \frac{2(a^2c^2 + a^2cd - 2a^2d^2) \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2c-2d) + \arctan\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2 + d^2}}\right) \right)}{(cd^2 + d^3) \sqrt{-c^2 + d^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.111 Problem number 202

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(40c^3 + 90c^2d + 78cd^2 + 23d^3) \operatorname{arctanh}(\sin(fx + e))}{16f} \\ & + \frac{a^3(40c^3 + 90c^2d + 78cd^2 + 23d^3) \tan(fx + e)}{16f} \\ & + \frac{(40c^3 + 90c^2d + 78cd^2 + 23d^3)(a^3 + a^3 \sec(fx + e)) \tan(fx + e)}{48f} \\ & + \frac{a(3c + 8d)(a + a \sec(fx + e))^2(c + d \sec(fx + e))^2 \tan(fx + e)}{30f} \\ & + \frac{a(a + a \sec(fx + e))^2(c + d \sec(fx + e))^3 \tan(fx + e)}{6f} \\ & + \frac{a(a + a \sec(fx + e))^2(8c^3 + 148c^2d + 132cd^2 + 42d^3 + d(6c^2 + 62cd + 31d^2) \sec(fx + e)) \tan(fx + e)}{120f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15(40a^3c^3 + 90a^3c^2d + 78a^3cd^2 + 23a^3d^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15(40a^3c^3 + 90a^3c^2d + 78a^3cd^2 + 23a^3d^3)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



**75.112 Problem number 203**

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3(20c^2 + 30cd + 13d^2) \operatorname{arctanh}(\sin(fx + e))}{8f} \\ & + \frac{a^3(2c^4 - 15c^3d + 72c^2d^2 + 180cd^3 + 76d^4) \tan(fx + e)}{30d^2f} \\ & + \frac{a^3(4c^3 - 30c^2d + 146cd^2 + 195d^3) \sec(fx + e) \tan(fx + e)}{120df} \\ & + \frac{a^3(2c^2 - 15cd + 76d^2) (c + d \sec(fx + e))^2 \tan(fx + e)}{60d^2f} \\ & - \frac{a^3(2c - 11d) (c + d \sec(fx + e))^3 \tan(fx + e)}{20d^2f} \\ & + \frac{(a^3 + a^3 \sec(fx + e)) (c + d \sec(fx + e))^3 \tan(fx + e)}{5df} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15(20a^3c^2 + 30a^3cd + 13a^3d^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 15(20a^3c^2 + 30a^3cd + 13a^3d^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.113 Problem number 204**

$$\int \sec(e + fx)(a + a \sec(e + fx))^3(c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5a^3(4c + 3d) \operatorname{arctanh}(\sin(fx + e))}{8f} + \frac{a^3(4c + 3d) \tan(fx + e)}{f} \\ & + \frac{3a^3(4c + 3d) \sec(fx + e) \tan(fx + e)}{8f} \\ & + \frac{d(a + a \sec(fx + e))^3 \tan(fx + e)}{4f} + \frac{a^3(4c + 3d) (\tan^3(fx + e))}{12f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$15 (4 a^3 c + 3 a^3 d) \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right) - 15 (4 a^3 c + 3 a^3 d) \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right) - \frac{2 \left( 60 a^3 c \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\dots}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.114 Problem number 205

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^3}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3 \operatorname{arctanh}(\sin(fx + e))}{2df} + \frac{a^3(c^2 - 3cd + 3d^2) \operatorname{arctanh}(\sin(fx + e))}{d^3 f} \\ & - \frac{2a^3(c - d)^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right)}{d^3 f \sqrt{c+d}} \\ & - \frac{a^3(c - 3d) \tan(fx + e)}{d^2 f} + \frac{a^3 \sec(fx + e) \tan(fx + e)}{2df} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2 a^3 c^2 - 6 a^3 c d + 7 a^3 d^2) \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) + 1 \right| \right)}{d^3} - \frac{(2 a^3 c^2 - 6 a^3 c d + 7 a^3 d^2) \log \left( \left| \tan \left( \frac{1}{2} f x + \frac{1}{2} e \right) - 1 \right| \right)}{d^3} + \frac{4 (a^3 c^3 - 3 a^3 c^2 d + 3 a^3 c d^2 - a^3 d^3) \left( \pi \left[ \dots \right] \right)}{\dots}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.115 Problem number 206

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^3}{(c + d \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^3(2c - 3d) \operatorname{arctanh}(\sin(fx + e))}{d^3 f} + \frac{2a^3(c - d)^{\frac{3}{2}}(2c + 3d) \operatorname{arctanh}\left(\frac{\sqrt{c - d} \tan\left(\frac{fx + e}{2}\right)}{\sqrt{c + d}}\right)}{d^3(c + d)^{\frac{3}{2}} f} \\ & + \frac{2a^3 c \tan(fx + e)}{d^2(c + d) f} - \frac{(c - d)(a^3 + a^3 \sec(fx + e)) \tan(fx + e)}{d(c + d) f (c + d \sec(fx + e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(2a^3c^3 - a^3c^2d - 4a^3cd^2 + 3a^3d^3) \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2c-2d) + \operatorname{arctan}\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2 + d^2}}\right) \right)}{(cd^3 + d^4) \sqrt{-c^2 + d^2}} + \frac{4(a^3c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - a^3c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5)}{(c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.116 Problem number 207

$$\int \frac{\sec(e + fx)(a + a \sec(e + fx))^3}{(c + d \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^3 \operatorname{arctanh}(\sin(fx + e))}{d^3 f} - \frac{a^3(2c^2 + 6cd + 7d^2) \operatorname{arctanh}\left(\frac{\sqrt{c - d} \tan\left(\frac{fx + e}{2}\right)}{\sqrt{c + d}}\right) \sqrt{c - d}}{d^3(c + d)^{\frac{5}{2}} f} \\ & - \frac{(c - d)(a^3 + a^3 \sec(fx + e)) \tan(fx + e)}{2d(c + d) f (c + d \sec(fx + e))^2} - \frac{a^3(c - d)(2c + 5d) \tan(fx + e)}{2d^2(c + d)^2 f (c + d \sec(fx + e))} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{d^3} - \frac{a^3 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{d^3} + \frac{(2a^3c^3 + 4a^3c^2d + a^3cd^2 - 7a^3d^3) \left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2c-2d) + \arctan\left(\frac{c \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{(c^2d^3 + 2cd^4 + d^5)}}\right) \right)}{(c^2d^3 + 2cd^4 + d^5) \sqrt{-c^2 + d^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.117 Problem number 210

$$\int \frac{\sec(e + fx)(c + d \sec(e + fx))^4}{a + a \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(8c^3 - 12c^2d + 12cd^2 - 3d^3) \operatorname{arctanh}(\sin(fx + e))}{2af} \\ & - \frac{(3c - 4d) d(c + d \sec(fx + e))^2 \tan(fx + e)}{3af} + \frac{(c - d)(c + d \sec(fx + e))^3 \tan(fx + e)}{f(a + a \sec(fx + e))} \\ & - \frac{d(12c^3 - 64c^2d + 48cd^2 - 16d^3 + d(6c^2 - 20cd + 9d^2) \sec(fx + e)) \tan(fx + e)}{6af} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(8c^3d - 12c^2d^2 + 12cd^3 - 3d^4) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{a} - \frac{3(8c^3d - 12c^2d^2 + 12cd^3 - 3d^4) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{a} + \frac{6(c^4 \tan(\frac{1}{2}fx + \frac{1}{2}e) - 4c^3d \sec(\frac{1}{2}fx + \frac{1}{2}e))}{a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.118 Problem number 211

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{3d(2c^2-2cd+d^2)\operatorname{arctanh}(\sin(fx+e))}{2af} + \frac{(c-d)(c+d\sec(fx+e))^2 \tan(fx+e)}{f(a+a\sec(fx+e))} - \frac{d(4c^2-12cd+4d^2+(2c-3d)d\sec(fx+e)) \tan(fx+e)}{2af}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(2c^2d-2cd^2+d^3)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{a} - \frac{3(2c^2d-2cd^2+d^3)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{a} + \frac{2(c^3\tan(\frac{1}{2}fx+\frac{1}{2}e)-3c^2d\tan(\frac{1}{2}fx+\frac{1}{2}e)+3cd^2\tan(\frac{1}{2}fx+\frac{1}{2}e))}{a}$$

---

$2f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.119 Problem number 212

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{(2c-d)d\operatorname{arctanh}(\sin(fx+e))}{af} + \frac{d^2 \tan(fx+e)}{af} + \frac{(c-d)^2 \tan(fx+e)}{f(a+a\sec(fx+e))}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2cd-d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{a} - \frac{(2cd-d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{a} - \frac{2d^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)}{(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2-1)a} + \frac{c^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)-2cd \tan(\frac{1}{2}fx+\frac{1}{2}e)}{a}$$


---

$f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.120 Problem number 213

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctanh}(\sin(fx+e))}{af} + \frac{(c-d)\tan(fx+e)}{f(a+a\sec(fx+e))}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{d \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a} - \frac{d \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a} + \frac{c \tan(\frac{1}{2}fx + \frac{1}{2}e) - d \tan(\frac{1}{2}fx + \frac{1}{2}e)}{a}}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.121 Problem number 217

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{5(2c-d)d^2(2c^2-3cd+2d^2)\operatorname{arctanh}(\sin(fx+e))}{2a^2f} \\ & - \frac{d(c^2+10cd-12d^2)(c+d\sec(fx+e))^2\tan(fx+e)}{3a^2f} \\ & + \frac{(c-d)(c+10d)(c+d\sec(fx+e))^3\tan(fx+e)}{3f(a^2+a^2\sec(fx+e))} + \frac{(c-d)(c+d\sec(fx+e))^4\tan(fx+e)}{3f(a+a\sec(fx+e))^2} \\ & - \frac{d(4c^4+40c^3d-176c^2d^2+160cd^3-48d^4+d(2c^3+20c^2d-57cd^2+30d^3)\sec(fx+e))\tan(fx+e)}{6a^2f} \end{aligned}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15(4c^3d^2-8c^2d^3+7cd^4-2d^5)\log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} - \frac{15(4c^3d^2-8c^2d^3+7cd^4-2d^5)\log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2} - \frac{2(60c^2d^3\tan(\frac{1}{2}fx + \frac{1}{2}e))}{a^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.122 Problem number 218

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2(12c^2 - 16cd + 7d^2) \operatorname{arctanh}(\sin(fx + e))}{2a^2f} \\ & + \frac{(c-d)(c+8d)(c+d\sec(fx+e))^2 \tan(fx+e)}{3f(a^2+a^2\sec(fx+e))} \\ & + \frac{(c-d)(c+d\sec(fx+e))^3 \tan(fx+e)}{3f(a+a\sec(fx+e))^2} \\ & - \frac{d(4c^3 + 32c^2d - 80cd^2 + 32d^3 + d(2c^2 + 16cd - 21d^2) \sec(fx+e)) \tan(fx+e)}{6a^2f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(12c^2d^2 - 16cd^3 + 7d^4) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^2} - \frac{3(12c^2d^2 - 16cd^3 + 7d^4) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^2} - \frac{6(8cd^3 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 5d^4 \tan(\frac{1}{2}fx + \frac{1}{2}e))}{(\tan(\frac{1}{2}fx + \frac{1}{2}e))^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.123 Problem number 219

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(3c-2d)d^2 \operatorname{arctanh}(\sin(fx+e))}{a^2f} + \frac{(c-d)(c+d\sec(fx+e))^2 \tan(fx+e)}{3f(a+a\sec(fx+e))^2} \\ & + \frac{(c^3 + 4c^2d - 12cd^2 + 10d^3 - (c-4d)d^2 \sec(fx+e)) \tan(fx+e)}{3f(a^2 + a^2 \sec(fx+e))} \end{aligned}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12 d^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1\right) a^2} - \frac{6 (3 c d^2 - 2 d^3) \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^2} + \frac{6 (3 c d^2 - 2 d^3) \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^2} + \frac{a^4 c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 3 a^4 c^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{a^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.124 Problem number 220

$$\int \frac{\sec(e + f x)(c + d \sec(e + f x))^2}{(a + a \sec(e + f x))^2} dx$$

Optimal antiderivative

$$\frac{d^2 \operatorname{arctanh}(\sin(f x + e))}{a^2 f} + \frac{(c - d)^2 \tan(f x + e)}{3 f (a + a \sec(f x + e))^2} + \frac{(c - d)(c + 5d) \tan(f x + e)}{3 f (a^2 + a^2 \sec(f x + e))}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6 d^2 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^2} - \frac{6 d^2 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^2} - \frac{a^4 c^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 2 a^4 c d \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + a^4 d^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 3 a^4 c^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{a^6}$$

$6 f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.125 Problem number 225

$$\int \frac{\sec(e + f x)(c + d \sec(e + f x))^6}{(a + a \sec(e + f x))^3} dx$$



Optimal antiderivative

$$\begin{aligned}
& \frac{d^3(40c^3 - 90c^2d + 78cd^2 - 23d^3) \operatorname{arctanh}(\sin(fx + e))}{2a^3f} \\
& - \frac{2d(2c^5 + 18c^4d + 107c^3d^2 - 472c^2d^3 + 456cd^4 - 136d^5) \tan(fx + e)}{15a^3f} \\
& - \frac{d^2(4c^4 + 36c^3d + 216c^2d^2 - 626cd^3 + 345d^4) \sec(fx + e) \tan(fx + e)}{30a^3f} \\
& - \frac{d(2c^3 + 18c^2d + 111cd^2 - 136d^3) (c + d \sec(fx + e))^2 \tan(fx + e)}{15a^3f} \\
& + \frac{(c - d) (2c^2 + 18cd + 115d^2) (c + d \sec(fx + e))^3 \tan(fx + e)}{15f(a^3 + a^3 \sec(fx + e))} \\
& + \frac{(c - d) (2c + 13d) (c + d \sec(fx + e))^4 \tan(fx + e)}{15af(a + a \sec(fx + e))^2} \\
& + \frac{(c - d) (c + d \sec(fx + e))^5 \tan(fx + e)}{5f(a + a \sec(fx + e))^3}
\end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{30(40c^3d^3 - 90c^2d^4 + 78cd^5 - 23d^6) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^3} - \frac{30(40c^3d^3 - 90c^2d^4 + 78cd^5 - 23d^6) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^3} - \frac{20(90c^2d^4 \tan(fx + e) \sec(fx + e) - 90c^2d^4 \tan(fx + e))}{a^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.126 Problem number 226**

$$\int \frac{\sec(e + fx)(c + d \sec(e + fx))^5}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{d^3(20c^2 - 30cd + 13d^2) \operatorname{arctanh}(\sin(fx + e))}{2a^3f} \\
& - \frac{2d(2c^4 + 15c^3d + 72c^2d^2 - 180cd^3 + 76d^4) \tan(fx + e)}{15a^3f} \\
& - \frac{d^2(4c^3 + 30c^2d + 146cd^2 - 195d^3) \sec(fx + e) \tan(fx + e)}{30a^3f} \\
& + \frac{(c - d)(2c^2 + 15cd + 76d^2)(c + d \sec(fx + e))^2 \tan(fx + e)}{15f(a^3 + a^3 \sec(fx + e))} \\
& + \frac{(c - d)(2c + 11d)(c + d \sec(fx + e))^3 \tan(fx + e)}{15af(a + a \sec(fx + e))^2} \\
& + \frac{(c - d)(c + d \sec(fx + e))^4 \tan(fx + e)}{5f(a + a \sec(fx + e))^3}
\end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{30(20c^2d^3 - 30cd^4 + 13d^5) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) + 1|)}{a^3} - \frac{30(20c^2d^3 - 30cd^4 + 13d^5) \log(|\tan(\frac{1}{2}fx + \frac{1}{2}e) - 1|)}{a^3} - \frac{60(10cd^4 \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 - 7d^5)}{a^3}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**75.127 Problem number 227**

$$\int \frac{\sec(e + fx)(c + d \sec(e + fx))^4}{(a + a \sec(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{(4c - 3d)d^3 \operatorname{arctanh}(\sin(fx + e))}{a^3f} + \frac{(c - d)(2c + 9d)(c + d \sec(fx + e))^2 \tan(fx + e)}{15af(a + a \sec(fx + e))^2} \\
& + \frac{(c - d)(c + d \sec(fx + e))^3 \tan(fx + e)}{5f(a + a \sec(fx + e))^3} \\
& + \frac{(2c^4 + 8c^3d + 21c^2d^2 - 88cd^3 + 72d^4 - d^2(2c^2 + 10cd - 27d^2) \sec(fx + e)) \tan(fx + e)}{15f(a^3 + a^3 \sec(fx + e))}
\end{aligned}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{120 d^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1\right) a^3} - \frac{60 (4 c d^3 - 3 d^4) \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^3} + \frac{60 (4 c d^3 - 3 d^4) \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^3} - \frac{3 a^{12} c^4 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5}{a^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.128 Problem number 228

$$\int \frac{\sec(e + f x)(c + d \sec(e + f x))^3}{(a + a \sec(e + f x))^3} dx$$

Optimal antiderivative

$$\frac{d^3 \operatorname{arctanh}(\sin(f x + e))}{a^3 f} + \frac{(c - d)(c + d \sec(f x + e))^2 \tan(f x + e)}{5 f (a + a \sec(f x + e))^3} + \frac{(c - d)(4 c^2 + 16 c d + 22 d^2 + (2 c^2 + 11 c d + 29 d^2) \sec(f x + e)) \tan(f x + e)}{15 a f (a + a \sec(f x + e))^2}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{60 d^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right|\right)}{a^3} - \frac{60 d^3 \log\left(\left|\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1\right|\right)}{a^3} + \frac{3 a^{12} c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 - 9 a^{12} c^2 d \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 9 a^{12} c d^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5}{a^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.129 Problem number 238

$$\int \frac{\sec(e + f x) \sqrt{a + a \sec(e + f x)}}{c + d \sec(e + f x)} dx$$

Optimal antiderivative

$$\frac{2 \arctan \left( \frac{\sqrt{a} \sqrt{d} \tan(fx+e)}{\sqrt{c+d} \sqrt{a+a \sec(fx+e)}} \right) \sqrt{a}}{f \sqrt{d} \sqrt{c+d}}$$

command

```
integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \sqrt{-a} \arctan \left( \frac{\sqrt{2} \left( \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right)^2 c - \left( \sqrt{-a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - \sqrt{-a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a} \right) \right)}{4 \sqrt{-cd - d^2} a} \right)}{\sqrt{-cd - d^2} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.130 Problem number 244

$$\int \sec(e+fx)(a+b \sec(e+fx))(c+d \sec(e+fx))^4 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^4 + 24a^2d^2 + 3a^4 + 16b^3c^3d + 12bc^3d^3) \operatorname{arctanh}(\sin(fx+e))}{8f} \\ & + \frac{(95a^3c^3d + 80ac^3d^3 + 12b^4c^4 + 112b^2c^2d^2 + 16b^4d^4) \tan(fx+e)}{30f} \\ & + \frac{d(130a^2c^2d + 45a^2d^3 + 24b^3c^3 + 116bc^2d^2) \sec(fx+e) \tan(fx+e)}{120f} \\ & + \frac{(35acd + 12b^2c^2 + 16b^2d^2) (c+d \sec(fx+e))^2 \tan(fx+e)}{60f} \\ & + \frac{(5ad + 4bc) (c+d \sec(fx+e))^3 \tan(fx+e)}{20f} + \frac{b(c+d \sec(fx+e))^4 \tan(fx+e)}{5f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.131 Problem number 245

$$\int \sec(e + fx)(a + b \sec(e + fx))(c + d \sec(e + fx))^3 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8ac^3 + 12acd^2 + 12bc^2d + 3bd^3) \operatorname{arctanh}(\sin(fx + e))}{8f} \\ & + \frac{(4ad(4c^2 + d^2) + 3b(c^3 + 4cd^2)) \tan(fx + e)}{6f} \\ & + \frac{d(20acd + 6bc^2 + 9bd^2) \sec(fx + e) \tan(fx + e)}{24f} \\ & + \frac{(4ad + 3bc)(c + d \sec(fx + e))^2 \tan(fx + e)}{12f} + \frac{b(c + d \sec(fx + e))^3 \tan(fx + e)}{4f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

---


$$3(8ac^3 + 12bc^2d + 12acd^2 + 3bd^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3(8ac^3 + 12bc^2d + 12acd^2 + 3bd^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.132 Problem number 246

$$\int \sec(e + fx)(a + b \sec(e + fx))(c + d \sec(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{(2bcd + a(2c^2 + d^2)) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{2(3acd + b(c^2 + d^2)) \tan(fx + e)}{3f} \\ + \frac{d(3ad + 2bc) \sec(fx + e) \tan(fx + e)}{6f} + \frac{b(c + d \sec(fx + e))^2 \tan(fx + e)}{3f}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(2ac^2 + 2bcd + ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - 3(2ac^2 + 2bcd + ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2(6bc^2 \tan(\frac{1}{2}fx + \frac{1}{2}e) + 3cd^2)}{3f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.133 Problem number 247

$$\int \sec(e + fx)(a + b \sec(e + fx))(c + d \sec(e + fx)) dx$$

Optimal antiderivative

$$\frac{(2ac + bd) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{(ad + bc) \tan(fx + e)}{f} + \frac{bd \sec(fx + e) \tan(fx + e)}{2f}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$(2ac + bd) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - (2ac + bd) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right) - \frac{2(2bc \tan(\frac{1}{2}fx + \frac{1}{2}e)^3 + 2ad \tan(\frac{1}{2}fx + \frac{1}{2}e))}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.134 Problem number 248

$$\int \frac{\sec(e + fx)(a + b \sec(e + fx))}{c + d \sec(e + fx)} dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}(\sin(fx + e))}{df} - \frac{2(-ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{c-d} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{c+d}}\right)}{df \sqrt{c-d} \sqrt{c+d}}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{d} - \frac{b \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{d} + \frac{2 \left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2c-2d) + \operatorname{arctan}\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-c^2 + d^2}}\right) \right) (bc-ad)}{f \sqrt{-c^2 + d^2} d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.135 Problem number 252

$$\int \frac{\sec(e + fx)(c + d \sec(e + fx))^4}{a + b \sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3(-ad + 4bc) \operatorname{arctanh}(\sin(fx + e))}{2b^2 f} \\ & + \frac{d(-ad + 2bc)(a^2 d^2 - 2abcd + 2b^2 c^2) \operatorname{arctanh}(\sin(fx + e))}{b^4 f} \\ & + \frac{2(-ad + bc)^4 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{b^4 f \sqrt{a-b} \sqrt{a+b}} \\ & + \frac{d^4 \tan(fx + e)}{bf} + \frac{d^2(a^2 d^2 - 4abcd + 6b^2 c^2) \tan(fx + e)}{b^3 f} \\ & + \frac{d^3(-ad + 4bc) \sec(fx + e) \tan(fx + e)}{2b^2 f} + \frac{d^4(\tan^3(fx + e))}{3bf} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(8b^3c^3d - 12ab^2c^2d^2 + 8a^2bcd^3 + 4b^3cd^3 - 2a^3d^4 - ab^2d^4)}{b^4} \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right) - \frac{3(8b^3c^3d - 12ab^2c^2d^2 + 8a^2bcd^3 + 4b^3cd^3 - 2a^3d^4 - ab^2d^4)}{b^4}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.136 Problem number 253

$$\int \frac{\sec(e + fx)(c + d\sec(e + fx))^3}{a + b\sec(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^3 \operatorname{arctanh}(\sin(fx + e))}{2bf} + \frac{d(a^2d^2 - 3abcd + 3b^2c^2) \operatorname{arctanh}(\sin(fx + e))}{b^3f} \\ & + \frac{2(-ad + bc)^3 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\sqrt{a+b}} \\ & + \frac{d^2(-ad + 3bc) \tan(fx + e)}{b^2f} + \frac{d^3 \sec(fx + e) \tan(fx + e)}{2bf} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(6b^2c^2d - 6abcd^2 + 2a^2d^3 + b^2d^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{b^3} - \frac{(6b^2c^2d - 6abcd^2 + 2a^2d^3 + b^2d^3) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{b^3} - \frac{4(b^3c^3 - 3ab^2c^2d + 3a^2cd^3 - ab^2d^4)}{b^4}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 75.137 Problem number 254

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{d(-ad+2bc) \operatorname{arctanh}(\sin(fx+e))}{b^2 f} + \frac{2(-ad+bc)^2 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx+e}{2}\right)}{\sqrt{a+b}}\right)}{b^2 f \sqrt{a-b} \sqrt{a+b}} + \frac{d^2 \tan(fx+e)}{bf}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1\right)b} - \frac{(2bcd-ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{b^2} + \frac{(2bcd-ad^2) \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{b^2} + \frac{2(b^2c^2 - 2abcd + a^2d^2)}{b^2} \left(\pi \left\lfloor \frac{fx+e}{2\pi} \right\rfloor\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.138 Problem number 255

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)} dx$$

Optimal antiderivative

$$\frac{d \operatorname{arctanh}(\sin(fx+e))}{bf} + \frac{2(-ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx+e}{2}\right)}{\sqrt{a+b}}\right)}{bf \sqrt{a-b} \sqrt{a+b}}$$

command

`integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{d \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{b} - \frac{d \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{b} - \frac{2 \left( \pi \left\lfloor \frac{fx+e}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2a-2b) + \operatorname{arctan}\left(\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-a^2 + b^2}}\right) \right)}{\sqrt{-a^2 + b^2} b} (bc-ad)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.139 Problem number 258

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^4(-2ad+5bc) \operatorname{arctanh}(\sin(fx+e))}{2b^3f} \\ & + \frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3) \operatorname{arctanh}(\sin(fx+e))}{b^5f} \\ & + \frac{2(-ad+bc)^5 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{\frac{3}{2}}b^3(a+b)^{\frac{3}{2}}f} - \frac{(-ad+bc)^5 \sin(fx+e)}{b^4(a^2-b^2)f(b+a\cos(fx+e))} \\ & + \frac{2(-ad+bc)^4(4ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{a+b}}\right)}{ab^5f\sqrt{a-b}\sqrt{a+b}} \\ & + \frac{d^5 \tan(fx+e)}{b^2f} + \frac{d^3(3a^2d^2-10abcd+10b^2c^2) \tan(fx+e)}{b^4f} \\ & + \frac{d^4(-2ad+5bc) \sec(fx+e) \tan(fx+e)}{2b^3f} + \frac{d^5(\tan^3(fx+e))}{3b^2f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.140 Problem number 259

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^4 \operatorname{arctanh}(\sin(fx + e))}{2b^2 f} + \frac{d^2(3a^2 d^2 - 8abcd + 6b^2 c^2) \operatorname{arctanh}(\sin(fx + e))}{b^4 f} \\ & + \frac{2(-ad + bc)^4 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{\frac{3}{2}} b^2 (a+b)^{\frac{3}{2}} f} - \frac{(-ad + bc)^4 \sin(fx + e)}{b^3 (a^2 - b^2) f (b + a \cos(fx + e))} \\ & + \frac{2(-ad + bc)^3 (3ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a b^4 f \sqrt{a-b} \sqrt{a+b}} \\ & + \frac{2d^3(-ad + 2bc) \tan(fx + e)}{b^3 f} + \frac{d^4 \sec(fx + e) \tan(fx + e)}{2b^2 f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4(ab^4c^4 - 4b^5c^3d - 6a^3b^2c^2d^2 + 12ab^4c^2d^2 + 8a^4bcd^3 - 12a^2b^3cd^3 - 3a^5d^4 + 4a^3b^2d^4) \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \operatorname{arctan}\left(\frac{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-a^2 + b^2}}\right) \right)}{(a^2b^4 - b^6) \sqrt{-a^2 + b^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 75.141 Problem number 260

$$\int \frac{\sec(e + fx)(c + d \sec(e + fx))^3}{(a + b \sec(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2(-2ad + 3bc) \operatorname{arctanh}(\sin(fx + e))}{b^3 f} \\ & + \frac{2(-ad + bc)^3 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{\frac{3}{2}} b (a+b)^{\frac{3}{2}} f} - \frac{(-ad + bc)^3 \sin(fx + e)}{b^2 (a^2 - b^2) f (b + a \cos(fx + e))} \\ & + \frac{2(-ad + bc)^2 (2ad + bc) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{\sqrt{a+b}}\right)}{a b^3 f \sqrt{a-b} \sqrt{a+b}} + \frac{d^3 \tan(fx + e)}{b^2 f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(ab^3c^3 - 3b^4c^2d - 3a^3bcd^2 + 6ab^3cd^2 + 2a^4d^3 - 3a^2b^2d^3) \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \arctan \left( \frac{a \tan(\frac{1}{2}fx + \frac{1}{2}e) - b \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^2b^3 - b^5) \sqrt{-a^2 + b^2}} - \frac{2(b^3c^3 + \dots)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 75.142 Problem number 261

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2} dx$$

Optimal antiderivative

$$\frac{d^2 \operatorname{arctanh}(\sin(fx+e))}{b^2 f} + \frac{2(-ad+bc)^2 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan(\frac{fx+e}{2})}{\sqrt{a+b}}\right)}{a(a-b)^{\frac{3}{2}}(a+b)^{\frac{3}{2}} f} - \frac{(-ad+bc)^2 \sin(fx+e)}{b(a^2-b^2)f(b+a\cos(fx+e))} + \frac{2(-a^2d^2+b^2c^2) \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tan(\frac{fx+e}{2})}{\sqrt{a+b}}\right)}{ab^2 f \sqrt{a-b} \sqrt{a+b}}$$

command

```
integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{d^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{b^2} - \frac{d^2 \log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{b^2} - \frac{2(ab^2c^2 - 2b^3cd - a^3d^2 + 2ab^2d^2) \left( \pi \left[ \frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \arctan \left( \frac{a \tan(\frac{1}{2}fx + \frac{1}{2}e) - b \tan(\frac{1}{2}fx + \frac{1}{2}e)}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^2b^2 - b^4) \sqrt{-a^2 + b^2} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76 Test file number 126

Test folder name:

test\_cases/4\_Trig\_functions/4.5\_Secant/126\_4.5.7-d\_trig-^m-a+b-c\_sec-^n-^p

### 76.1 Problem number 5

$$\int \csc(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{(a + b) \operatorname{arctanh}(\cos(fx + e))}{f} + \frac{b \sec(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a + b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) + \frac{4b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.2 Problem number 6

$$\int \csc^3(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{(a + 3b) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{(a + b) \cot(fx + e) \csc(fx + e)}{2f} + \frac{b \sec(fx + e)}{f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(a + 3b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a+b + \frac{14b(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + \frac{(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.3 Problem number 7

$$\int \csc^5(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a+5b) \operatorname{arctanh}(\cos(fx+e))}{8f} - \frac{(3a+7b) \cot(fx+e) \csc(fx+e)}{8f} \\ & - \frac{(a+b) \cot(fx+e) (\csc^3(fx+e))}{4f} + \frac{b \sec(fx+e)}{f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$12(a+5b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{\left(a+b - \frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{16b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{18a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{90b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^2}{(\cos(fx+e)-1)^2}$$

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.4 Problem number 18

$$\int \csc(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{(a+b)^2 \operatorname{arctanh}(\cos(fx+e))}{f} + \frac{b(2a+b) \sec(fx+e)}{f} + \frac{b^2 (\sec^3(fx+e))}{3f}$$

command

```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(a^2 + 2ab + b^2) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) + \frac{8\left(3ab+2b^2 + \frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{3ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{3b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{\left(\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right)^3}$$

6 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.5 Problem number 19

$$\int \csc^3(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a + b)(a + 5b) \operatorname{arctanh}(\cos(fx + e))}{2f} - \frac{(3a^2 + 6ab + 5b^2) \cot(fx + e) \csc(fx + e)}{6f} \\ & + \frac{b(6a + 5b) \sec(fx + e)}{3f} + \frac{b^2(\csc^2(fx + e))(\sec^3(fx + e))}{3f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{6ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{3b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - 6(a^2 + 6ab + 5b^2) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - \frac{3(a^2+2ab+b^2-2a^2)}{3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.6 Problem number 20

$$\int \csc^5(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a^2 + 30ab + 35b^2) \operatorname{arctanh}(\cos(fx + e))}{8f} - \frac{(3a + 7b)^2 \cot(fx + e) \csc(fx + e)}{24f} \\ & - \frac{(3a^2 + 6ab + 7b^2) \cot(fx + e) (\csc^3(fx + e))}{12f} \\ & + \frac{b(6a + 7b) \sec(fx + e)}{3f} + \frac{b^2(\csc^4(fx + e))(\sec^3(fx + e))}{3f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{24 a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{96 ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{72 b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{3 a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{6 ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{3 b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.7 Problem number 31

$$\int \frac{\csc(e+fx)}{a+b \sec^2(e+fx)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(fx+e))}{(a+b)f} + \frac{\operatorname{arctan}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{b}}\right)\sqrt{b}}{(a+b)f\sqrt{a}}$$

command

```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2b \operatorname{arctan}\left(-\frac{a \cos(fx+e)-b}{\sqrt{ab} \cos(fx+e)+\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.8 Problem number 32

$$\int \frac{\csc^3(e+fx)}{a+b \sec^2(e+fx)} dx$$

Optimal antiderivative

$$-\frac{(a-b) \operatorname{arctanh}(\cos(fx+e))}{2(a+b)^2 f} - \frac{\cot(fx+e) \csc(fx+e)}{2(a+b)f} + \frac{\operatorname{arctan}\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{b}}\right)\sqrt{a}\sqrt{b}}{(a+b)^2 f}$$



command

```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8ab \arctan\left(-\frac{a \cos(fx+e)-b}{\sqrt{ab} \cos(fx+e)+\sqrt{ab}}\right)}{(a^2+2ab+b^2)\sqrt{ab}} - \frac{2(a-b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2+2ab+b^2} - \frac{\left(a+b-\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{(a^2+2ab+b^2)(\cos(fx+e)-1)} + \frac{1}{af}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.9 Problem number 33

$$\int \frac{\csc^5(e+fx)}{a+b \sec^2(e+fx)} dx$$

Optimal antiderivative

$$\frac{(3a^2-6ab-b^2) \operatorname{arctanh}(\cos(fx+e))}{8(a+b)^3 f} - \frac{(3a-b) \cot(fx+e) \csc(fx+e)}{8(a+b)^2 f} - \frac{\cot(fx+e) (\csc^3(fx+e))}{4(a+b) f} + \frac{a^{\frac{3}{2}} \arctan\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{(a+b)^3 f}$$

command

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{64a^2b \arctan\left(-\frac{a \cos(fx+e)-b}{\sqrt{ab} \cos(fx+e)+\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{4(3a^2-6ab-b^2) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{\frac{8a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} - \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{a^2+2ab+b^2}$$

64 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.10 Problem number 44

$$\int \frac{\csc(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\cos(fx + e))}{(a + b)^2 f} - \frac{b \cos(fx + e)}{2a(a + b)f(b + a(\cos^2(fx + e)))} + \frac{(3a + b) \operatorname{arctan}\left(\frac{\cos(fx + e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{2a^{\frac{3}{2}}(a + b)^2 f}$$

command

`integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(3ab + b^2) \operatorname{arctan}\left(-\frac{a \cos(fx + e) - b}{\sqrt{ab} \cos(fx + e) + \sqrt{ab}}\right) - \frac{\log\left(\frac{|-\cos(fx + e) + 1|}{|\cos(fx + e) + 1|}\right)}{a^2 + 2ab + b^2} + \frac{2\left(ab + b^2 + \frac{ab(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{b^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1}\right)}{(a^3 + 2a^2b + ab^2)\left(a + b + \frac{2a(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{2b(\cos(fx + e) - 1)}{\cos(fx + e) + 1} + \frac{a(\cos(fx + e) - 1)}{\cos(fx + e) + 1}\right)}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.11 Problem number 45

$$\int \frac{\csc^3(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{(a - 3b) \operatorname{arctanh}(\cos(fx + e))}{2(a + b)^3 f} + \frac{(a - b) \cos(fx + e)}{2(a + b)^2 f(b + a(\cos^2(fx + e)))} - \frac{\cot(fx + e) \csc(fx + e)}{2(a + b)f(b + a(\cos^2(fx + e)))} + \frac{(3a - b) \operatorname{arctan}\left(\frac{\cos(fx + e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{2(a + b)^3 f \sqrt{a}}$$

command

`integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6(a - 3b) \log\left(\frac{|-\cos(fx + e) + 1|}{|\cos(fx + e) + 1|}\right)}{a^3 + 3a^2b + 3ab^2 + b^3} - \frac{12(3ab - b^2) \operatorname{arctan}\left(-\frac{a \cos(fx + e) - b}{\sqrt{ab} \cos(fx + e) + \sqrt{ab}}\right)}{(a^3 + 3a^2b + 3ab^2 + b^3)\sqrt{ab}} + \frac{3a^2 + 6ab + 3b^2 + \frac{4a^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{20ab(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{b^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1}}{(a^3 + 3a^2b + 3ab^2 + b^3)\left(\frac{a + b + \frac{2a(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{2b(\cos(fx + e) - 1)}{\cos(fx + e) + 1} + \frac{a(\cos(fx + e) - 1)}{\cos(fx + e) + 1}\right) \sqrt{a}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.12 Problem number 46

$$\int \frac{\csc^5(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a^2 - 6ab + b^2) \operatorname{arctanh}(\cos(fx + e))}{8(a + b)^4 f} + \frac{3a(a - 3b) \cos(fx + e)}{8(a + b)^3 f (b + a(\cos^2(fx + e)))} \\ & - \frac{(a - 5b) \cot(fx + e) \csc(fx + e)}{8(a + b)^2 f (b + a(\cos^2(fx + e)))} - \frac{\cot(fx + e) (\csc^3(fx + e))}{4(a + b) f (b + a(\cos^2(fx + e)))} \\ & + \frac{3(a - b) \arctan\left(\frac{\cos(fx + e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{a} \sqrt{b}}{2(a + b)^4 f} \end{aligned}$$

command

`integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12(a^2 - 6ab + b^2) \log\left(\frac{|-\cos(fx + e) + 1|}{|\cos(fx + e) + 1|}\right)}{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4} - \frac{96(a^2b - ab^2) \arctan\left(-\frac{a \cos(fx + e) - b}{\sqrt{ab} \cos(fx + e) + \sqrt{ab}}\right)}{(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \sqrt{ab}} - \frac{\frac{8a^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{8b^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1} - \frac{a^2(\cos(fx + e) - 1)}{\cos(fx + e) + 1}}{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.13 Problem number 57

$$\int \frac{\csc(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}(\cos(fx + e))}{(a + b)^3 f} - \frac{b(\cos^3(fx + e))}{4a(a + b) f (b + a(\cos^2(fx + e)))^2} \\ & - \frac{b(7a + 3b) \cos(fx + e)}{8a^2(a + b)^2 f (b + a(\cos^2(fx + e)))} + \frac{(15a^2 + 10ab + 3b^2) \arctan\left(\frac{\cos(fx + e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{8a^{\frac{5}{2}}(a + b)^3 f} \end{aligned}$$

command

`integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(15a^2b+10ab^2+3b^3) \arctan\left(-\frac{a \cos(fx+e)-b}{\sqrt{ab} \cos(fx+e)+\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}} - \frac{4 \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(9a^3b+21a^2b^2+15ab^3+3b^4+\frac{27a^3b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.14 Problem number 58

$$\int \frac{\csc^3(e+fx)}{(a+b \sec^2(e+fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{(a-5b) \operatorname{arctanh}(\cos(fx+e))}{2(a+b)^4 f} - \frac{(2a-b)b \cos(fx+e)}{4a(a+b)^2 f (b+a(\cos^2(fx+e)))^2} \\ & + \frac{(4a^2-9ab-b^2) \cos(fx+e)}{8a(a+b)^3 f (b+a(\cos^2(fx+e)))} - \frac{\cos(fx+e) (\cot^2(fx+e))}{2(a+b) f (b+a(\cos^2(fx+e)))^2} \\ & + \frac{(15a^2-10ab-b^2) \arctan\left(\frac{\cos(fx+e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{8a^{\frac{3}{2}}(a+b)^4 f} \end{aligned}$$

command

`integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(a-5b) \log\left(\frac{|\cos(fx+e)+1|}{|\cos(fx+e)-1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{(15a^2b-10ab^2-b^3) \arctan\left(-\frac{a \cos(fx+e)-b}{\sqrt{ab} \cos(fx+e)+\sqrt{ab}}\right)}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\sqrt{ab}} + \frac{\left(a+b-\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{10b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e))}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(fx+e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.15 Problem number 59

$$\int \frac{\csc^5(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3(a^2 - 10ab + 5b^2) \operatorname{arctanh}(\cos(fx + e))}{8(a + b)^5 f} + \frac{(a^2 - 9ab + 2b^2) \cos(fx + e)}{8(a + b)^3 f (b + a(\cos^2(fx + e)))^2} \\ & + \frac{3(a^2 - 6ab + b^2) \cos(fx + e)}{8(a + b)^4 f (b + a(\cos^2(fx + e)))} - \frac{(a - 7b) \cot(fx + e) \csc(fx + e)}{8(a + b)^2 f (b + a(\cos^2(fx + e)))^2} \\ & - \frac{(\cot^3(fx + e)) \csc(fx + e)}{4(a + b) f (b + a(\cos^2(fx + e)))^2} + \frac{3(5a^2 - 10ab + b^2) \arctan\left(\frac{\cos(fx + e)\sqrt{a}}{\sqrt{b}}\right) \sqrt{b}}{8(a + b)^5 f \sqrt{a}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.16 Problem number 67

$$\int \sqrt{a + b \sec^2(e + fx)} \sin^5(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2(5a + b) (\cos^3(fx + e)) (a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{15a^2 f} - \frac{(\cos^5(fx + e)) (a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{5a f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sec(fx + e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{f} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.17 Problem number 68

$$\int \sqrt{a + b \sec^2(e + fx)} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\frac{(\cos^3(fx + e)) (a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{3af} + \frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate(sin(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.18 Problem number 69

$$\int \sqrt{a + b \sec^2(e + fx)} \sin(e + fx) dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{f} - \frac{\cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate(sin(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{b \operatorname{arctan}\left(\frac{\sqrt{a \cos(fx + e)^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \sqrt{a \cos(fx + e)^2 + b} \right) \operatorname{sgn}(\cos(fx + e))}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.19 Problem number 70

$$\int \csc(e + fx) \sqrt{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)\sqrt{b}}{f} - \frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)\sqrt{a+b}}{f}$$

command

```
integrate(csc(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \begin{array}{l} 4b \operatorname{arctan} \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2bt}}{2\sqrt{-b}} \right) \\ \hline \sqrt{-b} \end{array} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.20 Problem number 71

$$\int \csc^3(e + fx) \sqrt{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)\sqrt{b}}{f} - \frac{(a+2b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)}{2f\sqrt{a+b}} - \frac{\cot(fx+e) \csc(fx+e) \sqrt{a+b(\sec^2(fx+e))}}{2f}$$

command

```
integrate(csc(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{4(a+2b) \arctan \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + \dots}}{\sqrt{-a-b}} \right)}{\sqrt{-a-b}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.21 Problem number 72

$$\int \csc^5(e+fx) \sqrt{a+b \sec^2(e+fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a^2 + 12ab + 8b^2) \operatorname{arctanh} \left( \frac{\sec(fx+e) \sqrt{a+b}}{\sqrt{a+b} (\sec^2(fx+e))} \right)}{8(a+b)^{\frac{3}{2}} f} \\ & + \frac{\operatorname{arctanh} \left( \frac{\sec(fx+e) \sqrt{b}}{\sqrt{a+b} (\sec^2(fx+e))} \right) \sqrt{b}}{f} \\ & - \frac{(3a+4b) \cot(fx+e) \csc(fx+e) \sqrt{a+b} (\sec^2(fx+e))}{8(a+b)f} \\ & - \frac{\cot(fx+e) (\csc^3(fx+e)) \sqrt{a+b} (\sec^2(fx+e))}{4f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



### 76.22 Problem number 80

$$\int (a + b \sec^2(e + fx))^{3/2} \sin^5(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a - 4b) \cos(fx + e) (a + b(\sec^2(fx + e)))^{3/2}}{3af} \\ & + \frac{2(\cos^3(fx + e)) (a + b(\sec^2(fx + e)))^{5/2}}{3af} - \frac{(\cos^5(fx + e)) (a + b(\sec^2(fx + e)))^{5/2}}{5af} \\ & + \frac{(3a - 4b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{2f} \\ & + \frac{(3a - 4b) b \sec(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{2af} \end{aligned}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.23 Problem number 81

$$\int (a + b \sec^2(e + fx))^{3/2} \sin^3(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a - 2b) \cos(fx + e) (a + b(\sec^2(fx + e)))^{3/2}}{3af} + \frac{(\cos^3(fx + e)) (a + b(\sec^2(fx + e)))^{5/2}}{3af} \\ & + \frac{(3a - 2b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{2f} \\ & + \frac{(3a - 2b) b \sec(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{2af} \end{aligned}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.24 Problem number 82

$$\int (a + b \sec^2(e + fx))^{3/2} \sin(e + fx) dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) (a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{f} + \frac{3a \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{b}}{\sqrt{a + b(\sec^2(fx + e))}}\right) \sqrt{b}}{2f} + \frac{3b \sec(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{2f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*sin(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\left( \frac{3b \operatorname{arctan}\left(\frac{\sqrt{a \cos(fx + e)^2 + b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2 \sqrt{a \cos(fx + e)^2 + b} - \frac{\sqrt{a \cos(fx + e)^2 + b} b}{a \cos(fx + e)^2} \right) \operatorname{asgn}(\cos(fx + e))$$


---

$2f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.25 Problem number 93

$$\int \frac{\sin^5(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(15a^2 + 20ab + 8b^2) \cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{15a^3 f} \\ & + \frac{2(5a + 2b) (\cos^3(fx + e)) \sqrt{a + b(\sec^2(fx + e))}}{15a^2 f} - \frac{(\cos^5(fx + e)) \sqrt{a + b(\sec^2(fx + e))}}{5af} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.26 Problem number 94

$$\int \frac{\sin^3(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$- \frac{(3a + 2b) \cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{3a^2 f} + \frac{(\cos^3(fx + e)) \sqrt{a + b(\sec^2(fx + e))}}{3af}$$

command

```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16 \left( 3 \left( \sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4} - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2 \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.27 Problem number 95

$$\int \frac{\sin(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e) \sqrt{a + b(\sec^2(fx + e))}}{af}$$

command

```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{a \cos(fx + e)^2 + b}}{af \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.28 Problem number 96

$$\int \frac{\csc(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)}{f\sqrt{a+b}}$$

command

```
integrate(csc(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(-\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.29 Problem number 98

$$\int \frac{\csc^5(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2 \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)}{8(a+b)^{\frac{5}{2}}f} \\ & - \frac{(5a+2b)\cot(fx+e)\csc(fx+e)\sqrt{a+b(\sec^2(fx+e))}}{8(a+b)^2f} \\ & - \frac{(\cot^3(fx+e))\csc(fx+e)\sqrt{a+b(\sec^2(fx+e))}}{4(a+b)f} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.30 Problem number 106

$$\int \frac{\sin^5(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(15a^2 + 40ab + 24b^2) \cos(fx + e)}{15a^3 f \sqrt{a + b(\sec^2(fx + e))}} + \frac{2(5a + 3b) (\cos^3(fx + e))}{15a^2 f \sqrt{a + b(\sec^2(fx + e))}} \\ & - \frac{\cos^5(fx + e)}{5a f \sqrt{a + b(\sec^2(fx + e))}} - \frac{2b(15a^2 + 40ab + 24b^2) \sec(fx + e)}{15a^4 f \sqrt{a + b(\sec^2(fx + e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.31 Problem number 107

$$\int \frac{\sin^3(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{(3a + 4b) \cos(fx + e)}{3a^2 f \sqrt{a + b(\sec^2(fx + e))}} + \frac{\cos^3(fx + e)}{3af \sqrt{a + b(\sec^2(fx + e))}} - \frac{2b(3a + 4b) \sec(fx + e)}{3a^3 f \sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sin(fx + e)^3}{(b \sec(fx + e)^2 + a)^{3/2}} dx$$

## 76.32 Problem number 108

$$\int \frac{\sin(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)}{af \sqrt{a + b(\sec^2(fx + e))}} - \frac{2b \sec(fx + e)}{a^2 f \sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{a \cos(fx + e)^2 + b} + \frac{b}{\sqrt{a \cos(fx + e)^2 + b}}}{a^2 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.33 Problem number 110

$$\int \frac{\csc^3(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(a - 2b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b}(\sec^2(fx+e))}\right)}{2(a+b)^{\frac{5}{2}}f} - \frac{\cot(fx+e) \csc(fx+e)}{2(a+b)f\sqrt{a+b}(\sec^2(fx+e))} - \frac{3b \sec(fx+e)}{2(a+b)^2 f \sqrt{a+b}(\sec^2(fx+e))}$$

command

```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.34 Problem number 111

$$\int \frac{\csc^5(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{3a(a - 4b) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b}(\sec^2(fx+e))}\right)}{8(a+b)^{\frac{7}{2}}f} - \frac{5a \cot(fx+e) \csc(fx+e)}{8(a+b)^2 f \sqrt{a+b}(\sec^2(fx+e))} - \frac{(\cot^3(fx+e)) \csc(fx+e)}{4(a+b)f\sqrt{a+b}(\sec^2(fx+e))} - \frac{(13a - 2b)b \sec(fx+e)}{8(a+b)^3 f \sqrt{a+b}(\sec^2(fx+e))}$$

command

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.35 Problem number 119

$$\int \frac{\sin^5(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(5a^2 + 20ab + 16b^2) \cos(fx + e)}{5a^3 f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} + \frac{2(5a + 4b) (\cos^3(fx + e))}{15a^2 f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} \\ & - \frac{\cos^5(fx + e)}{5a f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} - \frac{4b(5a^2 + 20ab + 16b^2) \sec(fx + e)}{15a^4 f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} \\ & - \frac{8b(5a^2 + 20ab + 16b^2) \sec(fx + e)}{15a^5 f \sqrt{a + b(\sec^2(fx + e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.36 Problem number 120

$$\int \frac{\sin^3(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a + 2b) \cos(fx + e)}{a^2 f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} + \frac{\cos^3(fx + e)}{3a f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} \\ & - \frac{4b(a + 2b) \sec(fx + e)}{3a^3 f (a + b(\sec^2(fx + e)))^{\frac{3}{2}}} - \frac{8b(a + 2b) \sec(fx + e)}{3a^4 f \sqrt{a + b(\sec^2(fx + e))}} \end{aligned}$$

command

```
integrate(sin(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sin(fx + e)^3}{(b \sec(fx + e)^2 + a)^{\frac{5}{2}}} dx$$



**76.37 Problem number 121**

$$\int \frac{\sin(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\cos(fx + e)}{af(a + b(\sec^2(fx + e)))^{3/2}} - \frac{4b \sec(fx + e)}{3a^2 f(a + b(\sec^2(fx + e)))^{3/2}} - \frac{8b \sec(fx + e)}{3a^3 f \sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(sin(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3 \sqrt{a \cos(fx + e)^2 + b} + \frac{6(a \cos(fx + e)^2 + b)b - b^2}{(a \cos(fx + e)^2 + b)^{3/2}}}{3a^3 f \operatorname{sgn}(\cos(fx + e))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.38 Problem number 123**

$$\int \frac{\csc^3(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{(a - 4b) \operatorname{arctanh}\left(\frac{\sec(fx + e) \sqrt{a + b}}{\sqrt{a + b(\sec^2(fx + e))}}\right)}{2(a + b)^{7/2} f} - \frac{\cot(fx + e) \csc(fx + e)}{2(a + b) f(a + b(\sec^2(fx + e)))^{3/2}} - \frac{5b \sec(fx + e)}{6(a + b)^2 f(a + b(\sec^2(fx + e)))^{3/2}} - \frac{(13a - 2b) b \sec(fx + e)}{6a(a + b)^3 f \sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(csc(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.39 Problem number 124

$$\int \frac{\csc^5(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(3a^2 - 24ab + 8b^2) \operatorname{arctanh}\left(\frac{\sec(fx+e)\sqrt{a+b}}{\sqrt{a+b(\sec^2(fx+e))}}\right)}{8(a+b)^{\frac{9}{2}}f} \\ & - \frac{(5a-2b)\cot(fx+e)\csc(fx+e)}{8(a+b)^2 f (a+b(\sec^2(fx+e)))^{\frac{3}{2}}} - \frac{(\cot^3(fx+e))\csc(fx+e)}{4(a+b)f(a+b(\sec^2(fx+e)))^{\frac{3}{2}}} \\ & - \frac{(23a-12b)b\sec(fx+e)}{24(a+b)^3 f (a+b(\sec^2(fx+e)))^{\frac{3}{2}}} - \frac{5(11a-10b)b\sec(fx+e)}{24(a+b)^4 f \sqrt{a+b(\sec^2(fx+e))}} \end{aligned}$$

command

```
integrate(csc(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.40 Problem number 152

$$\int \sec^5(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(6a+5b)\operatorname{arctanh}(\sin(fx+e))}{16f} + \frac{(6a+5b)\sec(fx+e)\tan(fx+e)}{16f} \\ & + \frac{(6a+5b)(\sec^3(fx+e))\tan(fx+e)}{24f} + \frac{b(\sec^5(fx+e))\tan(fx+e)}{6f} \end{aligned}$$

command

```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(6a+5b)\log(|\sin(fx+e)+1|) - 3(6a+5b)\log(|\sin(fx+e)-1|) - \frac{2(18a\sin(fx+e)^5 + 15b\sin(fx+e)^5 - 48a\sin(fx+e) - 15b)}{(\sin(fx+e))^5}}{96f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.41 Problem number 153

$$\int \sec^3(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{(4a + 3b) \operatorname{arctanh}(\sin(fx + e))}{8f} + \frac{(4a + 3b) \sec(fx + e) \tan(fx + e)}{8f} + \frac{b(\sec^3(fx + e)) \tan(fx + e)}{4f}$$

command

```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(4a + 3b) \log(|\sin(fx + e) + 1|) - (4a + 3b) \log(|\sin(fx + e) - 1|) - \frac{2(4a \sin(fx+e)^3 + 3b \sin(fx+e)^3 - 4a \sin(fx+e) - 5b)}{(\sin(fx+e)^2 - 1)^2}}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.42 Problem number 154

$$\int \sec(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{(2a + b) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{b \sec(fx + e) \tan(fx + e)}{2f}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2a + b) \log(|\sin(fx + e) + 1|) - (2a + b) \log(|\sin(fx + e) - 1|) - \frac{2b \sin(fx+e)}{\sin(fx+e)^2 - 1}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.43 Problem number 155**

$$\int \cos(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{b \operatorname{arctanh}(\sin(fx + e))}{f} + \frac{a \sin(fx + e)}{f}$$

command

```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \log(|\sin(fx + e) + 1|) - b \log(|\sin(fx + e) - 1|) + 2a \sin(fx + e)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.44 Problem number 165**

$$\int \sec^5(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(48a^2 + 80ab + 35b^2) \operatorname{arctanh}(\sin(fx + e))}{128f} \\ & + \frac{(48a^2 + 80ab + 35b^2) \sec(fx + e) \tan(fx + e)}{128f} \\ & + \frac{(48a^2 + 80ab + 35b^2) (\sec^3(fx + e)) \tan(fx + e)}{192f} \\ & + \frac{b(10a + 7b) (\sec^5(fx + e)) \tan(fx + e)}{48f} \\ & + \frac{b(\sec^7(fx + e)) (a + b - a(\sin^2(fx + e))) \tan(fx + e)}{8f} \end{aligned}$$

command

```
integrate(sec(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(48a^2 + 80ab + 35b^2) \log(|\sin(fx + e) + 1|) - 3(48a^2 + 80ab + 35b^2) \log(|\sin(fx + e) - 1|) - \frac{2(144a^2 \sin(fx + e) + \dots)}{\dots}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.45 Problem number 166**

$$\int \sec^3(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2 + 12ab + 5b^2) \operatorname{arctanh}(\sin(fx + e))}{16f} \\ & + \frac{(8a^2 + 12ab + 5b^2) \sec(fx + e) \tan(fx + e)}{16f} + \frac{b(8a + 5b) (\sec^3(fx + e)) \tan(fx + e)}{24f} \\ & + \frac{b(\sec^5(fx + e)) (a + b - a(\sin^2(fx + e))) \tan(fx + e)}{6f} \end{aligned}$$

command

```
integrate(sec(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3(8a^2 + 12ab + 5b^2) \log(|\sin(fx + e) + 1|) - 3(8a^2 + 12ab + 5b^2) \log(|\sin(fx + e) - 1|) - \frac{2(24a^2 \sin(fx+e)^5 + 36}{96f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.46 Problem number 167**

$$\int \sec(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(8a^2 + 8ab + 3b^2) \operatorname{arctanh}(\sin(fx + e))}{8f} + \frac{3b(2a + b) \sec(fx + e) \tan(fx + e)}{8f} \\ & + \frac{b(\sec^3(fx + e)) (a + b - a(\sin^2(fx + e))) \tan(fx + e)}{4f} \end{aligned}$$

command

```
integrate(sec(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$(8a^2 + 8ab + 3b^2) \log(|\sin(fx + e) + 1|) - (8a^2 + 8ab + 3b^2) \log(|\sin(fx + e) - 1|) - \frac{2(8ab \sin(fx+e)^3 + 3b^2 \sin(fx+e))}{16f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.47 Problem number 168**

$$\int \cos(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{b(4a + b) \operatorname{arctanh}(\sin(fx + e))}{2f} + \frac{a^2 \sin(fx + e)}{f} + \frac{b^2 \sec(fx + e) \tan(fx + e)}{2f}$$

command

```
integrate(cos(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^2 \sin(fx + e) + (4ab + b^2) \log(|\sin(fx + e) + 1|) - (4ab + b^2) \log(|\sin(fx + e) - 1|) - \frac{2b^2 \sin(fx + e)}{\sin(fx + e)^2 - 1}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.48 Problem number 169**

$$\int \cos^3(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{b^2 \operatorname{arctanh}(\sin(fx + e))}{f} + \frac{a(a + 2b) \sin(fx + e)}{f} - \frac{a^2 (\sin^3(fx + e))}{3f}$$

command

```
integrate(cos(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^2 \sin(fx + e)^3 - 3b^2 \log(|\sin(fx + e) + 1|) + 3b^2 \log(|\sin(fx + e) - 1|) - 6a^2 \sin(fx + e) - 12ab \sin(fx + e)}{6f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.49 Problem number 180

$$\int \frac{\sec^5(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{(2a - b) \operatorname{arctanh}(\sin(fx + e))}{2b^2 f} + \frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{a}}{\sqrt{a+b}}\right)}{b^2 f \sqrt{a+b}} + \frac{\sec(fx + e) \tan(fx + e)}{2bf}$$

command

```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4a^2 \arctan\left(\frac{a \sin(fx+e)}{\sqrt{-a^2 - ab}}\right) + \frac{(2a-b) \log(|\sin(fx+e)+1|)}{b^2} - \frac{(2a-b) \log(|\sin(fx+e)-1|)}{b^2} + \frac{2 \sin(fx+e)}{(\sin(fx+e)^2 - 1)b}}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.50 Problem number 181

$$\int \frac{\sec^3(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e))}{bf} - \frac{\operatorname{arctanh}\left(\frac{\sin(fx+e)\sqrt{a}}{\sqrt{a+b}}\right) \sqrt{a}}{bf \sqrt{a+b}}$$

command

```
integrate(sec(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a \arctan\left(\frac{a \sin(fx+e)}{\sqrt{-a^2 - ab}}\right) + \frac{\log(|\sin(fx+e)+1|)}{b} - \frac{\log(|\sin(fx+e)-1|)}{b}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.51 Problem number 193

$$\int \frac{\sec^5(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\sin(fx + e))}{b^2 f} - \frac{a \sin(fx + e)}{2b(a + b)f(a + b - a(\sin^2(fx + e)))}$$

$$- \frac{(2a + 3b) \operatorname{arctanh}\left(\frac{\sin(fx + e)\sqrt{a}}{\sqrt{a + b}}\right) \sqrt{a}}{2b^2(a + b)^{\frac{3}{2}} f}$$

command

```
integrate(sec(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2a^2 + 3ab) \operatorname{arctan}\left(\frac{a \sin(fx + e)}{\sqrt{-a^2 - ab}}\right)}{(ab^2 + b^3) \sqrt{-a^2 - ab}} + \frac{a \sin(fx + e)}{(a \sin(fx + e)^2 - a - b)(ab + b^2)} + \frac{\log(|\sin(fx + e) + 1|)}{b^2} - \frac{\log(|\sin(fx + e) - 1|)}{b^2}$$

$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.52 Problem number 278

$$\int \frac{\sec^2(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\tan(fx + e)}{(a + b)f \sqrt{a + b + b(\tan^2(fx + e))}}$$

command

```
integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^2 b^2 \operatorname{sgn}(\cos(fx + e)) \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{(a^3 b^2 + a^2 b^3) \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2} + a$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 76.53 Problem number 290

$$\int \frac{\sec^4(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \tan(fx + e)}{3(a + b)^2 f \sqrt{a + b + b(\tan^2(fx + e))}} + \frac{(\sec^2(fx + e)) \tan(fx + e)}{3(a + b) f (a + b + b(\tan^2(fx + e)))^{3/2}}$$

command

`integrate(sec(f*x+e)^4/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \left( \frac{3(a^6 b^4 \operatorname{sgn}(\cos(fx+e)) + 2a^5 b^5 \operatorname{sgn}(\cos(fx+e)) + a^4 b^6 \operatorname{sgn}(\cos(fx+e))) \tan(\frac{1}{2} fx + \frac{1}{2} e)^2}{a^7 b^4 + 3a^6 b^5 + 3a^5 b^6 + a^4 b^7} - \frac{2(a^6 b^4 \operatorname{sgn}(\cos(fx+e)) - 2a^5 b^5 \operatorname{sgn}(\cos(fx+e)) - 3a^4 b^6 \operatorname{sgn}(\cos(fx+e)))}{a^7 b^4 + 3a^6 b^5 + 3a^5 b^6 + a^4 b^7} \right) \right. \\ \left. 3 \left( a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 2a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.54 Problem number 291

$$\int \frac{\sec^2(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{2 \tan(fx + e)}{3(a + b)^2 f \sqrt{a + b + b(\tan^2(fx + e))}} + \frac{\tan(fx + e)}{3(a + b) f (a + b + b(\tan^2(fx + e)))^{3/2}}$$

command

`integrate(sec(f*x+e)^2/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \left( \frac{3(a^6 b^4 \operatorname{sgn}(\cos(fx+e)) + 2a^5 b^5 \operatorname{sgn}(\cos(fx+e)) + a^4 b^6 \operatorname{sgn}(\cos(fx+e))) \tan(\frac{1}{2} fx + \frac{1}{2} e)^2}{a^7 b^4 + 3a^6 b^5 + 3a^5 b^6 + a^4 b^7} - \frac{2(3a^6 b^4 \operatorname{sgn}(\cos(fx+e)) + 2a^5 b^5 \operatorname{sgn}(\cos(fx+e)) - 3a^4 b^6 \operatorname{sgn}(\cos(fx+e)))}{a^7 b^4 + 3a^6 b^5 + 3a^5 b^6 + a^4 b^7} \right) \right. \\ \left. 3 \left( a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 2a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.55 Problem number 311**

$$\int (a + b \sec^2(e + fx)) \tan^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{a \ln(\cos(fx + e))}{f} - \frac{(2a - b)(\sec^2(fx + e))}{2f} + \frac{(a - 2b)(\sec^4(fx + e))}{4f} + \frac{b(\sec^6(fx + e))}{6f}$$

command

```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$6a \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2 \right| \right) - 6a \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right| \right) + \frac{11a \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right)}{12f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.56 Problem number 312**

$$\int (a + b \sec^2(e + fx)) \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{a \ln(\cos(fx + e))}{f} + \frac{(a - b)(\sec^2(fx + e))}{2f} + \frac{b(\sec^4(fx + e))}{4f}$$

command

```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2a \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2 \right| \right) - 2a \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right| \right) + \frac{3a \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right)}{4f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.57 Problem number 313

$$\int (a + b \sec^2(e + fx)) \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{a \ln(\cos(fx + e))}{f} + \frac{b(\sec^2(fx + e))}{2f}$$

command

```
integrate((a+b*sec(f*x+e)^2)*tan(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - a \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) + 2a}{\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.58 Problem number 314

$$\int \cot(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{b \ln(\cos(fx + e))}{f} + \frac{(a + b) \ln(\sin(fx + e))}{f}$$

command

```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{a \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) + b \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.59 Problem number 315

$$\int \cot^3(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$-\frac{(a+b)(\csc^2(fx+e))}{2f} - \frac{a \ln(\sin(fx+e))}{f}$$

command

```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4a \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 8a \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right) - \frac{(a+b+\frac{4a(\cos(fx+e)-1)}{\cos(fx+e)+1})(\cos(fx+e)+1)}{\cos(fx+e)-1} - \frac{a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.60 Problem number 316

$$\int \cot^5(e + fx) (a + b \sec^2(e + fx)) dx$$

Optimal antiderivative

$$\frac{(2a+b)(\csc^2(fx+e))}{2f} - \frac{(a+b)(\csc^4(fx+e))}{4f} + \frac{a \ln(\sin(fx+e))}{f}$$

command

```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{32a \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 64a \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right) - \frac{(a+b+\frac{12a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{48a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2})(\cos(fx+e)+1)}{(\cos(fx+e)-1)^2}}{64f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.61 Problem number 324

$$\int (a + b \sec^2(e + fx))^2 \tan^5(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^2 \ln(\cos(fx + e))}{f} - \frac{a(a - b)(\sec^2(fx + e))}{f} + \frac{(a^2 - 4ab + b^2)(\sec^4(fx + e))}{4f} \\ & + \frac{(a - b)b(\sec^6(fx + e))}{3f} + \frac{b^2(\sec^8(fx + e))}{8f} \end{aligned}$$

command

```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$12 a^2 \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2 \right| \right) - 12 a^2 \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right| \right) + \frac{25 a^2 \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.62 Problem number 325

$$\int (a + b \sec^2(e + fx))^2 \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{a^2 \ln(\cos(fx + e))}{f} + \frac{a(a - 2b)(\sec^2(fx + e))}{2f} + \frac{(2a - b)b(\sec^4(fx + e))}{4f} + \frac{b^2(\sec^6(fx + e))}{6f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$6 a^2 \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2 \right| \right) - 6 a^2 \log \left( \left| -\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2 \right| \right) + \frac{11 a^2 \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.63 Problem number 326

$$\int (a + b \sec^2(e + fx))^2 \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{a^2 \ln(\cos(fx + e))}{f} + \frac{ab(\sec^2(fx + e))}{f} + \frac{b^2(\sec^4(fx + e))}{4f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^2*tan(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2a^2 \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) - 2a^2 \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) + \frac{3a^2\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{4f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.64 Problem number 327

$$\int \cot(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{b(2a + b) \ln(\cos(fx + e))}{f} + \frac{(a + b)^2 \ln(\sin(fx + e))}{f} + \frac{b^2(\sec^2(fx + e))}{2f}$$

command

```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right|\right) + (2ab + b^2) \log\left(\left|-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 2\right|\right) - \frac{2ab\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right)}{2f}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.65 Problem number 328**

$$\int \cot^3(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$-\frac{(a+b)^2 (\csc^2(fx+e))}{2f} - \frac{b^2 \ln(\cos(fx+e))}{f} - \frac{(a^2-b^2) \ln(\sin(fx+e))}{f}$$

command

```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right) + 2ab \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right) + b^2 \left( \frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1} \right) + 4a^2 \log \left( \left| -\frac{\cos}{\cos} \right. \right)}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.66 Problem number 329**

$$\int \cot^5(e + fx) (a + b \sec^2(e + fx))^2 dx$$

Optimal antiderivative

$$\frac{a(a+b) (\csc^2(fx+e))}{f} - \frac{(a+b)^2 (\csc^4(fx+e))}{4f} + \frac{a^2 \ln(\sin(fx+e))}{f}$$

command

```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32a^2 \log \left( \frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|} \right) - 64a^2 \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right| \right) - \frac{12a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{8ab(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4b^2(\cos(fx+e)-1)}{\cos(fx+e)+1}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.67 Problem number 337

$$\int \frac{\tan^5(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(a + 2b) \ln(\cos(fx + e))}{b^2 f} - \frac{(a + b)^2 \ln(b + a(\cos^2(fx + e)))}{2ab^2 f} + \frac{\sec^2(fx + e)}{2bf}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a^3 + 3a^2b + 3ab^2 + b^3) \log\left(-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a + 2b\right)}{a^2b^2 + ab^3} - \frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right)}{a}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.68 Problem number 338

$$\int \frac{\tan^3(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\ln(\cos(fx + e))}{bf} + \frac{(a + b) \ln(b + a(\cos^2(fx + e)))}{2abf}$$

command

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a^2 + 2ab + b^2) \log\left(-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a + 2b\right)}{a^2b + ab^2} - \frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right)}{a}$$


---


$$2f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 76.69 Problem number 339

$$\int \frac{\tan(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$-\frac{\ln(b + a(\cos^2(fx + e)))}{2af}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a}-\frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.70 Problem number 340

$$\int \frac{\cot(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{b \ln(b + a(\cos^2(fx + e)))}{2a(a + b)f} + \frac{\ln(\sin(fx + e))}{(a + b)f}$$

command

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b \log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2+ab} + \frac{\log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a+b} - \frac{2\log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a}}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.71 Problem number 341

$$\int \frac{\cot^3(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\csc^2(fx + e)}{2(a + b)f} - \frac{b^2 \ln(b + a(\cos^2(fx + e)))}{2a(a + b)^2 f} - \frac{(a + 2b) \ln(\sin(fx + e))}{(a + b)^2 f}$$

command

```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4b^2 \log\left(a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3 + 2a^2b + ab^2} + \frac{4(a+2b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^2 + 2ab + b^2} - \frac{\left(a + b + \frac{4a(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)}{a^2}$$

8 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.72 Problem number 342

$$\int \frac{\cot^5(e + fx)}{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{(2a + 3b) (\csc^2(fx + e))}{2(a + b)^2 f} - \frac{\csc^4(fx + e)}{4(a + b)f} + \frac{b^3 \ln(b + a(\cos^2(fx + e)))}{2a(a + b)^3 f} + \frac{(a^2 + 3ab + 3b^2) \ln(\sin(fx + e))}{(a + b)^3 f}$$

command

```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32b^3 \log\left(a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^4 + 3a^3b + 3a^2b^2 + ab^3} + \frac{32(a^2 + 3ab + 3b^2) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right)}{a^3 + 3a^2b + 3ab^2 + b^3} - \frac{12a(\cos(fx+e)-1)}{\cos(fx+e)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.73 Problem number 350

$$\int \frac{\tan^5(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{(a+b)^2}{2a^2bf(b+a(\cos^2(fx+e)))} - \frac{\ln(\cos(fx+e))}{b^2f} - \frac{(\frac{1}{a^2} - \frac{1}{b^2}) \ln(b+a(\cos^2(fx+e)))}{2f}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a^3+a^2b-ab^2-b^3) \log\left(-a\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - b\left(\frac{\cos(fx+e)+1}{\cos(fx+e)-1} + \frac{\cos(fx+e)-1}{\cos(fx+e)+1}\right) - 2a+2b\right)}{a^3b^2+a^2b^3} + \frac{\log\left(-\frac{\cos(fx+e)+1}{\cos(fx+e)-1} - \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 2\right)}{a^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.74 Problem number 351

$$\int \frac{\tan^3(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{a+b}{2a^2f(b+a(\cos^2(fx+e)))} + \frac{\ln(b+a(\cos^2(fx+e)))}{2a^2f}$$

command

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^2} - \frac{2 \log\left(-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right)}{a^2} - \frac{a+b+\frac{6a(\cos(fx+e)-1)}{\cos(fx+e)+1} - 2}{\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b}{c}\right)} \cdot \frac{1}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.75 Problem number 352

$$\int \frac{\tan(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$-\frac{b}{2a^2 f (b + a (\cos^2 (fx + e)))} - \frac{\ln (b + a (\cos^2 (fx + e)))}{2a^2 f}$$

command

`integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 + 2ab + b^2 + \frac{2a^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{4ab(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b^2(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{2ab(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b^2(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{(a^3 + a^2b) \left( a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} \right)} - \frac{\log \left( a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.76 Problem number 353

$$\int \frac{\cot(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\frac{b^2}{2a^2 (a + b) f (b + a (\cos^2 (fx + e)))} + \frac{b(2a + b) \ln (b + a (\cos^2 (fx + e)))}{2a^2 (a + b)^2 f} + \frac{\ln (\sin (fx + e))}{(a + b)^2 f}$$

command

`integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2ab + b^2) \log \left( a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} \right)}{a^4 + 2a^3b + a^2b^2} + \frac{\log \left( \frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|} \right)}{a^2 + 2ab + b^2} - \frac{2ab + b^2 + \frac{4ab(\cos(fx+e)-1)}{\cos(fx+e)+1}}{(a^3 + a^2b) \left( a + b + \frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} \right)}{2f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.77 Problem number 354

$$\int \frac{\cot^3(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b^3}{2a^2(a+b)^2 f(b+a(\cos^2(fx+e)))} - \frac{\csc^2(fx+e)}{2(a+b)^2 f} \\ & -\frac{b^2(3a+b)\ln(b+a(\cos^2(fx+e)))}{2a^2(a+b)^3 f} - \frac{(a+3b)\ln(\sin(fx+e))}{(a+b)^3 f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.78 Problem number 355

$$\int \frac{\cot^5(e + fx)}{(a + b \sec^2(e + fx))^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b^4}{2a^2(a+b)^3 f(b+a(\cos^2(fx+e)))} + \frac{(a+2b)(\csc^2(fx+e))}{(a+b)^3 f} - \frac{\csc^4(fx+e)}{4(a+b)^2 f} \\ & + \frac{b^3(4a+b)\ln(b+a(\cos^2(fx+e)))}{2a^2(a+b)^4 f} + \frac{(a^2+4ab+6b^2)\ln(\sin(fx+e))}{(a+b)^4 f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.79 Problem number 363

$$\int \frac{\tan^5(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{(a + b)^2}{4a^3 f (b + a (\cos^2(fx + e)))^2} + \frac{-a - b}{a^3 f (b + a (\cos^2(fx + e)))} - \frac{\ln(b + a (\cos^2(fx + e)))}{2a^3 f}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(a+b+\frac{2a(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}\right)}{a^3} - \frac{4 \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1}+1\right|\right)}{a^3} - \frac{3a^2+6ab+3b^2+\frac{20a^2(\cos(fx+e)-1)}{\cos(fx+e)+1}}{a^3}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.80 Problem number 364

$$\int \frac{\tan^3(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$-\frac{b(a + b)}{4a^3 f (b + a (\cos^2(fx + e)))^2} + \frac{a + 2b}{2a^3 f (b + a (\cos^2(fx + e)))} + \frac{\ln(b + a (\cos^2(fx + e)))}{2a^3 f}$$

command

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^3+9a^2b+9ab^2+3b^3+\frac{20a^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{28a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{4ab^2(\cos(fx+e)-1)}{\cos(fx+e)+1}-\frac{12b^3(\cos(fx+e)-1)}{\cos(fx+e)+1}+\frac{34a^3(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}+\frac{22a^2b(\cos(fx+e)-1)}{\cos(fx+e)+1}}{(a^4+a^3)}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.81 Problem number 365

$$\int \frac{\tan(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{b^2}{4a^3 f (b + a (\cos^2 (fx + e)))^2} - \frac{b}{a^3 f (b + a (\cos^2 (fx + e)))} - \frac{\ln (b + a (\cos^2 (fx + e)))}{2a^3 f}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.82 Problem number 366

$$\int \frac{\cot(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$-\frac{b^3}{4a^3 (a + b) f (b + a (\cos^2 (fx + e)))^2} + \frac{b^2(3a + 2b)}{2a^3 (a + b)^2 f (b + a (\cos^2 (fx + e)))} + \frac{b(3a^2 + 3ab + b^2) \ln (b + a (\cos^2 (fx + e)))}{2a^3 (a + b)^3 f} + \frac{\ln (\sin (fx + e))}{(a + b)^3 f}$$

command

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.83 Problem number 367**

$$\int \frac{\cot^3(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$\frac{b^4}{4a^3 (a + b)^2 f (b + a (\cos^2 (fx + e)))^2} - \frac{b^3(2a + b)}{a^3 (a + b)^3 f (b + a (\cos^2 (fx + e)))} - \frac{\csc^2 (fx + e)}{2(a + b)^3 f} - \frac{b^2(6a^2 + 4ab + b^2) \ln (b + a (\cos^2 (fx + e)))}{2a^3 (a + b)^4 f} - \frac{(a + 4b) \ln (\sin (fx + e))}{(a + b)^4 f}$$

command

```
integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.84 Problem number 368**

$$\int \frac{\cot^5(e + fx)}{(a + b \sec^2(e + fx))^3} dx$$

Optimal antiderivative

$$-\frac{b^5}{4a^3 (a + b)^3 f (b + a (\cos^2 (fx + e)))^2} + \frac{b^4(5a + 2b)}{2a^3 (a + b)^4 f (b + a (\cos^2 (fx + e)))} + \frac{(2a + 5b) (\csc^2 (fx + e))}{2(a + b)^4 f} - \frac{\csc^4 (fx + e)}{4(a + b)^3 f} + \frac{b^3(10a^2 + 5ab + b^2) \ln (b + a (\cos^2 (fx + e)))}{2a^3 (a + b)^5 f} + \frac{(a^2 + 5ab + 10b^2) \ln (\sin (fx + e))}{(a + b)^5 f}$$

command

```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



**76.85 Problem number 376**

$$\int \sqrt{a + b \sec^2(e + fx)} \tan^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{(a + 2b)(a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{3b^2 f} + \frac{(a + b(\sec^2(fx + e)))^{\frac{5}{2}}}{5b^2 f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} + \frac{\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.86 Problem number 377**

$$\int \sqrt{a + b \sec^2(e + fx)} \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{(a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{3bf} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} - \frac{\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.87 Problem number 378

$$\int \sqrt{a + b \sec^2(e + fx)} \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} + \frac{\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{a \operatorname{arctan}\left(\frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.88 Problem number 379

$$\int \cot(e + fx) \sqrt{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a + b}}\right) \sqrt{a + b}}{f}$$

command

`integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \begin{array}{l} 4a \arctan \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b}}{2\sqrt{-a}} \right) \\ \hline \sqrt{-a} \end{array} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.89 Problem number 380

$$\int \cot^3(e + fx) \sqrt{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\sec^2(fx+e))}}{\sqrt{a}}\right) \sqrt{a}}{f} + \frac{(2a+b) \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sec^2(fx+e))}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}} - \frac{(\cot^2(fx+e)) \sqrt{a+b(\sec^2(fx+e))}}{2f}$$

command

`integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \begin{array}{l} 16a \arctan \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b}}{2\sqrt{-a}} \right) \\ \hline \sqrt{-a} \end{array} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.90 Problem number 381

$$\int \cot^5(e + fx) \sqrt{a + b \sec^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(8a^2 + 12ab + 3b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a + b}}\right)}{8(a + b)^{\frac{3}{2}} f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right) \sqrt{a}}{f} \\ & + \frac{(4a + 3b) (\cot^2(fx + e)) \sqrt{a + b(\sec^2(fx + e))}}{8(a + b) f} \\ & - \frac{(\cot^4(fx + e)) \sqrt{a + b(\sec^2(fx + e))}}{4f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5*(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.91 Problem number 389

$$\int (a + b \sec^2(e + fx))^{3/2} \tan^5(e + fx) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f} + \frac{(a + b(\sec^2(fx + e)))^{\frac{3}{2}}}{3f} \\ & - \frac{(a + 2b) (a + b(\sec^2(fx + e)))^{\frac{5}{2}}}{5b^2 f} + \frac{(a + b(\sec^2(fx + e)))^{\frac{7}{2}}}{7b^2 f} + \frac{a \sqrt{a + b(\sec^2(fx + e))}}{f} \end{aligned}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.92 Problem number 390

$$\int (a + b \sec^2(e + fx))^{3/2} \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f} - \frac{(a + b(\sec^2(fx + e)))^{3/2}}{3f} + \frac{(a + b(\sec^2(fx + e)))^{5/2}}{5bf} - \frac{a\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.93 Problem number 391

$$\int (a + b \sec^2(e + fx))^{3/2} \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f} + \frac{(a + b(\sec^2(fx + e)))^{3/2}}{3f} + \frac{a\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate((a+b*sec(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.94 Problem number 392

$$\int \cot(e + fx) (a + b \sec^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f} - \frac{(a + b)^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a + b}}\right)}{f} + \frac{b\sqrt{a + b(\sec^2(fx + e))}}{f}$$

command

```
integrate(cot(f*x+e)*(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.95 Problem number 402

$$\int \frac{\tan^5(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(a + b(\sec^2(fx + e)))^{3/2}}{3b^2 f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}} - \frac{(a + 2b)\sqrt{a + b(\sec^2(fx + e))}}{b^2 f}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.96 Problem number 403

$$\int \frac{\tan^3(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}} + \frac{\sqrt{a + b(\sec^2(fx + e))}}{bf}$$

command

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \operatorname{arctan} \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2bt}{2\sqrt{-a}} \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.97 Problem number 404

$$\int \frac{\tan(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \operatorname{arctan} \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2bt}{2\sqrt{-a}} \right)$$


---


$$\sqrt{-a} \operatorname{sgn}(\cos(fx + e))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.98 Problem number 405

$$\int \frac{\cot(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a + b}}\right)}{f\sqrt{a + b}}$$

command

```
integrate(cot(f*x+e)/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$4 \arctan \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2}}{2\sqrt{-a}} \right)$$


---


$$\sqrt{-a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 76.99 Problem number 406

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sec^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{(2a+3b) \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sec^2(fx+e))}}{\sqrt{a+b}}\right)}{2(a+b)^{\frac{3}{2}}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\sec^2(fx+e))}}{\sqrt{a}}\right)}{f\sqrt{a}} - \frac{(\cot^2(fx+e))\sqrt{a+b(\sec^2(fx+e))}}{2(a+b)f}$$

command

`integrate(cot(f*x+e)^3/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$16 \arctan \left( \frac{\sqrt{a+b} \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - \sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2}}{2\sqrt{-a}} \right)$$


---


$$\sqrt{-a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 76.100 Problem number 407

$$\int \frac{\cot^5(e + fx)}{\sqrt{a + b \sec^2(e + fx)}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(8a^2 + 20ab + 15b^2) \operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a + b}}\right)}{8(a + b)^{\frac{5}{2}} f} \\ & + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{f\sqrt{a}} + \frac{(4a + 7b)(\cot^2(fx + e))\sqrt{a + b(\sec^2(fx + e))}}{8(a + b)^2 f} \\ & - \frac{(\cot^4(fx + e))\sqrt{a + b(\sec^2(fx + e))}}{4(a + b)f} \end{aligned}$$

command

```
integrate(cot(f*x+e)^5/(a+b*sec(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 76.101 Problem number 415

$$\int \frac{\tan^5(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} f} + \frac{(a + b)^2}{a b^2 f \sqrt{a + b(\sec^2(fx + e))}} + \frac{\sqrt{a + b(\sec^2(fx + e))}}{b^2 f} \end{aligned}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{a^4 b \operatorname{sgn}(\cos(fx+e)) + 2 a^3 b^2 \operatorname{sgn}(\cos(fx+e)) + a^2 b^3 \operatorname{sgn}(\cos(fx+e))}{a^3 b^3}\right) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - \frac{a^4 b \operatorname{sgn}(\cos(fx+e)) + 2 a^3 b^2 \operatorname{sgn}(\cos(fx+e)) + a^2 b^3 \operatorname{sgn}(\cos(fx+e))}{a^3 b^3}}{\sqrt{a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 2 a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 2 b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a + b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.102 Problem number 416

$$\int \frac{\tan^3(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} f} + \frac{-a - b}{abf \sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{a^3 \operatorname{sgn}(\cos(fx+e)) + a^2 b \operatorname{sgn}(\cos(fx+e))}{a^3 b}\right) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - \frac{a^3 \operatorname{sgn}(\cos(fx+e)) + a^2 b \operatorname{sgn}(\cos(fx+e))}{a^3 b}}{\sqrt{a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 2 a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 2 b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a + b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.103 Problem number 417

$$\int \frac{\tan(e + fx)}{(a + b \sec^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2}{\operatorname{asgn}(\cos(fx+e))} - \frac{1}{\operatorname{asgn}(\cos(fx+e))}}{\sqrt{a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 2a \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2b \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + a + b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.104 Problem number 428

$$\int \frac{\tan^5(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{(a + b)^2}{3ab^2f(a + b(\sec^2(fx + e)))^{3/2}} + \frac{\frac{1}{a^2} - \frac{1}{b^2}}{f\sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)^5/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \left( \frac{(2 a^{11} \operatorname{sgn}(\cos(fx+e)) + a^{10} b \operatorname{sgn}(\cos(fx+e)) - 4 a^9 b^2 \operatorname{sgn}(\cos(fx+e)) - 3 a^8 b^3 \operatorname{sgn}(\cos(fx+e))) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}{a^{10} b^2} - \frac{3 (2 a^{11} \operatorname{sgn}(\cos(fx+e)) - 3 a^{10} b \operatorname{sgn}(\cos(fx+e)) - 4 a^9 b^2 \operatorname{sgn}(\cos(fx+e)) - 3 a^8 b^3 \operatorname{sgn}(\cos(fx+e))) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)}{a^{10} b^2} \right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**76.105 Problem number 429**

$$\int \frac{\tan^3(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{5/2} f} + \frac{-a - b}{3abf(a + b(\sec^2(fx + e)))^{3/2}} - \frac{1}{a^2 f \sqrt{a + b(\sec^2(fx + e))}}$$

command

`integrate(tan(f*x+e)^3/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \left( \frac{(a^{10} b \operatorname{sgn}(\cos(fx+e)) + 4 a^9 b^2 \operatorname{sgn}(\cos(fx+e)) + 3 a^8 b^3 \operatorname{sgn}(\cos(fx+e))) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2}{a^{10} b^2} - \frac{3 (a^{10} b \operatorname{sgn}(\cos(fx+e)) + 4 a^9 b^2 \operatorname{sgn}(\cos(fx+e)) - a^8 b^3 \operatorname{sgn}(\cos(fx+e))) \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)}{a^{10} b^2} \right) \right)$$


---


$$(a \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + b \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right))$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.106 Problem number 430

$$\int \frac{\tan(e + fx)}{(a + b \sec^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\sec^2(fx + e))}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af(a + b(\sec^2(fx + e)))^{3/2}} + \frac{1}{a^2f\sqrt{a + b(\sec^2(fx + e))}}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\left(\frac{(4a^9b^2\operatorname{sgn}(\cos(fx+e))+3a^8b^3\operatorname{sgn}(\cos(fx+e)))\tan(\frac{1}{2}fx+\frac{1}{2}e)^2}{a^{10}b^2} - \frac{3(4a^9b^2\operatorname{sgn}(\cos(fx+e))-a^8b^3\operatorname{sgn}(\cos(fx+e)))}{a^{10}b^2}\right)\tan(\frac{1}{2}fx+\frac{1}{2}e)^2 + \frac{3(4a^9b^2\operatorname{sgn}(\cos(fx+e))-a^8b^3\operatorname{sgn}(\cos(fx+e)))}{a^{10}b^2}\right)}{(a\tan(\frac{1}{2}fx+\frac{1}{2}e)^4 + b\tan(\frac{1}{2}fx+\frac{1}{2}e)^4 - 2a\tan(\frac{1}{2}fx+\frac{1}{2}e)^2 + 2b\tan(\frac{1}{2}fx+\frac{1}{2}e)^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.107 Problem number 452

$$\int (a + b \sec^3(e + fx)) \tan^5(e + fx) dx$$

Optimal antiderivative

$$-\frac{a \ln(\cos(fx + e))}{f} - \frac{a(\sec^2(fx + e))}{f} + \frac{b(\sec^3(fx + e))}{3f} + \frac{a(\sec^4(fx + e))}{4f} - \frac{2b(\sec^5(fx + e))}{5f} + \frac{b(\sec^7(fx + e))}{7f}$$

command

```
integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$420 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right| \right) - 420 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 1 \right| \right) + \frac{1089 a + 64 b + \frac{8463 a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{448 b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{28749 a}{\cos(fx+e)+1}}{\cos(fx+e)+1}$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.108 Problem number 453

$$\int (a + b \sec^3(e + fx)) \tan^3(e + fx) dx$$

Optimal antiderivative

$$\frac{a \ln(\cos(fx + e))}{f} + \frac{a(\sec^2(fx + e))}{2f} - \frac{b(\sec^3(fx + e))}{3f} + \frac{b(\sec^5(fx + e))}{5f}$$

command

`integrate((a+b*sec(f*x+e)^3)*tan(f*x+e)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$60 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right| \right) - 60 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 1 \right| \right) + \frac{137 a + 16 b + \frac{805 a(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{80 b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{1730 a(\cos(fx+e)-1)}{\cos(fx+e)+1}}{\cos(fx+e)+1}$$


---

60 f

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.109 Problem number 454

$$\int (a + b \sec^3(e + fx)) \tan(e + fx) dx$$

Optimal antiderivative

$$-\frac{a \ln(\cos(fx + e))}{f} + \frac{b(\sec^3(fx + e))}{3f}$$

command

`integrate((a+b*sec(f*x+e)^3)*tan(f*x+e),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right| \right) - 6 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} - 1 \right| \right) + \frac{11 a + 4 b + \frac{33 a (\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{33 a (\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{12 b (\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2}}{\left( \frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right)^3}}{6 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.110 Problem number 455

$$\int \cot(e + fx) (a + b \sec^3(e + fx)) dx$$

Optimal antiderivative

$$\frac{(a + b) \ln(1 - \cos(fx + e))}{2f} + \frac{(a - b) \ln(1 + \cos(fx + e))}{2f} + \frac{b \sec(fx + e)}{f}$$

command

`integrate(cot(f*x+e)*(a+b*sec(f*x+e)^3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a + b) \log \left( \frac{1 - \cos(fx+e)+1}{|\cos(fx+e)+1|} \right) - 2 a \log \left( \left| -\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1 \right| \right) + \frac{4 b}{\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1}}{2 f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 76.111 Problem number 456

$$\int \cot^3(e + fx) (a + b \sec^3(e + fx)) dx$$

Optimal antiderivative

$$\frac{(a + b \cos(fx + e)) (\csc^2(fx + e))}{2f} - \frac{(2a - b) \ln(1 - \cos(fx + e))}{4f} - \frac{(2a + b) \ln(1 + \cos(fx + e))}{4f}$$



command

```
integrate(cot(f*x+e)^3*(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(2a - b) \log\left(\frac{|-\cos(fx+e)+1|}{|\cos(fx+e)+1|}\right) - 8a \log\left(\left|-\frac{\cos(fx+e)-1}{\cos(fx+e)+1} + 1\right|\right) - \frac{\left(a+b+\frac{4a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{2b(\cos(fx+e)-1)}{\cos(fx+e)+1}\right)(\cos(fx+e)+1)}{\cos(fx+e)-1}}{8f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 76.112 Problem number 459

$$\int \frac{\tan(e + fx)}{a + b \sec^3(e + fx)} dx$$

Optimal antiderivative

$$\frac{\ln(b + a(\cos^3(fx + e)))}{3af}$$

command

```
integrate(tan(f*x+e)/(a+b*sec(f*x+e)^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(a+b+\frac{3a(\cos(fx+e)-1)}{\cos(fx+e)+1} - \frac{3b(\cos(fx+e)-1)}{\cos(fx+e)+1} + \frac{3a(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{3b(\cos(fx+e)-1)^2}{(\cos(fx+e)+1)^2} + \frac{a(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3} - \frac{b(\cos(fx+e)-1)^3}{(\cos(fx+e)+1)^3}\right)}{3f} - \frac{3 \log\left(\left|-\frac{\cos(fx+e)}{\cos(fx+e)+1}\right|\right)}{a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 77 Test file number 129

Test folder name:

test\_cases/4\_Trig\_functions/4.6\_Cosecant/129\_4.6.1.2-d\_csc-<sup>n</sup>-a+b\_csc-<sup>m</sup>

### 77.1 Problem number 16

$$\int \frac{1}{\sqrt{a + a \csc(x)}} dx$$

Optimal antiderivative

$$-\frac{2 \arctan\left(\frac{\cot(x)\sqrt{a}}{\sqrt{a + a \csc(x)}}\right)}{\sqrt{a}} + \frac{\arctan\left(\frac{\cot(x)\sqrt{a} \sqrt{2}}{2\sqrt{a + a \csc(x)}}\right) \sqrt{2}}{\sqrt{a}}$$

command

`integrate(1/(a+a*csc(x))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \sqrt{2} \sqrt{a} \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} x\right)}}{\sqrt{a}}\right) - \frac{2 \left(a \sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|a|} + 2 \sqrt{a \tan\left(\frac{1}{2} x\right)}\right)}{2 \sqrt{|a|}}\right)}{a} - \frac{2 \left(a \sqrt{|a|}\right)}{a}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{a \csc(x) + a}} dx$$

### 77.2 Problem number 19

$$\int \sqrt{\csc(e + fx)} \sqrt{a + a \csc(e + fx)} dx$$

Optimal antiderivative

$$-\frac{2 \operatorname{arcsinh}\left(\frac{\cot(fx+e)\sqrt{a}}{\sqrt{a + a \csc(fx + e)}}\right) \sqrt{a}}{f}$$

command

`integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a \frac{\left( 2 \arctan \left( \frac{a^{\frac{3}{2}} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - \sqrt{a^3 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a^3}}{\sqrt{-a} a} \right) \right)}{\sqrt{-a}} - \frac{\log \left( \left( -a^{\frac{3}{2}} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + \sqrt{a^3 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a^3} \right)}{\sqrt{a}} \right)}{\sqrt{a}}$$


---

$f$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 77.3 Problem number 20

$$\int \sqrt{-\csc(e + fx)} \sqrt{a - a \csc(e + fx)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arcsinh} \left( \frac{\cot(fx+e)\sqrt{a}}{\sqrt{a - a \csc(fx+e)}} \right) \sqrt{a}}{f}$$

command

```
integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 a \arctan \left( \frac{a^{\frac{3}{2}} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + \sqrt{a^3 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a^3}}{\sqrt{-a} a} \right)}{\sqrt{-a}} - \frac{\sqrt{a} \log \left( \left( a^{\frac{3}{2}} \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + \sqrt{a^3 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + a^3} \right)}{\sqrt{a}} \right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 78 Test file number 134

Test folder name:

test\_cases/4\_Trig\_functions/4.6\_Cosecant/134\_4.6.7-d\_trig-<sup>m</sup>-a+b-c\_csc-<sup>n</sup>-<sup>p</sup>

### 78.1 Problem number 13

$$\int \frac{1}{(a + b \csc^2(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\cot(dx+c)\sqrt{a}}{\sqrt{a+b+b(\cot^2(dx+c))}}\right)}{a^{\frac{3}{2}}d} + \frac{b \cot(dx+c)}{a(a+b)d\sqrt{a+b+b(\cot^2(dx+c))}}$$

command

`integrate(1/(a+b*csc(d*x+c)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{a^2 b \operatorname{sgn}(\sin(dx+c)) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2}{a^4 + a^3 b} - \frac{a^2 b \operatorname{sgn}(\sin(dx+c))}{a^4 + a^3 b}}{\sqrt{b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 4 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 2 b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + b}} - \frac{2 \arctan\left(\frac{\sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - \sqrt{b t}}{d}\right)}{d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 78.2 Problem number 14

$$\int \frac{1}{(a + b \csc^2(c + dx))^{5/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\cot(dx+c)\sqrt{a}}{\sqrt{a+b+b(\cot^2(dx+c))}}\right)}{a^{\frac{5}{2}}d} + \frac{b \cot(dx+c)}{3a(a+b)d(a+b+b(\cot^2(dx+c)))^{\frac{3}{2}}} + \frac{b(5a+3b)\cot(dx+c)}{3a^2(a+b)^2d\sqrt{a+b+b(\cot^2(dx+c))}}$$

command

`integrate(1/(a+b*csc(d*x+c)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\left(\frac{(5a^9b^2\operatorname{sgn}(\sin(dx+c))+3a^8b^3\operatorname{sgn}(\sin(dx+c)))\tan(\frac{1}{2}dx+\frac{1}{2}c)^2}{a^{12}+2a^{11}b+a^{10}b^2} + \frac{3(8a^{10}b\operatorname{sgn}(\sin(dx+c))+7a^9b^2\operatorname{sgn}(\sin(dx+c))+a^8b^3\operatorname{sgn}(\sin(dx+c)))}{a^{12}+2a^{11}b+a^{10}b^2}\right)\tan(\frac{1}{2}dx+\frac{1}{2}c)^2\right)}{(b\tan(\frac{1}{2}dx+\frac{1}{2}c)^4+4a\tan(\frac{1}{2}dx+\frac{1}{2}c)^2+2a^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 78.3 Problem number 15

$$\int \frac{1}{(a+b\csc^2(c+dx))^{7/2}} dx$$

Optimal antiderivative

$$-\frac{\arctan\left(\frac{\cot(dx+c)\sqrt{a}}{\sqrt{a+b+b(\cot^2(dx+c))}}\right)}{a^{\frac{7}{2}}d} + \frac{b \cot(dx+c)}{5a(a+b)d(a+b+b(\cot^2(dx+c)))^{\frac{5}{2}}} + \frac{b(9a+5b)\cot(dx+c)}{15a^2(a+b)^2d(a+b+b(\cot^2(dx+c)))^{\frac{3}{2}}} + \frac{b(33a^2+40ab+15b^2)\cot(dx+c)}{15a^3(a+b)^3d\sqrt{a+b+b(\cot^2(dx+c))}}$$

command

`integrate(1/(a+b*csc(d*x+c)^2)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 79 Test file number 135

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/135\_4.7.1-c\_trig-<sup>^</sup>m-d\_trig-<sup>^</sup>n

### 79.1 Problem number 72

$$\int \csc^3(a + bx) \csc^4(2a + 2bx) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{105 \operatorname{arctanh}(\cos(bx + a))}{256b} + \frac{105 \sec(bx + a)}{256b} \\ & + \frac{35(\sec^3(bx + a))}{256b} - \frac{21(\csc^2(bx + a))(\sec^3(bx + a))}{256b} \\ & - \frac{3(\csc^4(bx + a))(\sec^3(bx + a))}{128b} - \frac{(\csc^6(bx + a))(\sec^3(bx + a))}{96b} \end{aligned}$$

command

`integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{285(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{21(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{225(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{2966(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{3513(\cos(bx+a)-1)}{(\cos(bx+a)+1)^4} - \frac{(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}$$


---

6144 b

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 79.2 Problem number 77

$$\int \frac{\sin(a + bx)}{\sin^{\frac{3}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$\frac{\sin(bx + a)}{b\sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 79.3 Problem number 78

$$\int \frac{\sin(a + bx)}{\sin^{\frac{5}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$\frac{\sin(bx + a)}{3b \sin(2bx + 2a)^{\frac{3}{2}}} - \frac{2 \cos(bx + a)}{3b \sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 79.4 Problem number 79

$$\int \frac{\sin(a + bx)}{\sin^{\frac{7}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$\frac{\sin(bx + a)}{5b \sin(2bx + 2a)^{\frac{5}{2}}} - \frac{4 \cos(bx + a)}{15b \sin(2bx + 2a)^{\frac{3}{2}}} + \frac{8 \sin(bx + a)}{15b \sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 79.5 Problem number 93

$$\int \frac{\sin^3(a + bx)}{\sin^{\frac{5}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$\frac{\sin^3(bx + a)}{3b \sin(2bx + 2a)^{\frac{3}{2}}}$$

command

```
integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



### 79.6 Problem number 165

$$\int \frac{\cos(a + bx)}{\sin^{\frac{3}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$-\frac{\cos(bx + a)}{b\sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(bx + a)}{\sin(2bx + 2a)^{\frac{3}{2}}} dx$$

### 79.7 Problem number 166

$$\int \frac{\cos(a + bx)}{\sin^{\frac{5}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$-\frac{\cos(bx + a)}{3b \sin(2bx + 2a)^{\frac{3}{2}}} + \frac{2 \sin(bx + a)}{3b \sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(bx + a)}{\sin(2bx + 2a)^{\frac{5}{2}}} dx$$

### 79.8 Problem number 167

$$\int \frac{\cos(a + bx)}{\sin^{\frac{7}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$-\frac{\cos(bx + a)}{5b \sin(2bx + 2a)^{\frac{5}{2}}} + \frac{4 \sin(bx + a)}{15b \sin(2bx + 2a)^{\frac{3}{2}}} - \frac{8 \cos(bx + a)}{15b \sqrt{\sin(2bx + 2a)}}$$

command

```
integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(bx + a)}{\sin(2bx + 2a)^{\frac{7}{2}}} dx$$

### 79.9 Problem number 181

$$\int \frac{\cos^3(a + bx)}{\sin^{\frac{5}{2}}(2a + 2bx)} dx$$

Optimal antiderivative

$$-\frac{\cos^3(bx + a)}{3b \sin(2bx + 2a)^{\frac{3}{2}}}$$

command

```
integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\cos(bx + a)^3}{\sin(2bx + 2a)^{\frac{5}{2}}} dx$$

### 79.10 Problem number 195

$$\int \csc(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$x \cos(a - c) + \frac{\ln(\sin(bx + c)) \sin(a - c)}{b}$$

command

```
integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^2 + 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}c\right)^2 + 1\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} - \frac{2\left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} b$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 79.11 Problem number 196

$$\int \csc^2(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{b} - \frac{\csc(bx + c) \sin(a - c)}{b}$$

command

```
integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^2 + 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}c\right)^2 + 1\right) \log\left(\left|\tan\left(\frac{1}{2}bx + \frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} - \frac{\tan\left(\frac{1}{2}bx + \frac{1}{2}c\right) \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}bx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.12 Problem number 198**

$$\int \csc^4(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{2b} - \frac{\cos(a - c) \cot(bx + c) \csc(bx + c)}{2b} - \frac{(\csc^3(bx + c)) \sin(a - c)}{3b}$$

command

```
integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.13 Problem number 200**

$$\int \csc^6(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{8b} - \frac{3 \cos(a - c) \cot(bx + c) \csc(bx + c)}{8b} - \frac{\cos(a - c) \cot(bx + c) (\csc^3(bx + c))}{4b} - \frac{(\csc^5(bx + c)) \sin(a - c)}{5b}$$

command

```
integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 79.14 Problem number 214

$$\int \sec^2(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\cos(a - c) \sec(bx + c)}{b} + \frac{\operatorname{arctanh}(\sin(bx + c)) \sin(a - c)}{b}$$

command

```
integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\left( \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right) \right) \log\left(\left|\tan\left(\frac{1}{2}bx + \frac{1}{2}c\right) + 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} - \frac{\left( \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right) \right)}{\tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 + \tan\left(\frac{1}{2}c\right)^2 + 1} \right) b$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 79.15 Problem number 216

$$\int \sec^4(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\cos(a - c) (\sec^3(bx + c))}{3b} + \frac{\operatorname{arctanh}(\sin(bx + c)) \sin(a - c)}{2b} + \frac{\sec(bx + c) \sin(a - c) \tan(bx + c)}{2b}$$

command

```
integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.16 Problem number 218**

$$\int \sec^6(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$\frac{\cos(a - c) (\sec^5(bx + c))}{5b} + \frac{3 \operatorname{arctanh}(\sin(bx + c)) \sin(a - c)}{8b} \\ + \frac{3 \sec(bx + c) \sin(a - c) \tan(bx + c)}{8b} + \frac{(\sec^3(bx + c)) \sin(a - c) \tan(bx + c)}{4b}$$

command

```
integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.17 Problem number 233**

$$\int \cot(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(bx + c)) \sin(a - c)}{b} + \frac{\sin(bx + a)}{b}$$

command

```
integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{(\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^2 - \tan(\frac{1}{2}a) \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}a) \tan(\frac{1}{2}c) - \tan(\frac{1}{2}c)^2) \log(|\tan(\frac{1}{2}bx) \tan(\frac{1}{2}c) - 1|)}{\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) + \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}a)} \right) - \frac{(\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) - \tan(\frac{1}{2}a))}{\tan(\frac{1}{2}a)}$$


---

$b$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.18 Problem number 234**

$$\int \cot^2(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{b} + \frac{\cos(bx + a)}{b} - \frac{\csc(bx + c) \sin(a - c)}{b}$$

command

```
integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.19 Problem number 235**

$$\int \cot^3(c + bx) \sin(a + bx) dx$$

Optimal antiderivative

$$-\frac{\cos(a - c) \csc(bx + c)}{b} + \frac{3 \operatorname{arctanh}(\cos(bx + c)) \sin(a - c)}{2b} - \frac{\cot(bx + c) \csc(bx + c) \sin(a - c)}{2b} - \frac{\sin(bx + a)}{b}$$

command

```
integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.20 Problem number 250**

$$\int \cos(a + bx) \cot(c + bx) dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{b} + \frac{\cos(bx + a)}{b}$$

command

```
integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^3 - \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) + 4 \tan(\frac{1}{2}a) \tan(\frac{1}{2}c)^2 - \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}c)) \log(|\tan(\frac{1}{2}bx) \tan(\frac{1}{2}c) - 1|)}{\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) + \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}c)} - \frac{(\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^2 - \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) + \tan(\frac{1}{2}c)^2 + \tan(\frac{1}{2}c)) \log(|\tan(\frac{1}{2}bx) \tan(\frac{1}{2}c) - 1|)}{\tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) + \tan(\frac{1}{2}c)^3 + \tan(\frac{1}{2}c)}$$

b

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**79.21 Problem number 251**

$$\int \cos(a + bx) \cot^2(c + bx) dx$$

Optimal antiderivative

$$-\frac{\cos(a - c) \csc(bx + c)}{b} + \frac{\operatorname{arctanh}(\cos(bx + c)) \sin(a - c)}{b} - \frac{\sin(bx + a)}{b}$$

command

```
integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 79.22 Problem number 252

$$\int \cos(a + bx) \cot^3(c + bx) dx$$

Optimal antiderivative

$$\frac{3 \operatorname{arctanh}(\cos(bx + c)) \cos(a - c)}{2b} - \frac{\cos(bx + a)}{b} - \frac{\cos(a - c) \cot(bx + c) \csc(bx + c)}{2b} + \frac{\csc(bx + c) \sin(a - c)}{b}$$

command

```
integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 80 Test file number 136

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/136\_4.7.2\_trig^m-a\_trig+b\_trig-^n

### 80.1 Problem number 229

$$\int \cos^3(c + dx)(a \sin(c + dx) + b \tan(c + dx)) dx$$

Optimal antiderivative

$$-\frac{b(\cos^3(dx + c))}{3d} - \frac{a(\cos^4(dx + c))}{4d}$$

command

```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 80.2 Problem number 236

$$\int \cos^3(c + dx)(a \sin(c + dx) + b \tan(c + dx))^2 dx$$

Optimal antiderivative

$$\frac{abx}{4} - \frac{ab \cos(dx + c) \sin(dx + c)}{4d} + \frac{(4a^2 + b^2) (\sin^3(dx + c))}{30d} + \frac{b(b + a \cos(dx + c)) (\sin^3(dx + c))}{10d} + \frac{(b + a \cos(dx + c))^2 (\sin^3(dx + c))}{5d}$$

command

```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81 Test file number 137

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/137\_4.7.3-c+d\_x-^m\_trig^n\_trig^p

### 81.1 Problem number 19

$$\int \frac{\cos(a + bx) \sin^2(a + bx)}{(c + dx)^2} dx$$

Optimal antiderivative

$$-\frac{\cos(bx + a)}{4d(dx + c)} + \frac{\cos(3bx + 3a)}{4d(dx + c)} - \frac{b \cos(a - \frac{bc}{d}) \operatorname{sinIntegral}(\frac{bc}{d} + bx)}{4d^2} + \frac{3b \cos(3a - \frac{3bc}{d}) \operatorname{sinIntegral}(\frac{3bc}{d} + 3bx)}{4d^2} + \frac{3b \operatorname{cosineIntegral}(\frac{3bc}{d} + 3bx) \sin(3a - \frac{3bc}{d})}{4d^2} - \frac{b \operatorname{cosineIntegral}(\frac{bc}{d} + bx) \sin(a - \frac{bc}{d})}{4d^2}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.2 Problem number 20

$$\int \frac{\cos(a + bx) \sin^2(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9b^2 \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{8d^3} - \frac{b^2 \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{8d^3} \\ & - \frac{\cos(bx + a)}{8d(dx + c)^2} + \frac{\cos(3bx + 3a)}{8d(dx + c)^2} - \frac{9b^2 \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^3} \\ & + \frac{b^2 \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^3} + \frac{b \sin(bx + a)}{8d^2(dx + c)} - \frac{3b \sin(3bx + 3a)}{8d^2(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.3 Problem number 21

$$\int \frac{\cos(a + bx) \sin^2(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cos(bx + a)}{12d(dx + c)^3} + \frac{b^2 \cos(bx + a)}{24d^3(dx + c)} + \frac{\cos(3bx + 3a)}{12d(dx + c)^3} \\ & - \frac{3b^2 \cos(3bx + 3a)}{8d^3(dx + c)} + \frac{b^3 \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right)}{24d^4} \\ & - \frac{9b^3 \cos\left(3a - \frac{3bc}{d}\right) \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right)}{8d^4} - \frac{9b^3 \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^4} \\ & + \frac{b^3 \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{24d^4} + \frac{b \sin(bx + a)}{24d^2(dx + c)^2} - \frac{b \sin(3bx + 3a)}{8d^2(dx + c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.4 Problem number 28

$$\int \frac{\cos(a+bx) \sin^3(a+bx)}{(c+dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \cos\left(4a - \frac{4bc}{d}\right)}{2d^2} \\ & + \frac{b \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \cos\left(2a - \frac{2bc}{d}\right)}{2d^2} + \frac{b \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{2d^2} \\ & - \frac{b \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d^2} - \frac{\sin(2bx+2a)}{4d(dx+c)} + \frac{\sin(4bx+4a)}{8d(dx+c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.5 Problem number 29

$$\int \frac{\cos(a+bx) \sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \cos(2bx+2a)}{4d^2(dx+c)} + \frac{b \cos(4bx+4a)}{4d^2(dx+c)} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} \\ & + \frac{b^2 \cos\left(4a - \frac{4bc}{d}\right) \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} + \frac{b^2 \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{d^3} \\ & - \frac{b^2 \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d^3} - \frac{\sin(2bx+2a)}{8d(dx+c)^2} + \frac{\sin(4bx+4a)}{16d(dx+c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.6 Problem number 30

$$\int \frac{\cos(a + bx) \sin^3(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{4b^3 \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \cos\left(4a - \frac{4bc}{d}\right)}{3d^4} - \frac{b^3 \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \cos\left(2a - \frac{2bc}{d}\right)}{3d^4} \\ & - \frac{b \cos(2bx + 2a)}{12d^2(dx + c)^2} + \frac{b \cos(4bx + 4a)}{12d^2(dx + c)^2} - \frac{4b^3 \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{3d^4} \\ & + \frac{b^3 \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{3d^4} - \frac{\sin(2bx + 2a)}{12d(dx + c)^3} \\ & + \frac{b^2 \sin(2bx + 2a)}{6d^3(dx + c)} + \frac{\sin(4bx + 4a)}{24d(dx + c)^3} - \frac{b^2 \sin(4bx + 4a)}{3d^3(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.7 Problem number 76

$$\int \frac{\cos^2(a + bx) \sin(a + bx)}{(c + dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3b \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{4d^2} + \frac{b \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{4d^2} \\ & - \frac{3b \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{4d^2} \\ & - \frac{b \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{4d^2} - \frac{\sin(bx + a)}{4d(dx + c)} - \frac{\sin(3bx + 3a)}{4d(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.8 Problem number 77

$$\int \frac{\cos^2(a + bx) \sin(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \cos(bx + a)}{8d^2(dx + c)} - \frac{3b \cos(3bx + 3a)}{8d^2(dx + c)} - \frac{b^2 \cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{bc}{d} + bx\right)}{8d^3} \\ & - \frac{9b^2 \cos\left(3a - \frac{3bc}{d}\right) \operatorname{Si}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} - \frac{9b^2 \operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^3} \\ & - \frac{b^2 \operatorname{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^3} - \frac{\sin(bx + a)}{8d(dx + c)^2} - \frac{\sin(3bx + 3a)}{8d(dx + c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.9 Problem number 78

$$\int \frac{\cos^2(a + bx) \sin(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{9b^3 \operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{8d^4} - \frac{b^3 \operatorname{Ci}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{24d^4} \\ & - \frac{b \cos(bx + a)}{24d^2(dx + c)^2} - \frac{b \cos(3bx + 3a)}{8d^2(dx + c)^2} + \frac{9b^3 \operatorname{Si}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{8d^4} \\ & + \frac{b^3 \operatorname{Si}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{24d^4} - \frac{\sin(bx + a)}{12d(dx + c)^3} \\ & + \frac{b^2 \sin(bx + a)}{24d^3(dx + c)} - \frac{\sin(3bx + 3a)}{12d(dx + c)^3} + \frac{3b^2 \sin(3bx + 3a)}{8d^3(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.10 Problem number 93

$$\int \frac{\cos^2(a + bx) \sin^3(a + bx)}{c + dx} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{Si}\left(\frac{bc}{d} + bx\right)}{8d} + \frac{\cos\left(3a - \frac{3bc}{d}\right) \operatorname{Si}\left(\frac{3bc}{d} + 3bx\right)}{16d} \\ & - \frac{\cos\left(5a - \frac{5bc}{d}\right) \operatorname{Si}\left(\frac{5bc}{d} + 5bx\right)}{16d} - \frac{\operatorname{Ci}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{16d} \\ & + \frac{\operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{16d} + \frac{\operatorname{Ci}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.11 Problem number 94

$$\int \frac{\cos^2(a + bx) \sin^3(a + bx)}{(c + dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{5b \operatorname{Ci}\left(\frac{5bc}{d} + 5bx\right) \cos\left(5a - \frac{5bc}{d}\right)}{16d^2} \\ & + \frac{3b \operatorname{Ci}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{16d^2} + \frac{b \operatorname{Ci}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{8d^2} \\ & + \frac{5b \operatorname{Si}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{16d^2} - \frac{3b \operatorname{Si}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{16d^2} \\ & - \frac{b \operatorname{Si}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^2} - \frac{\sin(bx + a)}{8d(dx + c)} - \frac{\sin(3bx + 3a)}{16d(dx + c)} + \frac{\sin(5bx + 5a)}{16d(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.12 Problem number 95

$$\int \frac{\cos^2(a + bx) \sin^3(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \cos (bx+a)}{16 d^2 (dx+c)}-\frac{3 b \cos (3 bx+3 a)}{32 d^2 (dx+c)}+\frac{5 b \cos (5 bx+5 a)}{32 d^2 (dx+c)}-\frac{b^2 \cos \left(a-\frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{bc}{d}+bx\right)}{16 d^3} \\ & -\frac{9 b^2 \cos \left(3 a-\frac{3 bc}{d}\right) \operatorname{sinIntegral}\left(\frac{3 bc}{d}+3 bx\right)}{32 d^3}+\frac{25 b^2 \cos \left(5 a-\frac{5 bc}{d}\right) \operatorname{sinIntegral}\left(\frac{5 bc}{d}+5 bx\right)}{32 d^3} \\ & +\frac{25 b^2 \operatorname{cosineIntegral}\left(\frac{5 bc}{d}+5 bx\right) \sin \left(5 a-\frac{5 bc}{d}\right)}{32 d^3}-\frac{9 b^2 \operatorname{cosineIntegral}\left(\frac{3 bc}{d}+3 bx\right) \sin \left(3 a-\frac{3 bc}{d}\right)}{32 d^3} \\ & -\frac{b^2 \operatorname{cosineIntegral}\left(\frac{bc}{d}+bx\right) \sin \left(a-\frac{bc}{d}\right)}{16 d^3}-\frac{\sin (bx+a)}{16 d(dx+c)^2}-\frac{\sin (3 bx+3 a)}{32 d(dx+c)^2}+\frac{\sin (5 bx+5 a)}{32 d(dx+c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.13 Problem number 96

$$\int \frac{\cos^2(a + bx) \sin^3(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{125 b^3 \operatorname{cosineIntegral}\left(\frac{5 bc}{d}+5 bx\right) \cos \left(5 a-\frac{5 bc}{d}\right)}{96 d^4} \\ & -\frac{9 b^3 \operatorname{cosineIntegral}\left(\frac{3 bc}{d}+3 bx\right) \cos \left(3 a-\frac{3 bc}{d}\right)}{32 d^4}-\frac{b^3 \operatorname{cosineIntegral}\left(\frac{bc}{d}+bx\right) \cos \left(a-\frac{bc}{d}\right)}{48 d^4} \\ & -\frac{b \cos (bx+a)}{48 d^2(dx+c)^2}-\frac{b \cos (3 bx+3 a)}{32 d^2(dx+c)^2}+\frac{5 b \cos (5 bx+5 a)}{96 d^2(dx+c)^2} \\ & -\frac{125 b^3 \operatorname{sinIntegral}\left(\frac{5 bc}{d}+5 bx\right) \sin \left(5 a-\frac{5 bc}{d}\right)}{96 d^4}+\frac{9 b^3 \operatorname{sinIntegral}\left(\frac{3 bc}{d}+3 bx\right) \sin \left(3 a-\frac{3 bc}{d}\right)}{32 d^4} \\ & +\frac{b^3 \operatorname{sinIntegral}\left(\frac{bc}{d}+bx\right) \sin \left(a-\frac{bc}{d}\right)}{48 d^4}-\frac{\sin (bx+a)}{24 d(dx+c)^3}+\frac{b^2 \sin (bx+a)}{48 d^3(dx+c)} \\ & -\frac{\sin (3 bx+3 a)}{48 d(dx+c)^3}+\frac{3 b^2 \sin (3 bx+3 a)}{32 d^3(dx+c)}+\frac{\sin (5 bx+5 a)}{48 d(dx+c)^3}-\frac{25 b^2 \sin (5 bx+5 a)}{96 d^3(dx+c)} \end{aligned}$$



command

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.14 Problem number 142

$$\int \frac{\cos^3(a + bx) \sin(a + bx)}{(c + dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{b \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \cos\left(4a - \frac{4bc}{d}\right)}{2d^2} \\ & + \frac{b \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \cos\left(2a - \frac{2bc}{d}\right)}{2d^2} - \frac{b \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{2d^2} \\ & - \frac{b \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d^2} - \frac{\sin(2bx + 2a)}{4d(dx + c)} - \frac{\sin(4bx + 4a)}{8d(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.15 Problem number 143

$$\int \frac{\cos^3(a + bx) \sin(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{b \cos(2bx + 2a)}{4d^2(dx + c)} - \frac{b \cos(4bx + 4a)}{4d^2(dx + c)} - \frac{b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right)}{2d^3} \\ & - \frac{b^2 \cos\left(4a - \frac{4bc}{d}\right) \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right)}{d^3} - \frac{b^2 \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{d^3} \\ & - \frac{b^2 \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{2d^3} - \frac{\sin(2bx + 2a)}{8d(dx + c)^2} - \frac{\sin(4bx + 4a)}{16d(dx + c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.16 Problem number 144

$$\int \frac{\cos^3(a + bx) \sin(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{4b^3 \operatorname{cosineIntegral}\left(\frac{4bc}{d} + 4bx\right) \cos\left(4a - \frac{4bc}{d}\right)}{3d^4} \\ & - \frac{b^3 \operatorname{cosineIntegral}\left(\frac{2bc}{d} + 2bx\right) \cos\left(2a - \frac{2bc}{d}\right)}{3d^4} - \frac{b \cos(2bx + 2a)}{12d^2(dx + c)^2} - \frac{b \cos(4bx + 4a)}{12d^2(dx + c)^2} \\ & + \frac{4b^3 \operatorname{sinIntegral}\left(\frac{4bc}{d} + 4bx\right) \sin\left(4a - \frac{4bc}{d}\right)}{3d^4} + \frac{b^3 \operatorname{sinIntegral}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{3d^4} \\ & - \frac{\sin(2bx + 2a)}{12d(dx + c)^3} + \frac{b^2 \sin(2bx + 2a)}{6d^3(dx + c)} - \frac{\sin(4bx + 4a)}{24d(dx + c)^3} + \frac{b^2 \sin(4bx + 4a)}{3d^3(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.17 Problem number 150

$$\int \frac{\cos^3(a + bx) \sin^2(a + bx)}{c + dx} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{cosineIntegral}\left(\frac{5bc}{d} + 5bx\right) \cos\left(5a - \frac{5bc}{d}\right)}{16d} - \frac{\operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{16d} \\ & + \frac{\operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{8d} + \frac{\operatorname{sinIntegral}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{16d} \\ & + \frac{\operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{16d} - \frac{\operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.18 Problem number 151

$$\int \frac{\cos^3(a + bx) \sin^2(a + bx)}{(c + dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\cos(bx + a)}{8d(dx + c)} + \frac{\cos(3bx + 3a)}{16d(dx + c)} + \frac{\cos(5bx + 5a)}{16d(dx + c)} \\ & - \frac{b \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right)}{8d^2} + \frac{3b \cos\left(3a - \frac{3bc}{d}\right) \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right)}{16d^2} \\ & + \frac{5b \cos\left(5a - \frac{5bc}{d}\right) \operatorname{sinIntegral}\left(\frac{5bc}{d} + 5bx\right)}{16d^2} + \frac{5b \operatorname{cosineIntegral}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{16d^2} \\ & + \frac{3b \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{16d^2} - \frac{b \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{8d^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.19 Problem number 152

$$\int \frac{\cos^3(a + bx) \sin^2(a + bx)}{(c + dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{25b^2 \operatorname{cosineIntegral}\left(\frac{5bc}{d} + 5bx\right) \cos\left(5a - \frac{5bc}{d}\right)}{32d^3} \\ & + \frac{9b^2 \operatorname{cosineIntegral}\left(\frac{3bc}{d} + 3bx\right) \cos\left(3a - \frac{3bc}{d}\right)}{32d^3} \\ & - \frac{b^2 \operatorname{cosineIntegral}\left(\frac{bc}{d} + bx\right) \cos\left(a - \frac{bc}{d}\right)}{16d^3} - \frac{\cos(bx + a)}{16d(dx + c)^2} + \frac{\cos(3bx + 3a)}{32d(dx + c)^2} \\ & + \frac{\cos(5bx + 5a)}{32d(dx + c)^2} - \frac{25b^2 \operatorname{sinIntegral}\left(\frac{5bc}{d} + 5bx\right) \sin\left(5a - \frac{5bc}{d}\right)}{32d^3} \\ & - \frac{9b^2 \operatorname{sinIntegral}\left(\frac{3bc}{d} + 3bx\right) \sin\left(3a - \frac{3bc}{d}\right)}{32d^3} + \frac{b^2 \operatorname{sinIntegral}\left(\frac{bc}{d} + bx\right) \sin\left(a - \frac{bc}{d}\right)}{16d^3} \\ & + \frac{b \sin(bx + a)}{16d^2(dx + c)} - \frac{3b \sin(3bx + 3a)}{32d^2(dx + c)} - \frac{5b \sin(5bx + 5a)}{32d^2(dx + c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.20 Problem number 153

$$\int \frac{\cos^3(a + bx) \sin^2(a + bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{\cos(bx+a)}{24d(dx+c)^3} + \frac{b^2 \cos(bx+a)}{48d^3(dx+c)} + \frac{\cos(3bx+3a)}{48d(dx+c)^3} \\
& -\frac{3b^2 \cos(3bx+3a)}{32d^3(dx+c)} + \frac{\cos(5bx+5a)}{48d(dx+c)^3} - \frac{25b^2 \cos(5bx+5a)}{96d^3(dx+c)} \\
& + \frac{b^3 \cos(a - \frac{bc}{d}) \operatorname{sinIntegral}(\frac{bc}{d} + bx)}{48d^4} - \frac{9b^3 \cos(3a - \frac{3bc}{d}) \operatorname{sinIntegral}(\frac{3bc}{d} + 3bx)}{32d^4} \\
& - \frac{125b^3 \cos(5a - \frac{5bc}{d}) \operatorname{sinIntegral}(\frac{5bc}{d} + 5bx)}{96d^4} \\
& - \frac{125b^3 \operatorname{cosineIntegral}(\frac{5bc}{d} + 5bx) \sin(5a - \frac{5bc}{d})}{96d^4} \\
& - \frac{9b^3 \operatorname{cosineIntegral}(\frac{3bc}{d} + 3bx) \sin(3a - \frac{3bc}{d})}{32d^4} + \frac{b^3 \operatorname{cosineIntegral}(\frac{bc}{d} + bx) \sin(a - \frac{bc}{d})}{48d^4} \\
& + \frac{b \sin(bx+a)}{48d^2(dx+c)^2} - \frac{b \sin(3bx+3a)}{32d^2(dx+c)^2} - \frac{5b \sin(5bx+5a)}{96d^2(dx+c)^2}
\end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.21 Problem number 160

$$\int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^2} dx$$

Optimal antiderivative

$$\begin{aligned}
& -\frac{3b \operatorname{cosineIntegral}(\frac{6bc}{d} + 6bx) \cos(6a - \frac{6bc}{d})}{16d^2} \\
& + \frac{3b \operatorname{cosineIntegral}(\frac{2bc}{d} + 2bx) \cos(2a - \frac{2bc}{d})}{16d^2} + \frac{3b \operatorname{sinIntegral}(\frac{6bc}{d} + 6bx) \sin(6a - \frac{6bc}{d})}{16d^2} \\
& - \frac{3b \operatorname{sinIntegral}(\frac{2bc}{d} + 2bx) \sin(2a - \frac{2bc}{d})}{16d^2} - \frac{3 \sin(2bx+2a)}{32d(dx+c)} + \frac{\sin(6bx+6a)}{32d(dx+c)}
\end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.22 Problem number 161

$$\int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b \cos(2bx+2a)}{32d^2(dx+c)} + \frac{3b \cos(6bx+6a)}{32d^2(dx+c)} - \frac{3b^2 \cos\left(2a - \frac{2bc}{d}\right) \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^3} \\ & + \frac{9b^2 \cos\left(6a - \frac{6bc}{d}\right) \operatorname{Si}\left(\frac{6bc}{d} + 6bx\right)}{16d^3} + \frac{9b^2 \operatorname{Ci}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right)}{16d^3} \\ & - \frac{3b^2 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{16d^3} - \frac{3 \sin(2bx+2a)}{64d(dx+c)^2} + \frac{\sin(6bx+6a)}{64d(dx+c)^2} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 81.23 Problem number 162

$$\int \frac{\cos^3(a+bx) \sin^3(a+bx)}{(c+dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{9b^3 \operatorname{Ci}\left(\frac{6bc}{d} + 6bx\right) \cos\left(6a - \frac{6bc}{d}\right)}{8d^4} - \frac{b^3 \operatorname{Ci}\left(\frac{2bc}{d} + 2bx\right) \cos\left(2a - \frac{2bc}{d}\right)}{8d^4} \\ & - \frac{b \cos(2bx+2a)}{32d^2(dx+c)^2} + \frac{b \cos(6bx+6a)}{32d^2(dx+c)^2} - \frac{9b^3 \operatorname{Si}\left(\frac{6bc}{d} + 6bx\right) \sin\left(6a - \frac{6bc}{d}\right)}{8d^4} \\ & + \frac{b^3 \operatorname{Si}\left(\frac{2bc}{d} + 2bx\right) \sin\left(2a - \frac{2bc}{d}\right)}{8d^4} - \frac{\sin(2bx+2a)}{32d(dx+c)^3} \\ & + \frac{b^2 \sin(2bx+2a)}{16d^3(dx+c)} + \frac{\sin(6bx+6a)}{96d(dx+c)^3} - \frac{3b^2 \sin(6bx+6a)}{16d^3(dx+c)} \end{aligned}$$

command

```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.24 Problem number 275

$$\int (c + dx) \csc^2(a + bx) \sec^2(a + bx) dx$$

Optimal antiderivative

$$-\frac{2(dx + c) \cot(2bx + 2a)}{b} + \frac{d \ln(\sin(2bx + 2a))}{b^2}$$

command

```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 81.25 Problem number 375

$$\int \frac{\csc(a + bx) \sin(3a + 3bx)}{(c + dx)^4} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b^2}{3d^3(dx+c)} - \frac{\cos^2(bx+a)}{d(dx+c)^3} + \frac{2b^2(\cos^2(bx+a))}{d^3(dx+c)} \\ & + \frac{8b^3 \cos(2a - \frac{2bc}{d}) \operatorname{Si}(\frac{2bc}{d} + 2bx)}{3d^4} + \frac{8b^3 \operatorname{Ci}(\frac{2bc}{d} + 2bx) \sin(2a - \frac{2bc}{d})}{3d^4} \\ & + \frac{4b \cos(bx+a) \sin(bx+a)}{3d^2(dx+c)^2} + \frac{\sin^2(bx+a)}{3d(dx+c)^3} - \frac{2b^2(\sin^2(bx+a))}{3d^3(dx+c)} \end{aligned}$$

command

```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82 Test file number 139

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/139\_4.7.5\_x^m\_trig-a+b\_log-c\_x^n-p

### 82.1 Problem number 13

$$\int x^2 \sin^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{2b^3n^3x^3 \cos(a + b \ln(cx^n))}{3(b^4n^4 + 10b^2n^2 + 9)} + \frac{2b^2n^2x^3 \sin(a + b \ln(cx^n))}{b^4n^4 + 10b^2n^2 + 9} \\ & -\frac{bnx^3 \cos(a + b \ln(cx^n)) (\sin^2(a + b \ln(cx^n)))}{3(b^2n^2 + 1)} + \frac{x^3 (\sin^3(a + b \ln(cx^n)))}{3b^2n^2 + 3} \end{aligned}$$

command

```
integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 82.2 Problem number 14

$$\int x \sin^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$-\frac{6b^3n^3x^2 \cos(a + b \ln(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{12b^2n^2x^2 \sin(a + b \ln(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} - \frac{3bnx^2 \cos(a + b \ln(cx^n)) (\sin^2(a + b \ln(cx^n)))}{9b^2n^2 + 4} + \frac{2x^2(\sin^3(a + b \ln(cx^n)))}{9b^2n^2 + 4}$$

command

```
integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.3 Problem number 15

$$\int \sin^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$-\frac{6b^3n^3x \cos(a + b \ln(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{6b^2n^2x \sin(a + b \ln(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} - \frac{3bnx \cos(a + b \ln(cx^n)) (\sin^2(a + b \ln(cx^n)))}{9b^2n^2 + 1} + \frac{x(\sin^3(a + b \ln(cx^n)))}{9b^2n^2 + 1}$$

command

```
integrate(sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.4 Problem number 19

$$\int x^2 \sin^4(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{8b^4n^4x^3}{64b^4n^4 + 180b^2n^2 + 81} - \frac{24b^3n^3x^3 \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{64b^4n^4 + 180b^2n^2 + 81} \\ & + \frac{36b^2n^2x^3(\sin^2(a + b \ln(cx^n)))}{64b^4n^4 + 180b^2n^2 + 81} \\ & - \frac{4bnx^3 \cos(a + b \ln(cx^n))(\sin^3(a + b \ln(cx^n)))}{16b^2n^2 + 9} + \frac{3x^3(\sin^4(a + b \ln(cx^n)))}{16b^2n^2 + 9} \end{aligned}$$

command

```
integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.5 Problem number 20

$$\int x \sin^4(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3b^4n^4x^2}{4(4b^4n^4 + 5b^2n^2 + 1)} - \frac{3b^3n^3x^2 \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{2(4b^4n^4 + 5b^2n^2 + 1)} \\ & + \frac{3b^2n^2x^2(\sin^2(a + b \ln(cx^n)))}{2(4b^4n^4 + 5b^2n^2 + 1)} \\ & - \frac{bnx^2 \cos(a + b \ln(cx^n))(\sin^3(a + b \ln(cx^n)))}{4b^2n^2 + 1} + \frac{x^2(\sin^4(a + b \ln(cx^n)))}{8b^2n^2 + 2} \end{aligned}$$

command

```
integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.6 Problem number 21

$$\int \sin^4(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24b^4n^4x}{64b^4n^4 + 20b^2n^2 + 1} - \frac{24b^3n^3x \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} \\ & + \frac{12b^2n^2x (\sin^2(a + b \ln(cx^n)))}{64b^4n^4 + 20b^2n^2 + 1} \\ & - \frac{4bnx \cos(a + b \ln(cx^n)) (\sin^3(a + b \ln(cx^n)))}{16b^2n^2 + 1} + \frac{x (\sin^4(a + b \ln(cx^n)))}{16b^2n^2 + 1} \end{aligned}$$

command

```
integrate(sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.7 Problem number 70

$$\int (ex)^m \sin^4(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24b^4d^4n^4(ex)^{1+m}}{e(1+m) \left( (1+m)^2 + 4b^2d^2n^2 \right) \left( (1+m)^2 + 16b^2d^2n^2 \right)} \\ & - \frac{24b^3d^3n^3(ex)^{1+m} \cos(d(a + b \ln(cx^n))) \sin(d(a + b \ln(cx^n)))}{e \left( (1+m)^2 + 4b^2d^2n^2 \right) \left( (1+m)^2 + 16b^2d^2n^2 \right)} \\ & + \frac{12b^2d^2(1+m)n^2(ex)^{1+m} (\sin^2(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 4b^2d^2n^2 \right) \left( (1+m)^2 + 16b^2d^2n^2 \right)} \\ & - \frac{4bdn(ex)^{1+m} \cos(d(a + b \ln(cx^n))) (\sin^3(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 16b^2d^2n^2 \right)} \\ & + \frac{(1+m)(ex)^{1+m} (\sin^4(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 16b^2d^2n^2 \right)} \end{aligned}$$

command

```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.8 Problem number 71

$$\int (ex)^m \sin^3(d(a + b \log(cx^n))) dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6b^3 d^3 n^3 (ex)^{1+m} \cos(d(a + b \ln(cx^n)))}{e \left( (1+m)^2 + b^2 d^2 n^2 \right) \left( (1+m)^2 + 9b^2 d^2 n^2 \right)} \\ & + \frac{6b^2 d^2 (1+m) n^2 (ex)^{1+m} \sin(d(a + b \ln(cx^n)))}{e \left( (1+m)^2 + b^2 d^2 n^2 \right) \left( (1+m)^2 + 9b^2 d^2 n^2 \right)} \\ & - \frac{3bdn(ex)^{1+m} \cos(d(a + b \ln(cx^n))) (\sin^2(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 9b^2 d^2 n^2 \right)} \\ & + \frac{(1+m) (ex)^{1+m} (\sin^3(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 9b^2 d^2 n^2 \right)} \end{aligned}$$

command

```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 82.9 Problem number 72

$$\int (ex)^m \sin^2 (d(a + b \log (cx^n))) dx$$

Optimal antiderivative

$$\frac{2b^2 d^2 n^2 (ex)^{1+m}}{e(1+m) \left( (1+m)^2 + 4b^2 d^2 n^2 \right)} - \frac{2bdn(ex)^{1+m} \cos(d(a + b \ln(cx^n))) \sin(d(a + b \ln(cx^n)))}{e \left( (1+m)^2 + 4b^2 d^2 n^2 \right)} + \frac{(1+m)(ex)^{1+m} (\sin^2(d(a + b \ln(cx^n))))}{e \left( (1+m)^2 + 4b^2 d^2 n^2 \right)}$$

command

```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 82.10 Problem number 96

$$\int x^2 \cos^3 (a + b \log (cx^n)) dx$$

Optimal antiderivative

$$\frac{2b^2 n^2 x^3 \cos(a + b \ln(cx^n))}{b^4 n^4 + 10b^2 n^2 + 9} + \frac{x^3 (\cos^3(a + b \ln(cx^n)))}{3b^2 n^2 + 3} + \frac{2b^3 n^3 x^3 \sin(a + b \ln(cx^n))}{3(b^4 n^4 + 10b^2 n^2 + 9)} + \frac{bn x^3 (\cos^2(a + b \ln(cx^n))) \sin(a + b \ln(cx^n))}{3b^2 n^2 + 3}$$

command

```
integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.11 Problem number 97

$$\int x \cos^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\frac{12b^2n^2x^2 \cos(a + b \ln(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{2x^2(\cos^3(a + b \ln(cx^n)))}{9b^2n^2 + 4} \\ + \frac{6b^3n^3x^2 \sin(a + b \ln(cx^n))}{9b^4n^4 + 40b^2n^2 + 16} + \frac{3bnx^2(\cos^2(a + b \ln(cx^n))) \sin(a + b \ln(cx^n))}{9b^2n^2 + 4}$$

command

```
integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.12 Problem number 98

$$\int \cos^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\frac{6b^2n^2x \cos(a + b \ln(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{x(\cos^3(a + b \ln(cx^n)))}{9b^2n^2 + 1} \\ + \frac{6b^3n^3x \sin(a + b \ln(cx^n))}{9b^4n^4 + 10b^2n^2 + 1} + \frac{3bnx(\cos^2(a + b \ln(cx^n))) \sin(a + b \ln(cx^n))}{9b^2n^2 + 1}$$

command

```
integrate(cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.13 Problem number 101

$$\int \cos^4(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24b^4n^4x}{64b^4n^4 + 20b^2n^2 + 1} + \frac{12b^2n^2x(\cos^2(a + b \ln(cx^n)))}{64b^4n^4 + 20b^2n^2 + 1} \\ & + \frac{x(\cos^4(a + b \ln(cx^n)))}{16b^2n^2 + 1} + \frac{24b^3n^3x \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{64b^4n^4 + 20b^2n^2 + 1} \\ & + \frac{4bnx(\cos^3(a + b \ln(cx^n))) \sin(a + b \ln(cx^n))}{16b^2n^2 + 1} \end{aligned}$$

command

```
integrate(cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 82.14 Problem number 123

$$\int x^m \cos^4(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{24b^4n^4x^{1+m}}{(1+m) \left( (1+m)^2 + 4b^2n^2 \right) \left( (1+m)^2 + 16b^2n^2 \right)} \\ & + \frac{12b^2(1+m)n^2x^{1+m}(\cos^2(a + b \ln(cx^n)))}{\left( (1+m)^2 + 4b^2n^2 \right) \left( (1+m)^2 + 16b^2n^2 \right)} + \frac{(1+m)x^{1+m}(\cos^4(a + b \ln(cx^n)))}{(1+m)^2 + 16b^2n^2} \\ & + \frac{24b^3n^3x^{1+m} \cos(a + b \ln(cx^n)) \sin(a + b \ln(cx^n))}{\left( (1+m)^2 + 4b^2n^2 \right) \left( (1+m)^2 + 16b^2n^2 \right)} \\ & + \frac{4bnx^{1+m}(\cos^3(a + b \ln(cx^n))) \sin(a + b \ln(cx^n))}{(1+m)^2 + 16b^2n^2} \end{aligned}$$

command

```
integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 82.15 Problem number 124

$$\int x^m \cos^3(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\frac{6b^2(1+m)n^2x^{1+m}\cos(a+b\ln(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)} + \frac{(1+m)x^{1+m}(\cos^3(a+b\ln(cx^n)))}{(1+m)^2+9b^2n^2}$$

$$+ \frac{6b^3n^3x^{1+m}\sin(a+b\ln(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)} + \frac{3bnx^{1+m}(\cos^2(a+b\ln(cx^n)))\sin(a+b\ln(cx^n))}{(1+m)^2+9b^2n^2}$$

command

`integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 82.16 Problem number 125

$$\int x^m \cos^2(a + b \log(cx^n)) dx$$

Optimal antiderivative

$$\frac{2b^2n^2x^{1+m}}{(1+m)((1+m)^2+4b^2n^2)} + \frac{(1+m)x^{1+m}(\cos^2(a+b\ln(cx^n)))}{(1+m)^2+4b^2n^2}$$

$$+ \frac{2bnx^{1+m}\cos(a+b\ln(cx^n))\sin(a+b\ln(cx^n))}{(1+m)^2+4b^2n^2}$$

command



```
integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 83 Test file number 141

Test folder name:

test\_cases/4\_Trig\_functions/4.7\_Miscellaneous/141\_4.7.7\_Trig\_functions

### 83.1 Problem number 139

$$\int \tan(a + bx) \tan(c + bx) dx$$

Optimal antiderivative

$$-x - \frac{\cot(a - c) \ln(\cos(bx + a))}{b} + \frac{\cot(a - c) \ln(\cos(bx + c))}{b}$$

command

```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2bx + \frac{\left(\tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^3 + 4 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)\right) \log\left(\left|2 \tan(bx) \tan\left(\frac{1}{2}a\right) + \tan\left(\frac{1}{2}a\right)^2 - 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^2 - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)} - \left(\frac{1}{2}a - \frac{1}{2}c\right)$$

2b

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**83.2 Problem number 140**

$$\int \tan(c - bx) \tan(a + bx) dx$$

Optimal antiderivative

$$x - \frac{\cot(a + c) \ln(\cos(bx - c))}{b} + \frac{\cot(a + c) \ln(\cos(bx + a))}{b}$$

command`integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")`Giac 1.9.0-11 via sagemath 9.6 output

$$2bx - \frac{\left(\tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^3 - 4 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)\right) \log\left(\left|2 \tan(bx) \tan\left(\frac{1}{2}a\right) + \tan\left(\frac{1}{2}a\right)^2 - 1\right|\right)}{\tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^2 - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)} + \frac{(\tan}{2b}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**83.3 Problem number 141**

$$\int \cot(a + bx) \cot(c + bx) dx$$

Optimal antiderivative

$$-x - \frac{\cot(a - c) \ln(\sin(bx + a))}{b} + \frac{\cot(a - c) \ln(\sin(bx + c))}{b}$$

command`integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")`Giac 1.9.0-11 via sagemath 9.6 output

$$2bx + \frac{\left(\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)^4 + 4 \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right) - 2 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right)^2 + 2 \tan\left(\frac{1}{2}a\right)^2 - 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}c\right)^2 - 1\right) \log\left(\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}a\right)^4 \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right)^3 \tan\left(\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}a\right)^3 - 2 \tan\left(\frac{1}{2}a\right)^2 \tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^2 - \tan\left(\frac{1}{2}a\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.4 Problem number 142

$$\int \cot(c - bx) \cot(a + bx) dx$$

Optimal antiderivative

$$x - \frac{\cot(a + c) \ln(-\sin(bx - c))}{b} + \frac{\cot(a + c) \ln(\sin(bx + a))}{b}$$

command

`integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2bx - \frac{(\tan(\frac{1}{2}a)^4 \tan(\frac{1}{2}c)^2 - \tan(\frac{1}{2}a)^4 - 4 \tan(\frac{1}{2}a)^3 \tan(\frac{1}{2}c) - 2 \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c)^2 + 2 \tan(\frac{1}{2}a)^2 + 4 \tan(\frac{1}{2}a) \tan(\frac{1}{2}c) + \tan(\frac{1}{2}c)^2 - 1) \log(|\tan(\frac{1}{2}a) \tan(\frac{1}{2}c) + \tan(\frac{1}{2}a) \tan(\frac{1}{2}c)|)}{\tan(\frac{1}{2}a)^4 \tan(\frac{1}{2}c) + \tan(\frac{1}{2}a)^3 \tan(\frac{1}{2}c)^2 - \tan(\frac{1}{2}a)^3 - 2 \tan(\frac{1}{2}a)^2 \tan(\frac{1}{2}c) - \tan(\frac{1}{2}a) \tan(\frac{1}{2}c)^2 + \tan(\frac{1}{2}a)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.5 Problem number 165

$$\int x^3 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{6 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{f^4} \\ & + \frac{3x^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{f^2} \\ & - \frac{6x \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{f^3} \\ & + \frac{x^3 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{f} \end{aligned}$$

command

`integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\sqrt{a} \sqrt{c} \left( \frac{3(f^2 x^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) - 2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{f^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.6 Problem number 166

$$\int x^2 \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2x \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{f^2} \\ & - \frac{2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{f^3} \\ & + \frac{x^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{f} \end{aligned}$$

command

```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$- \left( \frac{2x \cos(fx + e) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{f^2} + \frac{(f^2 x^2 \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)))}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.7 Problem number 167

$$\int x \sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)} dx$$

Optimal antiderivative

$$\frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{f^2} + \frac{x \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{f}$$

command

```
integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$- \left( \frac{x \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \operatorname{sgn}(\sin(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e)) \sin(fx + e)}{f} + \frac{\cos(fx + e) \operatorname{sgn}(\cos(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e))}{f} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.8 Problem number 168

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & \text{cosineIntegral}(fx) \cos(e) \sec(fx + e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - \sec(fx + e) \text{sinIntegral}(fx) \sin(e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \end{aligned}$$

command

```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \Re(\text{Ci}(fx)) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \tan\left(\frac{1}{2}e\right)^2 + \Re(\text{Ci}(-fx)) \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.9 Problem number 169

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{x} \\ & - f \cos(e) \sec(fx + e) \text{sinIntegral}(fx) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - f \text{cosineIntegral}(fx) \sec(fx + e) \sin(e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \end{aligned}$$

command

```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.10 Problem number 170

$$\int \frac{\sqrt{a - a \sin(e + fx)} \sqrt{c + c \sin(e + fx)}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2x^2} \\ & - \frac{f^2 \operatorname{cosineIntegral}(fx) \cos(e) \sec(fx + e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2} \\ & + \frac{f^2 \sec(fx + e) \operatorname{sinIntegral}(fx) \sin(e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2} \\ & + \frac{f \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{2x} \end{aligned}$$

command

```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.11 Problem number 171

$$\int x^3 \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{x^3 \sec (fx + e) (c + c \sin (fx + e))^{\frac{5}{2}} \sqrt{a - a \sin (fx + e)}}{2cf} \\
& - \frac{6c \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{f^4} \\
& + \frac{3cx^2 \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{f^2} \\
& + \frac{3cx \sec (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{8f^3} \\
& - \frac{3cx^3 \sec (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{4f} \\
& - \frac{3c \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{8f^4} \\
& + \frac{3cx^2 \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{4f^2} \\
& - \frac{6cx \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)} \tan (fx + e)}{f^3} \\
& - \frac{3cx \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)} \tan (fx + e)}{4f^3}
\end{aligned}$$

command

```
integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**83.12 Problem number 172**

$$\int x^2 \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x^2 \sec (fx + e) (c + c \sin (fx + e))^{\frac{5}{2}} \sqrt{a - a \sin (fx + e)}}{2cf} \\
 & + \frac{2cx \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{f^2} \\
 & - \frac{3cx^2 \sec (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{4f} \\
 & + \frac{cx \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{2f^2} \\
 & - \frac{2c \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)} \tan (fx + e)}{f^3} \\
 & - \frac{c \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)} \tan (fx + e)}{4f^3}
 \end{aligned}$$

command

```
integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.13 Problem number 173

$$\int x \sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{x \sec (fx + e) (c + c \sin (fx + e))^{\frac{5}{2}} \sqrt{a - a \sin (fx + e)}}{2cf} \\
 & + \frac{c \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{f^2} \\
 & - \frac{3cx \sec (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{4f} \\
 & + \frac{c \sin (fx + e) \sqrt{a - a \sin (fx + e)} \sqrt{c + c \sin (fx + e)}}{4f^2}
 \end{aligned}$$

command



`integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \frac{8 c \cos(fx+e) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right)}{f} + \frac{c \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \sin(2fx+2e)}{f} \right)}{x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.14 Problem number 174

$$\int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x} dx$$

Optimal antiderivative

$$\begin{aligned} & c \operatorname{cosineIntegral}(fx) \cos(e) \sec(fx+e) \sqrt{a - a \sin(fx+e)} \sqrt{c + c \sin(fx+e)} \\ & + \frac{c \cos(2e) \sec(fx+e) \operatorname{sinIntegral}(2fx) \sqrt{a - a \sin(fx+e)} \sqrt{c + c \sin(fx+e)}}{2} \\ & - c \sec(fx+e) \operatorname{sinIntegral}(fx) \sin(e) \sqrt{a - a \sin(fx+e)} \sqrt{c + c \sin(fx+e)} \\ & + \frac{c \operatorname{cosineIntegral}(2fx) \sec(fx+e) \sin(2e) \sqrt{a - a \sin(fx+e)} \sqrt{c + c \sin(fx+e)}}{2} \end{aligned}$$

command

`integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(2cf \cos(e) \operatorname{Ci}(fx) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e\right)\right) + cf \operatorname{Ci}(2fx) \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} e\right)\right))}{x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 83.15 Problem number 175

$$\int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x^2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{x} \\ & + cf \operatorname{cosineIntegral}(2fx) \cos(2e) \sec(fx + e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - cf \cos(e) \sec(fx + e) \operatorname{sinIntegral}(fx) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - cf \operatorname{cosineIntegral}(fx) \sec(fx + e) \sin(e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - cf \sec(fx + e) \operatorname{sinIntegral}(2fx) \sin(2e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - \frac{c \sec(fx + e) \sin(2fx + 2e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2x} \end{aligned}$$

command

```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 83.16 Problem number 176

$$\int \frac{\sqrt{a - a \sin(e + fx)} (c + c \sin(e + fx))^{3/2}}{x^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{c \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2x^2} \\ & - \frac{cf^2 \operatorname{cosineIntegral}(fx) \cos(e) \sec(fx + e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2} \\ & - \frac{cf \cos(2fx + 2e) \sec(fx + e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2x} \\ & - cf^2 \cos(2e) \sec(fx + e) \operatorname{sinIntegral}(2fx) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & + \frac{cf^2 \sec(fx + e) \operatorname{sinIntegral}(fx) \sin(e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{2} \\ & - cf^2 \operatorname{cosineIntegral}(2fx) \sec(fx + e) \sin(2e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \\ & - \frac{c \sec(fx + e) \sin(2fx + 2e) \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}}{4x^2} \\ & + \frac{cf \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)} \tan(fx + e)}{2x} \end{aligned}$$

command

```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 83.17 Problem number 182

$$\int \frac{x^2 \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{2ax}{c f^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{2a \operatorname{arctanh}(\sin(fx + e)) \cos(fx + e)}{c f^3 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{2a \cos(fx + e) \ln(\cos(fx + e))}{c f^3 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & - \frac{a x^2 \sec(fx + e)}{c f \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{2ax \sin(fx + e)}{c f^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{a x^2 \tan(fx + e)}{c f \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \end{aligned}$$

command

```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-a \sin(fx + e) + a} x^2}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 83.18 Problem number 183

$$\int \frac{x \sqrt{a - a \sin(e + fx)}}{(c + c \sin(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{a}{c f^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & - \frac{ax \sec(fx + e)}{c f \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{a \sin(fx + e)}{c f^2 \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \\ & + \frac{ax \tan(fx + e)}{c f \sqrt{a - a \sin(fx + e)} \sqrt{c + c \sin(fx + e)}} \end{aligned}$$

command

```
integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\sqrt{2} \left( \pi \sqrt{c} \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \tan \left( -\frac{1}{8} \pi + \frac{1}{4} fx + \frac{1}{4} e \right)^4 - (\pi - 2fx - 2e) \sqrt{c} \operatorname{sgn} \left( \sin \left( -\frac{1}{4} \pi + \frac{1}{2} fx + \frac{1}{2} e \right) \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\sqrt{-a \sin(fx + e) + a} x}{(c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

## 83.19 Problem number 337

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$$

Optimal antiderivative

$$\frac{\arctan(\sqrt{\cos(x)} \sin(x)) \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}} - \frac{\operatorname{arctanh}(\sqrt{\cos(x)} \sin(x)) \sin(x)}{\sqrt{\cos(x)} \sqrt{\sin(x) \tan(x)}}$$

command

```
integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \arcsin \left( \tan \left( \frac{1}{2} x \right)^2 \right) - \frac{1}{2} \log \left( - \frac{\sqrt{-\tan \left( \frac{1}{2} x \right)^4 + 1} - 1}{\tan \left( \frac{1}{2} x \right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\sqrt{-\cos(x) + \sec(x)}} dx$$

### 83.20 Problem number 605

$$\int \sec^2(2(a + bx)) \sqrt{c \tan(a + bx) \tan(2(a + bx))} dx$$

Optimal antiderivative

$$-\frac{c \tan(2bx + 2a)}{3b \sqrt{-c + c \sec(2bx + 2a)}} + \frac{\sqrt{-c + c \sec(2bx + 2a)} \tan(2bx + 2a)}{3b}$$

command

```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 83.21 Problem number 606

$$\int \sec(2(a + bx)) \sqrt{c \tan(a + bx) \tan(2(a + bx))} dx$$

Optimal antiderivative

$$\frac{c \tan(2bx + 2a)}{b \sqrt{-c + c \sec(2bx + 2a)}}$$

command

```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**83.22 Problem number 637**

$$\int (b \sec(c + dx) + a \sin(c + dx))^n (a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)) dx$$

Optimal antiderivative

$$\frac{(b \sec(dx + c) + a \sin(dx + c))^{1+n}}{d(1+n)}$$

command

```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( -\frac{b \tan(\frac{1}{2} dx + \frac{1}{2} c)^4 - 2a \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 2b \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 2a \tan(\frac{1}{2} dx + \frac{1}{2} c) + b}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^4 - 1} \right)^{n+1}}{d(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \sec(dx + c) \tan(dx + c) + a \cos(dx + c))(b \sec(dx + c) + a \sin(dx + c))^n dx$$

**83.23 Problem number 641**

$$\int \frac{a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)}{b \sec(c + dx) + a \sin(c + dx)} dx$$

Optimal antiderivative

$$\frac{\ln(b \sec(dx + c) + a \sin(dx + c))}{d}$$

command

```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log(b \tan(dx + c)^2 + a \tan(dx + c) + b) - \log(\tan(dx + c)^2 + 1)}{2d}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**83.24 Problem number 643**

$$\int \frac{a \cos(c + dx) + b \sec(c + dx) \tan(c + dx)}{(b \sec(c + dx) + a \sin(c + dx))^3} dx$$

Optimal antiderivative

$$-\frac{1}{2d(b \sec(dx + c) + a \sin(dx + c))^2}$$

command

```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c))^3,x, algorithm="
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\tan(dx + c)^2 + 1}{2(b \tan(dx + c)^2 + a \tan(dx + c) + b)^2} d$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**83.25 Problem number 717**

$$\int (a + b \cot(x))^n \csc^2(x) dx$$

Optimal antiderivative

$$-\frac{(a + b \cot(x))^{1+n}}{b(1+n)}$$

command

```
integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(-\frac{b \tan\left(\frac{1}{2}x\right)^2 - 2a \tan\left(\frac{1}{2}x\right) - b}{2 \tan\left(\frac{1}{2}x\right)}\right)^{n+1}}{b(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \cot(x) + a)^n \csc(x)^2 dx$$

**83.26 Problem number 751**

$$\int e^{\cos^2(x)+\sin^2(x)} dx$$

Optimal antiderivative

$$ex$$

command

```
integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$xe$$

Giac 1.7.0 via sagemath 9.3 output

$$sage_0x$$

**83.27 Problem number 900**

$$\int (1 + \cot^2(9x))^2 (1 + \tan^2(9x))^3 dx$$

Optimal antiderivative

$$-\frac{4 \cot(9x)}{9} - \frac{(\cot^3(9x))}{27} + \frac{2 \tan(9x)}{3} + \frac{4(\tan^3(9x))}{27} + \frac{(\tan^5(9x))}{45}$$

command

```
integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{45} \tan(9x)^5 + \frac{4}{27} \tan(9x)^3 - \frac{12 \tan(9x)^2 + 1}{27 \tan(9x)^3} + \frac{2}{3} \tan(9x)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 84 Test file number 156

Test folder name:

test\_cases/5\_Inverse\_trig\_functions/5.5\_Inverse\_secant/156\_5.5.1\_u-a+b\_arcsec-c\_x-  
^n

### 84.1 Problem number 69

$$\int x^4 (d + ex^2) (a + b \sec^{-1}(cx)) dx$$

Optimal antiderivative

$$\frac{dx^5(a + b \operatorname{arcsec}(cx))}{5} + \frac{ex^7(a + b \operatorname{arcsec}(cx))}{7} - \frac{b(42c^2d + 25e)x \operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{560c^6\sqrt{c^2x^2}} \\ - \frac{b(42c^2d + 25e)x^2\sqrt{c^2x^2 - 1}}{560c^5\sqrt{c^2x^2}} - \frac{b(42c^2d + 25e)x^4\sqrt{c^2x^2 - 1}}{840c^3\sqrt{c^2x^2}} - \frac{be x^6\sqrt{c^2x^2 - 1}}{42c\sqrt{c^2x^2}}$$

command

```
integrate(x^4*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 84.2 Problem number 76

$$\int x^5 (d + ex^2) (a + b \sec^{-1}(cx)) dx$$

Optimal antiderivative

$$\frac{dx^6(a + b \operatorname{arcsec}(cx))}{6} + \frac{ex^8(a + b \operatorname{arcsec}(cx))}{8} - \frac{b(8c^2d + 9e)x(c^2x^2 - 1)^{\frac{3}{2}}}{72c^7\sqrt{c^2x^2}} \\ - \frac{b(4c^2d + 9e)x(c^2x^2 - 1)^{\frac{5}{2}}}{120c^7\sqrt{c^2x^2}} - \frac{bex(c^2x^2 - 1)^{\frac{7}{2}}}{56c^7\sqrt{c^2x^2}} - \frac{b(4c^2d + 3e)x\sqrt{c^2x^2 - 1}}{24c^7\sqrt{c^2x^2}}$$

command

```
integrate(x^5*(e*x^2+d)*(a+b*arcsec(c*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 84.3 Problem number 81

$$\int x^2 (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2 x^3 (a + b \operatorname{arcsec}(cx))}{3} + \frac{2de x^5 (a + b \operatorname{arcsec}(cx))}{5} + \frac{e^2 x^7 (a + b \operatorname{arcsec}(cx))}{7} \\ & - \frac{b(280c^4 d^2 + 252c^2 de + 75e^2) x \operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2 x^2 - 1}}\right)}{1680c^6 \sqrt{c^2 x^2}} \\ & - \frac{b(280c^4 d^2 + 252c^2 de + 75e^2) x^2 \sqrt{c^2 x^2 - 1}}{1680c^5 \sqrt{c^2 x^2}} \\ & - \frac{be(84c^2 d + 25e) x^4 \sqrt{c^2 x^2 - 1}}{840c^3 \sqrt{c^2 x^2}} - \frac{be^2 x^6 \sqrt{c^2 x^2 - 1}}{42c \sqrt{c^2 x^2}} \end{aligned}$$

command

`integrate(x^2*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 84.4 Problem number 82

$$\int (d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$$

Optimal antiderivative

$$\begin{aligned} & d^2 x (a + b \operatorname{arcsec}(cx)) + \frac{2de x^3 (a + b \operatorname{arcsec}(cx))}{3} + \frac{e^2 x^5 (a + b \operatorname{arcsec}(cx))}{5} \\ & - \frac{b(120c^4 d^2 + 40c^2 de + 9e^2) x \operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2 x^2 - 1}}\right)}{120c^4 \sqrt{c^2 x^2}} \\ & - \frac{be(40c^2 d + 9e) x^2 \sqrt{c^2 x^2 - 1}}{120c^3 \sqrt{c^2 x^2}} - \frac{be^2 x^4 \sqrt{c^2 x^2 - 1}}{20c \sqrt{c^2 x^2}} \end{aligned}$$

command

```
integrate((e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 84.5 Problem number 87

$$\int x^3(d + ex^2)^2 (a + b \sec^{-1}(cx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d^2 x^4 (a + b \operatorname{arcsec}(cx))}{4} + \frac{de x^6 (a + b \operatorname{arcsec}(cx))}{3} + \frac{e^2 x^8 (a + b \operatorname{arcsec}(cx))}{8} \\ & - \frac{b(6c^4 d^2 + 16c^2 de + 9e^2) x (c^2 x^2 - 1)^{\frac{3}{2}}}{72c^7 \sqrt{c^2 x^2}} - \frac{be(8c^2 d + 9e) x (c^2 x^2 - 1)^{\frac{5}{2}}}{120c^7 \sqrt{c^2 x^2}} \\ & - \frac{be^2 x (c^2 x^2 - 1)^{\frac{7}{2}}}{56c^7 \sqrt{c^2 x^2}} - \frac{b(6c^4 d^2 + 8c^2 de + 3e^2) x \sqrt{c^2 x^2 - 1}}{24c^7 \sqrt{c^2 x^2}} \end{aligned}$$

command

```
integrate(x^3*(e*x^2+d)^2*(a+b*arcsec(c*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 85 Test file number 163

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.1\_Hyperbolic\_sine/163\_6.1.5\_Hyperbolic\_sine\_functions

## 85.1 Problem number 287

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a + \frac{2\log(cx^n)}{n}\right)} dx$$

Optimal antiderivative

$$\frac{x\left(1 - e^{-2a}(cx^n)^{-\frac{4}{n}}\right)}{2\sinh\left(a + \frac{2\ln(cx^n)}{n}\right)^{\frac{3}{2}}}$$

command

`integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2}}{\sqrt{c^{\frac{4}{n}}e^{(3a)} - \frac{e^a}{x^4}c^{\left(\frac{1}{n}\right)}x^2\operatorname{sgn}(x)}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 85.2 Problem number 288

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a + \frac{2\log(cx^n)}{n}\right)} dx$$

Optimal antiderivative

$$\frac{x\left(1 - e^{-2a}(cx^n)^{-\frac{4}{n}}\right)}{6\sinh\left(a + \frac{2\ln(cx^n)}{n}\right)^{\frac{7}{2}}} + \frac{x\left(1 - e^{-2a}(cx^n)^{-\frac{4}{n}}\right)e^{-2a}(cx^n)^{-\frac{4}{n}}}{15\sinh\left(a + \frac{2\ln(cx^n)}{n}\right)^{\frac{7}{2}}}$$

command

`integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\sqrt{2}c^{\frac{7}{n}}\left(\frac{5e^a}{c^{\frac{4}{n}}\operatorname{sgn}(x)} - \frac{2e^{(-a)}}{c^{\frac{8}{n}}x^4\operatorname{sgn}(x)}\right)e^{(3a)}}{15\left(c^{\frac{4}{n}}e^{(3a)} - \frac{e^a}{x^4}\right)^{\frac{5}{2}}x^6}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 86 Test file number 164

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.1\_Hyperbolic\_sine/164\_6.1.7\_hyper<sup>m</sup>-a+b\_sinh<sup>n</sup>-<sup>p</sup>

### 86.1 Problem number 66

$$\int \sinh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a - b)(a + 3b) \operatorname{arctanh}\left(\frac{\cosh(fx + e)\sqrt{b}}{\sqrt{a - b + b(\cosh^2(fx + e))}}\right)}{8b^{\frac{3}{2}}f} \\ & + \frac{\cosh(fx + e)(a - b + b(\cosh^2(fx + e)))^{\frac{3}{2}}}{4bf} \\ & - \frac{(a + 3b)\cosh(fx + e)\sqrt{a - b + b(\cosh^2(fx + e))}}{8bf} \end{aligned}$$

command

```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.2 Problem number 76

$$\int \sinh^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(a-b)^2 (a+5b) \operatorname{arctanh} \left( \frac{\cosh(fx+e) \sqrt{b}}{\sqrt{a-b+b(\cosh^2(fx+e))}} \right)}{16b^{\frac{3}{2}} f} \\
 & - \frac{(a+5b) \cosh(fx+e) (a-b+b(\cosh^2(fx+e)))^{\frac{3}{2}}}{24bf} \\
 & + \frac{\cosh(fx+e) (a-b+b(\cosh^2(fx+e)))^{\frac{5}{2}}}{6bf} \\
 & - \frac{(a-b)(a+5b) \cosh(fx+e) \sqrt{a-b+b(\cosh^2(fx+e))}}{16bf}
 \end{aligned}$$

command

```
integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.3 Problem number 77

$$\int \sinh(e+fx) (a+b\sinh^2(e+fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{\cosh(fx+e) (a-b+b(\cosh^2(fx+e)))^{\frac{3}{2}}}{4f} \\
 & + \frac{3(a-b)^2 \operatorname{arctanh} \left( \frac{\cosh(fx+e) \sqrt{b}}{\sqrt{a-b+b(\cosh^2(fx+e))}} \right)}{8f\sqrt{b}} \\
 & + \frac{3(a-b) \cosh(fx+e) \sqrt{a-b+b(\cosh^2(fx+e))}}{8f}
 \end{aligned}$$

command

```
integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 86.4 Problem number 80

$$\int \operatorname{csch}^5(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{(a - b + b(\cosh^2(fx + e)))^{\frac{3}{2}} \coth(fx + e) \operatorname{csch}(fx + e)^3}{4f} \\ & - \frac{3(a - b)^2 \operatorname{arctanh}\left(\frac{\cosh(fx + e)\sqrt{a}}{\sqrt{a - b + b(\cosh^2(fx + e))}}\right)}{8f\sqrt{a}} \\ & + \frac{3(a - b) \coth(fx + e) \operatorname{csch}(fx + e) \sqrt{a - b + b(\cosh^2(fx + e))}}{8f} \end{aligned}$$

command

```
integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 86.5 Problem number 81

$$\int \operatorname{csch}^7(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{(a-b)^2(5a+b) \operatorname{arctanh}\left(\frac{\cosh(fx+e)\sqrt{a}}{\sqrt{a-b+b(\cosh^2(fx+e))}}\right)}{16a^{\frac{3}{2}}f} + \frac{(5a+b)(a-b+b(\cosh^2(fx+e)))^{\frac{3}{2}} \operatorname{coth}(fx+e) \operatorname{csch}(fx+e)^3}{24af} - \frac{(a-b+b(\cosh^2(fx+e)))^{\frac{5}{2}} \operatorname{coth}(fx+e) \operatorname{csch}(fx+e)^5}{6af} - \frac{(a-b)(5a+b) \operatorname{coth}(fx+e) \operatorname{csch}(fx+e) \sqrt{a-b+b(\cosh^2(fx+e))}}{16af}$$

command

```
integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.6 Problem number 100

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\cosh(fx+e)\sqrt{a}}{\sqrt{a-b+b(\cosh^2(fx+e))}}\right)}{f\sqrt{a}}$$

command

```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \operatorname{arctan}\left(-\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} - \sqrt{b}}{2\sqrt{-a}}\right)}{\sqrt{-a} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 86.7 Problem number 101

$$\int \frac{\operatorname{csch}^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{(a + b) \operatorname{arctanh} \left( \frac{\cosh(fx + e) \sqrt{a}}{\sqrt{a - b + b (\cosh^2(fx + e))}} \right)}{2a^{\frac{3}{2}} f} - \frac{\coth(fx + e) \operatorname{csch}(fx + e) \sqrt{a - b + b (\cosh^2(fx + e))}}{2af}$$

command

```
integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(a+b) \operatorname{arctan} \left( -\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} - \sqrt{b}}{2\sqrt{-a}} \right) e^{(-4e)}}{\sqrt{-a} a} \right) - \frac{2 \left( \left( \sqrt{b} e^{(2fx+2e)} - \sqrt{b} \right) \right)}{2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.8 Problem number 102

$$\int \frac{\sinh^4(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Optimal antiderivative

$$\frac{\cosh (f x+e) \sinh (f x+e) \sqrt{a+b\left(\sinh ^2(f x+e)\right)}}{3 b f}$$

$$+ \frac{2(a+b) \sqrt{2} \sqrt{\frac{1}{1+\cosh (2 f x+2 e)}} \sqrt{1+\sinh ^2(f x+e)} \operatorname{EllipticE}\left(\frac{\sinh (f x+e)}{\sqrt{1+\sinh ^2(f x+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(f x)}{3 b^2 f \sqrt{\frac{\operatorname{sech}(f x+e)^2(a+b\left(\sinh ^2(f x+e)\right))}{a}}}$$

$$- \frac{\sqrt{2} \sqrt{\frac{1}{1+\cosh (2 f x+2 e)}} \sqrt{1+\sinh ^2(f x+e)} \operatorname{EllipticF}\left(\frac{\sinh (f x+e)}{\sqrt{1+\sinh ^2(f x+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(f x+e) \sqrt{\frac{\operatorname{sech}(f x+e)^2(a+b\left(\sinh ^2(f x+e)\right))}{a}}}{3 b f \sqrt{\frac{\operatorname{sech}(f x+e)^2(a+b\left(\sinh ^2(f x+e)\right))}{a}}}$$

$$- \frac{2(a+b) \sqrt{a+b\left(\sinh ^2(f x+e)\right)} \tanh (f x+e)}{3 b^2 f}$$

command

```
integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.9 Problem number 105

$$\int \frac{\operatorname{csch}^2(e+f x)}{\sqrt{a+b \sinh ^2(e+f x)}} d x$$

Optimal antiderivative

$$\frac{\operatorname{coth}(f x+e) \sqrt{a+b\left(\sinh ^2(f x+e)\right)}}{a f}$$

$$+ \frac{\sqrt{2} \sqrt{\frac{1}{1+\cosh (2 f x+2 e)}} \sqrt{1+\sinh ^2(f x+e)} \operatorname{EllipticE}\left(\frac{\sinh (f x+e)}{\sqrt{1+\sinh ^2(f x+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(f x+e) \sqrt{\frac{\operatorname{sech}(f x+e)^2(a+b\left(\sinh ^2(f x+e)\right))}{a}}}{a f \sqrt{\frac{\operatorname{sech}(f x+e)^2(a+b\left(\sinh ^2(f x+e)\right))}{a}}}$$

$$+ \frac{\sqrt{a+b\left(\sinh ^2(f x+e)\right)} \tanh (f x+e)}{a f}$$

command

```
integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\arctan\left(-\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} - \sqrt{b}}{2\sqrt{-a}}\right) e^{(-2e)}}{\sqrt{-a}} \right) - \frac{1}{\left(\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.10 Problem number 106

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{2(a+b)\coth(fx+e)\sqrt{a+b(\sinh^2(fx+e))}}{3a^2f} - \frac{\coth(fx+e)\operatorname{csch}(fx+e)^2\sqrt{a+b(\sinh^2(fx+e))}}{3af} + \frac{2(a+b)\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticE}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e)}{3a^2f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}}}$$

$$- \frac{b\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticF}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e)}{3a^2f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}}}$$

$$- \frac{2(a+b)\sqrt{a+b(\sinh^2(fx+e))}\tanh(fx+e)}{3a^2f}$$

command

`integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.11 Problem number 108

$$\int \frac{\sinh(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\cosh(fx + e)}{(a - b) f \sqrt{a - b + b (\cosh^2(fx + e))}}$$

command

`integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{ae^{(2fx+4e)}}{a^2e^{(2e)}-abe^{(2e)}} + \frac{ae^{(2e)}}{a^2e^{(2e)}-abe^{(2e)}}}{\sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.12 Problem number 109

$$\int \frac{\operatorname{csch}(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\cosh(fx+e)\sqrt{a}}{\sqrt{a-b+b(\cosh^2(fx+e))}}\right)}{a^{\frac{3}{2}}f} - \frac{b \cosh(fx + e)}{a(a - b) f \sqrt{a - b + b (\cosh^2(fx + e))}}$$

command

`integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{\frac{a^2 b e^{(2 f x+4 e)}}{a^4 e^{(6 e)}-a^3 b e^{(6 e)}} + \frac{a^2 b e^{(2 e)}}{a^4 e^{(6 e)}-a^3 b e^{(6 e)}}}{\sqrt{b e^{(4 f x+4 e)} + 4 a e^{(2 f x+2 e)} - 2 b e^{(2 f x+2 e)} + b}} - \frac{2 \arctan\left(-\frac{\sqrt{b} e^{(2 f x+2 e)} - \sqrt{b e^{(4 f x+4 e)} + 4 a e^{(2 f x+2 e)} - 2 b e^{(2 f x+2 e)}}}{2 \sqrt{-a}}\right)}{\sqrt{-a} a} \right) f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.13 Problem number 111

$$\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{a \cosh(fx+e) (\sinh^3(fx+e))}{(a-b) b f \sqrt{a+b (\sinh^2(fx+e))}} + \frac{(4a-b) \cosh(fx+e) \sinh(fx+e) \sqrt{a+b (\sinh^2(fx+e))}}{3(a-b) b^2 f} + \frac{(8a^2-3ab-2b^2) \sqrt{2} \sqrt{\frac{1}{1+\cosh(2fx+2e)}} \sqrt{1+\sinh^2(fx+e)} \operatorname{EllipticE}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}}, \sqrt{1-\frac{b}{a}}\right)}{3(a-b) b^3 f \sqrt{\frac{\operatorname{sech}(fx+e)^2 (a+b (\sinh^2(fx+e)))}{a}}} + \frac{(4a-b) \sqrt{2} \sqrt{\frac{1}{1+\cosh(2fx+2e)}} \sqrt{1+\sinh^2(fx+e)} \operatorname{EllipticF}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(fx+e)}{3(a-b) b^2 f \sqrt{\frac{\operatorname{sech}(fx+e)^2 (a+b (\sinh^2(fx+e)))}{a}}} - \frac{(8a^2-3ab-2b^2) \sqrt{a+b (\sinh^2(fx+e))} \tanh(fx+e)}{3(a-b) b^3 f}$$

command

`integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.14 Problem number 117

$$\int \frac{\sinh^3(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cosh(fx + e) (\sinh^2(fx + e))}{3(a - b) f (a - b + b (\cosh^2(fx + e)))^{3/2}} - \frac{2 \cosh(fx + e)}{3(a - b)^2 f \sqrt{a - b + b (\cosh^2(fx + e))}}$$

command

`integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( \frac{a^3 e^{12e} - 3 a^2 b e^{12e}}{a^4 e^{6e} - 2 a^3 b e^{6e} + a^2 b^2 e^{6e}} \right) e^{2fx} - \frac{3 (3 a^3 e^{10e} - a^2 b e^{10e})}{a^4 e^{6e} - 2 a^3 b e^{6e} + a^2 b^2 e^{6e}} \right) e^{2fx} - \frac{3 (3 a^3 e^{8e} - a^2 b e^{8e})}{a^4 e^{6e} - 2 a^3 b e^{6e} + a^2 b^2 e^{6e}} e^{2fx} + \frac{a^3 e^{6e} - 3 a^2 b e^{6e}}{a^4 e^{6e} - 2 a^3 b e^{6e} + a^2 b^2 e^{6e}}}{3 (b e^{4fx+4e} + 4 a e^{2fx+2e} - 2 b e^{2fx+2e} + b)^{3/2}} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.15 Problem number 118

$$\int \frac{\sinh(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\cosh(fx + e)}{3(a - b) f (a - b + b (\cosh^2(fx + e)))^{3/2}} + \frac{2 \cosh(fx + e)}{3(a - b)^2 f \sqrt{a - b + b (\cosh^2(fx + e))}}$$

command

```
integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{a^2 b e^{(6e)}}{a^4 e^{(6e)} - 2 a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}} + \left( \left( \frac{a^2 b e^{(2fx+12e)}}{a^4 e^{(6e)} - 2 a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}} + \frac{3(2a^3 e^{(10e)} - a^2 b e^{(10e)})}{a^4 e^{(6e)} - 2 a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}} \right) e^{(2fx)} + \frac{3(2a^3 e^{(8e)} - a^2 b e^{(8e)})}{a^4 e^{(6e)} - 2 a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}} \right) e^{(2fx)} + \frac{3}{3(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b)} \frac{3}{2} f \right)}{3(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b) \frac{3}{2} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.16 Problem number 119

$$\int \frac{\operatorname{csch}(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \frac{\cosh(fx+e) \sqrt{a}}{\sqrt{a-b+b(\cosh^2(fx+e))}} \right)}{a^{5/2} f} - \frac{b \cosh(fx+e)}{3a(a-b) f (a-b+b(\cosh^2(fx+e)))^{3/2}} - \frac{(5a-3b) b \cosh(fx+e)}{3a^2(a-b)^2 f \sqrt{a-b+b(\cosh^2(fx+e))}}$$

command

```
integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left( \left( \left( \frac{(5a^9 b^2 e^{(12e)} - 3a^8 b^3 e^{(12e)}) e^{(2fx)}}{a^{12} e^{(12e)} - 2a^{11} b e^{(12e)} + a^{10} b^2 e^{(12e)}} + \frac{3(8a^{10} b e^{(10e)} - 7a^9 b^2 e^{(10e)} + a^8 b^3 e^{(10e)})}{a^{12} e^{(12e)} - 2a^{11} b e^{(12e)} + a^{10} b^2 e^{(12e)}} \right) e^{(2fx)} + \frac{3(8a^{10} b e^{(8e)} - 7a^9 b^2 e^{(8e)} + a^8 b^3 e^{(8e)})}{a^{12} e^{(12e)} - 2a^{11} b e^{(12e)} + a^{10} b^2 e^{(12e)}} \right) e^{(2fx)} + \frac{3}{(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b)} \frac{3}{2} \right)}{3(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b) \frac{3}{2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.17 Problem number 275

$$\int \frac{1}{1 - \sinh^8(x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\sqrt{1-i} \tanh(x)\right)}{4\sqrt{1-i}} + \frac{\operatorname{arctanh}\left(\sqrt{1+i} \tanh(x)\right)}{4\sqrt{1+i}} + \frac{\operatorname{arctanh}\left(\sqrt{2} \tanh(x)\right) \sqrt{2}}{8} + \frac{\tanh(x)}{4}$$

command

```
integrate(1/(1-sinh(x)^8),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{16} \sqrt{2} \log\left(\frac{\left| -4\sqrt{2} + 2e^{(2x)} - 6 \right|}{\left| 4\sqrt{2} + 2e^{(2x)} - 6 \right|}\right) - \frac{1}{2(e^{(2x)} + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 86.18 Problem number 352

$$\int \cosh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal antiderivative

$$\frac{a(a-4b) \operatorname{arctanh}\left(\frac{\sinh(fx+e)\sqrt{b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{8b^{\frac{3}{2}}f} + \frac{\sinh(fx+e)(a+b(\sinh^2(fx+e)))^{\frac{3}{2}}}{4bf} - \frac{(a-4b) \sinh(fx+e) \sqrt{a+b(\sinh^2(fx+e))}}{8bf}$$

command

```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 86.19 Problem number 362

$$\int \cosh^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(a - 6b) \operatorname{arctanh} \left( \frac{\sinh(fx+e)\sqrt{b}}{\sqrt{a + b(\sinh^2(fx + e))}} \right)}{16b^{\frac{3}{2}}f} \\ & - \frac{(a - 6b) \sinh(fx + e) (a + b(\sinh^2(fx + e)))^{\frac{3}{2}}}{24bf} + \frac{\sinh(fx + e) (a + b(\sinh^2(fx + e)))^{\frac{5}{2}}}{6bf} \\ & - \frac{a(a - 6b) \sinh(fx + e) \sqrt{a + b(\sinh^2(fx + e))}}{16bf} \end{aligned}$$

command

```
integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.20 Problem number 363

$$\int \cosh(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sinh(fx + e) (a + b(\sinh^2(fx + e)))^{\frac{3}{2}}}{4f} + \frac{3a^2 \operatorname{arctanh} \left( \frac{\sinh(fx+e)\sqrt{b}}{\sqrt{a + b(\sinh^2(fx + e))}} \right)}{8f\sqrt{b}} \\ & + \frac{3a \sinh(fx + e) \sqrt{a + b(\sinh^2(fx + e))}}{8f} \end{aligned}$$

command

```
integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{24 a^2 \arctan\left(-\frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}}{\sqrt{-b}}\right) e^{(4e)}}{\sqrt{-b}} - \frac{12 a^2 e^{(4e)} \log\left(-\left(\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b\right)\right)}{\sqrt{-b}} \right)$$


---

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.21 Problem number 366

$$\int \operatorname{sech}^5(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{3a^2 \arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{8f\sqrt{a-b}} \\ & + \frac{\operatorname{sech}(fx+e)^3 (a+b(\sinh^2(fx+e)))^{3/2} \tanh(fx+e)}{4f} \\ & + \frac{3a \operatorname{sech}(fx+e) \sqrt{a+b(\sinh^2(fx+e))} \tanh(fx+e)}{8f} \end{aligned}$$

command

```
integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.22 Problem number 367

$$\int \operatorname{sech}^7(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a^2(5a - 6b) \arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{16(a-b)^{\frac{3}{2}}f} \\ & + \frac{(5a - 6b) \operatorname{sech}(fx+e)^3 (a + b(\sinh^2(fx+e)))^{\frac{3}{2}} \tanh(fx+e)}{24(a-b)f} \\ & + \frac{\operatorname{sech}(fx+e)^5 (a + b(\sinh^2(fx+e)))^{\frac{5}{2}} \tanh(fx+e)}{6(a-b)f} \\ & + \frac{a(5a - 6b) \operatorname{sech}(fx+e) \sqrt{a+b(\sinh^2(fx+e))} \tanh(fx+e)}{16(a-b)f} \end{aligned}$$

command

```
integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.23 Problem number 372

$$\int \operatorname{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal antiderivative

$$\frac{2(a+b)\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticE}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e)}{3f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}}}$$

$$-\frac{b\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticF}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e)}{3f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}}}$$

$$+\frac{(a-b)\operatorname{sech}(fx+e)^2\sqrt{a+b(\sinh^2(fx+e))}\tanh(fx+e)}{3f}$$

command

```
integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.24 Problem number 375

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{f\sqrt{a-b}}$$

command

```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan \left( \frac{-\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} + \sqrt{b}}{2\sqrt{a-b}} \right)}{\sqrt{a-b} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.25 Problem number 376

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{(a-2b) \arctan \left( \frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}} \right)}{2(a-b)^{\frac{3}{2}} f} + \frac{\operatorname{sech}(fx+e) \sqrt{a+b(\sinh^2(fx+e))} \tanh(fx+e)}{2(a-b) f}$$

command

```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( \frac{(a-2b) \arctan \left( \frac{-\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} + \sqrt{b}}{2\sqrt{a-b}} \right)}{(ae^{(4e)} - be^{(4e)})\sqrt{a-b}} \right) - \frac{2 \left( \left( \sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)}} \right) \right)}{2(a-b) f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.26 Problem number 377

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\cosh (fx+e) \sinh (fx+e) \sqrt{a+b\left(\sinh ^2 (fx+e)\right)}}{3bf}$$

$$+ \frac{2(a-2b) \sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh ^2 (fx+e)} \operatorname{EllipticE}\left(\frac{\sinh (fx+e)}{\sqrt{1+\sinh ^2 (fx+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(fx)}{3b^2 f \sqrt{\frac{\operatorname{sech}^2 (fx+e)^2 (a+b\left(\sinh ^2 (fx+e)\right))}{a}}}$$

$$- \frac{(a-3b) \sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh ^2 (fx+e)} \operatorname{EllipticF}\left(\frac{\sinh (fx+e)}{\sqrt{1+\sinh ^2 (fx+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(fx)}{3abf \sqrt{\frac{\operatorname{sech}^2 (fx+e)^2 (a+b\left(\sinh ^2 (fx+e)\right))}{a}}}$$

$$- \frac{2(a-2b) \sqrt{a+b\left(\sinh ^2 (fx+e)\right)} \tanh (fx+e)}{3b^2 f}$$

command

```
integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.27 Problem number 380

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b \sinh ^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{\sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh ^2 (fx+e)} \operatorname{EllipticE}\left(\frac{\sinh (fx+e)}{\sqrt{1+\sinh ^2 (fx+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(fx+e) \sqrt{a}}{(a-b) f \sqrt{\frac{\operatorname{sech}^2 (fx+e)^2 (a+b\left(\sinh ^2 (fx+e)\right))}{a}}}$$

$$- \frac{b \sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh ^2 (fx+e)} \operatorname{EllipticF}\left(\frac{\sinh (fx+e)}{\sqrt{1+\sinh ^2 (fx+e)}}, \sqrt{1-\frac{b}{a}}\right) \operatorname{sech}(fx+e) \sqrt{a}}{a(a-b) f \sqrt{\frac{\operatorname{sech}^2 (fx+e)^2 (a+b\left(\sinh ^2 (fx+e)\right))}{a}}}$$

command

```
integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{\arctan \left( \frac{\sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} + \sqrt{b}}{2\sqrt{a-b}} \right)}{\sqrt{a-b}} \right) e^{(-2e)} - \frac{1}{\left( \sqrt{b} e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.28 Problem number 381

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{2(a-2b)\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticE}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e) + 3(a-b)^2f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}}}{(a-3b)b\sqrt{2}\sqrt{\frac{1}{1+\cosh(2fx+2e)}}\sqrt{1+\sinh^2(fx+e)}\operatorname{EllipticF}\left(\frac{\sinh(fx+e)}{\sqrt{1+\sinh^2(fx+e)}},\sqrt{1-\frac{b}{a}}\right)\operatorname{sech}(fx+e) + 3a(a-b)^2f\sqrt{\frac{\operatorname{sech}(fx+e)^2(a+b(\sinh^2(fx+e)))}{a}} + \frac{\operatorname{sech}(fx+e)^2\sqrt{a+b(\sinh^2(fx+e))}\tanh(fx+e)}{3(a-b)f}}$$

command

```
integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.29 Problem number 383

$$\int \frac{\cosh(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\sinh(fx + e)}{af \sqrt{a + b (\sinh^2(fx + e))}}$$

command

```
integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(ae^{4e} - be^{4e})e^{2fx}}{a^2e^{2e} - abe^{2e}} - \frac{ae^{2e} - be^{2e}}{a^2e^{2e} - abe^{2e}}}{\sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b} f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 86.30 Problem number 384

$$\int \frac{\operatorname{sech}(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{(a-b)^{\frac{3}{2}}f} - \frac{b \sinh(fx+e)}{a(a-b)f \sqrt{a+b(\sinh^2(fx+e))}}$$

command

```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{(a^2be^{4e} - 2ab^2e^{4e} + b^3e^{4e})e^{2fx}}{a^4e^{6e} - 3a^3be^{6e} + 3a^2b^2e^{6e} - ab^3e^{6e}} - \frac{a^2be^{2e} - 2ab^2e^{2e} + b^3e^{2e}}{a^4e^{6e} - 3a^3be^{6e} + 3a^2b^2e^{6e} - ab^3e^{6e}}\right)}{\sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}} - \frac{2 \arctan\left(\frac{-\sqrt{b}e^{(2fx+2e)} - \sqrt{be^{4fx+4e} + 4a}}{2\sqrt{(ae^{4e} - be^{4e})}}\right)}{f}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



### 86.31 Problem number 385

$$\int \frac{\operatorname{sech}^3(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{(a - 4b) \arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{2(a-b)^{\frac{5}{2}}f} + \frac{b(a+2b)\sinh(fx+e)}{2a(a-b)^2f\sqrt{a+b(\sinh^2(fx+e))}} + \frac{\operatorname{sech}(fx+e)\tanh(fx+e)}{2(a-b)f\sqrt{a+b(\sinh^2(fx+e))}}$$

command

```
integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.32 Problem number 386

$$\int \frac{\cosh^6(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
 & - \frac{(a-b) (\cosh^3 (fx+e)) \sinh (fx+e)}{abf \sqrt{a+b (\sinh^2 (fx+e))}} \\
 & + \frac{(4a-3b) \cosh (fx+e) \sinh (fx+e) \sqrt{a+b (\sinh^2 (fx+e))}}{3ab^2 f} \\
 & + \frac{(8a^2-13ab+3b^2) \sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh^2 (fx+e)} \operatorname{EllipticE} \left( \frac{\sinh (fx+e)}{\sqrt{1+\sinh^2 (fx+e)}}, \sqrt{1-\frac{b}{a}} \right)}{3ab^3 f \sqrt{\frac{\operatorname{sech} (fx+e)^2 (a+b (\sinh^2 (fx+e)))}{a}}} \\
 & - \frac{2(2a-3b) \sqrt{2} \sqrt{\frac{1}{1+\cosh (2fx+2e)}} \sqrt{1+\sinh^2 (fx+e)} \operatorname{EllipticF} \left( \frac{\sinh (fx+e)}{\sqrt{1+\sinh^2 (fx+e)}}, \sqrt{1-\frac{b}{a}} \right) \operatorname{sech} (fx+e)}{3ab^2 f \sqrt{\frac{\operatorname{sech} (fx+e)^2 (a+b (\sinh^2 (fx+e)))}{a}}} \\
 & - \frac{(8a^2-13ab+3b^2) \sqrt{a+b (\sinh^2 (fx+e))} \tanh (fx+e)}{3ab^3 f}
 \end{aligned}$$

command

```
integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.33 Problem number 392

$$\int \frac{\cosh^3 (e+fx)}{(a+b \sinh^2 (e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{(\cosh^2 (fx+e)) \sinh (fx+e)}{3af (a+b (\sinh^2 (fx+e)))^{3/2}} + \frac{2 \sinh (fx+e)}{3a^2 f \sqrt{a+b (\sinh^2 (fx+e))}}$$

command

`integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\left(\frac{a^3 e^{(12e)} - 3ab^2 e^{(12e)} + 2b^3 e^{(12e)}}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}\right)e^{(2fx)} + \frac{3(3a^3 e^{(10e)} - 8a^2 b e^{(10e)} + 7ab^2 e^{(10e)} - 2b^3 e^{(10e)})}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}\right)e^{(2fx)} - \frac{3(3a^3 e^{(8e)} - 8a^2 b e^{(8e)} + 7ab^2 e^{(8e)})}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}}{3\left(b e^{(4fx+4e)} + 4a e^{(2fx+2e)} - 2b e^{(2fx+2e)} + b\right)^{\frac{3}{2}}} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.34 Problem number 393

$$\int \frac{\cosh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\sinh(fx+e)}{3af(a+b(\sinh^2(fx+e)))^{\frac{3}{2}}} + \frac{2\sinh(fx+e)}{3a^2 f \sqrt{a+b(\sinh^2(fx+e))}}$$

command

`integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\left(\left(\frac{a^2 b e^{(12e)} - 2ab^2 e^{(12e)} + b^3 e^{(12e)}}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}\right)e^{(2fx)} + \frac{3(2a^3 e^{(10e)} - 5a^2 b e^{(10e)} + 4ab^2 e^{(10e)} - b^3 e^{(10e)})}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}\right)e^{(2fx)} - \frac{3(2a^3 e^{(8e)} - 5a^2 b e^{(8e)} + 4ab^2 e^{(8e)})}{a^4 e^{(6e)} - 2a^3 b e^{(6e)} + a^2 b^2 e^{(6e)}}\right)}{3\left(b e^{(4fx+4e)} + 4a e^{(2fx+2e)} - 2b e^{(2fx+2e)} + b\right)^{\frac{3}{2}}} f$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.35 Problem number 394

$$\int \frac{\operatorname{sech}(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sinh(fx+e)\sqrt{a-b}}{\sqrt{a+b(\sinh^2(fx+e))}}\right)}{(a-b)^{\frac{5}{2}}f} - \frac{b \sinh(fx+e)}{3a(a-b)f(a+b(\sinh^2(fx+e)))^{\frac{3}{2}}} - \frac{(5a-2b)b \sinh(fx+e)}{3a^2(a-b)^2f\sqrt{a+b(\sinh^2(fx+e))}}$$

command

```
integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 86.36 Problem number 479

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx$$

Optimal antiderivative

$$\frac{(8a^2 - 8ab + 3b^2) \operatorname{arctanh}\left(\frac{\sqrt{a+b(\sinh^2(fx+e))}}{\sqrt{a-b}}\right)}{8(a-b)^{\frac{5}{2}}f} + \frac{(8a-5b) \operatorname{sech}(fx+e)^2 \sqrt{a+b(\sinh^2(fx+e))}}{8(a-b)^2f} - \frac{\operatorname{sech}(fx+e)^4 \sqrt{a+b(\sinh^2(fx+e))}}{4(a-b)f}$$

command

```
integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 87 Test file number 170

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.2\_Hyperbolic\_cosine/170\_6.2.7\_hyper<sup>m</sup>-a+b\_cosh<sup>n</sup>-<sup>p</sup>

### 87.1 Problem number 75

$$\int \frac{1}{1 - \cosh^8(x)} dx$$

Optimal antiderivative

$$\frac{\coth(x)}{4} + \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-i}}\right)}{4\sqrt{1-i}} + \frac{\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+i}}\right)}{4\sqrt{1+i}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{2}\right)\sqrt{2}}{8}$$

command

`integrate(1/(1-cosh(x)^8),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{16} \sqrt{2} \log\left(-\frac{2\sqrt{2} - e^{(2x)} - 3}{2\sqrt{2} + e^{(2x)} + 3}\right) + \frac{1}{2(e^{(2x)} - 1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 88 Test file number 173

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.3\_Hyperbolic\_tangent/173\_6.3.7-d\_hyper<sup>m</sup>-a+b-c\_tanh<sup>n</sup>-<sup>p</sup>

### 88.1 Problem number 229

$$\int \frac{\tanh^5(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b(\tanh^2(x))}}{\sqrt{a + b}}\right)}{\sqrt{a + b}} + \frac{(a - b)\sqrt{a + b(\tanh^2(x))}}{b^2} - \frac{(a + b(\tanh^2(x)))^{\frac{3}{2}}}{3b^2}$$

command

```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\left|-\left(\sqrt{a+b} e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b} e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b} e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{8\left(3\left(\sqrt{a+b} e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right)^5 + 3\left(\sqrt{a+b} e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right)\right)}{2\sqrt{a+b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.2 Problem number 230

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{(a-2b) \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}}\right)}{2b^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b(\tanh^2(x))} \tanh(x)}{2b}$$

command

```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{(a-2b) \arctan\left(-\frac{\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b} b} \\
& - \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \\
& - \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\
& + \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\
& - \frac{2\left(\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^3(a+2b) + \left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\right)}{\left(\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^3(a+2b) + \left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\right)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.3 Problem number 231

$$\int \frac{\tanh^3(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b \tanh^2(x)}}{b}$$

command

`integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \\
& + \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\
& - \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\
& - \frac{4\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right)^2 + 2\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)}{\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 + 2\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 88.4 Problem number 232

$$\int \frac{\tanh^2(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}}\right)}{\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}}\right)}{\sqrt{a + b}}$$

command

```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2 \operatorname{arctan}\left(\frac{-\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} \\ & - \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \\ & - \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\ & + \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



### 88.5 Problem number 233

$$\int \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a + b}}\right)}{\sqrt{a + b}}$$

command

```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \\ & + \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \\ & - \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.6 Problem number 235

$$\int \frac{\coth(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a + b}}\right)}{\sqrt{a + b}}$$

command

```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \arctan \left( -\frac{\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-a}} \right)}{\sqrt{-a}}$$

$$- \frac{\log \left( \left| -\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2\sqrt{a+b}}$$

$$+ \frac{\log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right| \right)}{2\sqrt{a+b}}$$

$$- \frac{\log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right| \right)}{2\sqrt{a+b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.7 Problem number 236

$$\int \frac{\coth^2(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b (\tanh^2(x))}} \right)}{\sqrt{a+b}} - \frac{\coth(x) \sqrt{a+b (\tanh^2(x))}}{a}$$

command

`integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$- \frac{\log \left( \left| -\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2\sqrt{a+b}}$$

$$- \frac{\log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right| \right)}{2\sqrt{a+b}}$$

$$+ \frac{\log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right| \right)}{2\sqrt{a+b}}$$

$$+ \frac{4 \left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right)}{\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right)^2 - 2 \left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.8 Problem number 237

$$\int \frac{\coth^3(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(2a - b) \operatorname{arctanh} \left( \frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a}} \right)}{2a^{\frac{3}{2}}} \\ & + \frac{\operatorname{arctanh} \left( \frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a + b}} \right)}{\sqrt{a + b}} - \frac{(\coth^2(x)) \sqrt{a + b (\tanh^2(x))}}{2a} \end{aligned}$$

command

```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(2a - b) \operatorname{arctan} \left( -\frac{\sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a + b}}{2\sqrt{-a}} \right)}{\sqrt{-a} a} \\ & \frac{\log \left( \left| -\left( \sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right) (a + b) - \sqrt{a + b} (a - b) \right| \right)}{2\sqrt{a + b}} \\ & + \frac{\log \left( \left| -\sqrt{a + b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a + b} \right| \right)}{2\sqrt{a + b}} \\ & - \frac{\log \left( \left| -\sqrt{a + b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a + b} \right| \right)}{2\sqrt{a + b}} \\ & + \frac{2 \left( \left( \sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right)^3 (2a + b) + \left( \sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right) \left( \left( \sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right) \right)}{\left( \left( \sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right) \right)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.9 Problem number 238

$$\int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}}\right)}{(a + b)^{\frac{3}{2}}} - \frac{a^2}{b^2 (a + b) \sqrt{a + b \tanh^2(x)}} - \frac{\sqrt{a + b \tanh^2(x)}}{b^2}$$

command

```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{\frac{(a^4 b + a^3 b^2) e^{(2x)}}{a^3 b^3 + 2 a^2 b^4 + a b^5} + \frac{a^4 b + a^3 b^2}{a^3 b^3 + 2 a^2 b^4 + a b^5}}{\sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b}} \\ & + \frac{\sqrt{a + b} \log\left(\left| -\sqrt{a + b} e^{(2x)} + \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b} + \sqrt{a + b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ & - \frac{\sqrt{a + b} \log\left(\left| -\sqrt{a + b} e^{(2x)} + \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b} - \sqrt{a + b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ & - \frac{\log\left(\left| -\left(\sqrt{a + b} e^{(2x)} - \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b}\right)(a + b) - \sqrt{a + b}(a - b) \right|\right)}{2(a + b)^{\frac{3}{2}}} \\ & - \frac{4\left(\sqrt{a + b} e^{(2x)} - \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b} - \sqrt{a + b}\right)}{\left(\left(\sqrt{a + b} e^{(2x)} - \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b}\right)^2 + 2\left(\sqrt{a + b} e^{(2x)} - \sqrt{a e^{(4x)} + b e^{(4x)} + 2 a e^{(2x)} - 2 b e^{(2x)} + a + b}\right)\right)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.10 Problem number 239

$$\int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b}(\tanh^2(x))}\right)}{b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b}(\tanh^2(x))}\right)}{(a+b)^{3/2}} \\ & + \frac{a \tanh(x)}{b(a+b)\sqrt{a+b}(\tanh^2(x))} \end{aligned}$$

command

```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5} - \frac{a^3b^2+a^2b^3}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} \\ & - \frac{\sqrt{a+b} \log\left(\left| -\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b) \right|\right)}{2(a^2 + 2ab + b^2)} \\ & - \frac{\sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ & + \frac{\sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ & - \frac{2 \operatorname{arctan}\left(\frac{-\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b} b} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.11 Problem number 240

$$\int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh^2(x)}{\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}} + \frac{a}{b(a+b)\sqrt{a+b}\tanh^2(x)}}$$

command

```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(a^3+a^2b)e^{(2x)}}{a^3b+2a^2b^2+ab^3} + \frac{a^3+a^2b}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} + \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)}} - \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.12 Problem number 241

$$\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b}\tanh^2(x)}\right)}{(a+b)^{\frac{3}{2}}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b}\tanh^2(x)}}$$

command

```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} \sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right|\right) \\ + \frac{\sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ \log\left(\left| -\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b) \right|\right) \\ \frac{2(a+b)^{\frac{3}{2}}}{2(a+b)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.13 Problem number 242

$$\int \frac{\tanh(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\tanh^2(x))}}{\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}} - \frac{1}{(a+b)\sqrt{a+b(\tanh^2(x))}}$$

command

```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3} + \frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} \sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right|\right) \\ + \frac{\sqrt{a+b} \log\left(\left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right|\right)}{2(a^2 + 2ab + b^2)} \\ \log\left(\left| -\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b) \right|\right) \\ \frac{2(a+b)^{\frac{3}{2}}}{2(a+b)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 88.14 Problem number 243

$$\int \frac{1}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}}\right)}{(a+b)^{3/2}} + \frac{b \tanh(x)}{a(a+b)\sqrt{a+b(\tanh^2(x))}}$$

command

```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(ab^2+b^3)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{ab^2+b^3}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} \sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right) - \frac{2(a^2 + 2ab + b^2)}{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)} + \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{3/2}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 88.15 Problem number 244

$$\int \frac{\coth(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\tanh^2(x))}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b(\tanh^2(x))}}{\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}} + \frac{b}{a(a+b)\sqrt{a+b(\tanh^2(x))}}$$

command

`integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} + \frac{a^3b^2+a^2b^3}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} \sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right) \\ - \frac{2(a^2 + 2ab + b^2)}{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)} \\ - \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)} \\ + \frac{2 \operatorname{arctan}\left(-\frac{\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a} a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.16 Problem number 245

$$\int \frac{\coth^2(x)}{(a + b \tanh^2(x))^{3/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}}\right)}{(a+b)^{\frac{3}{2}}} + \frac{b \coth(x)}{a(a+b) \sqrt{a+b(\tanh^2(x))}} - \frac{(a+2b) \coth(x) \sqrt{a+b(\tanh^2(x))}}{a^2(a+b)}$$

command

`integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} - \frac{a^2b^3+ab^4}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}} - \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)} + \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^2 + 2ab + b^2)} - \frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{4\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right)\left(\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 - 2\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\right)}{2(a+b)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.17 Problem number 246

$$\int \frac{\tanh^6(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}}\right)}{b^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b (\tanh^2(x))}}\right)}{(a + b)^{5/2}} \\ & + \frac{a(a + 2b) \tanh(x)}{b^2 (a + b)^2 \sqrt{a + b (\tanh^2(x))}} + \frac{a(\tanh^3(x))}{3b(a + b)(a + b (\tanh^2(x)))^{3/2}} \end{aligned}$$

command

`integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{a + b} \log\left(\left|-\left(\sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a + b) - \sqrt{a + b}(a - b)\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & - \frac{\sqrt{a + b} \log\left(\left|-\sqrt{a + b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a + b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{\sqrt{a + b} \log\left(\left|-\sqrt{a + b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a + b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{\left(\left(\frac{3a^9b^8 + 22a^8b^9 + 65a^7b^{10} + 100a^6b^{11} + 85a^5b^{12} + 38a^4b^{13} + 7a^3b^{14}}{a^8b^{10} + 6a^7b^{11} + 15a^6b^{12} + 20a^5b^{13} + 15a^4b^{14} + 6a^3b^{15} + a^2b^{16}}\right)e^{(2x)} + \frac{3(a^9b^8 + 2a^8b^9 - 9a^7b^{10} - 36a^6b^{11} - 49a^5b^{12} - 30a^4b^{13} - 7a^3b^{14} - 3a^2b^{15} - ab^{16})}{a^8b^{10} + 6a^7b^{11} + 15a^6b^{12} + 20a^5b^{13} + 15a^4b^{14} + 6a^3b^{15} + a^2b^{16}}\right)}{3(ae^{(4x)} + be^{(4x)} + a + b)} \\ & - \frac{2 \arctan\left(-\frac{\sqrt{a + b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a + b}}{2\sqrt{-b}}\right)}{\sqrt{-b} b^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.18 Problem number 247

$$\int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}}\right)}{(a + b)^{5/2}} + \frac{a(a + 2b)}{b^2 (a + b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{a^2}{3b^2 (a + b) (a + b \tanh^2(x))^{3/2}}$$

command

```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & - \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{2\left(\left(\frac{(a^9 + 8a^8b + 25a^7b^2 + 40a^6b^3 + 35a^5b^4 + 16a^4b^5 + 3a^3b^6)e^{(2x)}}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} + \frac{3(a^9 + 6a^8b + 13a^7b^2 + 12a^6b^3 + 3a^5b^4 - 2a^4b^5 - a^3b^6)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)e^{(2x)} + \frac{3(a^9 + 6a^8b + 13a^7b^2 + 12a^6b^3 + 3a^5b^4 - 2a^4b^5 - a^3b^6)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.19 Problem number 248

$$\int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a + b)^{5/2}} - \frac{(a + 4b) \tanh(x)}{3b(a + b)^2 \sqrt{a + b \tanh^2(x)}} + \frac{a \tanh(x)}{3b(a + b) (a + b \tanh^2(x))^{3/2}}$$

command

`integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a+b} \log \left( \left| -\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$- \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} + \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$+ \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} - \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$+ \frac{4 \left( \left( \left( \frac{a^7b^2 + 5a^6b^3 + 10a^5b^4 + 10a^4b^5 + 5a^3b^6 + a^2b^7}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} e^{(2x)} - \frac{3(a^6b^3 + 4a^5b^4 + 6a^4b^5 + 4a^3b^6 + a^2b^7)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} \right) e^{(2x)} + \frac{3(a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})} \right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.20 Problem number 249

$$\int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh} \left( \frac{\sqrt{a+b} \tanh^2(x)}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} - \frac{1}{(a+b)^2 \sqrt{a+b} \tanh^2(x)} + \frac{a}{3b(a+b) (a+b \tanh^2(x))^{3/2}}$$

command

`integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a+b} \log \left( \left| -\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$+ \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} + \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$- \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} - \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$+ \frac{\left( \left( \frac{a^8b^2 + 2a^7b^3 - 5a^6b^4 - 20a^5b^5 - 25a^4b^6 - 14a^3b^7 - 3a^2b^8}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} e^{(2x)} + \frac{3(a^8b^2 + 2a^7b^3 - a^6b^4 - 4a^5b^5 - a^4b^6 + 2a^3b^7 + a^2b^8)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} \right) e^{(2x)} + \frac{3(a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})} \right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.21 Problem number 250

$$\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}}\right)}{(a+b)^{\frac{5}{2}}} - \frac{(2a-b) \tanh(x)}{3a(a+b)^2 \sqrt{a+b(\tanh^2(x))}} - \frac{\tanh(x)}{3(a+b)(a+b(\tanh^2(x)))^{\frac{3}{2}}}$$

command

`integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & - \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} + \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} - \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & - \frac{\left(\left(\frac{3a^7b^2 + 14a^6b^3 + 25a^5b^4 + 20a^4b^5 + 5a^3b^6 - 2a^2b^7 - ab^8}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)e^{(2x)} + \frac{3(a^7b^2 + 2a^6b^3 - a^5b^4 - 4a^4b^5 - a^3b^6 + 2a^2b^7 + ab^8)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)e^{(2x)} - \frac{3(a^7b^2 + 2a^6b^3 - a^5b^4 - 4a^4b^5 - a^3b^6 + 2a^2b^7 + ab^8)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.22 Problem number 251

$$\int \frac{\tanh(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}}\right)}{(a + b)^{5/2}} - \frac{1}{(a + b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{1}{3(a + b)(a + b \tanh^2(x))^{3/2}}$$

command

```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$+ \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$- \frac{\sqrt{a+b} \log\left(\left|-\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)}$$

$$- \frac{4\left(\left(\frac{(a^7b^2 + 5a^6b^3 + 10a^5b^4 + 10a^4b^5 + 5a^3b^6 + a^2b^7)e^{(2x)}}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} + \frac{3(a^7b^2 + 4a^6b^3 + 6a^5b^4 + 4a^4b^5 + a^3b^6)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)e^{(2x)} + \frac{3(a^7b^2 + 4a^6b^3 + 6a^5b^4 + 4a^4b^5 + a^3b^6)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8}\right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.23 Problem number 252

$$\int \frac{1}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}}\right)}{(a + b)^{5/2}} + \frac{b(5a + 2b) \tanh(x)}{3a^2(a + b)^2 \sqrt{a + b \tanh^2(x)}} + \frac{b \tanh(x)}{3a(a + b)(a + b \tanh^2(x))^{3/2}}$$

command

```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & - \frac{\sqrt{a+b} \log \left( \left| - \left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & - \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\ & + \frac{2 \left( \left( \frac{(3a^6b^3 + 16a^5b^4 + 35a^4b^5 + 40a^3b^6 + 25a^2b^7 + 8ab^8 + b^9)e^{(2x)}}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} + \frac{3(a^6b^3 + 2a^5b^4 - 3a^4b^5 - 12a^3b^6 - 13a^2b^7 - 6ab^8 - b^9)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} \right) e^{(2x)} - \frac{3(a^6b^3 + 2a^5b^4 - 3a^4b^5 - 12a^3b^6 - 13a^2b^7 - 6ab^8 - b^9)}{a^8b^2 + 6a^7b^3 + 15a^6b^4 + 20a^5b^5 + 15a^4b^6 + 6a^3b^7 + a^2b^8} \right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)})} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.24 Problem number 253

$$\int \frac{\coth(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{\operatorname{arctanh} \left( \frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a}} \right)}{a^{5/2}} + \frac{\operatorname{arctanh} \left( \frac{\sqrt{a + b (\tanh^2(x))}}{\sqrt{a + b}} \right)}{(a + b)^{5/2}} \\ & + \frac{b(2a + b)}{a^2 (a + b)^2 \sqrt{a + b (\tanh^2(x))}} + \frac{b}{3a (a + b) (a + b (\tanh^2(x)))^{3/2}} \end{aligned}$$

command

```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\begin{aligned}
& - \frac{\sqrt{a+b} \log \left( \left| -\left( \sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} \right) (a+b) - \sqrt{a+b} (a-b) \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\
& + \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} + \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\
& - \frac{\sqrt{a+b} \log \left( \left| -\sqrt{a+b} e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} - \sqrt{a+b} \right| \right)}{2(a^3 + 3a^2b + 3ab^2 + b^3)} \\
& + \frac{\left( \left( \frac{7a^{14}b^3 + 38a^{13}b^4 + 85a^{12}b^5 + 100a^{11}b^6 + 65a^{10}b^7 + 22a^9b^8 + 3a^8b^9}{a^{16}b^2 + 6a^{15}b^3 + 15a^{14}b^4 + 20a^{13}b^5 + 15a^{12}b^6 + 6a^{11}b^7 + a^{10}b^8} \right) e^{(2x)} + \frac{3(7a^{14}b^3 + 30a^{13}b^4 + 49a^{12}b^5 + 36a^{11}b^6 + 9a^{10}b^7 - 2a^9b^8 - a^8b^9)}{a^{16}b^2 + 6a^{15}b^3 + 15a^{14}b^4 + 20a^{13}b^5 + 15a^{12}b^6 + 6a^{11}b^7 + a^{10}b^8} \right)}{3(ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b)} \\
& + \frac{2 \arctan \left( -\frac{\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a+b} - \sqrt{a+b}}{2\sqrt{-a}} \right)}{\sqrt{-a} a^2}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 88.25 Problem number 254

$$\int \frac{\coth^2(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{arctanh} \left( \frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b(\tanh^2(x))}} \right)}{(a+b)^{5/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a+b(\tanh^2(x))}} \\
& - \frac{(3a+2b)(a+4b) \coth(x) \sqrt{a+b(\tanh^2(x))}}{3a^3(a+b)^2} + \frac{b \coth(x)}{3a(a+b)(a+b(\tanh^2(x)))^{3/2}}
\end{aligned}$$

command

```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 89 Test file number 176

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.4\_Hyperbolic\_cotangent/176\_6.4.7-d\_hyper- $\hat{m}$ -a+b-c\_coth- $\hat{n}$ - $\hat{p}$

### 89.1 Problem number 34

$$\int \frac{\coth(x)}{\sqrt{a + b \coth^2(x)}} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a + b \coth^2(x)}}{\sqrt{a + b}}\right)}{\sqrt{a + b}}$$

command

```
integrate(coth(x)/(a+b*coth(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\left|-\left(\sqrt{a+b} e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} - 2ae^{(2x)} + 2be^{(2x)} + a + b}\right)_{(a+b)+\sqrt{a+b}} (a-b)\right|\right)}{\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b} e^{(2x)} + \dots\right|\right)}{\sqrt{a+b}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 90 Test file number 179

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.5\_Hyperbolic\_secant/179\_6.5.3\_Hyperbolic\_secant\_functions

### 90.1 Problem number 82

$$\int \frac{1}{(a + a \operatorname{sech}(c + dx))^{3/2}} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(dx+c)}{\sqrt{a + a \operatorname{sech}(dx+c)}}\right)}{a^{3/2} d} - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(dx+c) \sqrt{2}}{2\sqrt{a + a \operatorname{sech}(dx+c)}}\right) \sqrt{2}}{4a^{3/2} d} - \frac{\tanh(dx+c)}{2d(a + a \operatorname{sech}(dx+c))^{3/2}}$$

command

```
integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5\sqrt{2} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}e^{(dx+c)} - \sqrt{ae^{(2dx+2c)} + a} + \sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a} a} - \frac{4 \arctan\left(-\frac{\sqrt{a}e^{(dx+c)} - \sqrt{ae^{(2dx+2c)} + a}}{\sqrt{-a}}\right)}{\sqrt{-a} a} + \frac{2 \log\left(-\sqrt{-a}e^{(dx+c)} - \sqrt{ae^{(2dx+2c)} + a} + \sqrt{-a}\right)}{\sqrt{-a} a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 91 Test file number 180

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.5\_Hyperbolic\_secant/180\_6.5.7-d\_hyper-<sup>m</sup>-a+b-c\_sech-<sup>n</sup>-<sup>p</sup>

### 91.1 Problem number 139

$$\int \frac{\tanh^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{x}{a} - \frac{(a+b)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right)}{a b^{\frac{3}{2}} d} + \frac{\tanh(dx+c)}{bd}$$

command

```
integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{dx+c}{a} - \frac{(a^2+2ab+b^2) \operatorname{arctan}\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2} ab}}{d} - \frac{2}{b(e^{(2dx+2c)+1})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 91.2 Problem number 141

$$\int \frac{\tanh^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{x}{a} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right) \sqrt{a+b}}{ad\sqrt{b}}$$

command

```
integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(a+b) \operatorname{arctan}\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2} a} - \frac{dx+c}{a}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 91.3 Problem number 145

$$\int \frac{\coth^2(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{x}{a} - \frac{b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right)}{a(a+b)^{\frac{3}{2}}d} - \frac{\coth(dx+c)}{(a+b)d}$$

command

```
integrate(coth(d*x+c)^2/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{b^2 \operatorname{arctan}\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^2+ab)\sqrt{-ab-b^2}} - \frac{dx+c}{a} + \frac{2}{(a+b)(e^{(2dx+2c)}-1)}$$


---


$$d$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 91.4 Problem number 147

$$\int \frac{\coth^4(c + dx)}{a + b \operatorname{sech}^2(c + dx)} dx$$

Optimal antiderivative

$$\frac{x}{a} - \frac{b^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b} \tanh(dx+c)}{\sqrt{a+b}}\right)}{a(a+b)^{\frac{5}{2}}d} - \frac{(a+2b)\coth(dx+c)}{(a+b)^2d} - \frac{\coth^3(dx+c)}{3(a+b)d}$$

command

```
integrate(coth(d*x+c)^4/(a+b*sech(d*x+c)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3b^3 \operatorname{arctan}\left(\frac{ae^{(2dx+2c)+a+2b}}{2\sqrt{-ab-b^2}}\right)}{(a^3+2a^2b+ab^2)\sqrt{-ab-b^2}} - \frac{3(dx+c)}{a} + \frac{2(6ae^{(4dx+4c)}+9be^{(4dx+4c)}-6ae^{(2dx+2c)}-12be^{(2dx+2c)}+4a+7b)}{(a^2+2ab+b^2)(e^{(2dx+2c)}-1)^3}$$


---


$$3d$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 92 Test file number 185

Test folder name:

test\_cases/6\_Hyperbolic\_functions/6.7\_Miscellaneous/185\_6.7.1\_Hyperbolic\_functions

### 92.1 Problem number 3

$$\int \frac{\operatorname{sech}^2(2+3x)}{1+2\tanh^2(2+3x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)\sqrt{2}}{6}$$

command

```
integrate(sech(2+3*x)^2/(1+2*tanh(2+3*x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 92.2 Problem number 4

$$\int \frac{\operatorname{csch}^2(2+3x)}{2+\operatorname{coth}^2(2+3x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\sqrt{2}\tanh(2+3x)\right)\sqrt{2}}{6}$$

command

```
integrate(csch(2+3*x)^2/(2+coth(2+3*x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(3e^{(6x+4)}-1\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 92.3 Problem number 5

$$\int \frac{\operatorname{csch}^2(2+3x)}{2-\operatorname{coth}^2(2+3x)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\sqrt{2} \tanh(2+3x)\right) \sqrt{2}}{6}$$

command

```
integrate(csch(2+3*x)^2/(2-coth(2+3*x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{12} \sqrt{2} \log \left( \frac{\left| -4\sqrt{2} e^4 - 6e^4 + 2e^{(6x+8)} \right|}{\left| 4\sqrt{2} e^4 - 6e^4 + 2e^{(6x+8)} \right|} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 92.4 Problem number 6

$$\int \frac{\operatorname{csch}^2(2+3x)}{1+2\operatorname{coth}^2(2+3x)} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctan}\left(\frac{\sqrt{2} \tanh(2+3x)}{2}\right) \sqrt{2}}{6}$$

command

```
integrate(csch(2+3*x)^2/(1+2*coth(2+3*x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} \sqrt{2} \operatorname{arctan}\left(\frac{1}{4} \sqrt{2} \left(3e^{(6x+4)} + 1\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 92.5 Problem number 7

$$\int \frac{\operatorname{csch}^2(2+3x)}{1-2\operatorname{coth}^2(2+3x)} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(2+3x)}{2}\right)\sqrt{2}}{6}$$

command

```
integrate(csch(2+3*x)^2/(1-2*coth(2+3*x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{2\sqrt{2}e^4-3e^4-e^{(6x+8)}}{2\sqrt{2}e^4+3e^4+e^{(6x+8)}}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 92.6 Problem number 86

$$\int \operatorname{sech}^2(a+bx)\tanh^n(a+bx) dx$$

Optimal antiderivative

$$\frac{\tanh^{1+n}(bx+a)}{b(1+n)}$$

command

```
integrate(sech(b*x+a)^2*tanh(b*x+a)^n,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{e^{(2bx+2a)}-1}{e^{(2bx+2a)}+1}\right)^{n+1}}{b(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \tanh(bx+a)^n \operatorname{sech}(bx+a)^2 dx$$



### 92.7 Problem number 117

$$\int \coth^n(a + bx) \operatorname{csch}^2(a + bx) dx$$

Optimal antiderivative

$$-\frac{\coth^{1+n}(bx + a)}{b(1+n)}$$

command

`integrate(coth(b*x+a)^n*csch(b*x+a)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(\frac{e^{(2bx+2a)}+1}{e^{(2bx+2a)}-1}\right)^{n+1}}{b(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \coth(bx + a)^n \operatorname{csch}(bx + a)^2 dx$$

### 92.8 Problem number 827

$$\int \frac{\sinh(x)}{a + b \sinh(x) + c \sinh^2(x)} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\left(2ic - \left(ib + \sqrt{4ac - b^2}\right) \tanh\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2(a-c)c - ib\sqrt{4ac - b^2}}}\right) \sqrt{2} \left(i - \frac{b}{\sqrt{4ac - b^2}}\right)}{\sqrt{b^2 - 2(a-c)c - ib\sqrt{4ac - b^2}}} + \frac{\arctan\left(\frac{\left(2ic - ib \tanh\left(\frac{x}{2}\right) + \sqrt{4ac - b^2} \tanh\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2(a-c)c + ib\sqrt{4ac - b^2}}}\right) \sqrt{2} \left(i + \frac{b}{\sqrt{4ac - b^2}}\right)}{\sqrt{b^2 - 2(a-c)c + ib\sqrt{4ac - b^2}}}$$

command

`integrate(sinh(x)/(a+b*sinh(x)+c*sinh(x)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

0

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 92.9 Problem number 833

$$\int \frac{\cosh(x)}{a + b \cosh(x) + c \cosh^2(x)} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left( 1 - \frac{b}{\sqrt{-4ac+b^2}} \right)}{\sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} + \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left( 1 + \frac{b}{\sqrt{-4ac+b^2}} \right)}{\sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

command

```
integrate(cosh(x)/(a+b*cosh(x)+c*cosh(x)^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

0

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 92.10 Problem number 1000

$$\int \frac{\operatorname{sech}^2(x)}{\sqrt{-4 + \tanh^2(x)}} dx$$

Optimal antiderivative

$$\operatorname{arctanh} \left( \frac{\tanh(x)}{\sqrt{-4 + \tanh^2(x)}} \right)$$

command

```
integrate(sech(x)^2/(-4+tanh(x)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\frac{8}{3}\sqrt{3}\left(2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right) \\ -\log\left(\frac{8}{3}\sqrt{3}\left(-2i\sqrt{3}-3\right)-8\sqrt{3}e^{(2x)}+8\sqrt{3e^{(4x)}+10e^{(2x)}+3}\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{\tanh(x)^2-4}} dx$$

### 92.11 Problem number 1012

$$\int \operatorname{csch}(x) \log(\tanh(x)) \operatorname{sech}(x) dx$$

Optimal antiderivative

$$\frac{\ln(\tanh(x))^2}{2}$$

command

`integrate(csch(x)*log(tanh(x))*sech(x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \log\left(\frac{e^{(2x)}-1}{e^{(2x)}+1}\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{csch}(x) \log(\tanh(x)) \operatorname{sech}(x) dx$$

### 92.12 Problem number 1037

$$\int \frac{\tanh(c+dx)}{\sqrt{a \sinh^2(c+dx)}} dx$$

Optimal antiderivative

$$\frac{\arctan\left(\frac{\sqrt{a(\sinh^2(dx+c))}}{\sqrt{a}}\right)}{d\sqrt{a}}$$

command

```
integrate(tanh(d*x+c)/(a*sinh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \operatorname{arctan}\left(e^{(dx+c)}\right)}{\sqrt{a} \operatorname{dsgn}\left(e^{(3dx+3c)} - e^{(dx+c)}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 93 Test file number 194

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/194\_7.3.4\_u-a+b\_arctanh-c\_x-^p

### 93.1 Problem number 248

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{1 - a^2x^2} dx$$

Optimal antiderivative

$$\frac{2 \operatorname{arctanh}(ax)^{\frac{3}{2}}}{3a}$$

command

```
integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} \log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6a}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{\sqrt{\operatorname{artanh}(ax)}}{a^2x^2 - 1} dx$$

### 93.2 Problem number 258

$$\int \frac{\tanh^{-1}(ax)^p}{1 - a^2x^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arctanh}(ax)^{1+p}}{a(1+p)}$$

command

`integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{1}{2} \log\left(-\frac{ax+1}{ax-1}\right)\right)^{p+1}}{a(p+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{\operatorname{artanh}(ax)^p}{a^2x^2 - 1} dx$$

## 94 Test file number 196

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/196\_7.3.6\_Exponentia

### 94.1 Problem number 696

$$\int e^{2 \tanh^{-1}(ax)} \left(c - \frac{c}{a^2x^2}\right)^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{11a^3\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^4}{30(-ax+1)^3} + \frac{57a^6\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^7}{16(-ax+1)^3(ax+1)^3} - \frac{41a^5\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^6}{24(-ax+1)^3(ax+1)^2} \\ & - \frac{57a^4\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^5}{80(-ax+1)^3(ax+1)} + \frac{13a^2\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^3(ax+1)}{40(-ax+1)^3} - \frac{a\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^2(ax+1)}{15(-ax+1)^2} \\ & - \frac{\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x(ax+1)}{6(-ax+1)} - \frac{2a^6\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^7 \arcsin(ax)}{(-ax+1)^{\frac{7}{2}}(ax+1)^{\frac{7}{2}}} \\ & - \frac{25a^6\left(c - \frac{c}{a^2x^2}\right)^{\frac{7}{2}}x^7 \operatorname{arctanh}\left(\sqrt{-ax+1} \sqrt{ax+1}\right)}{16(-ax+1)^{\frac{7}{2}}(ax+1)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((a*x+1)^2/(-a^2*x^2+1)*(c-c/a^2/x^2)^(7/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{120} \left( \frac{375 c^{\frac{7}{2}} \arctan \left( -\frac{\sqrt{a^2 c} x - \sqrt{a^2 c x^2 - c}}{\sqrt{c}} \right) \operatorname{sgn}(x)}{a^2} + \frac{240 c^{\frac{7}{2}} \log \left( \left| -\sqrt{a^2 c} x + \sqrt{a^2 c x^2 - c} \right| \right) \operatorname{sgn}(x)}{a|a|} - \frac{120 \sqrt{c}}{a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 94.2 Problem number 724

$$\int e^{-2 \tanh^{-1}(ax)} \left( c - \frac{c}{a^2 x^2} \right)^{7/2} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{7a^6 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^7}{16(-ax+1)^3 (ax+1)^3} - \frac{3a^5 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^6}{8(-ax+1)^3 (ax+1)^2} + \frac{a \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^2}{15ax+15} \\ & + \frac{19a^4 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^5}{16(-ax+1)^3 (ax+1)} - \frac{2a^3 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^4}{3(-ax+1)^2 (ax+1)} + \frac{23a^2 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^3}{120(-ax+1)(ax+1)} \\ & - \frac{\left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x(-ax+1)}{6(ax+1)} + \frac{2a^6 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^7 \arcsin(ax)}{(-ax+1)^{\frac{7}{2}} (ax+1)^{\frac{7}{2}}} \\ & - \frac{25a^6 \left( c - \frac{c}{a^2 x^2} \right)^{\frac{7}{2}} x^7 \operatorname{arctanh} \left( \sqrt{-ax+1} \sqrt{ax+1} \right)}{16(-ax+1)^{\frac{7}{2}} (ax+1)^{\frac{7}{2}}} \end{aligned}$$

command

```
integrate((c-c/a^2/x^2)^(7/2)/(a*x+1)^2*(-a^2*x^2+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{120} \left( \frac{375 c^{\frac{7}{2}} \arctan \left( -\frac{\sqrt{a^2 c} x - \sqrt{a^2 c x^2 - c}}{\sqrt{c}} \right) \operatorname{sgn}(x)}{a^2} - \frac{240 c^{\frac{7}{2}} \log \left( \left| -\sqrt{a^2 c} x + \sqrt{a^2 c x^2 - c} \right| \right) \operatorname{sgn}(x)}{a|a|} - \frac{120 \sqrt{c}}{a} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 95 Test file number 197

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.3\_Inverse\_hyperbolic\_tangent/197\_7.3.7\_Inverse\_hy

### 95.1 Problem number 12

$$\int \tanh^{-1} \left( \frac{\sqrt{e} x}{\sqrt{d + ex^2}} \right) dx$$

Optimal antiderivative

$$x \operatorname{arctanh} \left( \frac{x\sqrt{e}}{\sqrt{ex^2 + d}} \right) - \frac{\sqrt{ex^2 + d}}{\sqrt{e}}$$

command

```
integrate(arctanh(x*e^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} x \log \left( \frac{\frac{\sqrt{e} x}{\sqrt{ex^2 + d}} + 1}{\frac{\sqrt{e} x}{\sqrt{ex^2 + d}} - 1} \right) - \frac{\sqrt{e^2 x^2 + de}}{e}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 95.2 Problem number 274

$$\int x^m \coth^{-1}(\tanh(a + bx)) dx$$

Optimal antiderivative

$$-\frac{bx^{2+m}}{m^2 + 3m + 2} + \frac{x^{1+m} \operatorname{arccoth}(\tanh(bx + a))}{1 + m}$$

command

```
integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^{m+1} \log\left(-\frac{e^{\frac{(2bx+2a)+1}{e^{(2bx+2a)-1}}+1}}{e^{\frac{(2bx+2a)+1}{e^{(2bx+2a)-1}}-1}}\right)}{2(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^m \operatorname{arccoth}(\tanh(bx+a)) dx$$

## 96 Test file number 198

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.4\_Inverse\_hyperbolic\_cotangent/198\_7.4.1\_Inverse\_

### 96.1 Problem number 1

$$\int x^5 \operatorname{coth}^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{x}{6a^5} + \frac{x^3}{18a^3} + \frac{x^5}{30a} + \frac{x^6 \operatorname{arccoth}(ax)}{6} - \frac{\operatorname{arctanh}(ax)}{6a^6}$$

command

`integrate(x^5*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{45} a \left( \frac{\frac{45(ax+1)^4}{(ax-1)^4} - \frac{90(ax+1)^3}{(ax-1)^3} + \frac{140(ax+1)^2}{(ax-1)^2} - \frac{70(ax+1)}{ax-1} + 23}{a^7 \left(\frac{ax+1}{ax-1} - 1\right)^5} + \frac{15 \left(\frac{3(ax+1)^5}{(ax-1)^5} + \frac{10(ax+1)^3}{(ax-1)^3} + \frac{3(ax+1)}{ax-1}\right) \log\left(-\frac{\frac{(ax+1)a}{ax-1} - a}{a \left(\frac{ax+1}{ax-1} + 1\right)} - \frac{(ax+1)a - a}{a \left(\frac{ax+1}{ax-1} + 1\right)}\right)}{a^7 \left(\frac{ax+1}{ax-1} - 1\right)^6} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^5 \operatorname{arccoth}(ax) dx$$



## 96.2 Problem number 2

$$\int x^4 \coth^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{x^2}{10a^3} + \frac{x^4}{20a} + \frac{x^5 \operatorname{arccoth}(ax)}{5} + \frac{\ln(-a^2 x^2 + 1)}{10a^5}$$

command

```
integrate(x^4*arccoth(a*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{5} a \left( \frac{\log\left(\frac{|ax+1|}{|ax-1|}\right)}{a^6} - \frac{\log\left(\left|\frac{ax+1}{ax-1} - 1\right|\right)}{a^6} + \frac{4\left(\frac{(ax+1)^3}{(ax-1)^3} - \frac{(ax+1)^2}{(ax-1)^2} + \frac{ax+1}{ax-1}\right)}{a^6\left(\frac{ax+1}{ax-1} - 1\right)^4} + \frac{\left(\frac{5(ax+1)^4}{(ax-1)^4} + \frac{10(ax+1)^2}{(ax-1)^2} + 1\right) \log\left(-\frac{\frac{ax+1}{ax-1}}{a\left(\frac{ax+1}{ax-1} - 1\right)}\right)}{a^6\left(\frac{ax+1}{ax-1} - 1\right)^5} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^4 \operatorname{arccoth}(ax) dx$$

## 96.3 Problem number 3

$$\int x^3 \coth^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{x}{4a^3} + \frac{x^3}{12a} + \frac{x^4 \operatorname{arccoth}(ax)}{4} - \frac{\operatorname{arctanh}(ax)}{4a^4}$$

command

```
integrate(x^3*arccoth(a*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} a \left( \frac{\frac{3(ax+1)^2}{(ax-1)^2} - \frac{3(ax+1)}{ax-1} + 2}{a^5 \left( \frac{ax+1}{ax-1} - 1 \right)^3} + \frac{3 \left( \frac{(ax+1)^3}{(ax-1)^3} + \frac{ax+1}{ax-1} \right) \log \left( -\frac{\frac{(ax+1)a - a}{\frac{ax-1}{ax-1+1}} + 1}{\frac{(ax+1)a - a}{\frac{ax-1}{ax-1+1}} - 1} \right)}{a^5 \left( \frac{ax+1}{ax-1} - 1 \right)^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(ax) dx$$

#### 96.4 Problem number 4

$$\int x^2 \operatorname{coth}^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{x^2}{6a} + \frac{x^3 \operatorname{arccoth}(ax)}{3} + \frac{\ln(-a^2 x^2 + 1)}{6a^3}$$

command

`integrate(x^2*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} a \left( \frac{\log \left( \left| \frac{ax+1}{ax-1} \right| \right)}{a^4} - \frac{\log \left( \left| \frac{ax+1}{ax-1} - 1 \right| \right)}{a^4} + \frac{\left( \frac{3(ax+1)^2}{(ax-1)^2} + 1 \right) \log \left( -\frac{\frac{(ax+1)a - a}{\frac{ax-1}{ax-1+1}} + 1}{\frac{(ax+1)a - a}{\frac{ax-1}{ax-1+1}} - 1} \right)}{a^4 \left( \frac{ax+1}{ax-1} - 1 \right)^3} + \frac{2(ax+1)}{(ax-1)a^4 \left( \frac{ax+1}{ax-1} - 1 \right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(ax) dx$$

## 96.5 Problem number 5

$$\int x \coth^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{x}{2a} + \frac{x^2 \operatorname{arccoth}(ax)}{2} - \frac{\operatorname{arctanh}(ax)}{2a^2}$$

command

`integrate(x*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a \left( \frac{1}{a^3 \left( \frac{ax+1}{ax-1} - 1 \right)} + \frac{(ax+1) \log \left( -\frac{\frac{(ax+1)a-a}{ax-1} + 1}{\frac{(ax+1)a-a}{ax-1} - 1} \right)}{(ax-1)a^3 \left( \frac{ax+1}{ax-1} - 1 \right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(ax) dx$$

## 96.6 Problem number 6

$$\int \coth^{-1}(ax) dx$$

Optimal antiderivative

$$x \operatorname{arccoth}(ax) + \frac{\ln(-a^2x^2 + 1)}{2a}$$

command

`integrate(arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a \left( \frac{\log \left( \left| \frac{ax+1}{ax-1} \right| \right)}{a^2} - \frac{\log \left( \left| \frac{ax+1}{ax-1} - 1 \right| \right)}{a^2} + \frac{\log \left( -\frac{\frac{(ax+1)a-a}{ax-1} + 1}{\frac{(ax+1)a-a}{ax-1} - 1} \right)}{a^2 \left( \frac{ax+1}{ax-1} - 1 \right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arcoth}(ax) dx$$

### 96.7 Problem number 8

$$\int \frac{\operatorname{coth}^{-1}(ax)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arccoath}(ax)}{x} + a \ln(x) - \frac{a \ln(-a^2x^2 + 1)}{2}$$

command

`integrate(arccoath(a*x)/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a \left( \frac{\log \left( -\frac{\frac{(ax+1)a-a}{ax-1} + 1}{\frac{(ax+1)a-a}{ax-1} - 1} \right)}{\frac{ax+1}{ax-1} + 1} - \log \left( \frac{|ax+1|}{|ax-1|} \right) + \log \left( \left| \frac{ax+1}{ax-1} + 1 \right| \right) \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arcoth}(ax)}{x^2} dx$$

### 96.8 Problem number 9

$$\int \frac{\operatorname{coth}^{-1}(ax)}{x^3} dx$$

Optimal antiderivative

$$-\frac{a}{2x} - \frac{\operatorname{arccoath}(ax)}{2x^2} + \frac{a^2 \operatorname{arctanh}(ax)}{2}$$

command

`integrate(arccoth(a*x)/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$a \left( \frac{a}{\frac{ax+1}{ax-1} + 1} + \frac{(ax+1)a \log \left( -\frac{\frac{(ax+1)a}{ax-1} - a}{a \left( \frac{ax+1}{ax-1} + 1 \right)} \right)}{(ax-1) \left( \frac{ax+1}{ax-1} + 1 \right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{x^3} dx$$

## 96.9 Problem number 10

$$\int \frac{\operatorname{coth}^{-1}(ax)}{x^4} dx$$

Optimal antiderivative

$$-\frac{a}{6x^2} - \frac{\operatorname{arccoth}(ax)}{3x^3} + \frac{a^3 \ln(x)}{3} - \frac{a^3 \ln(-a^2x^2 + 1)}{6}$$

command

`integrate(arccoth(a*x)/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{3} \left( a^2 \log \left( \left| \frac{ax+1}{ax-1} \right| \right) - a^2 \log \left( \left| \frac{ax+1}{ax-1} + 1 \right| \right) - \frac{2(ax+1)a^2}{(ax-1) \left( \frac{ax+1}{ax-1} + 1 \right)^2} - \frac{\left( \frac{3(ax+1)^2 a^2}{(ax-1)^2} + a^2 \right) \log \left( -\frac{\frac{(ax+1)a}{ax-1} - a}{a \left( \frac{ax+1}{ax-1} + 1 \right)} \right)}{\left( \frac{ax+1}{ax-1} + 1 \right)^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{x^4} dx$$

## 96.10 Problem number 11

$$\int \frac{\coth^{-1}(ax)}{x^5} dx$$

Optimal antiderivative

$$-\frac{a}{12x^3} - \frac{a^3}{4x} - \frac{\operatorname{arccoth}(ax)}{4x^4} + \frac{a^4 \operatorname{arctanh}(ax)}{4}$$

command

`integrate(arccoth(a*x)/x^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} a \left( \frac{\frac{3(ax+1)^2 a^3}{(ax-1)^2} + \frac{3(ax+1)a^3}{ax-1} + 2a^3}{\left(\frac{ax+1}{ax-1} + 1\right)^3} + \frac{3 \left( \frac{(ax+1)^3 a^3}{(ax-1)^3} + \frac{(ax+1)a^3}{ax-1} \right) \log \left( -\frac{\frac{(ax+1)a}{ax-1} - a}{a \left( \frac{ax+1}{ax-1} + 1 \right) + 1} \right)}{\left(\frac{ax+1}{ax-1} + 1\right)^4} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{x^5} dx$$

## 96.11 Problem number 12

$$\int x^5 \coth^{-1}(ax)^2 dx$$

Optimal antiderivative

$$\frac{4x^2}{45a^4} + \frac{x^4}{60a^2} + \frac{x \operatorname{arccoth}(ax)}{3a^5} + \frac{x^3 \operatorname{arccoth}(ax)}{9a^3} + \frac{x^5 \operatorname{arccoth}(ax)}{15a} - \frac{\operatorname{arccoth}(ax)^2}{6a^6} + \frac{x^6 \operatorname{arccoth}(ax)^2}{6} + \frac{23 \ln(-a^2 x^2 + 1)}{90a^6}$$

command

`integrate(x^5*arccoth(a*x)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{90} \left( \frac{15 \left( \frac{3(ax+1)^5}{(ax-1)^5} + \frac{10(ax+1)^3}{(ax-1)^3} + \frac{3(ax+1)}{ax-1} \right) \log \left( \frac{ax+1}{ax-1} \right)^2}{\frac{(ax+1)^6 a^7}{(ax-1)^6} - \frac{6(ax+1)^5 a^7}{(ax-1)^5} + \frac{15(ax+1)^4 a^7}{(ax-1)^4} - \frac{20(ax+1)^3 a^7}{(ax-1)^3} + \frac{15(ax+1)^2 a^7}{(ax-1)^2} - \frac{6(ax+1) a^7}{ax-1} + a^7} + \frac{2 \left( \frac{45(ax+1)^4}{(ax-1)^4} - \frac{90(ax+1)^3}{(ax-1)^3} \right)}{\frac{(ax+1)^5 a^7}{(ax-1)^5} - \frac{5(ax+1)^4 a^7}{(ax-1)^4} + \dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^5 \operatorname{arccoth}(ax)^2 dx$$

## 96.12 Problem number 14

$$\int x^3 \coth^{-1}(ax)^2 dx$$

Optimal antiderivative

$$\frac{x^2}{12a^2} + \frac{x \operatorname{arccoth}(ax)}{2a^3} + \frac{x^3 \operatorname{arccoth}(ax)}{6a} - \frac{\operatorname{arccoth}(ax)^2}{4a^4} + \frac{x^4 \operatorname{arccoth}(ax)^2}{4} + \frac{\ln(-a^2 x^2 + 1)}{3a^4}$$

command

`integrate(x^3*arccoth(a*x)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} \left( \frac{3 \left( \frac{(ax+1)^3}{(ax-1)^3} + \frac{ax+1}{ax-1} \right) \log \left( \frac{ax+1}{ax-1} \right)^2}{\frac{(ax+1)^4 a^5}{(ax-1)^4} - \frac{4(ax+1)^3 a^5}{(ax-1)^3} + \frac{6(ax+1)^2 a^5}{(ax-1)^2} - \frac{4(ax+1) a^5}{ax-1} + a^5} + \frac{2 \left( \frac{3(ax+1)^2}{(ax-1)^2} - \frac{3(ax+1)}{ax-1} + 2 \right) \log \left( \frac{ax+1}{ax-1} \right)}{\frac{(ax+1)^3 a^5}{(ax-1)^3} - \frac{3(ax+1)^2 a^5}{(ax-1)^2} + \frac{3(ax+1) a^5}{ax-1} - a^5} + \frac{\left( \frac{(ax+1)^2 a^5}{(ax-1)^2} - \dots \right)}{\dots} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(ax)^2 dx$$

## 96.13 Problem number 16

$$\int x \coth^{-1}(ax)^2 dx$$

Optimal antiderivative

$$\frac{x \operatorname{arccoth}(ax)}{a} - \frac{\operatorname{arccoth}(ax)^2}{2a^2} + \frac{x^2 \operatorname{arccoth}(ax)^2}{2} + \frac{\ln(-a^2 x^2 + 1)}{2a^2}$$

command

`integrate(x*arccoth(a*x)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} a \left( \frac{(ax+1) \log\left(\frac{ax+1}{ax-1}\right)^2}{\left(\frac{(ax+1)^2 a^3}{(ax-1)^2} - \frac{2(ax+1)a^3}{ax-1} + a^3\right)(ax-1)} + \frac{2 \log\left(\frac{ax+1}{ax-1}\right)}{\frac{(ax+1)a^3}{ax-1} - a^3} - \frac{2 \log\left(\frac{ax+1}{ax-1} - 1\right)}{a^3} + \frac{2 \log\left(\frac{ax+1}{ax-1}\right)}{a^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(ax)^2 dx$$

## 96.14 Problem number 20

$$\int \frac{\operatorname{coth}^{-1}(ax)^2}{x^3} dx$$

Optimal antiderivative

$$-\frac{a \operatorname{arccoth}(ax)}{x} + \frac{a^2 \operatorname{arccoth}(ax)^2}{2} - \frac{\operatorname{arccoth}(ax)^2}{2x^2} + a^2 \ln(x) - \frac{a^2 \ln(-a^2 x^2 + 1)}{2}$$

command

`integrate(arccoth(a*x)^2/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \left( 2a \log\left(\frac{ax+1}{ax-1} + 1\right) - 2a \log\left(\frac{ax+1}{ax-1}\right) + \frac{(ax+1)a \log\left(\frac{ax+1}{ax-1}\right)^2}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2} + \frac{2(ax+1)}{ax-1} + 1\right)} + \frac{2a \log\left(\frac{ax+1}{ax-1}\right)}{\frac{ax+1}{ax-1} + 1} \right) a$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^3} dx$$



## 96.15 Problem number 22

$$\int \frac{\coth^{-1}(ax)^2}{x^5} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{a^2}{12x^2} - \frac{a \operatorname{arccoth}(ax)}{6x^3} - \frac{a^3 \operatorname{arccoth}(ax)}{2x} + \frac{a^4 \operatorname{arccoth}(ax)^2}{4} \\ & - \frac{\operatorname{arccoth}(ax)^2}{4x^4} + \frac{2a^4 \ln(x)}{3} - \frac{a^4 \ln(-a^2x^2 + 1)}{3} \end{aligned}$$

command

```
integrate(arccoth(a*x)^2/x^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} \left( 4a^3 \log\left(\frac{ax+1}{ax-1} + 1\right) - 4a^3 \log\left(\frac{ax+1}{ax-1}\right) + \frac{2(ax+1)a^3}{(ax-1)\left(\frac{(ax+1)^2}{(ax-1)^2} + \frac{2(ax+1)}{ax-1} + 1\right)} + \frac{3\left(\frac{(ax+1)^3 a^3}{(ax-1)^3} + \frac{(ax+1)a^3}{ax-1}\right)}{\frac{(ax+1)^4}{(ax-1)^4} + \frac{4(ax+1)^3}{(ax-1)^3} + \frac{6(ax+1)}{ax-1}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)^2}{x^5} dx$$

## 96.16 Problem number 35

$$\int (c + dx^2)^4 \coth^{-1}(ax) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(420a^6c^3 + 378a^4c^2d + 180a^2cd^2 + 35d^3)x^2}{630a^7} + \frac{d^2(378a^4c^2 + 180a^2cd + 35d^2)x^4}{1260a^5} \\ & + \frac{d^3(36a^2c + 7d)x^6}{378a^3} + \frac{d^4x^8}{72a} + c^4x \operatorname{arccoth}(ax) + \frac{4c^3dx^3 \operatorname{arccoth}(ax)}{3} \\ & + \frac{6c^2d^2x^5 \operatorname{arccoth}(ax)}{5} + \frac{4cd^3x^7 \operatorname{arccoth}(ax)}{7} + \frac{d^4x^9 \operatorname{arccoth}(ax)}{9} \\ & + \frac{(315a^8c^4 + 420a^6c^3d + 378a^4c^2d^2 + 180a^2cd^3 + 35d^4) \ln(-a^2x^2 + 1)}{630a^9} \end{aligned}$$

command

```
integrate((d*x^2+c)^4*arccoth(a*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (dx^2 + c)^4 \operatorname{arccoth}(ax) dx$$

### 96.17 Problem number 36

$$\int (c + dx^2)^3 \operatorname{coth}^{-1}(ax) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(35a^4c^2 + 21a^2cd + 5d^2)x^2}{70a^5} + \frac{d^2(21a^2c + 5d)x^4}{140a^3} + \frac{d^3x^6}{42a} \\ & + c^3x \operatorname{arccoth}(ax) + c^2dx^3 \operatorname{arccoth}(ax) + \frac{3cd^2x^5 \operatorname{arccoth}(ax)}{5} \\ & + \frac{d^3x^7 \operatorname{arccoth}(ax)}{7} + \frac{(35a^6c^3 + 35a^4c^2d + 21a^2cd^2 + 5d^3) \ln(-a^2x^2 + 1)}{70a^7} \end{aligned}$$

command

`integrate((d*x^2+c)^3*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (dx^2 + c)^3 \operatorname{arccoth}(ax) dx$$

### 96.18 Problem number 37

$$\int (c + dx^2)^2 \operatorname{coth}^{-1}(ax) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{d(10a^2c + 3d)x^2}{30a^3} + \frac{d^2x^4}{20a} + c^2x \operatorname{arccoth}(ax) + \frac{2cdx^3 \operatorname{arccoth}(ax)}{3} \\ & + \frac{d^2x^5 \operatorname{arccoth}(ax)}{5} + \frac{(15a^4c^2 + 10a^2cd + 3d^2) \ln(-a^2x^2 + 1)}{30a^5} \end{aligned}$$

command

`integrate((d*x^2+c)^2*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{15} a \left( \frac{(15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log\left(\left|\frac{ax+1}{ax-1}\right|\right)}{a^6} - \frac{(15 a^4 c^2 + 10 a^2 c d + 3 d^2) \log\left(\left|\frac{ax+1}{ax-1} - 1\right|\right)}{a^6} + \frac{4 \left(\frac{5 a^2 c d + 3 d^2}{(ax-1)^3}\right)(ax+1)}{(ax-1)^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (dx^2 + c)^2 \operatorname{arccoth}(ax) dx$$

### 96.19 Problem number 38

$$\int (c + dx^2) \operatorname{coth}^{-1}(ax) dx$$

Optimal antiderivative

$$\frac{dx^2}{6a} + cx \operatorname{arccoth}(ax) + \frac{dx^3 \operatorname{arccoth}(ax)}{3} + \frac{(3a^2c + d) \ln(-a^2x^2 + 1)}{6a^3}$$

command

`integrate((d*x^2+c)*arccoth(a*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} a \left( \frac{(3 a^2 c + d) \log\left(\left|\frac{ax+1}{ax-1}\right|\right)}{a^4} - \frac{(3 a^2 c + d) \log\left(\left|\frac{ax+1}{ax-1} - 1\right|\right)}{a^4} + \frac{2 (ax + 1) d}{(ax - 1) a^4 \left(\frac{ax+1}{ax-1} - 1\right)^2} + \frac{\left(\frac{3 (ax+1)^2 a^2 c}{(ax-1)^2} - \frac{6 (ax+1) a^2 d}{ax-1}\right)}{(ax-1)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (dx^2 + c) \operatorname{arccoth}(ax) dx$$

## 96.20 Problem number 44

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{dx^2+c}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}} + \frac{x \operatorname{arccoth}(ax)}{c\sqrt{dx^2+c}}$$

command

`integrate(arccoth(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x \log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{2\sqrt{dx^2+c}c} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-d}c}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2+c)^{\frac{3}{2}}} dx$$

## 96.21 Problem number 45

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{5/2}} dx$$

Optimal antiderivative

$$\frac{x \operatorname{arccoth}(ax)}{3c(dx^2+c)^{\frac{3}{2}}} - \frac{(3a^2c+2d) \operatorname{arctanh}\left(\frac{a\sqrt{dx^2+c}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{\frac{3}{2}}} + \frac{a}{3c(a^2c+d)\sqrt{dx^2+c}} + \frac{2x \operatorname{arccoth}(ax)}{3c^2\sqrt{dx^2+c}}$$

command

`integrate(arccoth(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} a \left( \frac{(3a^2c + 2d) \arctan\left(\frac{\sqrt{dx^2 + ca}}{\sqrt{-a^2c - d}}\right)}{(a^2c^3 + c^2d)\sqrt{-a^2c - d} a} + \frac{1}{(a^2c^2 + cd)\sqrt{dx^2 + c}} \right) + \frac{x\left(\frac{2dx^2}{c^2} + \frac{3}{c}\right) \log\left(-\frac{\frac{1}{ax} + 1}{\frac{1}{ax} - 1}\right)}{6(dx^2 + c)^{\frac{3}{2}}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{5}{2}}} dx$$

## 96.22 Problem number 46

$$\int \frac{\coth^{-1}(ax)}{(c + dx^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a}{15c(a^2c + d)(dx^2 + c)^{\frac{3}{2}}} + \frac{x \operatorname{arccoth}(ax)}{5c(dx^2 + c)^{\frac{5}{2}}} + \frac{4x \operatorname{arccoth}(ax)}{15c^2(dx^2 + c)^{\frac{3}{2}}} \\ & - \frac{(15a^4c^2 + 20a^2cd + 8d^2) \operatorname{arctanh}\left(\frac{a\sqrt{dx^2 + c}}{\sqrt{a^2c + d}}\right)}{15c^3(a^2c + d)^{\frac{5}{2}}} \\ & + \frac{a(7a^2c + 4d)}{15c^2(a^2c + d)^2\sqrt{dx^2 + c}} + \frac{8x \operatorname{arccoth}(ax)}{15c^3\sqrt{dx^2 + c}} \end{aligned}$$

command

`integrate(arccoth(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{15} a \left( \frac{(15a^4c^2 + 20a^2cd + 8d^2) \arctan\left(\frac{\sqrt{dx^2 + ca}}{\sqrt{-a^2c - d}}\right)}{(a^4c^5 + 2a^2c^4d + c^3d^2)\sqrt{-a^2c - d} a} + \frac{7(dx^2 + c)a^2c + a^2c^2 + 4(dx^2 + c)d + cd}{(a^4c^4 + 2a^2c^3d + c^2d^2)(dx^2 + c)^{\frac{3}{2}}} \right) \\ & + \frac{\left(4x^2\left(\frac{2d^2x^2}{c^3} + \frac{5d}{c^2}\right) + \frac{15}{c}\right)x \log\left(-\frac{\frac{1}{ax} + 1}{\frac{1}{ax} - 1}\right)}{30(dx^2 + c)^{\frac{5}{2}}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2 + c)^{\frac{7}{2}}} dx$$

## 96.23 Problem number 47

$$\int \frac{\coth^{-1}(ax)}{(c+dx^2)^{9/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{a}{35c(a^2c+d)(dx^2+c)^{5/2}} + \frac{a(11a^2c+6d)}{105c^2(a^2c+d)^2(dx^2+c)^{3/2}} + \frac{x \operatorname{arccoth}(ax)}{7c(dx^2+c)^{7/2}} + \frac{6x \operatorname{arccoth}(ax)}{35c^2(dx^2+c)^{5/2}} \\ & + \frac{8x \operatorname{arccoth}(ax)}{35c^3(dx^2+c)^{3/2}} - \frac{(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3) \operatorname{arctanh}\left(\frac{a\sqrt{dx^2+c}}{\sqrt{a^2c+d}}\right)}{35c^4(a^2c+d)^{7/2}} \\ & + \frac{a(19a^4c^2+22a^2cd+8d^2)}{35c^3(a^2c+d)^3\sqrt{dx^2+c}} + \frac{16x \operatorname{arccoth}(ax)}{35c^4\sqrt{dx^2+c}} \end{aligned}$$

command

`integrate(arccoth(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{105} a \left( \frac{3(35a^6c^3+70a^4c^2d+56a^2cd^2+16d^3) \operatorname{arctan}\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^6c^7+3a^4c^6d+3a^2c^5d^2+c^4d^3)\sqrt{-a^2c-d}} a \right) + \frac{57(dx^2+c)^2a^4c^2+11(dx^2+c)a^4c^3}{70(dx^2+c)^{7/2}} \\ & + \frac{\left(2\left(4x^2\left(\frac{2d^3x^2}{c^4}+\frac{7d^2}{c^3}\right)+\frac{35d}{c^2}\right)x^2+\frac{35}{c}\right)x \log\left(-\frac{\frac{1}{ax}+1}{\frac{1}{ax}-1}\right)}{70(dx^2+c)^{7/2}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(ax)}{(dx^2+c)^{9/2}} dx$$

## 96.24 Problem number 50

$$\int \frac{\coth^{-1}(x)}{(a-ax^2)^{3/2}} dx$$

Optimal antiderivative

$$-\frac{1}{a\sqrt{-ax^2+a}} + \frac{x \operatorname{arccoth}(x)}{a\sqrt{-ax^2+a}}$$

command

`integrate(arccoth(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-ax^2+a} x \log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{2(ax^2-a)a} - \frac{1}{\sqrt{-ax^2+a} a}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2+a)^{\frac{3}{2}}} dx$$

## 96.25 Problem number 51

$$\int \frac{\operatorname{coth}^{-1}(x)}{(a-ax^2)^{5/2}} dx$$

Optimal antiderivative

$$-\frac{1}{9a(-ax^2+a)^{\frac{3}{2}}} + \frac{x \operatorname{arccoth}(x)}{3a(-ax^2+a)^{\frac{3}{2}}} - \frac{2}{3a^2\sqrt{-ax^2+a}} + \frac{2x \operatorname{arccoth}(x)}{3a^2\sqrt{-ax^2+a}}$$

command

`integrate(arccoth(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-ax^2+a} x \left(\frac{2x^2}{a} - \frac{3}{a}\right) \log\left(-\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\right)}{6(ax^2-a)^2} - \frac{6ax^2-7a}{9(ax^2-a)\sqrt{-ax^2+a} a^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2+a)^{\frac{5}{2}}} dx$$

## 96.26 Problem number 52

$$\int \frac{\coth^{-1}(x)}{(a - ax^2)^{7/2}} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{1}{25a(-ax^2+a)^{\frac{5}{2}}} - \frac{4}{45a^2(-ax^2+a)^{\frac{3}{2}}} + \frac{x \operatorname{arccoth}(x)}{5a(-ax^2+a)^{\frac{5}{2}}} \\ & + \frac{4x \operatorname{arccoth}(x)}{15a^2(-ax^2+a)^{\frac{3}{2}}} - \frac{8}{15a^3\sqrt{-ax^2+a}} + \frac{8x \operatorname{arccoth}(x)}{15a^3\sqrt{-ax^2+a}} \end{aligned}$$

command

`integrate(arccoth(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\sqrt{-ax^2+a} \left(4x^2 \left(\frac{2x^2}{a} - \frac{5}{a}\right) + \frac{15}{a}\right) x \log\left(-\frac{\frac{1}{x}+1}{x-1}\right)}{30(ax^2-a)^3} - \frac{120(ax^2-a)^2 - 20(ax^2-a)a + 9a^2}{225(ax^2-a)^2\sqrt{-ax^2+a}a^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(x)}{(-ax^2+a)^{\frac{7}{2}}} dx$$

## 96.27 Problem number 53

$$\int \frac{1}{(1-x^2)\coth^{-1}(x)} dx$$

Optimal antiderivative

$$\ln(\operatorname{arccoth}(x))$$

command

`integrate(1/(-x^2+1)/arccoth(x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\left|\log\left(\frac{x+1}{x-1}\right)\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{1}{(x^2-1)\operatorname{arccoth}(x)} dx$$



## 96.28 Problem number 55

$$\int \frac{\coth^{-1}(x)^2}{(1-x^2)^2} dx$$

Optimal antiderivative

$$\frac{x}{-4x^2+4} - \frac{\operatorname{arccoth}(x)}{2(-x^2+1)} + \frac{x\operatorname{arccoth}(x)^2}{-2x^2+2} + \frac{\operatorname{arccoth}(x)^3}{6} + \frac{\operatorname{arctanh}(x)}{4}$$

command

`integrate(arccoth(x)^2/(-x^2+1)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(x-1)\log\left(\frac{x+1}{x-1}\right)^2}{16(x+1)} - \frac{(x-1)\log\left(\frac{x+1}{x-1}\right)}{8(x+1)} - \frac{x-1}{8(x+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(x)^2}{(x^2-1)^2} dx$$

## 96.29 Problem number 57

$$\int \frac{\coth^{-1}(x)}{1-x^2} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(x)^2}{2}$$

command

`integrate(arccoth(x)/(-x^2+1),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{8} \log\left(\frac{x+1}{x-1}\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{\operatorname{arccoth}(x)}{x^2-1} dx$$

## 96.30 Problem number 58

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^2} dx$$

Optimal antiderivative

$$-\frac{x}{4(-x^2+1)} + \frac{\operatorname{arccoth}(x)}{-2x^2+2} - \frac{\operatorname{arctanh}(x)}{4}$$

command

`integrate(x*arccoth(x)/(-x^2+1)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{16} \left( \frac{x+1}{x-1} + \frac{x-1}{x+1} \right) \log \left( -\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} + 1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} - 1} \right) + \frac{x+1}{16(x-1)} - \frac{x-1}{16(x+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x \operatorname{arccoth}(x)}{(x^2-1)^2} dx$$

## 96.31 Problem number 59

$$\int \frac{\coth^{-1}(x)}{(1-x^2)^2} dx$$

Optimal antiderivative

$$-\frac{1}{4(-x^2+1)} + \frac{x \operatorname{arccoth}(x)}{-2x^2+2} + \frac{\operatorname{arccoth}(x)^2}{4}$$

command

`integrate(arccoth(x)/(-x^2+1)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(x-1) \log \left( -\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} + 1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} - 1} \right)}{8(x+1)} - \frac{x-1}{8(x+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(x)}{(x^2-1)^2} dx$$

## 96.32 Problem number 60

$$\int \frac{x \coth^{-1}(x)}{(1-x^2)^3} dx$$

Optimal antiderivative

$$-\frac{x}{16(-x^2+1)^2} - \frac{3x}{32(-x^2+1)} + \frac{\operatorname{arccoth}(x)}{4(-x^2+1)^2} - \frac{3 \operatorname{arctanh}(x)}{32}$$

command

`integrate(x*arccoth(x)/(-x^2+1)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{128} \left( \frac{(x-1)^2 \left( \frac{4(x+1)}{x-1} - 1 \right)}{(x+1)^2} - \frac{(x+1)^2}{(x-1)^2} + \frac{4(x+1)}{x-1} \right) \log \left( -\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} + 1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1} - 1} \right) \\ - \frac{(x-1)^2 \left( \frac{8(x+1)}{x-1} - 1 \right)}{256(x+1)^2} - \frac{(x+1)^2}{256(x-1)^2} + \frac{x+1}{32(x-1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{x \operatorname{arccoth}(x)}{(x^2-1)^3} dx$$

## 96.33 Problem number 62

$$\int x^3 \coth^{-1}(a+bx) dx$$

Optimal antiderivative

$$\frac{(6a^2+1)x}{4b^3} - \frac{a(bx+a)^2}{2b^4} + \frac{(bx+a)^3}{12b^4} + \frac{x^4 \operatorname{arccoth}(bx+a)}{4} \\ + \frac{(1-a)^4 \ln(-bx-a+1)}{8b^4} - \frac{(1+a)^4 \ln(bx+a+1)}{8b^4}$$

command

`integrate(x^3*arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6}((a+1)b - (a-1)b) \left( \frac{3(a^3 + a) \log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{b^5} - \frac{3(a^3 + a) \log\left(\left|\frac{bx+a+1}{bx+a-1} - 1\right|\right)}{b^5} - \frac{9a^2 + \frac{3(3a^2 - 2a + 1)(bx+a+1)}{(bx+a-1)^2}}{b^5 \left(\frac{bx+a+1}{bx+a-1}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(bx + a) dx$$

### 96.34 Problem number 63

$$\int x^2 \operatorname{coth}^{-1}(a + bx) dx$$

Optimal antiderivative

$$-\frac{ax}{b^2} + \frac{(bx+a)^2}{6b^3} + \frac{x^3 \operatorname{arccoth}(bx+a)}{3} + \frac{(1-a)^3 \ln(-bx-a+1)}{6b^3} + \frac{(1+a)^3 \ln(bx+a+1)}{6b^3}$$

command

`integrate(x^2*arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6}((a+1)b - (a-1)b) \left( \frac{(3a^2 + 1) \log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{b^4} - \frac{(3a^2 + 1) \log\left(\left|\frac{bx+a+1}{bx+a-1} - 1\right|\right)}{b^4} - \frac{2\left(\frac{(bx+a+1)(3a-1)}{bx+a-1} - 3a\right)}{b^4 \left(\frac{bx+a+1}{bx+a-1} - 1\right)^2} + \dots \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(bx + a) dx$$

### 96.35 Problem number 64

$$\int x \coth^{-1}(a + bx) dx$$

Optimal antiderivative

$$\frac{x}{2b} + \frac{x^2 \operatorname{arccoth}(bx + a)}{2} + \frac{(1 - a)^2 \ln(-bx - a + 1)}{4b^2} - \frac{(1 + a)^2 \ln(bx + a + 1)}{4b^2}$$

command

`integrate(x*arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2}((a+1)b - (a-1)b) \left( \frac{a \log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{b^3} - \frac{a \log\left(\left|\frac{bx+a+1}{bx+a-1} - 1\right|\right)}{b^3} + \frac{\left(\frac{(bx+a+1)a}{bx+a-1} - a - \frac{bx+a+1}{bx+a-1}\right) \log\left(\frac{\frac{(bx+a+1)}{bx+a} - \frac{(bx+a+1)}{bx}}{a - \frac{(bx+a+1)}{bx+a}}\right)}{b^3 \left(\frac{bx+a+1}{bx+a-1} - 1\right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(bx + a) dx$$

### 96.36 Problem number 65

$$\int \coth^{-1}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a) \operatorname{arccoth}(bx + a)}{b} + \frac{\ln(1 - (bx + a)^2)}{2b}$$

command

`integrate(arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} ((a+1)b - (a-1)b) \left( \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{b^2} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1} - 1\right|\right)}{b^2} + \frac{\log\left(\frac{\frac{1}{a - \frac{\frac{1}{(bx+a+1)(a-1)} - a - 1} b} - b}}{\frac{1}{a - \frac{\frac{1}{(bx+a+1)(a-1)} - a - 1} b} - b} - 1\right)}{b^2 \left(\frac{bx+a+1}{bx+a-1} - 1\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(bx+a) dx$$

### 96.37 Problem number 67

$$\int \frac{\coth^{-1}(a+bx)}{x^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arccoth}(bx+a)}{x} + \frac{b \ln(x)}{-a^2+1} - \frac{b \ln(-bx-a+1)}{2(1-a)} - \frac{b \ln(bx+a+1)}{2(1+a)}$$

command

`integrate(arccoth(b*x+a)/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} ((a+1)b - (a-1)b) \left( \frac{(a-1) \log\left(\left|\frac{(bx+a+1)a}{bx+a-1} - a - \frac{bx+a+1}{bx+a-1} - 1\right|\right)}{a^3 - a^2 - a + 1} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{a^2 - 1} - \frac{\log\left(\frac{\frac{1}{a - \frac{\frac{1}{(bx+a+1)(a-1)} - a - 1} b} - b}}{\frac{1}{a - \frac{\frac{1}{(bx+a+1)(a-1)} - a - 1} b} - b} - 1\right)}{\left(\frac{(bx+a+1)a}{bx+a-1} - a - \frac{bx+a+1}{bx+a-1}\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(bx+a)}{x^2} dx$$

## 96.38 Problem number 68

$$\int \frac{\coth^{-1}(a + bx)}{x^3} dx$$

Optimal antiderivative

$$-\frac{b}{2(-a^2 + 1)x} - \frac{\operatorname{arccoth}(bx + a)}{2x^2} + \frac{ab^2 \ln(x)}{(-a^2 + 1)^2} - \frac{b^2 \ln(-bx - a + 1)}{4(1 - a)^2} + \frac{b^2 \ln(bx + a + 1)}{4(1 + a)^2}$$

command

`integrate(arccoth(b*x+a)/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2}((a+1)b - (a-1)b) \left( \frac{ab \log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{a^4 - 2a^2 + 1} - \frac{ab \log\left(\left|\frac{(bx+a+1)a}{bx+a-1} - a - \frac{bx+a+1}{bx+a-1} - 1\right|\right)}{a^4 - 2a^2 + 1} + \frac{\left(\frac{(bx+a+1)ab}{bx+a-1} - ab - \frac{(bx+a+1)}{bx+a-1}\right)}{(a^2 - 2a + 1)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(bx + a)}{x^3} dx$$

## 96.39 Problem number 83

$$\int x^2 \coth^{-1}(\sqrt{x}) dx$$

Optimal antiderivative

$$\frac{x^{\frac{3}{2}}}{9} + \frac{x^{\frac{5}{2}}}{15} + \frac{x^3 \operatorname{arccoth}(\sqrt{x})}{3} - \frac{\operatorname{arctanh}(\sqrt{x})}{3} + \frac{\sqrt{x}}{3}$$

command

`integrate(x^2*arccoth(x^(1/2)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{45(\sqrt{x}+1)^4}{(\sqrt{x}-1)^4} - \frac{90(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3} + \frac{140(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} - \frac{70(\sqrt{x}+1)}{\sqrt{x}-1} + 23 \right) \\ + \frac{45 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^5}{3 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^6} \\ + \frac{2 \left( \frac{3(\sqrt{x}+1)^5}{(\sqrt{x}-1)^5} + \frac{10(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3} + \frac{3(\sqrt{x}+1)}{\sqrt{x}-1} \right) \log \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} \right)}{3 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^6}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(\sqrt{x}) \, dx$$

#### 96.40 Problem number 84

$$\int x \operatorname{coth}^{-1}(\sqrt{x}) \, dx$$

Optimal antiderivative

$$\frac{x^{\frac{3}{2}}}{6} + \frac{x^2 \operatorname{arccoth}(\sqrt{x})}{2} - \frac{\operatorname{arctanh}(\sqrt{x})}{2} + \frac{\sqrt{x}}{2}$$

command

`integrate(x*arccoth(x^(1/2)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{3(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} - \frac{3(\sqrt{x}+1)}{\sqrt{x}-1} + 2 \right)}{3 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^3} + \frac{2 \left( \frac{(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3} + \frac{\sqrt{x}+1}{\sqrt{x}-1} \right) \log \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} \right)}{\left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(\sqrt{x}) \, dx$$



## 96.41 Problem number 85

$$\int \coth^{-1}(\sqrt{x}) dx$$

Optimal antiderivative

$$x \operatorname{arccoth}(\sqrt{x}) - \operatorname{arctanh}(\sqrt{x}) + \sqrt{x}$$

command

`integrate(arccoth(x^(1/2)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1} + \frac{2(\sqrt{x}+1) \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{(\sqrt{x}-1)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\sqrt{x}) dx$$

## 96.42 Problem number 87

$$\int \frac{\coth^{-1}(\sqrt{x})}{x^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arccoth}(\sqrt{x})}{x} + \operatorname{arctanh}(\sqrt{x}) - \frac{1}{\sqrt{x}}$$

command

`integrate(arccoth(x^(1/2))/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1} + \frac{2(\sqrt{x}+1) \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{(\sqrt{x}-1)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^2} dx$$

## 96.43 Problem number 88

$$\int \frac{\coth^{-1}(\sqrt{x})}{x^3} dx$$

Optimal antiderivative

$$-\frac{1}{6x^{\frac{3}{2}}} - \frac{\operatorname{arccoth}(\sqrt{x})}{2x^2} + \frac{\operatorname{arctanh}(\sqrt{x})}{2} - \frac{1}{2\sqrt{x}}$$

command

`integrate(arccoth(x^(1/2))/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{3(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} + \frac{3(\sqrt{x}+1)}{\sqrt{x}-1} + 2 \right)}{3 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} + 1 \right)^3} + \frac{2 \left( \frac{(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3} + \frac{\sqrt{x}+1}{\sqrt{x}-1} \right) \log \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} \right)}{\left( \frac{\sqrt{x}+1}{\sqrt{x}-1} + 1 \right)^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^3} dx$$

## 96.44 Problem number 89

$$\int x^{3/2} \coth^{-1}(\sqrt{x}) dx$$

Optimal antiderivative

$$\frac{x}{5} + \frac{x^2}{10} + \frac{2x^{\frac{5}{2}} \operatorname{arccoth}(\sqrt{x})}{5} + \frac{\ln(1-x)}{5}$$

command

`integrate(x^(3/2)*arccoth(x^(1/2)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8 \left( \frac{(\sqrt{x}+1)^3}{(\sqrt{x}-1)^3} - \frac{(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} + \frac{\sqrt{x}+1}{\sqrt{x}-1} \right)}{5 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^4} + \frac{2 \left( \frac{5(\sqrt{x}+1)^4}{(\sqrt{x}-1)^4} + \frac{10(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} + 1 \right) \log \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} \right)}{5 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^5} \\ + \frac{2}{5} \log \left( \left| \frac{\sqrt{x}+1}{\sqrt{x}-1} \right| \right) - \frac{2}{5} \log \left( \left| \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right| \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x}) dx$$

### 96.45 Problem number 90

$$\int \sqrt{x} \operatorname{coth}^{-1}(\sqrt{x}) dx$$

Optimal antiderivative

$$\frac{x}{3} + \frac{2x^{\frac{3}{2}} \operatorname{arccoth}(\sqrt{x})}{3} + \frac{\ln(1-x)}{3}$$

command

`integrate(arccoth(x^(1/2))*x^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{3(\sqrt{x}+1)^2}{(\sqrt{x}-1)^2} + 1 \right) \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3 \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^3} + \frac{4(\sqrt{x}+1)}{3(\sqrt{x}-1) \left( \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right)^2} + \frac{2}{3} \log\left(\left| \frac{\sqrt{x}+1}{\sqrt{x}-1} \right|\right) - \frac{2}{3} \log\left(\left| \frac{\sqrt{x}+1}{\sqrt{x}-1} - 1 \right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \sqrt{x} \operatorname{arccoth}(\sqrt{x}) dx$$

### 96.46 Problem number 91

$$\int \frac{\operatorname{coth}^{-1}(\sqrt{x})}{\sqrt{x}} dx$$

Optimal antiderivative

$$\ln(1-x) + 2 \operatorname{arccoth}(\sqrt{x}) \sqrt{x}$$

command

`integrate(arccoth(x^(1/2))/x^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1} + 2 \log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) - 2 \log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{\sqrt{x}} dx$$

**96.47 Problem number 92**

$$\int \frac{\operatorname{coth}^{-1}(\sqrt{x})}{x^{3/2}} dx$$

Optimal antiderivative

$$-\ln(1-x) + \ln(x) - \frac{2 \operatorname{arccoth}(\sqrt{x})}{\sqrt{x}}$$

command

`integrate(arccoth(x^(1/2))/x^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1} + 1} - 2 \log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) + 2 \log\left(\left|\frac{\sqrt{x}+1}{\sqrt{x}-1} + 1\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\sqrt{x})}{x^{\frac{3}{2}}} dx$$

## 96.48 Problem number 94

$$\int \coth^{-1}\left(\frac{1}{x}\right) dx$$

Optimal antiderivative

$$x \operatorname{arccoth}\left(\frac{1}{x}\right) + \frac{\ln(-x^2 + 1)}{2}$$

command

`integrate(arccoth(1/x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(\frac{\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}}{\frac{x+1}{x-1}-1}\right)}{\frac{x+1}{x-1}-1} + \log\left(\frac{|-x-1|}{|x-1|}\right) - \log\left(\left|-\frac{x+1}{x-1}+1\right|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}\left(\frac{1}{x}\right) dx$$

## 96.49 Problem number 96

$$\int (a + bx) \coth^{-1}(a + bx) dx$$

Optimal antiderivative

$$\frac{x}{2} + \frac{(bx + a)^2 \operatorname{arccoth}(bx + a)}{2b} - \frac{\operatorname{arctanh}(bx + a)}{2b}$$

command

`integrate((b*x+a)*arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} ((a+1)b - (a-1)b) \left( \frac{1}{b^2 \left(\frac{bx+a+1}{bx+a-1} - 1\right)} + \frac{(bx+a+1) \log\left(\frac{a - \frac{\frac{1}{\left(\frac{(bx+a+1)(a-1)}{bx+a-1} - a-1\right)b} + 1}}{\frac{(bx+a+1)b}{bx+a-1} - b}}{a - \frac{\frac{1}{\left(\frac{(bx+a+1)(a-1)}{bx+a-1} - a-1\right)b} - 1}}{\frac{(bx+a+1)b}{bx+a-1} - b}}\right)}{(bx+a-1)b^2 \left(\frac{bx+a+1}{bx+a-1} - 1\right)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (bx + a) \operatorname{arccoth}(bx + a) dx$$

**96.50 Problem number 97**

$$\int (a + bx)^2 \coth^{-1}(a + bx) dx$$

Optimal antiderivative

$$\frac{(bx + a)^2}{6b} + \frac{(bx + a)^3 \operatorname{arccoth}(bx + a)}{3b} + \frac{\ln(1 - (bx + a)^2)}{6b}$$

command

`integrate((b*x+a)^2*arccoth(b*x+a),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} ((a + 1)b - (a - 1)b) \left( \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1}\right|\right)}{b^2} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1} - 1\right|\right)}{b^2} + \frac{\left(\frac{3(bx+a+1)^2}{(bx+a-1)^2} + 1\right) \log\left(\frac{\frac{1}{\frac{(bx+a+1)(a-1)-a-1}{bx+a-1}b} - \frac{(bx+a+1)b-b}{bx+a-1}}{\frac{1}{\frac{(bx+a+1)(a-1)-a-1}{bx+a-1}b} - \frac{(bx+a+1)b-b}{bx+a-1}} - 1\right)}{b^2 \left(\frac{bx+a+1}{bx+a-1} - 1\right)^3} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (bx + a)^2 \operatorname{arccoth}(bx + a) dx$$

**96.51 Problem number 99**

$$\int \frac{\coth^{-1}(a + bx)}{(a + bx)^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arccoth}(bx + a)}{b(bx + a)} + \frac{\ln(bx + a)}{b} - \frac{\ln(1 - (bx + a)^2)}{2b}$$

command

`integrate(arccoth(b*x+a)/(b*x+a)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2}((a+1)b - (a-1)b) \left( \frac{\log\left(\frac{|bx+a+1|}{|bx+a-1|}\right)}{b^2} - \frac{\log\left(\left|\frac{bx+a+1}{bx+a-1} + 1\right|\right)}{b^2} - \frac{\log\left(\frac{\frac{1}{\frac{(bx+a+1)(a-1)}{bx+a-1} - a - 1} + 1}}{\frac{1}{\frac{(bx+a+1)(a-1)}{bx+a-1} - a - 1} - 1}}{b^2 \left(\frac{bx+a+1}{bx+a-1} + 1\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(bx+a)}{(bx+a)^2} dx$$

## 96.52 Problem number 102

$$\int (e+fx)^3 (a+b \operatorname{coth}^{-1}(c+dx)) dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{bf(6d^2e^2 - 12cdef + (6c^2 + 1)f^2)x}{4d^3} + \frac{bf^2(-cf + de)(dx + c)^2}{2d^4} \\ & + \frac{bf^3(dx + c)^3}{12d^4} + \frac{(fx + e)^4(a + b \operatorname{arccoth}(dx + c))}{4f} \\ & + \frac{b(-cf + de + f)^4 \ln(-dx - c + 1)}{8d^4f} - \frac{b(-cf + de - f)^4 \ln(dx + c + 1)}{8d^4f} \end{aligned}$$

command

`integrate((f*x+e)^3*(a+b*arccoth(d*x+c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (fx+e)^3 (b \operatorname{arccoth}(dx+c) + a) dx$$

## 96.53 Problem number 103

$$\int (e + fx)^2 (a + b \coth^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{bf(-cf + de)x}{d^2} + \frac{bf^2(dx + c)^2}{6d^3} + \frac{(fx + e)^3(a + b \operatorname{arccoth}(dx + c))}{3f} \\ + \frac{b(-cf + de + f)^3 \ln(-dx - c + 1)}{6d^3 f} - \frac{b(de - (1 + c)f)^3 \ln(dx + c + 1)}{6d^3 f}$$

command

```
integrate((f*x+e)^2*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int (fx + e)^2 (b \operatorname{arccoth}(dx + c) + a) dx$$

## 96.54 Problem number 104

$$\int (e + fx) (a + b \coth^{-1}(c + dx)) dx$$

Optimal antiderivative

$$\frac{bfx}{2d} + \frac{(fx + e)^2(a + b \operatorname{arccoth}(dx + c))}{2f} \\ + \frac{b(-cf + de + f)^2 \ln(-dx - c + 1)}{4d^2 f} - \frac{b(de - (1 + c)f)^2 \ln(dx + c + 1)}{4d^2 f}$$

command

```
integrate((f*x+e)*(a+b*arccoth(d*x+c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} ((c + 1)d - (c - 1)d) \left( \frac{\left( \frac{(dx+c+1)bde}{dx+c-1} - bde - \frac{(dx+c+1)bcf}{dx+c-1} + bcf + \frac{(dx+c+1)bf}{dx+c-1} \right) \log\left(\frac{dx+c+1}{dx+c-1}\right)}{\frac{(dx+c+1)^2 d^3}{(dx+c-1)^2} - \frac{2(dx+c+1)d^3}{dx+c-1} + d^3} + \frac{2(dx+c+1)ade}{dx+c-1} - 2ad \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (fx + e)(b \operatorname{arccoth}(dx + c) + a) dx$$



## 96.55 Problem number 105

$$\int (a + b \coth^{-1}(c + dx)) dx$$

Optimal antiderivative

$$ax + \frac{b(dx + c) \operatorname{arccoth}(dx + c)}{d} + \frac{b \ln(1 - (dx + c)^2)}{2d}$$

command

`integrate(a+b*arccoth(d*x+c),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} ((c + 1)d - (c - 1)d)b \left( \frac{\log\left(\left|\frac{dx+c+1}{dx+c-1}\right|\right)}{d^2} - \frac{\log\left(\left|\frac{dx+c+1}{dx+c-1} - 1\right|\right)}{d^2} + \frac{\log\left(\frac{\frac{1}{\frac{(dx+c+1)(c-1)}{dx+c-1} - c - 1} + 1}}{\frac{\frac{(dx+c+1)d}{dx+c-1} - d}{\frac{1}{\frac{(dx+c+1)(c-1)}{dx+c-1} - c - 1} - 1}}}\right)}{d^2 \left(\frac{dx+c+1}{dx+c-1} - 1\right)} \right)$$

+ ax

Giac 1.7.0 via sagemath 9.3 output

$$\int b \operatorname{arccoth}(dx + c) + a dx$$

## 96.56 Problem number 107

$$\int \frac{a + b \coth^{-1}(c + dx)}{(e + fx)^2} dx$$

Optimal antiderivative

$$\frac{-a - b \operatorname{arccoth}(dx + c)}{f(fx + e)} - \frac{bd \ln(-dx - c + 1)}{2f(-cf + de + f)} + \frac{bd \ln(dx + c + 1)}{2f(-cf + de - f)} - \frac{bd \ln(fx + e)}{(-cf + de - f)(-cf + de + f)}$$

command

`integrate((a+b*arccoth(d*x+c))/(f*x+e)^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2}((c+1)d - (c-1)d) \left( \frac{b \log \left( -\frac{(dx+c+1)de}{dx+c-1} + de + \frac{(dx+c+1)cf}{dx+c-1} - cf - \frac{(dx+c+1)f}{dx+c-1} - f \right)}{d^2e^2 - 2cdef + c^2f^2 - f^2} - \frac{\frac{(dx+c+1)d^2e^2}{dx+c-1} - d^2e^2 - 2c}{d^2e^2 - 2cdef + c^2f^2 - f^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{b \operatorname{arccoth}(dx+c) + a}{(fx+e)^2} dx$$

**96.57 Problem number 108**

$$\int \frac{a + b \operatorname{coth}^{-1}(c+dx)}{(e+fx)^3} dx$$

Optimal antiderivative

$$\frac{bd}{2(-cf+de-f)(-cf+de+f)(fx+e)} + \frac{-a - b \operatorname{arccoth}(dx+c)}{2f(fx+e)^2} - \frac{bd^2 \ln(-dx-c+1)}{4f(-cf+de+f)^2} + \frac{bd^2 \ln(dx+c+1)}{4f(-cf+de-f)^2} - \frac{bd^2(-cf+de) \ln(fx+e)}{(-cf+de+f)^2(de-(1+c)f)^2}$$

command

`integrate((a+b*arccoth(d*x+c))/(f*x+e)^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{b \operatorname{arccoth}(dx+c) + a}{(fx+e)^3} dx$$

**96.58 Problem number 128**

$$\int x^m \operatorname{coth}^{-1}(\tanh(a+bx)) dx$$

Optimal antiderivative

$$-\frac{bx^{2+m}}{m^2+3m+2} + \frac{x^{1+m} \operatorname{arccoth}(\tanh(bx+a))}{1+m}$$

command

`integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^{m+1} \log\left(-\frac{\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}}}{\frac{e^{(2bx+2a)+1}-1}{e^{(2bx+2a)-1}}}\right)}{2(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^m \operatorname{arccoth}(\tanh(bx+a)) dx$$

### 96.59 Problem number 129

$$\int x^2 \operatorname{coth}^{-1}(\tanh(a+bx)) dx$$

Optimal antiderivative

$$-\frac{bx^4}{12} + \frac{x^3 \operatorname{arccoth}(\tanh(bx+a))}{3}$$

command

`integrate(x^2*arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{12} bx^4 + \frac{1}{6} x^3 \log\left(-\frac{\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}}}{\frac{e^{(2bx+2a)+1}-1}{e^{(2bx+2a)-1}}}\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(\tanh(bx+a)) dx$$

**96.60 Problem number 130**

$$\int x \coth^{-1}(\tanh(a + bx)) dx$$

Optimal antiderivative

$$-\frac{bx^3}{6} + \frac{x^2 \operatorname{arccoth}(\tanh(bx + a))}{2}$$

command

`integrate(x*arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{6}bx^3 + \frac{1}{4}x^2 \log\left(-\frac{\frac{e^{(2bx+2a)+1}}{e^{(2bx+2a)-1}} + 1}{\frac{e^{(2bx+2a)+1}}{e^{(2bx+2a)-1}} - 1}\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(\tanh(bx + a)) dx$$

**96.61 Problem number 131**

$$\int \coth^{-1}(\tanh(a + bx)) dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^2}{2b}$$

command

`integrate(arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2}bx^2 + \frac{1}{2}x \log\left(-\frac{\frac{e^{(2bx+2a)+1}}{e^{(2bx+2a)-1}} + 1}{\frac{e^{(2bx+2a)+1}}{e^{(2bx+2a)-1}} - 1}\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\tanh(bx + a)) dx$$

### 96.62 Problem number 132

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x} dx$$

Optimal antiderivative

$$bx - (bx - \operatorname{arccoth}(\tanh(bx + a))) \ln(x)$$

command

```
integrate(arccoth(tanh(b*x+a))/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$bx + \frac{1}{2} (i\pi + 2a) \log(x)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x} dx$$

### 96.63 Problem number 133

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^2} dx$$

Optimal antiderivative

$$-\frac{\operatorname{arccoth}(\tanh(bx + a))}{x} + b \ln(x)$$

command

```
integrate(arccoth(tanh(b*x+a))/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$b \log(|x|) - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}}}{\frac{e^{(2bx+2a)+1}-1}{e^{(2bx+2a)-1}}}\right)}{2x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^2} dx$$

## 96.64 Problem number 134

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^3} dx$$

Optimal antiderivative

$$-\frac{b}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{2x^2}$$

command

`integrate(arccoth(tanh(b*x+a))/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{b}{2x} - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}}}{\frac{e^{(2bx+2a)+1}-1}{e^{(2bx+2a)-1}}}\right)}{4x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^3} dx$$

## 96.65 Problem number 135

$$\int \frac{\coth^{-1}(\tanh(a + bx))}{x^4} dx$$

Optimal antiderivative

$$-\frac{b}{6x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))}{3x^3}$$

command

`integrate(arccoth(tanh(b*x+a))/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{b}{6x^2} - \frac{\log\left(-\frac{\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}}}{\frac{e^{(2bx+2a)+1}-1}{e^{(2bx+2a)-1}}}\right)}{6x^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))}{x^4} dx$$

**96.66 Problem number 137**

$$\int x^3 \coth^{-1}(\tanh(a + bx))^2 dx$$

Optimal antiderivative

$$\frac{b^2 x^6}{60} - \frac{b x^5 \operatorname{arccoth}(\tanh(bx + a))}{10} + \frac{x^4 \operatorname{arccoth}(\tanh(bx + a))^2}{4}$$

command

```
integrate(x^3*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} b^2 x^6 - \frac{1}{5} (-i \pi b - 2 ab) x^5 - \frac{1}{16} (\pi^2 - 4i \pi a - 4 a^2) x^4$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

**96.67 Problem number 138**

$$\int x^2 \coth^{-1}(\tanh(a + bx))^2 dx$$

Optimal antiderivative

$$\frac{b^2 x^5}{30} - \frac{b x^4 \operatorname{arccoth}(\tanh(bx + a))}{6} + \frac{x^3 \operatorname{arccoth}(\tanh(bx + a))^2}{3}$$

command

```
integrate(x^2*arccoth(tanh(b*x+a))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{5} b^2 x^5 - \frac{1}{4} (-i \pi b - 2 ab) x^4 - \frac{1}{12} (\pi^2 - 4i \pi a - 4 a^2) x^3$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

**96.68 Problem number 139**

$$\int x \coth^{-1}(\tanh(a + bx))^2 dx$$

Optimal antiderivative

$$\frac{x \operatorname{arccoth}(\tanh(bx + a))^3}{3b} - \frac{\operatorname{arccoth}(\tanh(bx + a))^4}{12b^2}$$

command

`integrate(x*arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} b^2 x^4 - \frac{1}{3} (-i \pi b - 2 ab) x^3 - \frac{1}{8} (\pi^2 - 4i \pi a - 4 a^2) x^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(\tanh(bx + a))^2 dx$$

**96.69 Problem number 140**

$$\int \coth^{-1}(\tanh(a + bx))^2 dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3b}$$

command

`integrate(arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} b^2 x^3 - \frac{1}{2} (-i \pi b - 2 ab) x^2 - \frac{1}{4} (\pi^2 - 4i \pi a - 4 a^2) x$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\tanh(bx + a))^2 dx$$



### 96.70 Problem number 141

$$\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x} dx$$

Optimal antiderivative

$$-bx(bx - \operatorname{arccoth}(\tanh(bx + a))) + \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{2} + (bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^2/x,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} b^2 x^2 + (i \pi b + 2 ab)x - \frac{1}{4} (\pi^2 - 4i \pi a - 4 a^2) \log(x)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x} dx$$

### 96.71 Problem number 142

$$\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^2} dx$$

Optimal antiderivative

$$2b^2x - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x} - 2b(bx - \operatorname{arccoth}(\tanh(bx + a))) \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^2/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^2x + (i \pi b + 2 ab) \log(x) + \frac{\pi^2 - 4i \pi a - 4 a^2}{4x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^2} dx$$

### 96.72 Problem number 143

$$\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^3} dx$$

Optimal antiderivative

$$-\frac{b \operatorname{arccoth}(\tanh(bx + a))}{x} - \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{2x^2} + b^2 \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^2/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^2 \log(x) - \frac{8i\pi bx + 16abx - \pi^2 + 4i\pi a + 4a^2}{8x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^3} dx$$

### 96.73 Problem number 144

$$\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^4} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3x^3 (bx - \operatorname{arccoth}(\tanh(bx + a)))}$$

command

`integrate(arccoth(tanh(b*x+a))^2/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12b^2x^2 + 6i\pi bx + 12abx - \pi^2 + 4i\pi a + 4a^2}{12x^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^4} dx$$

## 96.74 Problem number 145

$$\int \frac{\coth^{-1}(\tanh(a + bx))^2}{x^5} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^3}{12x^3 (bx - \operatorname{arccoth}(\tanh(bx + a)))^2} + \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{4x^4 (bx - \operatorname{arccoth}(\tanh(bx + a)))}$$

command

`integrate(arccoth(tanh(b*x+a))^2/x^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{24b^2x^2 + 16i\pi bx + 32abx - 3\pi^2 + 12i\pi a + 12a^2}{48x^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^2}{x^5} dx$$

## 96.75 Problem number 147

$$\int x^4 \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal antiderivative

$$-\frac{b^3x^8}{280} + \frac{b^2x^7 \operatorname{arccoth}(\tanh(bx + a))}{35} - \frac{bx^6 \operatorname{arccoth}(\tanh(bx + a))^2}{10} + \frac{x^5 \operatorname{arccoth}(\tanh(bx + a))^3}{5}$$

command

`integrate(x^4*arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{8}b^3x^8 - \frac{3}{14}(-i\pi b^2 - 2ab^2)x^7 - \frac{1}{8}(\pi^2b - 4i\pi ab - 4a^2b)x^6 - \frac{1}{40}(i\pi^3 + 6\pi^2a - 12i\pi a^2 - 8a^3)x^5$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^4 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

**96.76 Problem number 148**

$$\int x^3 \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal antiderivative

$$-\frac{b^3 x^7}{140} + \frac{b^2 x^6 \operatorname{arccoth}(\tanh(bx + a))}{20} - \frac{3b x^5 \operatorname{arccoth}(\tanh(bx + a))^2}{20} + \frac{x^4 \operatorname{arccoth}(\tanh(bx + a))^3}{4}$$

command

```
integrate(x^3*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{7} b^3 x^7 - \frac{1}{4} (-i \pi b^2 - 2 a b^2) x^6 - \frac{3}{20} (\pi^2 b - 4i \pi a b - 4 a^2 b) x^5 - \frac{1}{32} (i \pi^3 + 6 \pi^2 a - 12i \pi a^2 - 8 a^3) x^4$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

**96.77 Problem number 149**

$$\int x^2 \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal antiderivative

$$\frac{x^2 \operatorname{arccoth}(\tanh(bx + a))^4}{4b} - \frac{x \operatorname{arccoth}(\tanh(bx + a))^5}{10b^2} + \frac{\operatorname{arccoth}(\tanh(bx + a))^6}{60b^3}$$

command

```
integrate(x^2*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{6} b^3 x^6 - \frac{3}{10} (-i \pi b^2 - 2 a b^2) x^5 - \frac{3}{16} (\pi^2 b - 4i \pi a b - 4 a^2 b) x^4 - \frac{1}{24} (i \pi^3 + 6 \pi^2 a - 12i \pi a^2 - 8 a^3) x^3$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^2 \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

**96.78 Problem number 150**

$$\int x \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal antiderivative

$$\frac{x \operatorname{arccoth}(\tanh(bx + a))^4}{4b} - \frac{\operatorname{arccoth}(\tanh(bx + a))^5}{20b^2}$$

command

```
integrate(x*arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{5} b^3 x^5 - \frac{3}{8} (-i \pi b^2 - 2 a b^2) x^4 - \frac{1}{4} (\pi^2 b - 4 i \pi a b - 4 a^2 b) x^3 - \frac{1}{16} (i \pi^3 + 6 \pi^2 a - 12 i \pi a^2 - 8 a^3) x^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

**96.79 Problem number 151**

$$\int \coth^{-1}(\tanh(a + bx))^3 dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^4}{4b}$$

command

```
integrate(arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} b^3 x^4 - \frac{1}{2} (-i \pi b^2 - 2 a b^2) x^3 - \frac{3}{8} (\pi^2 b - 4 i \pi a b - 4 a^2 b) x^2 - \frac{1}{8} (i \pi^3 + 6 \pi^2 a - 12 i \pi a^2 - 8 a^3) x$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\tanh(bx + a))^3 dx$$

### 96.80 Problem number 152

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x} dx$$

Optimal antiderivative

$$bx(bx - \operatorname{arccoth}(\tanh(bx + a)))^2 - \frac{(bx - \operatorname{arccoth}(\tanh(bx + a))) \operatorname{arccoth}(\tanh(bx + a))^2}{2} \\ + \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3} - (bx - \operatorname{arccoth}(\tanh(bx + a)))^3 \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^3/x,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3} b^3 x^3 - \frac{3}{4} (-i \pi b^2 - 2 a b^2) x^2 - \frac{3}{4} (\pi^2 b - 4 i \pi a b - 4 a^2 b) x + \frac{1}{8} (-i \pi^3 - 6 \pi^2 a + 12 i \pi a^2 + 8 a^3) \log(x)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x} dx$$

### 96.81 Problem number 153

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^2} dx$$

Optimal antiderivative

$$-3b^2x(bx - \operatorname{arccoth}(\tanh(bx + a))) + \frac{3b \operatorname{arccoth}(\tanh(bx + a))^2}{2} \\ - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x} + 3b(bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^3/x^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} b^3 x^2 - \frac{3}{2} (-i \pi b^2 - 2 a b^2) x - \frac{3}{4} (\pi^2 b - 4 i \pi a b - 4 a^2 b) \log(x) - \frac{-i \pi^3 - 6 \pi^2 a + 12 i \pi a^2 + 8 a^3}{8 x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^2} dx$$

### 96.82 Problem number 154

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^3} dx$$

Optimal antiderivative

$$3b^3x - \frac{3b \operatorname{arccoth}(\tanh(bx + a))^2}{2x} - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{2x^2} - 3b^2(bx - \operatorname{arccoth}(\tanh(bx + a))) \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^3/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^3x + \frac{3}{2}(i\pi b^2 + 2ab^2) \log(x) + \frac{12\pi^2bx - 48i\pi abx - 48a^2bx + i\pi^3 + 6\pi^2a - 12i\pi a^2 - 8a^3}{16x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^3} dx$$

### 96.83 Problem number 155

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^4} dx$$

Optimal antiderivative

$$-\frac{b^2 \operatorname{arccoth}(\tanh(bx + a))}{x} - \frac{b \operatorname{arccoth}(\tanh(bx + a))^2}{2x^2} - \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{3x^3} + b^3 \ln(x)$$

command

`integrate(arccoth(tanh(b*x+a))^3/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$b^3 \log(x) - \frac{36i\pi b^2x^2 + 72ab^2x^2 - 9\pi^2bx + 36i\pi abx + 36a^2bx - i\pi^3 - 6\pi^2a + 12i\pi a^2 + 8a^3}{24x^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^4} dx$$

## 96.84 Problem number 156

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^5} dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^4}{4x^4(bx - \operatorname{arccoth}(\tanh(bx + a)))}$$

command

`integrate(arccoth(tanh(b*x+a))^3/x^5,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32b^3x^3 + 24i\pi b^2x^2 + 48ab^2x^2 - 8\pi^2bx + 32i\pi abx + 32a^2bx - i\pi^3 - 6\pi^2a + 12i\pi a^2 + 8a^3}{32x^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^5} dx$$

## 96.85 Problem number 157

$$\int \frac{\coth^{-1}(\tanh(a + bx))^3}{x^6} dx$$

Optimal antiderivative

$$\frac{\operatorname{barccoth}(\tanh(bx + a))^4}{20x^4(bx - \operatorname{arccoth}(\tanh(bx + a)))^2} + \frac{\operatorname{arccoth}(\tanh(bx + a))^4}{5x^5(bx - \operatorname{arccoth}(\tanh(bx + a)))}$$

command

`integrate(arccoth(tanh(b*x+a))^3/x^6,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{40b^3x^3 + 40i\pi b^2x^2 + 80ab^2x^2 - 15\pi^2bx + 60i\pi abx + 60a^2bx - 2i\pi^3 - 12\pi^2a + 24i\pi a^2 + 16a^3}{80x^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\operatorname{arccoth}(\tanh(bx + a))^3}{x^6} dx$$



## 96.86 Problem number 159

$$\int \frac{x^3}{\coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{x^3}{3b} + \frac{x^2(bx - \operatorname{arccoth}(\tanh(bx + a)))}{2b^2} + \frac{x(bx - \operatorname{arccoth}(\tanh(bx + a)))^2}{b^3} + \frac{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3 \ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^4}$$

command

`integrate(x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3}{3b} - \frac{(i\pi + 2a)x^2}{4b^2} - \frac{(\pi^2 - 4i\pi a - 4a^2)x}{4b^3} + \frac{(i\pi^3 + 6\pi^2 a - 12i\pi a^2 - 8a^3) \log(\pi - 2ibx - 2ia)}{8b^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

## 96.87 Problem number 160

$$\int \frac{x^2}{\coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{x^2}{2b} + \frac{x(bx - \operatorname{arccoth}(\tanh(bx + a)))}{b^2} + \frac{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^3}$$

command

`integrate(x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2}{2b} - \frac{(i\pi + 2a)x}{2b^2} - \frac{(\pi^2 - 4i\pi a - 4a^2) \log(\pi - 2ibx - 2ia)}{4b^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))} dx$$

**96.88 Problem number 161**

$$\int \frac{x}{\coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{x}{b} + \frac{(bx - \operatorname{arccoth}(\tanh(bx + a))) \ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^2}$$

command

```
integrate(x/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{b} - \frac{(i\pi + 2a) \log(\pi - 2ibx - 2ia)}{2b^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x}{\operatorname{arcoth}(\tanh(bx + a))} dx$$

**96.89 Problem number 162**

$$\int \frac{1}{\coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{b}$$

command

```
integrate(1/arccoth(tanh(b*x+a)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(\pi - 2ibx - 2ia)}{b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\operatorname{arcoth}(\tanh(bx + a))} dx$$

### 96.90 Problem number 163

$$\int \frac{1}{x \coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$-\frac{\ln(x)}{bx - \operatorname{arccoth}(\tanh(bx + a))} + \frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{bx - \operatorname{arccoth}(\tanh(bx + a))}$$

command

`integrate(1/x/arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \log(\pi - 2i bx - 2i a)}{-i \pi - 2 a} - \frac{2i \log(x)}{\pi - 2i a}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))} dx$$

### 96.91 Problem number 164

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{1}{x(bx - \operatorname{arccoth}(\tanh(bx + a)))} - \frac{b \ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2} + \frac{b \ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2}$$

command

`integrate(1/x^2/arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4i b \log(\pi - 2i bx - 2i a)}{i \pi^2 + 4 \pi a - 4i a^2} + \frac{4 b \log(x)}{\pi^2 - 4i \pi a - 4 a^2} + \frac{2}{-i \pi x - 2 a x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))} dx$$

**96.92 Problem number 165**

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))} dx$$

Optimal antiderivative

$$\frac{b}{x (bx - \operatorname{arccoth}(\tanh(bx + a)))^2} + \frac{1}{2x^2 (bx - \operatorname{arccoth}(\tanh(bx + a)))} \\ - \frac{b^2 \ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3} + \frac{b^2 \ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3}$$

command

`integrate(1/x^3/arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8b^2 \log(\pi - 2ibx - 2ia)}{-i\pi^3 - 6\pi^2a + 12i\pi a^2 + 8a^3} + \frac{8ib^2 \log(x)}{\pi^3 - 6i\pi^2a - 12\pi a^2 + 8ia^3} - \frac{-i\pi + 4bx - 2a}{\pi^2x^2 - 4i\pi ax^2 - 4a^2x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))} dx$$

**96.93 Problem number 167**

$$\int \frac{x^4}{\coth^{-1}(\tanh(a + bx))^2} dx$$

Optimal antiderivative

$$\frac{4x^3}{3b^2} + \frac{2x^2(bx - \operatorname{arccoth}(\tanh(bx + a)))}{b^3} + \frac{4x(bx - \operatorname{arccoth}(\tanh(bx + a)))^2}{b^4} \\ - \frac{x^4}{b \operatorname{arccoth}(\tanh(bx + a))} + \frac{4(bx - \operatorname{arccoth}(\tanh(bx + a)))^3 \ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^5}$$

command

`integrate(x^4/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3}{3b^2} - \frac{\pi^4 - 8i\pi^3a - 24\pi^2a^2 + 32i\pi a^3 + 16a^4}{8(2b^6x + i\pi b^5 + 2ab^5)} - \frac{(i\pi + 2a)x^2}{2b^3} \\ - \frac{3(\pi^2 - 4i\pi a - 4a^2)x}{4b^4} + \frac{(i\pi^3 + 6\pi^2a - 12i\pi a^2 - 8a^3) \log(i\pi + 2bx + 2a)}{2b^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

## 96.94 Problem number 168

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal antiderivative

$$\frac{3x^2}{2b^2} + \frac{3x(bx - \operatorname{arccoth}(\tanh(bx+a)))}{b^3} - \frac{x^3}{b \operatorname{arccoth}(\tanh(bx+a))} + \frac{3(bx - \operatorname{arccoth}(\tanh(bx+a)))^2 \ln(\operatorname{arccoth}(\tanh(bx+a)))}{b^4}$$

command

`integrate(x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{i\pi^3 + 6\pi^2a - 12i\pi a^2 - 8a^3}{4(2b^5x + i\pi b^4 + 2ab^4)} + \frac{x^2}{2b^2} + \frac{(-i\pi - 2a)x}{b^3} - \frac{3(\pi^2 - 4i\pi a - 4a^2) \log(i\pi + 2bx + 2a)}{4b^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx+a))^2} dx$$

## 96.95 Problem number 169

$$\int \frac{x^2}{\coth^{-1}(\tanh(a+bx))^2} dx$$

Optimal antiderivative

$$\frac{2x}{b^2} - \frac{x^2}{b \operatorname{arccoth}(\tanh(bx+a))} + \frac{2(bx - \operatorname{arccoth}(\tanh(bx+a))) \ln(\operatorname{arccoth}(\tanh(bx+a)))}{b^3}$$

command

`integrate(x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\pi^2 - 4i\pi a - 4a^2}{2(2b^4x + i\pi b^3 + 2ab^3)} + \frac{x}{b^2} - \frac{(i\pi + 2a) \log(i\pi + 2bx + 2a)}{b^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx+a))^2} dx$$

## 96.96 Problem number 170

$$\int \frac{x}{\coth^{-1}(\tanh(a + bx))^2} dx$$

Optimal antiderivative

$$-\frac{x}{b \operatorname{arccoth}(\tanh(bx + a))} + \frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^2}$$

command

`integrate(x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{-i\pi - 2a}{2b^3x + i\pi b^2 + 2ab^2} + \frac{\log(i\pi + 2bx + 2a)}{b^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

## 96.97 Problem number 171

$$\int \frac{1}{\coth^{-1}(\tanh(a + bx))^2} dx$$

Optimal antiderivative

$$-\frac{1}{b \operatorname{arccoth}(\tanh(bx + a))}$$

command

`integrate(1/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{2b^2x + i\pi b + 2ab}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx + a))^2} dx$$

### 96.98 Problem number 172

$$\int \frac{1}{x \coth^{-1}(\tanh(a + bx))^2} dx$$

Optimal antiderivative

$$-\frac{1}{(bx - \operatorname{arccoth}(\tanh(bx + a))) \operatorname{arccoth}(\tanh(bx + a))} + \frac{\ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2} - \frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2}$$

command

`integrate(1/x/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \log(i \pi + 2bx + 2a)}{\pi^2 - 4i \pi a - 4a^2} - \frac{4 \log(x)}{\pi^2 - 4i \pi a - 4a^2} + \frac{4}{2i \pi bx + 4abx - \pi^2 + 4i \pi a + 4a^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^2} dx$$

### 96.99 Problem number 173

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^2} dx$$

Optimal antiderivative

$$-\frac{2b}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \operatorname{arccoth}(\tanh(bx + a))} + \frac{1}{x(bx - \operatorname{arccoth}(\tanh(bx + a))) \operatorname{arccoth}(\tanh(bx + a))} + \frac{2b \ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3} - \frac{2b \ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3}$$

command

`integrate(1/x^2/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 b \log (i \pi + 2 b x + 2 a)}{-i \pi^3 - 6 \pi^2 a + 12 i \pi a^2 + 8 a^3} - \frac{16 b \log (x)}{-i \pi^3 - 6 \pi^2 a + 12 i \pi a^2 + 8 a^3}$$

$$+ \frac{8 b}{2 \pi^2 b x - 8 i \pi a b x - 8 a^2 b x + i \pi^3 + 6 \pi^2 a - 12 i \pi a^2 - 8 a^3} + \frac{4}{\pi^2 x - 4 i \pi a x - 4 a^2 x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh (b x + a))^2} dx$$

### 96.100 Problem number 174

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a + b x))^2} dx$$

Optimal antiderivative

$$-\frac{3 b^2}{(b x - \operatorname{arccoth}(\tanh (b x + a)))^3 \operatorname{arccoth}(\tanh (b x + a))}$$

$$+ \frac{3 b}{2 x (b x - \operatorname{arccoth}(\tanh (b x + a)))^2 \operatorname{arccoth}(\tanh (b x + a))}$$

$$+ \frac{1}{2 x^2 (b x - \operatorname{arccoth}(\tanh (b x + a))) \operatorname{arccoth}(\tanh (b x + a))}$$

$$+ \frac{3 b^2 \ln (x)}{(b x - \operatorname{arccoth}(\tanh (b x + a)))^4} - \frac{3 b^2 \ln (\operatorname{arccoth}(\tanh (b x + a)))}{(b x - \operatorname{arccoth}(\tanh (b x + a)))^4}$$

command

`integrate(1/x^3/arccoth(tanh(b*x+a))^2,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{48 b^2 \log (i \pi + 2 b x + 2 a)}{\pi^4 - 8 i \pi^3 a - 24 \pi^2 a^2 + 32 i \pi a^3 + 16 a^4} + \frac{48 b^2 \log (x)}{\pi^4 - 8 i \pi^3 a - 24 \pi^2 a^2 + 32 i \pi a^3 + 16 a^4}$$

$$+ \frac{16 b^2}{-2 i \pi^3 b x - 12 \pi^2 a b x + 24 i \pi a^2 b x + 16 a^3 b x + \pi^4 - 8 i \pi^3 a - 24 \pi^2 a^2 + 32 i \pi a^3 + 16 a^4}$$

$$- \frac{4 (i \pi - 8 b x + 2 a)}{-2 i \pi^3 x^2 - 12 \pi^2 a x^2 + 24 i \pi a^2 x^2 + 16 a^3 x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh (b x + a))^2} dx$$



## 96.101 Problem number 176

$$\int \frac{x^4}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal antiderivative

$$\frac{3x^2}{b^3} + \frac{6x(bx - \operatorname{arccoth}(\tanh(bx+a)))}{b^4} - \frac{x^4}{2b \operatorname{arccoth}(\tanh(bx+a))^2} - \frac{2x^3}{b^2 \operatorname{arccoth}(\tanh(bx+a))} + \frac{6(bx - \operatorname{arccoth}(\tanh(bx+a)))^2 \ln(\operatorname{arccoth}(\tanh(bx+a)))}{b^5}$$

command

```
integrate(x^4/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16\pi^3bx - 96i\pi^2abx - 192\pi a^2bx + 128i a^3bx + 7i\pi^4 + 56\pi^3a - 168i\pi^2a^2 - 224\pi a^3 + 112i a^4}{-32i b^7x^2 + 32\pi b^6x - 64i ab^6x + 8i\pi^2b^5 + 32\pi ab^5 - 32i a^2b^5} + \frac{x^2}{2b^3} - \frac{3(i\pi + 2a)x}{2b^4} - \frac{3(\pi^2 - 4i\pi a - 4a^2) \log(i\pi + 2bx + 2a)}{2b^5}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^4}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

## 96.102 Problem number 177

$$\int \frac{x^3}{\coth^{-1}(\tanh(a+bx))^3} dx$$

Optimal antiderivative

$$\frac{3x}{b^3} - \frac{x^3}{2b \operatorname{arccoth}(\tanh(bx+a))^2} - \frac{3x^2}{2b^2 \operatorname{arccoth}(\tanh(bx+a))} + \frac{3(bx - \operatorname{arccoth}(\tanh(bx+a))) \ln(\operatorname{arccoth}(\tanh(bx+a)))}{b^4}$$

command

```
integrate(x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12\pi^2bx - 48i\pi abx - 48a^2bx + 5i\pi^3 + 30\pi^2a - 60i\pi a^2 - 40a^3}{4(4b^6x^2 + 4i\pi b^5x + 8ab^5x - \pi^2b^4 + 4i\pi ab^4 + 4a^2b^4)} + \frac{x}{b^3} + \frac{3(-i\pi - 2a)\log(i\pi + 2bx + 2a)}{2b^4}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^3}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

### 96.103 Problem number 178

$$\int \frac{x^2}{\coth^{-1}(\tanh(a + bx))^3} dx$$

Optimal antiderivative

$$-\frac{x^2}{2b\operatorname{arccoth}(\tanh(bx + a))^2} - \frac{x}{b^2\operatorname{arccoth}(\tanh(bx + a))} + \frac{\ln(\operatorname{arccoth}(\tanh(bx + a)))}{b^3}$$

command

`integrate(x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\pi bx - 16i abx + 3i\pi^2 + 12\pi a - 12i a^2}{-8i b^5 x^2 + 8\pi b^4 x - 16i ab^4 x + 2i\pi^2 b^3 + 8\pi ab^3 - 8i a^2 b^3} + \frac{\log(i\pi + 2bx + 2a)}{b^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x^2}{\operatorname{arccoth}(\tanh(bx + a))^3} dx$$

### 96.104 Problem number 179

$$\int \frac{x}{\coth^{-1}(\tanh(a + bx))^3} dx$$

Optimal antiderivative

$$-\frac{x}{2b\operatorname{arccoth}(\tanh(bx + a))^2} - \frac{1}{2b^2\operatorname{arccoth}(\tanh(bx + a))}$$

command

`integrate(x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{i\pi + 4bx + 2a}{4b^4x^2 + 4i\pi b^3x + 8ab^3x - \pi^2b^2 + 4i\pi ab^2 + 4a^2b^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{x}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

### 96.105 Problem number 180

$$\int \frac{1}{\operatorname{coth}^{-1}(\tanh(a+bx))^3} dx$$

Optimal antiderivative

$$-\frac{1}{2b\operatorname{arccoth}(\tanh(bx+a))^2}$$

command

`integrate(1/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2i}{4ib^3x^2 - 4\pi b^2x + 8iab^2x - i\pi^2b - 4\pi ab + 4ia^2b}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{\operatorname{arccoth}(\tanh(bx+a))^3} dx$$

### 96.106 Problem number 181

$$\int \frac{1}{x \operatorname{coth}^{-1}(\tanh(a+bx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} &-\frac{1}{2(bx - \operatorname{arccoth}(\tanh(bx+a))) \operatorname{arccoth}(\tanh(bx+a))^2} \\ &+ \frac{1}{(bx - \operatorname{arccoth}(\tanh(bx+a)))^2 \operatorname{arccoth}(\tanh(bx+a))} \\ &- \frac{\ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx+a)))^3} + \frac{\ln(\operatorname{arccoth}(\tanh(bx+a)))}{(bx - \operatorname{arccoth}(\tanh(bx+a)))^3} \end{aligned}$$

command

`integrate(1/x/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4(-3i\pi - 4bx - 6a)}{4\pi^2 b^2 x^2 - 16i\pi ab^2 x^2 - 16a^2 b^2 x^2 + 4i\pi^3 bx + 24\pi^2 abx - 48i\pi a^2 bx - 32a^3 bx - \pi^4 + 8i\pi^3 a + 24\pi^2 a^2 - 32i\pi a^3} - \frac{8i \log(i\pi + 2bx + 2a)}{\pi^3 - 6i\pi^2 a - 12\pi a^2 + 8i a^3} + \frac{8i \log(x)}{\pi^3 - 6i\pi^2 a - 12\pi a^2 + 8i a^3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

### 96.107 Problem number 182

$$\int \frac{1}{x^2 \coth^{-1}(\tanh(a + bx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & -\frac{3b}{2(bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \operatorname{arccoth}(\tanh(bx + a))^2} \\ & + \frac{1}{x(bx - \operatorname{arccoth}(\tanh(bx + a))) \operatorname{arccoth}(\tanh(bx + a))^2} \\ & + \frac{3b}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3 \operatorname{arccoth}(\tanh(bx + a))} \\ & - \frac{3b \ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^4} + \frac{3b \ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^4} \end{aligned}$$

command

`integrate(1/x^2/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{48ib \log(i\pi + 2bx + 2a)}{i\pi^4 + 8\pi^3 a - 24i\pi^2 a^2 - 32\pi a^3 + 16i a^4} - \frac{48ib \log(x)}{i\pi^4 + 8\pi^3 a - 24i\pi^2 a^2 - 32\pi a^3 + 16i a^4} \\ & + \frac{16(8b^2 x + 5i\pi b + 10ab)}{8i\pi^3 b^2 x^2 + 48\pi^2 ab^2 x^2 - 96i\pi a^2 b^2 x^2 - 64a^3 b^2 x^2 - 8\pi^4 bx + 64i\pi^3 abx + 192\pi^2 a^2 bx - 256i\pi a^3 bx - 128a^4 bx -} \\ & + \frac{i\pi^3 x + 6\pi^2 ax - 12i\pi a^2 x - 8a^3 x}{8} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^2 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

## 96.108 Problem number 183

$$\int \frac{1}{x^3 \coth^{-1}(\tanh(a + bx))^3} dx$$

Optimal antiderivative

$$\begin{aligned} & - \frac{3b^2}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^3 \operatorname{arccoth}(\tanh(bx + a))^2} \\ & + \frac{2b}{x (bx - \operatorname{arccoth}(\tanh(bx + a)))^2 \operatorname{arccoth}(\tanh(bx + a))^2} \\ & + \frac{1}{2x^2 (bx - \operatorname{arccoth}(\tanh(bx + a))) \operatorname{arccoth}(\tanh(bx + a))^2} \\ & + \frac{6b^2}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^4 \operatorname{arccoth}(\tanh(bx + a))} \\ & - \frac{6b^2 \ln(x)}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^5} + \frac{6b^2 \ln(\operatorname{arccoth}(\tanh(bx + a)))}{(bx - \operatorname{arccoth}(\tanh(bx + a)))^5} \end{aligned}$$

command

`integrate(1/x^3/arccoth(tanh(b*x+a))^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{192i b^2 \log(i \pi + 2bx + 2a)}{\pi^5 - 10i \pi^4 a - 40 \pi^3 a^2 + 80i \pi^2 a^3 + 80 \pi a^4 - 32i a^5} \\ & - \frac{192i b^2 \log(x)}{\pi^5 - 10i \pi^4 a - 40 \pi^3 a^2 + 80i \pi^2 a^3 + 80 \pi a^4 - 32i a^5} \\ & - \frac{4(i \pi - 12bx + 2a)}{\pi^4 x^2 - 8i \pi^3 a x^2 - 24 \pi^2 a^2 x^2 + 32i \pi a^3 x^2 + 16 a^4 x^2} \\ & + \frac{16(12b^3 x + 7i \pi b^2 + 1}{4 \pi^4 b^2 x^2 - 32i \pi^3 a b^2 x^2 - 96 \pi^2 a^2 b^2 x^2 + 128i \pi a^3 b^2 x^2 + 64 a^4 b^2 x^2 + 4i \pi^5 b x + 40 \pi^4 a b x - 160i \pi^3 a^2 b x - 320 \pi^2 a^3} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{x^3 \operatorname{arccoth}(\tanh(bx + a))^3} dx$$

## 96.109 Problem number 189

$$\int \coth^{-1}(\tanh(a + bx))^n dx$$

Optimal antiderivative

$$\frac{\operatorname{arccoth}(\tanh(bx + a))^{1+n}}{b(1+n)}$$

command

`integrate(arccoth(tanh(b*x+a))^n,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\frac{1}{2} \log(-e^{(2bx+2a)})\right)^{n+1}}{b(n+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\tanh(bx + a))^n dx$$

## 96.110 Problem number 193

$$\int x^m \coth^{-1}(\tanh(a + bx)) dx$$

Optimal antiderivative

$$-\frac{bx^{2+m}}{m^2 + 3m + 2} + \frac{x^{1+m} \operatorname{arccoth}(\tanh(bx + a))}{1 + m}$$

command

`integrate(x^m*arccoth(tanh(b*x+a)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^{m+1} \log\left(-\frac{e^{(2bx+2a)+1}+1}{e^{(2bx+2a)-1}-1}\right)}{2(m+1)} - \frac{bx^{m+2}}{(m+2)(m+1)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^m \operatorname{arccoth}(\tanh(bx + a)) dx$$

## 96.111 Problem number 268

$$\int x(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx$$

Optimal antiderivative

$$\frac{b(d-e)x}{2c} - \frac{bex}{c} + \frac{dx^2(a + b \operatorname{arccoth}(cx))}{2} - \frac{ex^2(a + b \operatorname{arccoth}(cx))}{2} - \frac{b(d-e) \operatorname{arctanh}(cx)}{2c^2}$$

$$+ \frac{be \operatorname{arctanh}(cx)}{c^2} + \frac{bex \ln(-c^2 x^2 + 1)}{2c} - \frac{e(-c^2 x^2 + 1)(a + b \operatorname{arccoth}(cx)) \ln(-c^2 x^2 + 1)}{2c^2}$$

command

`integrate(x*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{4} bex^2 \log(-cx + 1)^2 - \frac{1}{4} (-i\pi bd + i\pi be - 2ad + 2ae)x^2 + \frac{1}{4} \left( bex^2 - \frac{be}{c^2} \right) \log(cx + 1)^2$$

$$- \frac{1}{4} \left( (-i\pi be - bd - 2ae + be)x^2 - \frac{2bex}{c} \right) \log(cx + 1) - \frac{be \log(cx - 1)^2}{4c^2}$$

$$- \frac{1}{4} \left( (-i\pi be + bd - 2ae - be)x^2 - \frac{2bex}{c} - \frac{2be \log(cx - 1)}{c^2} \right) \log(-cx + 1) + \frac{(bd - 3be)x}{2c}$$

$$+ \frac{(-i\pi be - bd - 2ae + 3be) \log(cx + 1)}{4c^2} + \frac{(-i\pi be + bd - 2ae - 3be) \log(cx - 1)}{4c^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \operatorname{arccoth}(cx) + a)(e \log(-c^2 x^2 + 1) + d)x dx$$

## 96.112 Problem number 272

$$\int x^4(a + b \coth^{-1}(cx)) (d + e \log(1 - c^2 x^2)) dx$$

Optimal antiderivative

$$\frac{2aex}{5c^4} - \frac{77be x^2}{300c^3} - \frac{2aex^3}{15c^2} - \frac{9be x^4}{200c} - \frac{2aex^5}{25} - \frac{2bex \operatorname{arccoth}(cx)}{5c^4}$$

$$- \frac{2be x^3 \operatorname{arccoth}(cx)}{15c^2} - \frac{2be x^5 \operatorname{arccoth}(cx)}{25} + \frac{be \operatorname{arccoth}(cx)^2}{5c^5}$$

$$- \frac{(4a + 3b)e \ln(-cx + 1)}{20c^5} + \frac{(4a - 3b)e \ln(cx + 1)}{20c^5} - \frac{23be \ln(-c^2 x^2 + 1)}{75c^5}$$

$$- \frac{be \ln(-c^2 x^2 + 1)^2}{20c^5} + \frac{bx^2(d + e \ln(-c^2 x^2 + 1))}{10c^3} + \frac{bx^4(d + e \ln(-c^2 x^2 + 1))}{20c}$$

$$+ \frac{x^5(a + b \operatorname{arccoth}(cx))(d + e \ln(-c^2 x^2 + 1))}{5} + \frac{b \ln(-c^2 x^2 + 1)(d + e \ln(-c^2 x^2 + 1))}{10c^5}$$

command

```
integrate(x^4*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
 & -\frac{1}{10} b e x^5 \log(-c x+1)^2 - \frac{1}{50} (-5 i \pi b d+2 i \pi b e-10 a d+4 a e) x^5 \\
 & + \frac{(10 b d-9 b e) x^4}{200 c} + \frac{1}{10} \left( b e x^5 + \frac{b e}{c^5} \right) \log(c x+1)^2 - \frac{(i \pi b e+2 a e) x^3}{15 c^2} \\
 & - \frac{1}{300} \left( 6(-5 i \pi b e-5 b d-10 a e+2 b e) x^5 - \frac{15 b e x^4}{c} + \frac{20 b e x^3}{c^2} - \frac{30 b e x^2}{c^3} + \frac{60 b e x}{c^4} \right) \log(c x+1) \\
 & - \frac{1}{300} \left( 6(-5 i \pi b e+5 b d-10 a e-2 b e) x^5 - \frac{15 b e x^4}{c} - \frac{20 b e x^3}{c^2} - \frac{30 b e x^2}{c^3} - \frac{60 b e x}{c^4} - \frac{60 b e \log(c x-1)}{c^5} \right) \log(-c x \\
 & + 1) + \frac{(30 b d-77 b e) x^2}{300 c^3} - \frac{b e \log(c x-1)^2}{10 c^5} - \frac{(i \pi b e+2 a e) x}{5 c^4} \\
 & + \frac{(30 i \pi b e+30 b d+60 a e-137 b e) \log(c x+1)}{300 c^5} + \frac{(-30 i \pi b e+30 b d-60 a e-137 b e) \log(c x-1)}{300 c^5}
 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \operatorname{arccoth}(c x)+a)(e \log(-c^2 x^2+1)+d) x^4 d x$$

### 96.113 Problem number 273

$$\int x^2(a+b \operatorname{coth}^{-1}(c x))(d+e \log(1-c^2 x^2)) d x$$

Optimal antiderivative

$$\begin{aligned}
 & -\frac{2 a e x}{3 c^2} - \frac{5 b e x^2}{18 c} - \frac{2 a e x^3}{9} - \frac{2 b e x \operatorname{arccoth}(c x)}{3 c^2} - \frac{2 b e x^3 \operatorname{arccoth}(c x)}{9} \\
 & + \frac{b e \operatorname{arccoth}(c x)^2}{3 c^3} - \frac{(2 a+b) e \ln(-c x+1)}{6 c^3} + \frac{(2 a-b) e \ln(c x+1)}{6 c^3} \\
 & - \frac{4 b e \ln(-c^2 x^2+1)}{9 c^3} - \frac{b e \ln(-c^2 x^2+1)^2}{12 c^3} + \frac{b x^2(d+e \ln(-c^2 x^2+1))}{6 c} \\
 & + \frac{x^3(a+b \operatorname{arccoth}(c x))(d+e \ln(-c^2 x^2+1))}{3} + \frac{b \ln(-c^2 x^2+1)(d+e \ln(-c^2 x^2+1))}{6 c^3}
 \end{aligned}$$

command

```
integrate(x^2*(a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\begin{aligned}
& -\frac{1}{6} b e x^3 \log(-c x+1)^2 - \frac{1}{18} (-3 i \pi b d+2 i \pi b e-6 a d+4 a e) x^3 \\
& + \frac{1}{6} \left( b e x^3 + \frac{b e}{c^3} \right) \log(c x+1)^2 + \frac{(3 b d-5 b e) x^2}{18 c} \\
& - \frac{1}{18} \left( (-3 i \pi b e-3 b d-6 a e+2 b e) x^3 - \frac{3 b e x^2}{c} + \frac{6 b e x}{c^2} \right) \log(c x+1) \\
& - \frac{1}{18} \left( (-3 i \pi b e+3 b d-6 a e-2 b e) x^3 - \frac{3 b e x^2}{c} - \frac{6 b e x}{c^2} - \frac{6 b e \log(c x-1)}{c^3} \right) \log(-c x \\
& + 1) - \frac{b e \log(c x-1)^2}{6 c^3} - \frac{(i \pi b e+2 a e) x}{3 c^2} + \frac{(3 i \pi b e+3 b d+6 a e-11 b e) \log(c x+1)}{18 c^3} \\
& + \frac{(-3 i \pi b e+3 b d-6 a e-11 b e) \log(c x-1)}{18 c^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \operatorname{arccoth}(c x)+a)(e \log(-c^2 x^2+1)+d) x^2 d x$$

### 96.114 Problem number 274

$$\int (a+b \operatorname{coth}^{-1}(c x))(d+e \log(1-c^2 x^2)) d x$$

Optimal antiderivative

$$\begin{aligned}
& -2 a e x-2 b e x \operatorname{arccoth}(c x)+\frac{e(a+b \operatorname{arccoth}(c x))^2}{b c}-\frac{b e \ln(-c^2 x^2+1)}{c} \\
& +x(a+b \operatorname{arccoth}(c x))(d+e \ln(-c^2 x^2+1))+\frac{b(d+e \ln(-c^2 x^2+1))^2}{4 c e}
\end{aligned}$$

command

`integrate((a+b*arccoth(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& -\frac{1}{2} b e x \log(-c x+1)^2 - \frac{1}{2} (-i \pi b e-b d-2 a e+2 b e) x \log(c x+1) \\
& + \frac{1}{2} \left( b e x + \frac{b e}{c} \right) \log(c x+1)^2 - \frac{b e \log(c x-1)^2}{2 c} - \frac{1}{2} (-i \pi b d+2 i \pi b e-2 a d+4 a e) x \\
& - \frac{1}{2} \left( (-i \pi b e+b d-2 a e-2 b e) x - \frac{2 b e \log(c x-1)}{c} \right) \log(-c x+1) \\
& + \frac{(i \pi b e+b d+2 a e-2 b e) \log(c x+1)}{2 c} + \frac{(-i \pi b e+b d-2 a e-2 b e) \log(c x-1)}{2 c}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int (b \operatorname{arccoth}(c x)+a)(e \log(-c^2 x^2+1)+d) d x$$

**96.115 Problem number 292**

$$\int \frac{1}{(a - ax^2)(b - 2b \coth^{-1}(x))} dx$$

Optimal antiderivative

$$-\frac{\ln(1 - 2 \operatorname{arccoth}(x))}{2ab}$$

command

```
integrate(1/(-a*x^2+a)/(b-2*b*arccoth(x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\log\left(\frac{1}{4}\pi^2(\operatorname{sgn}(x+1)\operatorname{sgn}(x-1)-1)^2 + \left(\log\left(\frac{|x+1|}{|x-1|}\right) - 1\right)^2\right)}{4ab}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{1}{(ax^2 - a)(2b \operatorname{arccoth}(x) - b)} dx$$

**96.116 Problem number 293**

$$\int x^3 \coth^{-1}(a + bx^4) dx$$

Optimal antiderivative

$$\frac{(bx^4 + a) \operatorname{arccoth}(bx^4 + a)}{4b} + \frac{\ln(1 - (bx^4 + a)^2)}{8b}$$

command

```
integrate(x^3*arccoth(b*x^4+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{8} ((a+1)b - (a-1)b) \left( \frac{\log\left(\left|\frac{bx^4+a+1}{bx^4+a-1}\right|\right)}{b^2} - \frac{\log\left(\left|\frac{bx^4+a+1}{bx^4+a-1} - 1\right|\right)}{b^2} + \frac{\log\left(\frac{\frac{\frac{1}{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1} - a-1\right)b} + 1}}{\frac{(bx^4+a+1)b}{bx^4+a-1} - b}}{\frac{\frac{1}{\left(\frac{(bx^4+a+1)(a-1)}{bx^4+a-1} - a-1\right)b} - 1}}{\frac{(bx^4+a+1)b}{bx^4+a-1} - b}}\right)}{b^2\left(\frac{bx^4+a+1}{bx^4+a-1} - 1\right)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^3 \operatorname{arccoth}(bx^4 + a) dx$$

### 96.117 Problem number 294

$$\int x^{-1+n} \operatorname{coth}^{-1}(a + bx^n) dx$$

Optimal antiderivative

$$\frac{(a + bx^n) \operatorname{arccoth}(a + bx^n)}{bn} + \frac{\ln\left(1 - (a + bx^n)^2\right)}{2bn}$$

command

`integrate(x^(-1+n)*arccoth(a+b*x^n),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{((a+1)b - (a-1)b) \left( \frac{\log\left(\left|\frac{bx^n+a+1}{bx^n+a-1}\right|\right)}{b^2} - \frac{\log\left(\left|\frac{bx^n+a+1}{bx^n+a-1} - 1\right|\right)}{b^2} + \frac{\log\left(\frac{bx^n+a+1}{bx^n+a-1}\right)}{b^2\left(\frac{bx^n+a+1}{bx^n+a-1} - 1\right)} \right)}{2n}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int x^{n-1} \operatorname{arccoth}(bx^n + a) dx$$

**96.118 Problem number 295**

$$\int e^{c(a+bx)} \coth^{-1}(\sinh(ac + bcx)) dx$$

Optimal antiderivative

$$\frac{e^{bcx+ac} \operatorname{arccoth}(\sinh(c(bx+a)))}{bc} + \frac{\ln(3 - e^{2c(bx+a)} - 2\sqrt{2})(1 - \sqrt{2})}{2bc}$$

$$+ \frac{\ln(3 - e^{2c(bx+a)} + 2\sqrt{2})(1 + \sqrt{2})}{2bc}$$

command

`integrate(exp(c*(b*x+a))*arccoth(sinh(b*c*x+a*c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{((bx+a)c)} \log\left(-\frac{e^{\frac{(bcx+ac)-e^{(-bcx-ac)}}{2}}+1}{e^{\frac{(bcx+ac)-e^{(-bcx-ac)}}{2}}-1}\right)}{2bc}$$

$$+ \frac{\sqrt{2} \log\left(\left|\frac{-4\sqrt{2}+2e^{(2bcx+2ac)}-6}{4\sqrt{2}+2e^{(2bcx+2ac)}-6}\right|\right) + \log(|e^{(4bcx+4ac)} - 6e^{(2bcx+2ac)} + 1|)}{2bc}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\sinh(bcx + ac)) e^{((bx+a)c)} dx$$

**96.119 Problem number 296**

$$\int e^{c(a+bx)} \coth^{-1}(\cosh(ac + bcx)) dx$$

Optimal antiderivative

$$\frac{e^{bcx+ac} \operatorname{arccoth}(\cosh(c(bx+a)))}{bc} + \frac{\ln(1 - e^{2c(bx+a)})}{bc}$$

command

`integrate(exp(c*(b*x+a))*arccoth(cosh(b*c*x+a*c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{(bx+a)c} \log\left(-\frac{\frac{e^{(bcx+ac)} + e^{(-bcx-ac)}}{2} + 1}{\frac{e^{(bcx+ac)} + e^{(-bcx-ac)}}{2} - 1}\right)}{2bc} + \frac{\log(|e^{(2bcx+2ac)} - 1|)}{bc}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arcoth}(\cosh(bc x + ac)) e^{(bx+a)c} dx$$

### 96.120 Problem number 297

$$\int e^{c(a+bx)} \operatorname{coth}^{-1}(\tanh(ac + bcx)) dx$$

Optimal antiderivative

$$-\frac{e^{bcx+ac}}{bc} + \frac{e^{bcx+ac} \operatorname{arccoth}(\tanh(c(bx+a)))}{bc}$$

command

`integrate(exp(c*(b*x+a))*arccoth(tanh(b*c*x+a*c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(e^{(bcx)} \log(-e^{(2bcx+2ac)}) - 2e^{(bcx)})e^{(ac)}}{2bc}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arcoth}(\tanh(bc x + ac)) e^{(bx+a)c} dx$$

### 96.121 Problem number 299

$$\int e^{c(a+bx)} \operatorname{coth}^{-1}(\operatorname{sech}(ac + bcx)) dx$$

Optimal antiderivative

$$\frac{e^{bcx+ac} \operatorname{arccoth}(\operatorname{sech}(c(bx+a)))}{bc} + \frac{\ln(1 - e^{2c(bx+a)})}{bc}$$

command

`integrate(exp(c*(b*x+a))*arccoth(sech(b*c*x+a*c)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(e^{(bcx)} \log\left(-\frac{e^{(2bcx+2ac)}}{e^{(2bcx+2ac)} - 2e^{(bcx+ac)} + 1} - \frac{2e^{(bcx+ac)}}{e^{(2bcx+2ac)} - 2e^{(bcx+ac)} + 1} - \frac{1}{e^{(2bcx+2ac)} - 2e^{(bcx+ac)} + 1}\right) + 2e^{(-ac)} \log(|e^{(2bcx+2ac)} - 1|)\right)}{2bc}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arcoth}(\operatorname{sech}(bc x + ac)) e^{(bx+a)c} dx$$

### 96.122 Problem number 300

$$\int e^{c(a+bx)} \coth^{-1}(\operatorname{csch}(ac + bcx)) dx$$

Optimal antiderivative

$$\frac{e^{bcx+ac} \operatorname{arccoth}(\operatorname{csch}(c(bx+a)))}{bc} + \frac{\ln\left(3 - e^{2c(bx+a)} - 2\sqrt{2}\right) (1 - \sqrt{2})}{2bc}$$

$$+ \frac{\ln\left(3 - e^{2c(bx+a)} + 2\sqrt{2}\right) (1 + \sqrt{2})}{2bc}$$

command

```
integrate(exp(c*(b*x+a))*arccoth(csch(b*c*x+a*c)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{((bx+a)c)} \log\left(-\frac{e^{(bcx+ac)} - e^{(-bcx-ac)} + 2}{e^{(bcx+ac)} - e^{(-bcx-ac)} - 2}\right)}{2bc}$$

$$+ \frac{\sqrt{2} \log\left(\frac{|-4\sqrt{2} + 2e^{(2bcx+2ac)} - 6|}{|4\sqrt{2} + 2e^{(2bcx+2ac)} - 6|}\right) + \log(|e^{(4bcx+4ac)} - 6e^{(2bcx+2ac)} + 1|)}{2bc}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \operatorname{arccoth}(\operatorname{csch}(bcx + ac)) e^{((bx+a)c)} dx$$

## 97 Test file number 203

Test folder name:

test\_cases/7\_Inverse\_hyperbolic\_functions/7.6\_Inverse\_hyperbolic\_cosecant/203\_7.6.2\_Inverse\_h

### 97.1 Problem number 49

$$\int e^{2\operatorname{csch}^{-1}(ax)} x^4 dx$$

Optimal antiderivative

$$\frac{2x^3}{3a^2} + \frac{x^5}{5} - \frac{\operatorname{arctanh}\left(\sqrt{1 + \frac{1}{a^2x^2}}\right)}{4a^5} + \frac{x^2 \sqrt{1 + \frac{1}{a^2x^2}}}{4a^3} + \frac{x^4 \sqrt{1 + \frac{1}{a^2x^2}}}{2a}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} \sqrt{a^2 x^2 + 1} x \left( \frac{2 x^2 |a| \operatorname{sgn}(x)}{a^3} + \frac{|a| \operatorname{sgn}(x)}{a^5} \right) + \frac{3 a^2 x^5 + 10 x^3}{15 a^2} + \frac{\log(-x|a| + \sqrt{a^2 x^2 + 1}) \operatorname{sgn}(x)}{4 a^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 97.2 Problem number 50

$$\int e^{2\operatorname{csch}^{-1}(ax)} x^3 dx$$

Optimal antiderivative

$$\frac{x^2}{a^2} + \frac{2 \left(1 + \frac{1}{a^2 x^2}\right)^{\frac{3}{2}} x^3}{3a} + \frac{x^4}{4}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^2 x^2 + 1}{2 a^4} - \frac{2 |a| \operatorname{sgn}(x)}{3 a^5} + \frac{8 (a^2 x^2 + 1)^{\frac{3}{2}} a^2 |a| \operatorname{sgn}(x) + 3 (a^2 x^2 + 1)^2 a^3}{12 a^7}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 97.3 Problem number 51

$$\int e^{2\operatorname{csch}^{-1}(ax)} x^2 dx$$

Optimal antiderivative

$$\frac{2x}{a^2} + \frac{x^3}{3} + \frac{\operatorname{arctanh}\left(\sqrt{1 + \frac{1}{a^2 x^2}}\right)}{a^3} + \frac{x^2 \sqrt{1 + \frac{1}{a^2 x^2}}}{a}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{a^2x^2 + 1} x|a|\operatorname{sgn}(x)}{a^3} + \frac{a^2x^3 + 6x}{3a^2} - \frac{\log(-x|a| + \sqrt{a^2x^2 + 1}) \operatorname{sgn}(x)}{a^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 97.4 Problem number 52

$$\int e^{2\operatorname{csch}^{-1}(ax)} x dx$$

Optimal antiderivative

$$\frac{x^2}{2} - \frac{2 \operatorname{arccsch}(ax)}{a^2} + \frac{2 \ln(x)}{a^2} + \frac{2x \sqrt{1 + \frac{1}{a^2x^2}}}{a}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2*x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4 \sqrt{a^2x^2 + 1} |a|\operatorname{sgn}(x) + (a^2x^2 + 1)a - 2(|a|\operatorname{sgn}(x) - a) \log(\sqrt{a^2x^2 + 1} + 1) + 2(|a|\operatorname{sgn}(x) + a) \log(\sqrt{a^2x^2 + 1} - 1)}{2a^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 97.5 Problem number 54

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x} dx$$

Optimal antiderivative

$$-\frac{1}{a^2x^2} - \operatorname{arccsch}(ax) + \ln(x) - \frac{\sqrt{1 + \frac{1}{a^2x^2}}}{ax}$$



command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(a^4|a|\operatorname{sgn}(x) - a^5) \log\left(\sqrt{a^2x^2 + 1} + 1\right) - (a^4|a|\operatorname{sgn}(x) + a^5) \log\left(\sqrt{a^2x^2 + 1} - 1\right) + \frac{2\left(\sqrt{a^2x^2 + 1} a^4|a|\operatorname{sgn}(x)\right)}{\left(\sqrt{a^2x^2 + 1} + 1\right)\left(\sqrt{a^2x^2 + 1} - 1\right)}}{2a^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 97.6 Problem number 55

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^2} dx$$

Optimal antiderivative

$$-\frac{2a\left(1 + \frac{1}{a^2x^2}\right)^{\frac{3}{2}}}{3} - \frac{2}{3a^2x^3} - \frac{1}{x}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4\left(3\left(x|a| - \sqrt{a^2x^2 + 1}\right)^4 a\operatorname{sgn}(x) + a\operatorname{sgn}(x)\right)}{3\left(\left(x|a| - \sqrt{a^2x^2 + 1}\right)^2 - 1\right)^3} - \frac{3a^2x^2 + 2}{3a^2x^3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 97.7 Problem number 56

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^3} dx$$

Optimal antiderivative

$$-\frac{1}{2a^2x^4} - \frac{1}{2x^2} + \frac{a^2 \operatorname{arccsch}(ax)}{4} - \frac{\sqrt{1 + \frac{1}{a^2x^2}}}{2x^3a} - \frac{a\sqrt{1 + \frac{1}{a^2x^2}}}{4x}$$

command

`integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^3,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{a^6|a| \log\left(\sqrt{a^2x^2+1}+1\right) \operatorname{sgn}(x) - a^6|a| \log\left(\sqrt{a^2x^2+1}-1\right) \operatorname{sgn}(x) - \frac{2\left((a^2x^2+1)^{\frac{3}{2}} a^6|a| \operatorname{sgn}(x) + \sqrt{a^2x^2+1} a^6|a| \operatorname{sgn}(x)\right)}{a^4x^4}}{8a^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 97.8 Problem number 57

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^4} dx$$

Optimal antiderivative

$$\frac{2a^3\left(1 + \frac{1}{a^2x^2}\right)^{\frac{3}{2}}}{3} - \frac{2a^3\left(1 + \frac{1}{a^2x^2}\right)^{\frac{5}{2}}}{5} - \frac{2}{5a^2x^5} - \frac{1}{3x^3}$$

command

`integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^4,x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8\left(15\left(x|a| - \sqrt{a^2x^2+1}\right)^6 a^3 \operatorname{sgn}(x) + 5\left(x|a| - \sqrt{a^2x^2+1}\right)^4 a^3 \operatorname{sgn}(x) + 5\left(x|a| - \sqrt{a^2x^2+1}\right)^2 a^3 \operatorname{sgn}(x) - a^3\right)}{15\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^5} - \frac{5a^2x^2+6}{15a^2x^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 97.9 Problem number 58

$$\int \frac{e^{2\operatorname{csch}^{-1}(ax)}}{x^5} dx$$

Optimal antiderivative

$$-\frac{1}{3a^2x^6} - \frac{1}{4x^4} - \frac{a^4 \operatorname{arccsch}(ax)}{8} - \frac{\sqrt{1 + \frac{1}{a^2x^2}}}{3ax^5} - \frac{a\sqrt{1 + \frac{1}{a^2x^2}}}{12x^3} + \frac{a^3\sqrt{1 + \frac{1}{a^2x^2}}}{8x}$$

command

```
integrate((1/a/x+(1+1/a^2/x^2)^(1/2))^2/x^5,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3a^8|a|\log\left(\sqrt{a^2x^2+1}+1\right)\operatorname{sgn}(x)-3a^8|a|\log\left(\sqrt{a^2x^2+1}-1\right)\operatorname{sgn}(x)-\frac{2\left(3(a^2x^2+1)^{\frac{5}{2}}a^8|a|\operatorname{sgn}(x)-8(a^2x^2+1)^{\frac{3}{2}}a\right)}{48a^5}}{48a^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 98 Test file number 206

Test folder name:

test\_cases/8\_Special\_functions/206\_8.4\_Trig\_integral\_functions

### 98.1 Problem number 63

$$\int x \sin(ax + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} \\
& + \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{\cos\left(a - c + (b-d)x\right)}{2b(b-d)} \\
& - \frac{\cos\left(a + c + (b+d)x\right)}{2b(b+d)} + \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2bd} \\
& - \frac{x \cos(bx+a) \operatorname{sinIntegral}(dx+c)}{b} - \frac{c \cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2bd} \\
& + \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& - \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{\operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{\operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} + \frac{\operatorname{sinIntegral}(dx+c) \sin(bx+a)}{b^2}
\end{aligned}$$

command

```
integrate(x*sin_integral(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**98.2 Problem number 66**

$$\int x \cos(a + bx) \operatorname{Si}(c + dx) dx$$

Optimal antiderivative

$$\begin{aligned}
& \frac{c \operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\
& - \frac{c \operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \cos\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right)}{2b^2} + \frac{\cos(bx+a) \operatorname{sinIntegral}(dx+c)}{b^2} \\
& - \frac{\cos\left(a - \frac{bc}{d}\right) \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right)}{2b^2} \\
& + \frac{\operatorname{cosineIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{\operatorname{cosineIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2b^2} \\
& - \frac{c \operatorname{sinIntegral}\left(\frac{c(b-d)}{d} + (b-d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} \\
& + \frac{c \operatorname{sinIntegral}\left(\frac{c(b+d)}{d} + (b+d)x\right) \sin\left(a - \frac{bc}{d}\right)}{2bd} + \frac{x \operatorname{sinIntegral}(dx+c) \sin(bx+a)}{b} \\
& - \frac{\sin\left(a - c + (b-d)x\right)}{2b(b-d)} + \frac{\sin\left(a + c + (b+d)x\right)}{2b(b+d)}
\end{aligned}$$

command

```
integrate(x*cos(b*x+a)*sin_integral(d*x+c),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

**99 Test file number 209**

Test folder name:

test\_cases/209\_Blake\_problems

### 99.1 Problem number 521

$$\int \frac{-1+x}{\sqrt{-7+4x+14x^2-12x^3+x^4}} dx$$

Optimal antiderivative

$$\ln(-1+x) - \ln\left(-5+6x-x^2 + \sqrt{x^4-12x^3+14x^2+4x-7}\right)$$

command

```
integrate((-1+x)/(x^4-12*x^3+14*x^2+4*x-7)^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(4\sqrt{2}\right) \operatorname{sgn}(x-1) - \frac{\log\left(\left|-x + \sqrt{x^2 - 10x - 7} + 5\right|\right)}{\operatorname{sgn}(x-1)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

### 99.2 Problem number 1760

$$\int \frac{1}{(f+ex)\sqrt{d+\sqrt{c+\sqrt{b+ax}}}} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate(1/(e*x+f)/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 99.3 Problem number 2024

$$\int \frac{1}{(-1+x)(-2x^2-3x^3+x^4)^{3/2}} dx$$

Optimal antiderivative

$$\frac{(-453x^3 + 1555x^2 + 238x - 136) \sqrt{x^4 - 3x^3 - 2x^2}}{544x^3(x^2 - 3x - 2)} + \frac{\arctan\left(\frac{-\frac{x}{2} + \frac{x^2}{2} - \frac{\sqrt{x^4 - 3x^3 - 2x^2}}{2}}{x}\right)}{4} - \frac{119 \arctan\left(\frac{\frac{x^2\sqrt{2}}{2} - \frac{\sqrt{x^4 - 3x^3 - 2x^2}\sqrt{2}}{2}}{x}\right)\sqrt{2}}{64}$$

command

`integrate(1/(-1+x)/(x^4-3*x^3-2*x^2)^(3/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{29x}{\operatorname{sgn}(x)} - \frac{103}{\operatorname{sgn}(x)}}{68\sqrt{x^2 - 3x - 2}} + \frac{119\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x - \sqrt{x^2 - 3x - 2}\right)\right)}{64\operatorname{sgn}(x)} - \frac{\arctan\left(-\frac{1}{2}x + \frac{1}{2}\sqrt{x^2 - 3x - 2} + \frac{1}{2}\right)}{4\operatorname{sgn}(x)} - \frac{47\left(x - \sqrt{x^2 - 3x - 2}\right)^3 + 16\left(x - \sqrt{x^2 - 3x - 2}\right)^2 + 98x - 98\sqrt{x^2 - 3x - 2} + 128}{32\left(\left(x - \sqrt{x^2 - 3x - 2}\right)^2 + 2\right)^2\operatorname{sgn}(x)}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

### 99.4 Problem number 2308

$$\int \frac{\sqrt[3]{bx+ax^3}(b+ax^4)}{x^4} dx$$

Optimal antiderivative

$$\frac{(x^3a+bx)^{\frac{1}{3}}(4ax^4-3ax^2-3b)}{8x^3} - \frac{a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}a^{\frac{1}{3}}x}{a^{\frac{1}{3}}x+2(x^3a+bx)^{\frac{1}{3}}}\right)\sqrt{3}}{6} - \frac{a^{\frac{1}{3}}b\ln\left(-a^{\frac{1}{3}}x+(x^3a+bx)^{\frac{1}{3}}\right)}{6} + \frac{a^{\frac{1}{3}}b\ln\left(a^{\frac{2}{3}}x^2+a^{\frac{1}{3}}x(x^3a+bx)^{\frac{1}{3}}+(x^3a+bx)^{\frac{2}{3}}\right)}{12}$$

command

```
integrate((a*x^3+b*x)^(1/3)*(a*x^4+b)/x^4,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$12 \left(a + \frac{b}{x^2}\right)^{\frac{1}{3}} abx^2 + 4 \sqrt{3} a^{\frac{1}{3}} b^2 \arctan \left( \frac{\sqrt{3} \left(2 \left(a + \frac{b}{x^2}\right)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}} \right) + 2 a^{\frac{1}{3}} b^2 \log \left( \left(a + \frac{b}{x^2}\right)^{\frac{2}{3}} + \left(a + \frac{b}{x^2}\right)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}} \right) - 4$$


---


$$24 b$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 99.5 Problem number 2358

$$\int \frac{\sqrt[4]{bx^3 + ax^4}}{x^2(-d + cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4+b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left(\frac{ad + \sqrt{cd} b}{d}\right)^{\frac{1}{4}} \arctan \left( \frac{\left(a + \frac{b}{x}\right)^{\frac{1}{4}} d}{\left(ad^4 + \sqrt{cd} bd^3\right)^{\frac{1}{4}}} \right) + 2 \left(\frac{ad - \sqrt{cd} b}{d}\right)^{\frac{1}{4}} \arctan \left( \frac{\left(a + \frac{b}{x}\right)^{\frac{1}{4}} d}{\left(ad^4 - \sqrt{cd} bd^3\right)^{\frac{1}{4}}} \right) + \left(\frac{ad + \sqrt{cd} b}{d}\right)^{\frac{1}{4}} \log$$


---


$$+ \frac{4 \left(a + \frac{b}{x}\right)^{\frac{1}{4}}}{d}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*



### 99.6 Problem number 2359

$$\int \frac{\sqrt[4]{bx^3 + ax^4}}{x^2(-d + cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4+b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + 2 \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \frac{ad + \sqrt{cd} b}{d} \right) + \frac{4 \left( a + \frac{b}{x} \right)^{\frac{1}{4}}}{d}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

### 99.7 Problem number 2384

$$\int \frac{\sqrt[4]{-bx^3 + ax^4}}{x^2(-d + cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4-b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a - \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + 2 \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a - \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \frac{ad + \sqrt{cd} b}{d} \right) + \frac{4 \left( a - \frac{b}{x} \right)^{\frac{1}{4}}}{d}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

## 99.8 Problem number 2385

$$\int \frac{\sqrt[4]{-bx^3 + ax^4}}{x^2(-d + cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4-b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a - \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + 2 \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a - \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) + \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log$$


---


$$+ \frac{4 \left( a - \frac{b}{x} \right)^{\frac{1}{4}}}{d}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

## 99.9 Problem number 2406

$$\int \frac{x^4(2 + x^5)}{\sqrt{1 + x^5}(-1 - x^5 + ax^{10})} dx$$

Optimal antiderivative

$$\frac{\sqrt{2} \left( 1 + 4a + \sqrt{1 + 4a} \right) \arctan \left( \frac{\sqrt{2} \sqrt{a} \sqrt{x^5 + 1}}{\sqrt{-1 - 2a - \sqrt{1 + 4a}}} \right)}{5\sqrt{a} \sqrt{1 + 4a} \sqrt{-1 - 2a - \sqrt{1 + 4a}}} + \frac{\sqrt{2} \left( -1 - 4a + \sqrt{1 + 4a} \right) \arctan \left( \frac{\sqrt{2} \sqrt{a} \sqrt{x^5 + 1}}{\sqrt{-1 - 2a + \sqrt{1 + 4a}}} \right)}{5\sqrt{a} \sqrt{1 + 4a} \sqrt{-1 - 2a + \sqrt{1 + 4a}}}$$

command

`integrate(x^4*(x^5+2)/(x^5+1)^(1/2)/(a*x^10-x^5-1),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\pi - 2 \arctan\left(\frac{(x^5+1)^{a-a}}{\sqrt{x^5+1} \sqrt{-a}}\right)}{5 \sqrt{-a}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 99.10 Problem number 2407

$$\int \frac{x^4(-2+x^5)}{\sqrt{-1+x^5}(1-x^5+ax^{10})} dx$$

Optimal antiderivative

$$\frac{\sqrt{2} (1 + \sqrt{1-4a} - 4a) \arctan\left(\frac{\sqrt{2} \sqrt{a} \sqrt{x^5-1}}{\sqrt{-1-\sqrt{1-4a}+2a}}\right)}{5\sqrt{1-4a} \sqrt{a} \sqrt{-1-\sqrt{1-4a}+2a}} + \frac{\sqrt{2} (-1 + \sqrt{1-4a} + 4a) \arctan\left(\frac{\sqrt{2} \sqrt{a} \sqrt{x^5-1}}{\sqrt{-1+\sqrt{1-4a}+2a}}\right)}{5\sqrt{1-4a} \sqrt{a} \sqrt{-1+\sqrt{1-4a}+2a}}$$

command

`integrate(x^4*(x^5-2)/(x^5-1)^(1/2)/(a*x^10-x^5+1),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\pi + 2 \arctan\left(\frac{(x^5-1)^{a+a}}{\sqrt{x^5-1} \sqrt{-a}}\right)}{5 \sqrt{-a}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.11 Problem number 2408

$$\int \frac{x^4(3+x^5)}{\sqrt{1+x^5}(-1+a-(1+2a)x^5+ax^{10})} dx$$

Optimal antiderivative

$$\frac{\sqrt{2} (1 + 8a + \sqrt{1 + 8a}) \arctan\left(\frac{\sqrt{2} \sqrt{a} \sqrt{x^5 + 1}}{\sqrt{-1 - 4a - \sqrt{1 + 8a}}}\right)}{5\sqrt{a} \sqrt{1 + 8a} \sqrt{-1 - 4a - \sqrt{1 + 8a}}} + \frac{\sqrt{2} (-1 - 8a + \sqrt{1 + 8a}) \arctan\left(\frac{\sqrt{2} \sqrt{a} \sqrt{x^5 + 1}}{\sqrt{-1 - 4a + \sqrt{1 + 8a}}}\right)}{5\sqrt{a} \sqrt{1 + 8a} \sqrt{-1 - 4a + \sqrt{1 + 8a}}}$$

command

```
integrate(x^4*(x^5+3)/(x^5+1)^(1/2)/(-1+a-(1+2*a)*x^5+a*x^10),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\sqrt{-a} \arctan\left(\frac{2\sqrt{\frac{1}{2}} \sqrt{x^5 + 1}}{\sqrt{\frac{4a + \sqrt{(4a+1)^2 - 16a^2} + 1}{a}}}\right)}{5|a|}$$

$$(8\sqrt{-8a^2 - a} \sqrt{-a} a^2 \operatorname{sgn}(a) + 8\sqrt{-8a^2 - a} \sqrt{8a + 1} a^2 \operatorname{sgn}(a) - 8\sqrt{-a} a^3 - 8\sqrt{8a + 1} a^3 + 3\sqrt{-8a^2 - a})$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.12 Problem number 2525

$$\int \frac{(-b + ax^2) \sqrt[4]{-bx^2 + ax^4}}{b - ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(x^4-a*x^2+b),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} a \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} a \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \frac{1}{4} \sqrt{2} (-a)^{\frac{1}{4}} a \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right) \\ & - \frac{1}{4} \sqrt{2} (-a)^{\frac{1}{4}} a \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.13 Problem number 2526

$$\int \frac{(-b + ax^2) \sqrt[4]{-bx^2 + ax^4}}{b - ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^2-b)*(a*x^4-b*x^2)^(1/4)/(x^4-a*x^2+b),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} a \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\
& + \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} a \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\
& + \frac{1}{4} \sqrt{2} (-a)^{\frac{1}{4}} a \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right) \\
& - \frac{1}{4} \sqrt{2} (-a)^{\frac{1}{4}} a \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right)
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

#### 99.14 Problem number 2651

$$\int \frac{\sqrt[4]{-bx^3 + ax^4}}{d + cx^2} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4-b*x^3)^(1/4)/(c*x^2+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{\sqrt{2}(-a)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{c} \\
& + \frac{\sqrt{2}(-a)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{c} \\
& + \frac{\sqrt{2}(-a)^{\frac{1}{4}} \log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2c} \\
& - \frac{\sqrt{2}(-a)^{\frac{1}{4}} \log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2c} \\
& - \frac{2\left(\frac{ad+\sqrt{-cd}b}{d}\right)^{\frac{1}{4}} \arctan\left(\frac{\left(a-\frac{b}{x}\right)^{\frac{1}{4}}d}{\left(ad^4+\sqrt{-cd}bd^3\right)^{\frac{1}{4}}}\right)+2\left(\frac{ad-\sqrt{-cd}b}{d}\right)^{\frac{1}{4}} \arctan\left(\frac{\left(a-\frac{b}{x}\right)^{\frac{1}{4}}d}{\left(ad^4-\sqrt{-cd}bd^3\right)^{\frac{1}{4}}}\right)+\left(\frac{ad+\sqrt{-cd}b}{d}\right)^{\frac{1}{4}}}{c}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

### 99.15 Problem number 2652

$$\int \frac{\sqrt[4]{-bx^3+ax^4}}{d+cx^2} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4-b*x^3)^(1/4)/(c*x^2+d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{\sqrt{2}(-a)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{c} \\
& + \frac{\sqrt{2}(-a)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(a-\frac{b}{x}\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{c} \\
& + \frac{\sqrt{2}(-a)^{\frac{1}{4}} \log\left(\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2c} \\
& - \frac{\sqrt{2}(-a)^{\frac{1}{4}} \log\left(-\sqrt{2}(-a)^{\frac{1}{4}}\left(a-\frac{b}{x}\right)^{\frac{1}{4}}+\sqrt{-a}+\sqrt{a-\frac{b}{x}}\right)}{2c} \\
& - \frac{2\left(\frac{ad+\sqrt{-cd}b}{d}\right)^{\frac{1}{4}} \arctan\left(\frac{\left(a-\frac{b}{x}\right)^{\frac{1}{4}}d}{\left(ad^4+\sqrt{-cd}bd^3\right)^{\frac{1}{4}}}\right)+2\left(\frac{ad-\sqrt{-cd}b}{d}\right)^{\frac{1}{4}} \arctan\left(\frac{\left(a-\frac{b}{x}\right)^{\frac{1}{4}}d}{\left(ad^4-\sqrt{-cd}bd^3\right)^{\frac{1}{4}}}\right)+\left(\frac{ad+\sqrt{-cd}b}{d}\right)^{\frac{1}{4}}}{c}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

### 99.16 Problem number 2672

$$\int \frac{(d+cx^2)^{\frac{1}{4}} \sqrt[4]{bx^3+ax^4}}{x^2(-d+cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((c*x^2+d)*(a*x^4+b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output



$$\begin{aligned}
& \sqrt{2} (-a)^{\frac{1}{4}} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\
& + \sqrt{2} (-a)^{\frac{1}{4}} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\
& + \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right) \\
& - \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right) \\
& - 2 \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) \\
& - 2 \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) \\
& - \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| \left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\
& - \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| \left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\
& + \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| -\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\
& + \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| -\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) + 4 \left( a + \frac{b}{x} \right)^{\frac{1}{4}}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

## 99.17 Problem number 2673

$$\int \frac{(d + cx^2)^4 \sqrt[4]{bx^3 + ax^4}}{x^2(-d + cx^2)} dx$$

Optimal antiderivative

*Unintegrable*

command

`integrate((c*x^2+d)*(a*x^4+b*x^3)^(1/4)/x^2/(c*x^2-d),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \sqrt{2} (-a)^{\frac{1}{4}} \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \sqrt{2} (-a)^{\frac{1}{4}} \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right) \\ & - \frac{1}{2} \sqrt{2} (-a)^{\frac{1}{4}} \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right) \\ & - 2 \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) \\ & - 2 \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \arctan \left( \frac{\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d}{\left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}}} \right) \\ & - \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| \left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\ & - \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| \left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\ & + \left( \frac{ad + \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| -\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 + \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) \\ & + \left( \frac{ad - \sqrt{cd} b}{d} \right)^{\frac{1}{4}} \log \left( \left| -\left( a + \frac{b}{x} \right)^{\frac{1}{4}} d + \left( ad^4 - \sqrt{cd} bd^3 \right)^{\frac{1}{4}} \right| \right) + 4 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>2</sub>*

### 99.18 Problem number 2706

$$\int \frac{x^4 \sqrt{bx^2 + ax^4}}{b + ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate(x^4*(a*x^4+b*x^2)^(1/4)/(x^4+a*x^2+b),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} x^2 + \frac{\sqrt{2} (4a^2 - b) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2(-a)^{\frac{1}{4}}} \right)}{8 (-a)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2} (4a^2 - b) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2(-a)^{\frac{1}{4}}} \right)}{8 (-a)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2} (4a^2 - b) \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}} \right)}{16 (-a)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2} (4a^2 - b) \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}} \right)}{16 (-a)^{\frac{3}{4}}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 99.19 Problem number 2707

$$\int \frac{x^4 \sqrt[4]{bx^2 + ax^4}}{b + ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate(x^4*(a*x^4+b*x^2)^(1/4)/(x^4+a*x^2+b),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} x^2 + \frac{\sqrt{2} (4a^2 - b) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2(-a)^{\frac{1}{4}}} \right)}{8 (-a)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2} (4a^2 - b) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2(-a)^{\frac{1}{4}}} \right)}{8 (-a)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2} (4a^2 - b) \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}} \right)}{16 (-a)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2} (4a^2 - b) \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x^2}} \right)}{16 (-a)^{\frac{3}{4}}} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.20 Problem number 2724

$$\int \frac{\sqrt{1+x^5} (2+x^5)}{x^6 (-1-x^5+ax^{10})} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^5+1}}{5x^5} + \frac{\left(\sqrt{2}\sqrt{a} + 4\sqrt{2}a^{\frac{3}{2}} + \sqrt{2}\sqrt{a}\sqrt{1+4a}\right) \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-1-2a-\sqrt{1+4a}}}\right)}{5\sqrt{1+4a}\sqrt{-1-2a-\sqrt{1+4a}}} \\ + \frac{\left(-\sqrt{2}\sqrt{a} - 4\sqrt{2}a^{\frac{3}{2}} + \sqrt{2}\sqrt{a}\sqrt{1+4a}\right) \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5+1}}{\sqrt{-1-2a+\sqrt{1+4a}}}\right)}{5\sqrt{1+4a}\sqrt{-1-2a+\sqrt{1+4a}}}$$

command

```
integrate((x^5+1)^(1/2)*(x^5+2)/x^6/(a*x^10-x^5-1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{\left(\pi - 2 \arctan\left(\frac{(x^5+1)a-a}{\sqrt{x^5+1}\sqrt{-a}}\right)\right)a}{5\sqrt{-a}} + \frac{2}{5\left(\sqrt{x^5+1} - \frac{1}{\sqrt{x^5+1}}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.21 Problem number 2725

$$\int \frac{(-2+x^5)\sqrt{-1+x^5}}{x^6(1-x^5+ax^{10})} dx$$

Optimal antiderivative

$$\frac{2\sqrt{x^5-1}}{5x^5} + \frac{\left(\sqrt{2}\sqrt{a} + \sqrt{2}\sqrt{1-4a}\sqrt{a} - 4\sqrt{2}a^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-1-\sqrt{1-4a}+2a}}\right)}{5\sqrt{1-4a}\sqrt{-1-\sqrt{1-4a}+2a}} \\ + \frac{\left(-\sqrt{2}\sqrt{a} + \sqrt{2}\sqrt{1-4a}\sqrt{a} + 4\sqrt{2}a^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2}\sqrt{a}\sqrt{x^5-1}}{\sqrt{-1+\sqrt{1-4a}+2a}}\right)}{5\sqrt{1-4a}\sqrt{-1+\sqrt{1-4a}+2a}}$$

command

`integrate((x^5-2)*(x^5-1)^(1/2)/x^6/(a*x^10-x^5+1),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\left(\pi + 2 \arctan\left(\frac{(x^5-1)a+a}{\sqrt{x^5-1}\sqrt{-a}}\right)\right)a}{5\sqrt{-a}} + \frac{2}{5\left(\sqrt{x^5-1} + \frac{1}{\sqrt{x^5-1}}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.22 Problem number 2893

$$\int \frac{f + ex}{(h + gx)\sqrt{d + \sqrt{c + \sqrt{b + ax}}}} dx$$

Optimal antiderivative

*Unintegrable*

command

`integrate((e*x+f)/(g*x+h)/(d+(c+(a*x+b)^(1/2))^(1/2))^(1/2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.23 Problem number 2897

$$\int \frac{1}{(-b + ax)\sqrt[3]{-b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

$$\frac{\sqrt{-3 + 3i\sqrt{3}} \arctan\left(\frac{\sqrt{3} a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}x}{a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}x - 2(-1)^{\frac{1}{3}}(a^3x^3 - b^2x^2)^{\frac{1}{3}}}\right) \sqrt{2}}{2a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}b} - \frac{i(-i + \sqrt{3}) \ln\left(a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}x + (-1)^{\frac{1}{3}}(a^3x^3 - b^2x^2)^{\frac{1}{3}}\right)}{2a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}b} + \frac{(1 + i\sqrt{3}) \ln\left(a^{\frac{2}{3}}(a^2-b)^{\frac{2}{3}}x^2 - (-1)^{\frac{1}{3}}a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}x(a^3x^3 - b^2x^2)^{\frac{1}{3}} + (-1)^{\frac{2}{3}}(a^3x^3 - b^2x^2)^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}(a^2-b)^{\frac{1}{3}}b}$$

command

`integrate(1/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3(a^3 - ab)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left((a^3 - ab)^{\frac{1}{3}} + 2\left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3(a^3 - ab)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3b - \sqrt{3}ab^2} - \frac{(a^3 - ab)^{\frac{2}{3}} \log\left((a^3 - ab)^{\frac{2}{3}} + (a^3 - ab)^{\frac{1}{3}}\left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}} + \left(a^3 - \frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2(a^3b - ab^2)} + \frac{(a^3 - ab)^{\frac{2}{3}} \log\left(\left|-(a^3 - ab)^{\frac{1}{3}} + \left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a^3b - ab^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 99.24 Problem number 2936

$$\int \frac{(-b + ax^4) \sqrt[4]{-bx^2 + ax^4}}{-b - ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

`integrate((a*x^4-b)*(a*x^4-b*x^2)^(1/4)/(x^4-a*x^2-b),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{2} \left(a - \frac{b}{x^2}\right)^{\frac{1}{4}} ax^2 + \frac{1}{8} \sqrt{2} \left(4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} (-a)^{\frac{1}{4}} + 2 \left(a - \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right) + \frac{1}{8} \sqrt{2} \left(4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b\right) \arctan\left(-\frac{\sqrt{2} \left(\sqrt{2} (-a)^{\frac{1}{4}} - 2 \left(a - \frac{b}{x^2}\right)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right) + \frac{1}{16} \sqrt{2} \left(4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b\right) \log\left(\sqrt{2} (-a)^{\frac{1}{4}} \left(a - \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}}\right) - \frac{1}{16} \sqrt{2} \left(4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b\right) \log\left(-\sqrt{2} (-a)^{\frac{1}{4}} \left(a - \frac{b}{x^2}\right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}}\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 99.25 Problem number 2937

$$\int \frac{(-b + ax^4) \sqrt[4]{-bx^2 + ax^4}}{-b - ax^2 + x^4} dx$$

Optimal antiderivative

*Unintegrable*

command

```
integrate((a*x^4-b)*(a*x^4-b*x^2)^(1/4)/(x^4-a*x^2-b),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{1}{2} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} ax^2 \\ & + \frac{1}{8} \sqrt{2} \left( 4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b \right) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \frac{1}{8} \sqrt{2} \left( 4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b \right) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right) \\ & + \frac{1}{16} \sqrt{2} \left( 4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b \right) \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right) \\ & - \frac{1}{16} \sqrt{2} \left( 4(-a)^{\frac{1}{4}} a^2 - (-a)^{\frac{1}{4}} b \right) \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a - \frac{b}{x^2} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a - \frac{b}{x^2}} \right) \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



## 99.26 Problem number 2962

$$\int \frac{(bx + ax^2) \sqrt[4]{bx^3 + ax^4}}{-b + ax + x^2} dx$$

Optimal antiderivative

*Unintegrable*

command

`integrate((a*x^2+b*x)*(a*x^4+b*x^3)^(1/4)/(a*x+x^2-b),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right)}{32 (-a)^{\frac{3}{4}}}$$

$$- \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right)}{32 (-a)^{\frac{3}{4}}}$$

$$- \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right)}{64 (-a)^{\frac{3}{4}}}$$

$$+ \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right)}{64 (-a)^{\frac{3}{4}}}$$

$$- \frac{\left( 8 \left( a + \frac{b}{x} \right)^{\frac{5}{4}} a^2 b - 8 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} a^3 b - 9 \left( a + \frac{b}{x} \right)^{\frac{5}{4}} b^2 + 5 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} a b^2 \right) x^2}{8 b^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.27 Problem number 2963

$$\int \frac{(bx + ax^2) \sqrt[4]{bx^3 + ax^4}}{-b + ax + x^2} dx$$

Optimal antiderivative

*Unintegrable*

command

`integrate((a*x^2+b*x)*(a*x^4+b*x^3)^(1/4)/(a*x+x^2-b),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \arctan \left( \frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} + 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right)}{32 (-a)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \arctan \left( -\frac{\sqrt{2} \left( \sqrt{2} (-a)^{\frac{1}{4}} - 2 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} \right)}{2 (-a)^{\frac{1}{4}}} \right)}{32 (-a)^{\frac{3}{4}}} \\ & - \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \log \left( \sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right)}{64 (-a)^{\frac{3}{4}}} \\ & + \frac{\sqrt{2} (32 a^4 - 8 a^2 b + 5 b^2) \log \left( -\sqrt{2} (-a)^{\frac{1}{4}} \left( a + \frac{b}{x} \right)^{\frac{1}{4}} + \sqrt{-a} + \sqrt{a + \frac{b}{x}} \right)}{64 (-a)^{\frac{3}{4}}} \\ & - \frac{\left( 8 \left( a + \frac{b}{x} \right)^{\frac{5}{4}} a^2 b - 8 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} a^3 b - 9 \left( a + \frac{b}{x} \right)^{\frac{5}{4}} b^2 + 5 \left( a + \frac{b}{x} \right)^{\frac{1}{4}} a b^2 \right) x^2}{8 b^2} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 99.28 Problem number 3016

$$\int \frac{b + ax}{(-b + ax)\sqrt[3]{b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} ax}{ax + 2(a^3x^3 + b^2x^2)^{\frac{1}{3}}}\right)}{a} + \frac{\sqrt{-6 + 6i\sqrt{3}} \arctan\left(\frac{\sqrt{3} a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x}{a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x - 2(-1)^{\frac{1}{3}}(a^3x^3 + b^2x^2)^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}} \\ & - \frac{\ln\left(-ax + (a^3x^3 + b^2x^2)^{\frac{1}{3}}\right)}{a} - \frac{i(-i + \sqrt{3}) \ln\left(a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x + (-1)^{\frac{1}{3}}(a^3x^3 + b^2x^2)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}} \\ & + \frac{\ln\left(a^2x^2 + ax(a^3x^3 + b^2x^2)^{\frac{1}{3}} + (a^3x^3 + b^2x^2)^{\frac{2}{3}}\right)}{2a} \\ & + \frac{(1 + i\sqrt{3}) \ln\left(a^{\frac{2}{3}}(a^2 + b)^{\frac{2}{3}}x^2 - (-1)^{\frac{1}{3}}a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x(a^3x^3 + b^2x^2)^{\frac{1}{3}} + (-1)^{\frac{2}{3}}(a^3x^3 + b^2x^2)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}} \end{aligned}$$

command

`integrate((a*x+b)/(a*x-b)/(a^3*x^3+b^2*x^2)^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left((a^3+ab)^{\frac{1}{3}}+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3(a^3+ab)^{\frac{1}{3}}}\right)}{(a^3+ab)^{\frac{1}{3}}} \\ & - \frac{\log\left(\left(a^3+ab\right)^{\frac{2}{3}}+\left(a^3+ab\right)^{\frac{1}{3}}\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{(a^3+ab)^{\frac{1}{3}}} \\ & + \frac{2\log\left(\left|-\left(a^3+ab\right)^{\frac{1}{3}}+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{(a^3+ab)^{\frac{1}{3}}} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(a+2\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} \\ & + \frac{\log\left(a^2+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}}a+\left(a^3+\frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(\left| -a+\left(a^3+\frac{b^2}{x}\right)^{\frac{1}{3}} \right|\right)}{a} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 99.29 Problem number 3033

$$\int \frac{b + ax}{(-b + ax)\sqrt[3]{-b^2x^2 + a^3x^3}} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} ax}{ax + 2(a^3x^3 - b^2x^2)^{\frac{1}{3}}}\right)}{a} + \frac{\sqrt{-6 + 6i\sqrt{3}} \arctan\left(\frac{\sqrt{3} a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}x}{a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}x - 2(-1)^{\frac{1}{3}}(a^3x^3 - b^2x^2)^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}} \\ & - \frac{\ln\left(-ax + (a^3x^3 - b^2x^2)^{\frac{1}{3}}\right)}{a} - \frac{i(-i + \sqrt{3}) \ln\left(a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}x + (-1)^{\frac{1}{3}}(a^3x^3 - b^2x^2)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}} \\ & + \frac{\ln\left(a^2x^2 + ax(a^3x^3 - b^2x^2)^{\frac{1}{3}} + (a^3x^3 - b^2x^2)^{\frac{2}{3}}\right)}{2a} \\ & + \frac{(1 + i\sqrt{3}) \ln\left(a^{\frac{2}{3}}(a^2 - b)^{\frac{2}{3}}x^2 - (-1)^{\frac{1}{3}}a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}x(a^3x^3 - b^2x^2)^{\frac{1}{3}} + (-1)^{\frac{2}{3}}(a^3x^3 - b^2x^2)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}(a^2 - b)^{\frac{1}{3}}} \end{aligned}$$

command

`integrate((a*x+b)/(a*x-b)/(a^3*x^3-b^2*x^2)^(1/3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left((a^3-ab)^{\frac{1}{3}} + 2\left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3(a^3-ab)^{\frac{1}{3}}}\right)}{(a^3 - ab)^{\frac{1}{3}}} \\ & - \frac{\log\left(\left(a^3 - ab\right)^{\frac{2}{3}} + \left(a^3 - ab\right)^{\frac{1}{3}}\left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}} + \left(a^3 - \frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{(a^3 - ab)^{\frac{1}{3}}} \\ & + \frac{2 \log\left(\left| -\left(a^3 - ab\right)^{\frac{1}{3}} + \left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}} \right|\right)}{(a^3 - ab)^{\frac{1}{3}}} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(a + 2\left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} \\ & + \frac{\log\left(a^2 + \left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}}a + \left(a^3 - \frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(\left| -a + \left(a^3 - \frac{b^2}{x}\right)^{\frac{1}{3}} \right|\right)}{a} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 99.30 Problem number 3078

$$\int \frac{\sqrt[3]{b^2x^2 + a^3x^3}}{-b + ax} dx$$

Optimal antiderivative

$$\begin{aligned} & \frac{(a^3x^3 + b^2x^2)^{\frac{1}{3}}}{a} - \frac{(3a^2b + b^2) \arctan\left(\frac{\sqrt{3} ax}{ax + 2(a^3x^3 + b^2x^2)^{\frac{1}{3}}}\right) \sqrt{3}}{3a^3} \\ & - \frac{\sqrt{-3 - 3i\sqrt{3}} b(a^2 + b)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3} a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x}{a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x - 2(-1)^{\frac{1}{3}}(a^3x^3 + b^2x^2)^{\frac{1}{3}}}\right) \sqrt{2}}{2a^{\frac{5}{3}}} \\ & + \frac{(-3a^2b - b^2) \ln\left(-ax + (a^3x^3 + b^2x^2)^{\frac{1}{3}}\right)}{3a^3} \\ & + \frac{i\left(ib(a^2 + b)^{\frac{1}{3}} + \sqrt{3} b(a^2 + b)^{\frac{1}{3}}\right) \ln\left(a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x + (-1)^{\frac{1}{3}}(a^3x^3 + b^2x^2)^{\frac{1}{3}}\right)}{2a^{\frac{5}{3}}} \\ & + \frac{(3a^2b + b^2) \ln\left(a^2x^2 + ax(a^3x^3 + b^2x^2)^{\frac{1}{3}} + (a^3x^3 + b^2x^2)^{\frac{2}{3}}\right)}{6a^3} \\ & + \frac{\left(b(a^2 + b)^{\frac{1}{3}} - i\sqrt{3} b(a^2 + b)^{\frac{1}{3}}\right) \ln\left(a^{\frac{2}{3}}(a^2 + b)^{\frac{2}{3}}x^2 - (-1)^{\frac{1}{3}}a^{\frac{1}{3}}(a^2 + b)^{\frac{1}{3}}x(a^3x^3 + b^2x^2)^{\frac{1}{3}} + (-1)^{\frac{2}{3}}(a^3x^3 + b^2x^2)^{\frac{1}{3}}\right)}{4a^{\frac{5}{3}}} \end{aligned}$$

command

`integrate((a^3*x^3+b^2*x^2)^(1/3)/(a*x-b),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned}
& \frac{(a^3 + ab)^{\frac{1}{3}}(a^2b + b^2) \log\left(\left|-(a^3 + ab)^{\frac{1}{3}} + \left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{a^4 + a^2b} \\
& - \frac{\sqrt{3}(a^3 + ab)^{\frac{1}{3}}b \arctan\left(\frac{\sqrt{3}\left((a^3 + ab)^{\frac{1}{3}} + 2\left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3(a^3 + ab)^{\frac{1}{3}}}\right)}{a^2} + \frac{\left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}x}{a} \\
& - \frac{(a^3 + ab)^{\frac{1}{3}}b \log\left(\left(a^3 + ab\right)^{\frac{2}{3}} + (a^3 + ab)^{\frac{1}{3}}\left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}} + \left(a^3 + \frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{2a^2} \\
& + \frac{\sqrt{3}(3a^2b + b^2) \arctan\left(\frac{\sqrt{3}\left(a + 2\left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}\right)}{3a}\right)}{3a^3} \\
& + \frac{(3a^2b + b^2) \log\left(a^2 + \left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}a + \left(a^3 + \frac{b^2}{x}\right)^{\frac{2}{3}}\right)}{6a^3} \\
& - \frac{(3a^2b + b^2) \log\left(\left|-a + \left(a^3 + \frac{b^2}{x}\right)^{\frac{1}{3}}\right|\right)}{3a^3}
\end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 99.31 Problem number 3138

$$\int \frac{\sqrt[6]{\frac{1-bx}{c+x}}(1+dx^2)}{(1+bx)(1+cx)} dx$$

Optimal antiderivative

$$\begin{aligned}
 & \frac{d(c+x) \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{bc} - \frac{(bc^2 + 6b + 7c) d \arctan \left(\frac{\left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{b^{\frac{1}{6}}}\right)}{3b^{\frac{11}{6}} c^2} \\
 & - \frac{2(b+c)^{\frac{1}{6}} (c^2 + d) \arctan \left(\frac{(-c^2+1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{(b+c)^{\frac{1}{6}}}\right)}{c^2 (-b+c) (-c^2+1)^{\frac{1}{6}}} \\
 & - \frac{2^{\frac{1}{6}} \sqrt{3} (b^2 + d) \arctan \left(\frac{\left(-b^{\frac{1}{6}} + 2^{\frac{5}{6}} (bc-1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}\right) \sqrt{3}}{3b^{\frac{1}{6}}}\right)}{b^{\frac{11}{6}} (b-c) (bc-1)^{\frac{1}{6}}} \\
 & - \frac{2^{\frac{1}{6}} \sqrt{3} (b^2 + d) \arctan \left(\frac{\left(b^{\frac{1}{6}} + 2^{\frac{5}{6}} (bc-1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}\right) \sqrt{3}}{3b^{\frac{1}{6}}}\right)}{b^{\frac{11}{6}} (b-c) (bc-1)^{\frac{1}{6}}} \\
 & + \frac{(bc^2 + 6b + 7c) d \arctan \left(\frac{b^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{-b^{\frac{1}{3}} + \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{3}}}\right)}{6b^{\frac{11}{6}} c^2} \\
 & - \frac{(b+c)^{\frac{1}{6}} (c^2 + d) \arctan \left(\frac{(b+c)^{\frac{1}{6}} (-c^2+1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{(b+c)^{\frac{1}{3}} - (-c^2+1)^{\frac{1}{3}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{3}}}\right)}{c^2 (-b+c) (-c^2+1)^{\frac{1}{6}}} \\
 & - \frac{2 \cdot 2^{\frac{1}{6}} (b^2 + d) \operatorname{arctanh} \left(\frac{(bc-1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}} 2^{\frac{5}{6}}}{2b^{\frac{1}{6}}}\right)}{b^{\frac{11}{6}} (b-c) (bc-1)^{\frac{1}{6}}} \\
 & - \frac{(bc^2 + 6b + 7c) d \operatorname{arctanh} \left(\frac{\sqrt{3} b^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{b^{\frac{1}{3}} + \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{3}}}\right) \sqrt{3}}{6b^{\frac{11}{6}} c^2} \\
 & - \frac{2^{\frac{1}{6}} (b^2 + d) \operatorname{arctanh} \left(\frac{2^{\frac{5}{6}} b^{\frac{1}{6}} (bc-1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}{2b^{\frac{1}{3}} + 2^{\frac{2}{3}} (bc-1)^{\frac{1}{3}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{3}}}\right)}{b^{\frac{11}{6}} (b-c) (bc-1)^{\frac{1}{6}}} \\
 & - \frac{\sqrt{3} (b+c)^{\frac{1}{6}} (c^2 + d) \operatorname{arctanh} \left(\frac{\left((b+c)^{\frac{1}{3}} + (-c^2+1)^{\frac{1}{3}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{3}}\right) \sqrt{3}}{3(b+c)^{\frac{1}{6}} (-c^2+1)^{\frac{1}{6}} \left(\frac{-bx+1}{c+x}\right)^{\frac{1}{6}}}\right)}{c^2 (-b+c) (-c^2+1)^{\frac{1}{6}}}
 \end{aligned}$$

command

```
integrate(((b*x+1)/(c+x))^(1/6)*(d*x^2+1)/(b*x+1)/(c*x+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100 Test file number 210

Test folder name:

test\_cases/210\_Hebisch

### 100.1 Problem number 16

$$\int \frac{648 + 8e^8 - 8x^2}{6561 + e^{16} + 4e^{12}x + 162x^2 + x^4 + e^8(162 + 6x^2) + e^4(324x + 4x^3)} dx$$

Optimal antiderivative

$$\frac{8x}{81 + (x + e^4)^2}$$

command

```
integrate((8*exp(4)^2-8*x^2+648)/(exp(4)^4+4*x*exp(4)^3+(6*x^2+162)*exp(4)^2+(4*x^3+324*x)*exp(4)+6561),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8x}{x^2 + 2xe^4 + e^8 + 81}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{8(x^2 - e^8 - 81)}{x^4 + 162x^2 + 4xe^{12} + 6(x^2 + 27)e^8 + 4(x^3 + 81x)e^4 + e^{16} + 6561} dx$$



### 100.2 Problem number 76

$$\int \frac{e^5(-432ex^2 - 64e^2x^6) + 288e^6x^2 \log(2) - 48e^6x^2 \log^2(2)}{81 - 72ex^4 + 16e^2x^8 + (-108 + 48ex^4) \log(2) + (54 - 8ex^4) \log^2(2) - 12 \log^3(2) + \log^4(2)} dx$$

Optimal antiderivative

$$\frac{4e^5}{x - \frac{(\ln(2)-3)^2 e^{-1}}{4x^3}}$$

command

```
integrate((-48*x^2*exp(1)*exp(5)*log(2)^2+288*x^2*exp(1)*exp(5)*log(2)+(-64*x^6*exp(1)^2-432*x^2*exp(1))*exp(5))/(log(2)^4-12*log(2)^3+(-8*x^4*exp(1)+54)*log(2)^2+(48*x^4*exp(1)-108)*log(2)+16*x^8*exp(1)^2-72*x^4*exp(1)+81),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16x^3e^6}{4x^4e - \log(2)^2 + 6\log(2) - 9}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.3 Problem number 88

$$\int \frac{12 + 71e^{25/4} + \log(2)}{e^{25/2} - 2e^{25/4}x + x^2} dx$$

Optimal antiderivative

$$\frac{71x + 12 + \ln(2)}{e^{\frac{25}{4}} - x}$$

command

```
integrate((log(2)+71*exp(25/4)+12)/(exp(25/4)^2-2*x*exp(25/4)+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{71e^{\frac{25}{4}} + \log(2) + 12}{x - e^{\frac{25}{4}}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.4 Problem number 159

$$\int \frac{(-120x^2 + 240x^3 - 40x^4 + e(24 - 24x^2 - 16x^3)) \log(2)}{225x^2 - 150x^3 + 25x^4 + e^2(1 + 2x + x^2) + e(-30x - 20x^2 + 10x^3)} dx$$

Optimal antiderivative

$$\frac{8x(-x^2 + 3) \ln(2)}{e - (15 - 5x - e)x}$$

command

```
integrate((( -16*x^3-24*x^2+24)*exp(1)-40*x^4+240*x^3-120*x^2)*log(2)/((x^2+2*x+1)*exp(1)^2+(1
20*x^2-30*x)*exp(1)+25*x^4-150*x^3+225*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8}{25} \left( 5x + \frac{xe^2 - 35xe + 150x + e^2 - 15e}{5x^2 + xe - 15x + e} \right) \log(2)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{8(5x^4 - 30x^3 + 15x^2 + (2x^3 + 3x^2 - 3)e) \log(2)}{25x^4 - 150x^3 + 225x^2 + (x^2 + 2x + 1)e^2 + 10(x^3 - 2x^2 - 3x)e} dx$$

### 100.5 Problem number 182

$$\int \frac{-32x^7 - 12x^8 + 16x^9 + e^2(144x^3 + 576x^5 + 576x^7) + e(240x^4 + 72x^5 + 192x^6 + 96x^7 - 192x^8)}{x^6 + e^2(9 + 36x^2 + 36x^4) + e(-6x^3 - 12x^5)} dx$$

Optimal antiderivative

$$4 \left( x - \frac{4 + x}{x - (6 + \frac{3}{x^2})e} \right) x^3$$

command

```
integrate(((576*x^7+576*x^5+144*x^3)*exp(1)^2+(-192*x^8+96*x^7+192*x^6+72*x^5+240*x^4)*exp(1)
12*x^8-32*x^7)/((36*x^4+36*x^2+9)*exp(1)^2+(-12*x^5-6*x^3)*exp(1)+x^6),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4x^4 - 4x^3 - 24x^2e - 16x^2 - 144xe^2 - 96xe}{12(432x^2e^4 + 288x^2e^3 + 12x^2e^2 + 4x^2e + 36xe^3 + 24xe^2 + 216e^4 + 144e^3 + 3e^2)} x^3$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4(4x^9 - 3x^8 - 8x^7 + 36(4x^7 + 4x^5 + x^3)e^2 - 6(8x^8 - 4x^7 - 8x^6 - 3x^5 - 10x^4)e)}{x^6 + 9(4x^4 + 4x^2 + 1)e^2 - 6(2x^5 + x^3)e} dx$$

### 100.6 Problem number 206

$$\int -\frac{15e^2}{1550 - 620ex + 62e^2x^2} dx$$

Optimal antiderivative

$$\frac{15}{62(x - 5e^{-1})}$$

command

```
integrate(-15*exp(1)^2/(62*x^2*exp(1)^2-620*x*exp(1)+1550),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15e}{62(xe - 5)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.7 Problem number 253

$$\int \frac{409600 + e^x(-384000 - 15360x) + 16384x + e^{3x}(-12500 - 16500x - 640x^2) + e^{2x}(120000 + 56000x + 49152x^2)}{102400 + e^x(-96000 - 38400x)}$$

Optimal antiderivative

$$\frac{64x^2}{\left(\left(\ln\left(\frac{x}{5} + 5\right) + 2\right)^4 e^{-x} - 5\right)^2} + 4x$$

command

```
integrate(((4*x+100)*log(1/5*x+5)^12+(96*x+2400)*log(1/5*x+5)^11+(1056*x+26400)*log(1/5*x+5)^10+
60*x-1500)*exp(x)+31680*x+792000)*log(1/5*x+5)^8+((-960*x-24000)*exp(x)+101376*x+2534400)*log
6720*x-168000)*exp(x)+236544*x+5913600)*log(1/5*x+5)^6+((-26880*x-672000)*exp(x)+405504*x+101
67200*x-1680000)*exp(x)+506880*x+12672000)*log(1/5*x+5)^4+((1024*x^3+26112*x^2+28000*x+60000)
107520*x-2688000)*exp(x)+450560*x+11264000)*log(1/5*x+5)^3+((3072*x^3+76800*x^2+84000*x+18000
107520*x-2688000)*exp(x)+270336*x+6758400)*log(1/5*x+5)^2+((4096*x^3+100352*x^2+112000*x+2400
61440*x-1536000)*exp(x)+98304*x+2457600)*log(1/5*x+5)+(-640*x^2-16500*x-12500)*exp(x)^3+(2048
15360*x-384000)*exp(x)+16384*x+409600)/(x+25)*log(1/5*x+5)^12+(24*x+600)*log(1/5*x+5)^11+(26
15*x-375)*exp(x)+7920*x+198000)*log(1/5*x+5)^8+((-240*x-6000)*exp(x)+25344*x+633600)*log(1/5*
1680*x-42000)*exp(x)+59136*x+1478400)*log(1/5*x+5)^6+((-6720*x-168000)*exp(x)+101376*x+253440
16800*x-420000)*exp(x)+126720*x+3168000)*log(1/5*x+5)^4+((600*x+15000)*exp(x)^2+(-
26880*x-672000)*exp(x)+112640*x+2816000)*log(1/5*x+5)^3+((1800*x+45000)*exp(x)^2+(-
26880*x-672000)*exp(x)+67584*x+1689600)*log(1/5*x+5)^2+((2400*x+60000)*exp(x)^2+(-
15360*x-384000)*exp(x)+24576*x+614400)*log(1/5*x+5)+(-125*x-3125)*exp(x)^3+(1200*x+30000)*exp
3840*x-96000)*exp(x)+4096*x+102400),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 \left( (x+25)e^{50} \log\left(\frac{1}{5}x+5\right)^8 + 16(x+25)e^{50} \log\left(\frac{1}{5}x+5\right)^7 - 400e^{50} \log\left(\frac{1}{5}x+5\right)^8 + 112(x+25)e^{50} \log\left(\frac{1}{5}x+5\right)^6 \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.8 Problem number 351

$$\int \frac{625 + 9225x + 46950x^2 + 78550x^3 + 753x^4 + 3798x^5 + 6266x^6 + 15x^7 + 75x^8 + 125x^9 + e^8(x^4 + 15x^5 + 75x^6)}{625 + 9375x + 46950x^2 + 78550x^3 + 750x^4 + 3753x^5 + 6266x^6 + 15x^7 + 75x^8 + 125x^9 + e^8(x^4 + 15x^5 + 75x^6)}$$

Optimal antiderivative

$$x - \ln\left(\frac{3}{\left(x + \frac{25}{x^2} - e^4\right)(1+5x)^2} + 1\right)$$

command

```
integrate(((125*x^7+75*x^6+15*x^5+x^4)*exp(4)^2+(-250*x^8-150*x^7-30*x^6-6267*x^5-3783*x^4-750*x^3-50*x^2)*exp(4)+125*x^9+75*x^8+15*x^7+6266*x^6+3798*x^5+753*x^4+78550*x^3+46950*x^2+250*x^8-150*x^7-30*x^6-6267*x^5-3753*x^4-750*x^3-50*x^2)*exp(4)+125*x^9+75*x^8+15*x^7+6266*x^6
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x - \log\left(|25x^5 - 25x^4e^4 + 10x^4 - 10x^3e^4 + x^3 - x^2e^4 + 628x^2 + 250x + 25|\right) + \log\left(|x^3 - x^2e^4 + 25|\right) + 2 \log\left(|5x + 1|\right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{125x^9 + 75x^8 + 15x^7 + 6266x^6 + 3798x^5 + 753x^4 + 78550x^3 + 46950x^2 + (125x^7 + 75x^6 + 15x^5 + x^4)e^8}{125x^9 + 75x^8 + 15x^7 + 6266x^6 + 3753x^5 + 750x^4 + 78550x^3 + 46950x^2 + (125x^7 + 75x^6 + 15x^5 + x^4)e^8}$$

### 100.9 Problem number 379

$$\int \frac{-4x^6 + 2x^6 \log(2) + e^{4x^2}(-4x^2 + 2x^2 \log(2)) + e^{3x^2}(-16x^3 + 8x^3 \log(2)) + (-4x^3 - 3x^2 \log(2)) \log(4) + e^{2x^2}}{16x^6 + 16x^7 + 4x^8 + e^{4x^2}(16x^2 + 16x^3 + 4x^4) + e^{3x^2}(64x^3 + 64x^4 + 16x^5) + (8x^3 + 4x^4)}$$

Optimal antiderivative

$$\frac{-x - \ln(2)}{2x + \frac{2 \ln(2)}{(e^{x^2} + x)^2} + 4}$$

command

```
integrate(((2*x^2*log(2)-4*x^2)*exp(x^2)^4+(8*x^3*log(2)-16*x^3)*exp(x^2)^3+(2*((-
4*x^2-1)*log(2)-4*x^3-2*x)*log(2)+12*x^4*log(2)-24*x^4)*exp(x^2)^2+(2*((-4*x^3-4*x)*log(2)-
4*x^4-6*x^2)*log(2)+8*x^5*log(2)-16*x^5)*exp(x^2)+2*(-3*x^2*log(2)-4*x^3)*log(2)+2*x^6*log(2)
4*x^6)/((4*x^4+16*x^3+16*x^2)*exp(x^2)^4+(16*x^5+64*x^4+64*x^3)*exp(x^2)^3+(2*(4*x^2+8*x)*log(2)
4*x^6))
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^4 \log(2) + 2x^3 e^{(x^2)} \log(2) - 2x^4 - 4x^3 e^{(x^2)} + 2x^3 \log(2) + x^2 e^{(2x^2)} \log(2) + 4x^2 e^{(x^2)} \log(2) - 4x^3 - 2x^2 e^{(2x^2)}}{2(x^5 + 2x^4 e^{(x^2)} + 4x^4 + x^3 e^{(2x^2)} + 8x^3 e^{(x^2)} + 4x^3 + 4x^2 e^{(2x^2)} + 8x^2 e^{(x^2)})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 100.10 Problem number 539

$$\int \frac{(2+4x) \log\left(\frac{1}{16}(x+x^2)\right)}{x+x^2} dx$$

Optimal antiderivative

$$\ln\left(\frac{(1+x)x}{16}\right)^2$$

command

```
integrate((4*x+2)*log(1/16*x^2+1/16*x)/(x^2+x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\frac{1}{16}x^2 + \frac{1}{16}x\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(2x+1) \log\left(\frac{1}{16}x^2 + \frac{1}{16}x\right)}{x^2+x} dx$$

### 100.11 Problem number 549

$$\int \frac{-156 - 87x - 6x^2 + 2x^3 + e(-6x - 105x^2 - 24x^3 + 4x^4) + e^2(-18x^4 + 2x^5)}{-171 - 99x - 8x^2 + 2x^3 + e(-117x^2 - 28x^3 + 4x^4) + e^2(-20x^4 + 2x^5)} dx$$

Optimal antiderivative

$$x + \ln \left( \frac{1}{x + 3 + x^2 e} + \frac{2x}{3} - \frac{20}{3} \right)$$

command

```
integrate(((2*x^5-18*x^4)*exp(1)^2+(4*x^4-24*x^3-105*x^2-6*x)*exp(1)+2*x^3-6*x^2-87*x-156)/((2*x^5-20*x^4)*exp(1)^2+(4*x^4-28*x^3-117*x^2)*exp(1)+2*x^3-8*x^2-99*x-171),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x - \log(x^2 e + x + 3) + \log(|2x^3 e - 20x^2 e + 2x^2 - 14x - 57|)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2x^3 - 6x^2 + 2(x^5 - 9x^4)e^2 + (4x^4 - 24x^3 - 105x^2 - 6x)e - 87x - 156}{2x^3 - 8x^2 + 2(x^5 - 10x^4)e^2 + (4x^4 - 28x^3 - 117x^2)e - 99x - 171} dx$$

### 100.12 Problem number 567

$$\int \frac{-7 + 7x - 3x^2 + e^{-4-2e^x-2x}(-4 + 3x + e^x(-4 + 3x)) + e^{-2-e^x-x}(-4 + 7x - 3x^2 + e^x(4x - 3x^2))}{-4 + 3x} dx$$

Optimal antiderivative

$$x - \ln(-4 + 3x) - \frac{(x - e^{-e^x-x-2})^2}{2}$$

command

```
integrate(((((-4+3*x)*exp(x)+3*x-4)*exp(-exp(x)-x-2)^2+((-3*x^2+4*x)*exp(x)-3*x^2+7*x-4)*exp(-exp(x)-x-2)-3*x^2+7*x-7)/(-4+3*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} \left( x^2 e^{(2x+4)} - 2x e^{(2x+4)} - 2x e^{(x-e^x+2)} + 2e^{(2x+4)} \log(3x-4) + e^{(-2e^x)} \right) e^{(-2x-4)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{3x^2 + (3x^2 + (3x^2 - 4x)e^x - 7x + 4)e^{(-x-e^x-2)} - ((3x-4)e^x + 3x-4)e^{(-2x-2e^x-4)} - 7x + 7}{3x-4} dx$$

**100.13 Problem number 614**

$$\int \frac{-30e^3x + e^6x^4 + (-15 + 2e^3x^3) \log(\log(2)) + x^2 \log^2(\log(2))}{e^6x^4 + 2e^3x^3 \log(\log(2)) + x^2 \log^2(\log(2))} dx$$

Optimal antiderivative

$$x - \frac{15 \left( 2x - \frac{1}{x e^3 + \ln(\ln(2))} \right)}{x}$$

command

```
integrate((x^2*log(log(2))^2+(2*x^3*exp(3)-15)*log(log(2))+x^4*exp(3)^2-30*x*exp(3))/(x^2*log
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x + \frac{15}{x^2 e^3 + x \log(\log(2))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.14 Problem number 636**

$$\int \frac{e^{\frac{3}{50x-25x^3+e^x(-10+5x^2)}} (-90 + 135x^2 + e^x(18 - 18x - 9x^2))}{500x^2 - 500x^4 + 125x^6 + e^{2x}(20 - 20x^2 + 5x^4) + e^x(-200x + 200x^3 - 50x^5)} dx$$

Optimal antiderivative

$$3 e^{\frac{3}{(5x-e^x)(-5x^2+10)}} - 4$$

command

```
integrate((( -9*x^2-18*x+18)*exp(x)+135*x^2-90)*exp(3/((5*x^2-10)*exp(x)-25*x^3+50*x))/((5*x^4
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3 e^{\left( -\frac{3}{5(5x^3-x^2e^x-10x+2e^x)} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: RuntimeError

### 100.15 Problem number 802

$$\int \frac{-20x^5 + 120x^9 - 160x^{11} + 60x^{13} + e^{25}(-320 + 480x^4 - 320x^6 + 60x^8) + e^{20}(-1280x + 480x^3 + 1680x^5 - 140x^7)}{x^2} dx$$

Optimal antiderivative

$$4 + \frac{10 \left( \frac{x}{e^5 + x} + x^2 - 2 \right)^4}{x^2}$$

command

```
integrate(((60*x^8-320*x^6+480*x^4-320)*exp(5)^5+(300*x^9-1400*x^7+1680*x^5+480*x^3-1280*x)*exp(5)^4+(600*x^10-2440*x^8+2400*x^6+960*x^4-1600*x^2)*exp(5)^3+(600*x^11-2120*x^9+1800*x^7+600*x^5-880*x^3)*exp(5)^2+(300*x^12-920*x^10+720*x^8+120*x^6-220*x^4)*exp(5)+160*x^11+120*x^9-20*x^5)/(x^3*exp(5)^5+5*x^4*exp(5)^4+10*x^5*exp(5)^3+10*x^6*exp(5)^2+5*x^7*exp(5)^1)
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10x^6 - 40x^4 - 40x^3e^5 + 40x^2e^{10} + 60x^2 - 40xe^{15} + 120xe^5 - \frac{160(2x - e^5)e^{(-5)}}{x^2}}{(x + e^5)^4} - \frac{10(4x^3e^{30} + 12x^3e^{10} - 32x^3 + 12x^2e^{35} - 6x^2e^{25} + 48x^2e^{15} - 113x^2e^5 + 12xe^{40} - 12xe^{30} + 64xe^{20} - 136xe^{10})}{(x + e^5)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.16 Problem number 809

$$\int \frac{-156 - 65x + e^{2x/3}(-36 + 12x) + e^{x/3}(-150 - 6x + 10x^2) + (25 + e^{x/3}(12 - 2x) + 5x) \log(x) - \log^2(x)}{2x^3} dx$$

Optimal antiderivative

$$\frac{\left( 12 + 6e^{\frac{x}{3}} + 5x - \ln(x) \right)^2}{4x^2}$$

command

```
integrate(1/2*(-log(x)^2+((-2*x+12)*exp(1/3*x)+25+5*x)*log(x)+(12*x-36)*exp(1/3*x)^2+(10*x^2-6*x-150)*exp(1/3*x)-65*x-156)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{60xe^{\frac{1}{3}x} - 10x \log(3) - 12e^{\frac{1}{3}x} \log(3) + \log(3)^2 - 10x \log\left(\frac{1}{3}x\right) - 12e^{\frac{1}{3}x} \log\left(\frac{1}{3}x\right) + 2 \log(3) \log\left(\frac{1}{3}x\right) + \log^2\left(\frac{1}{3}x\right)}{4x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{12(x-3)e^{\frac{2}{3}x} + 2(5x^2 - 3x - 75)e^{\frac{1}{3}x} - \left( 2(x-6)e^{\frac{1}{3}x} - 5x - 25 \right) \log(x) - \log(x)^2 - 65x - 156}{2x^3} dx$$



## 100.17 Problem number 817

$$\int \frac{-300 + 25e^3 - 200x}{144x^2 + e^6x^2 + 96x^3 + 16x^4 + e^3(-24x^2 - 8x^3)} dx$$

Optimal antiderivative

$$\frac{25}{(12 + 4x - e^3)x}$$

command

```
integrate((25*exp(3)-200*x-300)/(x^2*exp(3)^2+(-8*x^3-24*x^2)*exp(3)+16*x^4+96*x^3+144*x^2),x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25}{4x^2 - xe^3 + 12x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.18 Problem number 850

$$\int \frac{-3 + 2401e^8}{e^4(2401e^8 + 4802e^4x + 2401x^2)} dx$$

Optimal antiderivative

$$\frac{x + \frac{3e^{-4}}{2401}}{x + e^4}$$

command

```
integrate((2401*exp(2)^2*exp(4)-3)/(2401*exp(2)^4+4802*x*exp(2)^2+2401*x^2)/exp(4),x, algorithm="sympy")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(2401e^8 - 3)e^{(-4)}}{2401(x + e^4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.19 Problem number 886

$$\int \frac{-64 - 768x^2 + 256x^3 + x^5 + 20x^7 - 5x^8 + 160x^9 - 80x^{10} + 650x^{11} - 480x^{12} + 1400x^{13} - 1290x^{14} + 1504x^{15} - 136x^{16}}{x^5 + 20x^7 - 5x^8 + 160x^9 - 80x^{10} + 650x^{11} - 480x^{12} + 1400x^{13} - 1290x^{14} + 1504x^{15} - 136x^{16}} dx$$

Optimal antiderivative

$$\frac{16}{((-4 + x)(xe^4 - x)x^2 + x)^4} + x$$

command

```
integrate((x^20-20*x^19+160*x^18-640*x^17+1280*x^16-1024*x^15)*exp(4)^5+(-5*x^20+100*x^19-800*x^18+3205*x^17-6480*x^16+5600*x^15-1280*x^14+1280*x^13)*exp(4)^4+(10*x^20-200*x^19+1600*x^18-6420*x^17+13120*x^16-12160*x^15+5130*x^14-5240*x^13+480*x^12-640*x^11)*exp(4)^3+(-10*x^20+200*x^19-1600*x^18+6430*x^17-13280*x^16+13120*x^15-7710*x^14+8040*x^13-1440*x^12+193080*x^10+160*x^9)*exp(4)^2+(5*x^20-100*x^19+800*x^18-3220*x^17+6720*x^16-7040*x^15+5150*x^14-5480*x^13+1440*x^12-1940*x^11+160*x^10-320*x^9+5*x^8-20*x^7-256*x^3+768*x^2)*exp(4)-x^20+20*x^19-160*x^18+645*x^17-1360*x^16+1504*x^15-1290*x^14+1400*x^13-480*x^12+650*x^11-80*x^10+160*x^9-5*x^8+20*x^7+x^5+256*x^3-768*x^2-64)/((x^20-20*x^19+160*x^18-640*x^17+1280*x^16-1024*x^15)*exp(4)^5+(-5*x^20+100*x^19-800*x^18+3205*x^17-6480*x^16+5600*x^15-1280*x^14+1280*x^13-200*x^19+1600*x^18-6420*x^17+13120*x^16-12160*x^15+5130*x^14-5240*x^13+480*x^12-640*x^11)*exp(4)^3+(-10*x^20+200*x^19-1600*x^18+6430*x^17-13280*x^16+13120*x^15-7710*x^14+8040*x^13-1440*x^12+193080*x^10+160*x^9)*exp(4)^2+(5*x^20-100*x^19+800*x^18-3220*x^17+6720*x^16-7040*x^15+5150*x^14-5480*x^13+1440*x^12-1940*x^11+160*x^10-320*x^9+5*x^8-20*x^7)*exp(4)-x^20+20*x^19-160*x^18+645*x^17-1360*x^16+1504*x^15-1290*x^14+1400*x^13-480*x^12+650*x^11-80*x^10+160*x^9-5*x^8+20*x^7+x^5), x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^{20} - 5xe^{16} + 10xe^{12} - 10xe^8 + 5xe^4 - x}{e^{20} - 5e^{16} + 10e^{12} - 10e^8 + 5e^4 - 1} + \frac{16}{(x^4e^4 - x^4 - 4x^3e^4 + 4x^3 + x)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.20 Problem number 962

$$\int \frac{12 + 16x^3 + e^6(2 - 24x^2) - 8 \log(2)}{e^{12} - 2e^6x + x^2} dx$$

Optimal antiderivative

$$\frac{4x^3 - 6 + 4 \ln(2) - x}{\frac{x}{2} - \frac{e^6}{2}}$$

command

`integrate((-8*log(2)+(-24*x^2+2)*exp(3)^2+16*x^3+12)/(exp(3)^4-2*x*exp(3)^2+x^2),x, algorithm  
Giac 1.9.0-11 via sagemath 9.6 output`

$$8x^2 + 8xe^6 + \frac{2(4e^{18} - e^6 + 4\log(2) - 6)}{x - e^6}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.21 Problem number 1011

$$\int \frac{e^x(-625x + 625x^2) + e^{\frac{-4+e^x(2x+x^2)}{x}}(2500 + 2500x + 625e^x x^2)}{e^{x+\frac{2e^{-x}(-4+e^x(2x+x^2))}{x}}x^2 + e^x(4x^2 + 4x^3 + x^4) + e^x(-4x^2 - 2x^3)\log(5x) + e^x x^2 \log^2(5x) + e^{\frac{-4+e^x(2x+x^2)}{x}}}$$

Optimal antiderivative

$$\frac{625}{\ln(5x) - e^{x+2-\frac{4e^{-x}}{x}} - 2 - x}$$

command

`integrate(((625*exp(x))*x^2+2500*x+2500)*exp(((x^2+2*x)*exp(x)-4)/exp(x)/x)+(625*x^2-  
625*x)*exp(x))/(x^2*exp(x)*exp(((x^2+2*x)*exp(x)-4)/exp(x)/x)^2+(-2*x^2*exp(x)*log(5*x)+(2*x^  
4)/exp(x)/x)+x^2*exp(x)*log(5*x)^2+(-2*x^3-4*x^2)*exp(x)*log(5*x)+(x^4+4*x^3+4*x^2)*exp(x)),x`

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.22 Problem number 1030

$$\int \frac{1}{2097152000 + 2516582400x + 1515192320x^2 + 626524160x^3 + 201646080x^4 + 52879360x^5 + 11408640x^6 + 203}$$

Optimal antiderivative

$$\frac{1}{4 + \frac{\left(x+5\left(e^4 + \frac{4x}{4+x} + x\right)\right)^2}{x^2}}$$

command

```
integrate(((10*x^9+320*x^8+4480*x^7+35840*x^6+179200*x^5+573440*x^4+1146880*x^3+1310720*x^2+6
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2x^5e^8 + 4x^5e^4 + 2x^5 + 5x^4e^{16} + 20x^4e^{12} + 62x^4e^8 + 100x^4e^4 + 53x^4 + 80x^3e^{16} + 400x^3e^{12} + 912x^3e^8 + 504x^3e^4 + 149x^3 + 80x^2e^{16} + 400x^2e^{12} + 912x^2e^8 + 504x^2e^4 + 149x^2 + 80xe^{16} + 400xe^{12} + 912xe^8 + 504xe^4 + 149x + 80e^{16} + 400e^{12} + 912e^8 + 504e^4 + 149}{5(x^6 + 2x^5e^8 + 4x^5e^4 + 18x^5 + 5x^4e^{16} + 20x^4e^{12} + 62x^4e^8 + 100x^4e^4 + 149x^4 + 80x^3e^{16} + 400x^3e^{12} + 912x^3e^8 + 504x^3e^4 + 149x^3 + 80x^2e^{16} + 400x^2e^{12} + 912x^2e^8 + 504x^2e^4 + 149x^2 + 80xe^{16} + 400xe^{12} + 912xe^8 + 504xe^4 + 149x + 80e^{16} + 400e^{12} + 912e^8 + 504e^4 + 149)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.23 Problem number 1093

$$\int e^{158-1000080x+1562750010x^2} (2000160 + e^{-158+1000080x-1562750010x^2} - 6251000040x) dx$$

Optimal antiderivative

$$x - 2e^{-2+10(4-12501x)^2}$$

command

```
integrate((exp(-781375005*x^2+500040*x-79)^2-6251000040*x+2000160)/exp(-781375005*x^2+500040*x-79)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x - 2e^{(1562750010x^2 - 1000080x + 158)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 100.24 Problem number 1123

$$\int \frac{4 + (-4x + 2e^5x^3 - 6e^8x^5) \log(x) + (-2e^5x^2 + 6e^8x^4) \log^2(x)}{(-e^{10}x^5 + 2e^{13}x^7 - e^{16}x^9) \log(x) + (e^{10}x^4 - 2e^{13}x^6 + e^{16}x^8) \log(x) \log(\log(x)) + ((2e^5x^3 - 2e^8x^5) \log(x) + (-2e^5x^2 + 6e^8x^4) \log^2(x))}$$

Optimal antiderivative

$$\frac{2x}{x^2(e^5 - x^2e^8) - \ln((x - \ln(\ln(x)))^2)}$$

command

```
integrate((( -2*log(x)*log(log(x))+2*x*log(x))*log(log(log(x)))^2-2*x*log(log(x))+x^2)+(-
2*x^2*exp(5)+6*x^4*exp(4)^2)*log(x)*log(log(x))+(2*x^3*exp(5)-6*x^5*exp(4)^2-4*x)*log(x)+4)/(
x*log(x))*log(log(log(x)))^2-2*x*log(log(x))+x^2)^2+((-2*x^2*exp(5)+2*x^4*exp(4)^2)*log(x)*log
2*x^5*exp(4)^2)*log(x))*log(log(log(x)))^2-2*x*log(log(x))+x^2)+(x^4*exp(5)^2-2*x^6*exp(4)^2*
x^5*exp(5)^2+2*x^7*exp(4)^2*exp(5)-x^9*exp(4)^4)*log(x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2x}{x^4e^8 - x^2e^5 + \log\left(x^2 - 2x\log(\log(x)) + \log(\log(x))^2\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.25 Problem number 1164

$$\int \frac{8e^4x + e^5(4 - 16x^3)}{-x^{10} + e(-5x^9 + 5x^{12}) + e^2(-10x^8 + 20x^{11} - 10x^{14}) + e^3(-10x^7 + 30x^{10} - 30x^{13} + 10x^{16}) + e^4(-5x^6 + 20x^{12} - 10x^{15}) + e^5(-5x^5 + 20x^{11} - 10x^{14})} dx$$

Optimal antiderivative

$$\frac{1}{(x^2(e^{-1} - x^2) + x)^4}$$

command

```
integrate((( -16*x^3+4)*exp(1)^5+8*x*exp(1)^4)/((x^20-5*x^17+10*x^14-10*x^11+5*x^8-
x^5)*exp(1)^5+(-5*x^18+20*x^15-30*x^12+20*x^9-5*x^6)*exp(1)^4+(10*x^16-30*x^13+30*x^10-
10*x^7)*exp(1)^3+(-10*x^14+20*x^11-10*x^8)*exp(1)^2+(5*x^12-5*x^9)*exp(1)-x^10),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^{20}}{(x^2e^4 - (x^4 - x)e^5)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.26 Problem number 1174

$$\int \frac{-32 - 2592e^8 - 630x - 10368e^6x - 2340x^2 + 5346x^3 - 2592x^4 + e^4(576 + 5670x - 15552x^2) + e^2(11664x^3 + 5184e^8x^3 + 1296x^4 + 20736e^6x^4 + 5409x^5 - 11664x^6 + 5184x^7 + e^4(-1152x^3 - 11664x^4 + 31104x^5) +$$

Optimal antiderivative

$$\frac{2}{\left(\frac{x}{x-(x+e^2)^2+\frac{1}{9}} + 8\right)} x^2$$

command

```
integrate((-2592*exp(2)^4-10368*x*exp(2)^3+(-15552*x^2+5670*x+576)*exp(2)^2+(-10368*x^3+11016
2592*x^4+5346*x^3-2340*x^2-630*x-32)/(5184*x^3*exp(2)^4+20736*x^4*exp(2)^3+(31104*x^5-
11664*x^4-1152*x^3)*exp(2)^2+(20736*x^6-23328*x^5-2304*x^4)*exp(2)+5184*x^7-11664*x^6+5409*x^
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{81(8x + 16e^2 - 9)}{32(72x^2 + 144xe^2 - 81x + 72e^4 - 8)(9e^4 - 1)} + \frac{9x + 72e^4 - 8}{32x^2(9e^4 - 1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.27 Problem number 1236

$$\int \frac{8e^2 - 768x + 288x^2}{2304x^4 - 1152x^5 + 144x^6 + e^4(16 - 8x + x^2) + e^2(384x^2 - 192x^3 + 24x^4)} dx$$

Optimal antiderivative

$$1 - \frac{1}{\left(x^2 + \frac{x\left(x + \frac{e^2}{4x}\right)}{2}\right)(-4 + x)}$$

command

```
integrate((8*exp(2)+288*x^2-768*x)/((x^2-8*x+16)*exp(2)^2+(24*x^4-192*x^3+384*x^2)*exp(2)+144
1152*x^5+2304*x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8}{12x^3 - 48x^2 + xe^2 - 4e^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.28 Problem number 1241

$$\int \frac{20x^5 + e^{15}(-800 - 640x^2 + 2x^5 - 20x^6) + e^{30}(500x + 480x^3 + 64x^5 + 5x^7)}{4x^5 - 4e^{15}x^6 + e^{30}x^7} dx$$

Optimal antiderivative

$$\frac{4\left(4 + \frac{5}{x^2}\right)^2 + x}{2e^{-15} - x} + 5x$$

command

```
integrate(((5*x^7+64*x^5+480*x^3+500*x)*exp(5)^6+(-20*x^6+2*x^5-640*x^2-800)*exp(5)^3+20*x^5+
4*x^6*exp(5)^3+4*x^5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$5x - \frac{25e^{75} + 160e^{45} + 256e^{15} + 8}{4(xe^{15} - 2)} + \frac{5(5x^3e^{60} + 32x^3e^{30} + 10x^2e^{45} + 64x^2e^{15} + 20xe^{30} + 40e^{15})}{4x^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.29 Problem number 1269

$$\int \frac{8e^3x}{64 + 192x^2 + 144x^4 + e^2(1 + 4x^2 + 4x^4) + e^{20}(1 + 4x^2 + 4x^4) + e(16 + 56x^2 + 48x^4) + e^{10}(-16 - 56x^2 - 48x^4)}$$

Optimal antiderivative

$$\frac{e^3}{\frac{1}{\frac{1}{2} + x^2} + 6 - e^{10} + e}$$

command

```
integrate(8*x*exp(3)/((4*x^4+4*x^2+1)*exp(5)^4+((-8*x^4-8*x^2-2)*exp(1)-48*x^4-56*x^2-
16)*exp(5)^2+(4*x^4+4*x^2+1)*exp(1)^2+(48*x^4+56*x^2+16)*exp(1)+144*x^4+192*x^2+64),x, algori
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2e^3}{(2x^2e^{10} - 2x^2e - 12x^2 + e^{10} - e - 8)(e^{10} - e - 6)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.30 Problem number 1280

$$\int -\frac{5000e^{5+625e^{4x}+4x}}{e^{10+1250e^{4x}} + 2e^{5+625e^{4x}} \log(2) + \log^2(2)} dx$$

Optimal antiderivative

$$\frac{2}{e^{625e^{4x}} e^5 + \ln(2)} - 5$$

command

```
integrate(-5000*exp(5)*exp(x)^4*exp(625*exp(x)^4)/(exp(5)^2*exp(625*exp(x)^4)^2+2*exp(5)*log(
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{e^{(625e^{4x}+5)} + \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.31 Problem number 1290

$$\int \frac{-264 + 192x}{256 + 16e^6 - 352x + 249x^2 - 88x^3 + 16x^4 + e^3(-128 + 88x - 32x^2)} dx$$

Optimal antiderivative

$$\frac{6}{\frac{11x}{4} - x^2 - 4 + e^3}$$

command

```
integrate((192*x-264)/(16*exp(3)^2+(-32*x^2+88*x-128)*exp(3)+16*x^4-88*x^3+249*x^2-352*x+256),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{24}{4x^2 - 11x - 4e^3 + 16}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 100.32 Problem number 1366

$$\int \frac{8x^2 + e^4(64 - 32x + 4x^2)}{4x^4 + e^8(256 - 32x^2 + x^4) + e^4(-64x^2 + 4x^4)} dx$$

Optimal antiderivative

$$\frac{x}{e^4(4+x) - \frac{2x^2}{4-x}}$$

command

`integrate(((4*x^2-32*x+64)*exp(4)+8*x^2)/((x^4-32*x^2+256)*exp(4)^2+(4*x^4-64*x^2)*exp(4)+4*x`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4(xe^4 + 2x - 4e^4)}{(x^2e^4 + 2x^2 - 16e^4)(e^4 + 2)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.33 Problem number 1380

$$\int \frac{e^{-e^{-x}(e^x(-3-x)+x)} - x \left( e^{e^{-x}(e^x(-3-x)+x)} (4x^2 - 4e^x x^2 - 4x^3) + e^{e^{-x}(e^x(-3-x)+x)} (e^{2x}(-1+x) + e^x(4-x^2)) \right)}{x^2} dx$$

Optimal antiderivative

$$-x - \frac{x - e^x + 4}{x} - 4e^{-e^x e^{-x} - 3 - x}$$

command

`integrate(((((-1+x)*exp(x)^2+(-x^2+4)*exp(x))*exp(exp(((3-x)*exp(x)+x)/exp(x)))+(-4*exp(x)*x^2-4*x^3+4*x^2)*exp(((3-x)*exp(x)+x)/exp(x)))/x^2/exp(x)/exp(exp(((3-x)*exp(x)+x)`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{x^2 - e^x + 4}{x} - 4e^{-e^{(xe^{(-x)} - x - 3)}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left( 4(x^3 + x^2 e^x - x^2) e^{-((x+3)e^x - x)e^{(-x)}} - ((x-1)e^{(2x)} - (x^2 - 4)e^x) e^{e^{-((x+3)e^x - x)e^{(-x)}}} \right) e^{-x - e^{-((x+3)e^x - x)}}}{x^2} dx$$

### 100.34 Problem number 1492

$$\int \frac{e^3(160 - 40x^4) + e^6(-48x + 13x^3 + 4x^5 + e^2(-32 + 8x^4))}{25x^3 + e^3(-10e^2x^3 - 10x^4) + e^6(e^4x^3 + 2e^2x^4 + x^5)} dx$$

Optimal antiderivative

$$\frac{3 + \left(2x - \frac{4}{x}\right)^2}{x + e^2 - 5e^{-3}}$$

command

```
integrate((((8*x^4-32)*exp(2)+4*x^5+13*x^3-48*x)*exp(3)^2+(-40*x^4+160)*exp(3))/((x^3*exp(2)-10*x^3*exp(2)-10*x^4)*exp(3)+25*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4x + \frac{4e^{20} - 13e^{16} - 80e^{15} + 16e^{12} + 130e^{11} + 600e^{10} - 325e^6 - 2000e^5 + 2500}{(xe^3 + e^5 - 5)(e^{13} - 10e^8 + 25e^3)} - \frac{16(xe^6 - e^8 + 5e^3)}{x^2(e^{10} - 10e^5 + 25)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.35 Problem number 1522

$$\int \frac{e^{2x}(120 - 480x + 400x^2 - 40x^4 + e^{2/3}(-8 + 2e^{2/3} - 2e^{4/3}))}{64000 - 72000x + 3000x^2 + 14625x^3 - 375x^4 - 1125x^5 - 125x^6 + e^2(-1 + 3x - 3x^2 + x^3) + e^{4/3}(120 - 285x)}$$

Optimal antiderivative

$$\frac{4e^{2x}}{\left(e^{\frac{2}{3}} - 15 + \frac{-5x+25}{-1+x} - 5x\right)^2}$$

command

```
integrate((((8*x^3-24*x^2+24*x-8)*exp(2/3)-40*x^4+400*x^2-480*x+120)*exp(x)^2/((x^3-3*x^2+3*x-1)*exp(2/3)^3+(-15*x^4-15*x^3+195*x^2-285*x+120)*exp(2/3)^2+(75*x^5+375*x^4-975*x^3-3075*x^2+8400*x-4800)*exp(2/3)-125*x^6-1125*x^5-375*x^4+14625*x^3+3000*x^2-72000*x+64000)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8(x^2e^{(2x)} - 2xe^{(2x)} + e^{(2x)})}{25x^4 - 10x^3e^{\frac{2}{3}} + 150x^3 + x^2e^{\frac{4}{3}} - 20x^2e^{\frac{2}{3}} - 175x^2 - 2xe^{\frac{4}{3}} + 110xe^{\frac{2}{3}} - 1200x + e^{\frac{4}{3}} - 80e^{\frac{2}{3}} + 1600}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.36 Problem number 1598

$$\int \frac{e^8(144x^2 - 96x^3 + 84x^4 - 27x^6) + e^{12}(64 - 96x + 192x^2 - 152x^3 + 144x^4 - 54x^5 - 135x^6 + e^8(-720x^2 + 720x^3 - 1260x^4 + 540x^5 - 405x^6) + e^{12}(320 - 480x + 960x^2 - 760x^3 + 720x^4 - 270x^5))}{-135x^6 + e^8(-720x^2 + 720x^3 - 1260x^4 + 540x^5 - 405x^6) + e^{12}(320 - 480x + 960x^2 - 760x^3 + 720x^4 - 270x^5)}$$

Optimal antiderivative

$$\frac{x}{5 \left( \frac{x e^{-4}}{x + \frac{4}{3x} - \frac{2}{3}} - 1 \right)^2}$$

command

```
integrate(((27*x^6-54*x^5+144*x^4-152*x^3+192*x^2-96*x+64)*exp(4)^3+(-27*x^6+84*x^4-96*x^3+144*x^2)*exp(4)^2)/((135*x^6-270*x^5+720*x^4-760*x^3+960*x^2-480*x+320)*exp(4)^3+(-405*x^6+540*x^5-1260*x^4+720*x^3-720*x^2)*exp(4)^2+(405*x^6-270*x^5+540*x^4)*exp(4)-135*x^6),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\frac{x e^{12} - x e^8}{5(e^{12} - 3e^8 + 3e^4 - 1)}}{4(12x^3e^{16} - 33x^3e^{12} + 21x^3e^8 + 4x^2e^{16} + 12x^2e^{12} - 12x^2e^8 + 8xe^{16} - 36xe^{12} + 12xe^8 + 16e^{16})} \frac{1}{15(3x^2e^4 - 3x^2 - 2xe^4 + 4e^4)^2(e^{12} - 3e^8 + 3e^4 - 1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.37 Problem number 1622

$$\int \frac{-12x^2 - 1203x^4 + e^5(12 - 6x - 3609x^2)}{e^{10} + 2e^5x^2 + x^4} dx$$

Optimal antiderivative

$$\frac{3x(-401x^2 - x + 4)}{x^2 + e^5}$$

command

```
integrate((-3609*x^2-6*x+12)*exp(5)-1203*x^4-12*x^2)/(exp(5)^2+2*x^2*exp(5)+x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-1203x + \frac{3(401xe^5 + 4x + e^5)}{x^2 + e^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.38 Problem number 1628

$$\int \frac{-54x - 6e^6x + 162x^2 - 108x^3 + e^3(-36x + 54x^2) + 2^{60/x}x^{60/x}(2x - 6x^2 + 4x^3) + 2^{20/x}x^{20/x}(54x - 162x^2 + 108x^3)}{x^4} dx$$

Optimal antiderivative

$$\left( \frac{e^3}{3 - e^{\frac{20 \ln(2x)}{x}}} - x + 1 \right)^2 x^2$$

command

```
integrate(((4*x^3-6*x^2+2*x)*exp(20*log(2*x)/x)^3+((40*x-40)*exp(3)*log(2*x)+(6*x^2-44*x+40)*exp(3)-36*x^3+54*x^2-18*x)*exp(20*log(2*x)/x)^2+((40*exp(3)^2+(-120*x+120)*exp(3))*140)*exp(3)^2+(-36*x^2+144*x-120)*exp(3)+108*x^3-162*x^2+54*x)*exp(20*log(2*x)/x)-6*x*exp(3)^2-36*x*exp(3)-108*x^3+162*x^2-54*x)/(exp(20*log(2*x)/x)^3-9*exp(20*log(2*x)/x)^2+27*exp(20*log(2*x)/x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} + \frac{2(2x)^{\frac{20}{x}}e^3}{x \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} \\ & - \frac{2(2x)^{\frac{40}{x}}}{x \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} + \frac{12(2x)^{\frac{20}{x}}}{x \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} + \frac{9}{\frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4}} \\ & - \frac{2(2x)^{\frac{20}{x}}e^3}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} - \frac{6e^3}{x \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} \\ & + \frac{(2x)^{\frac{40}{x}}}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} - \frac{6(2x)^{\frac{20}{x}}}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} \\ & - \frac{18}{x \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} + \frac{e^6}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} \\ & + \frac{6e^3}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} + \frac{9}{x^2 \left( \frac{(2x)^{\frac{40}{x}}}{x^4} - \frac{6(2x)^{\frac{20}{x}}}{x^4} + \frac{9}{x^4} \right)} \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**100.39 Problem number 1666**

$$\int \frac{135x + 40x^6 + 15x^7 + e^4(40x^4 + 15x^5) + e^2(90 + 80x^5 + 30x^6)}{e^4x^3 + 2e^2x^4 + x^5} dx$$

Optimal antiderivative

$$5x^2(4 + x) - \frac{45}{x^2(x + e^2)} - 20$$

command

```
integrate(((15*x^5+40*x^4)*exp(2)^2+(30*x^6+80*x^5+90)*exp(2)+15*x^7+40*x^6+135*x)/(x^3*exp(2
```

Giac 1.9.0-11 via sagemath 9.6 output

$$5x^3 + 20x^2 - \frac{45e^{(-4)}}{x + e^2} + \frac{45(x - e^2)e^{(-4)}}{x^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.40 Problem number 1680**

$$\int \frac{-432 - 288x - 81x^3 + 3e^{12}x^3 - 81x^4 - 27x^5 - 3x^6 + e^8(-27x^3 - 9x^4) + e^4(144 + 81x^3 + 54x^4 + 9x^5)}{-27x^3 + e^{12}x^3 - 27x^4 - 9x^5 - x^6 + e^8(-9x^3 - 3x^4) + e^4(27x^3 + 18x^4 + 3x^5)} dx$$

Optimal antiderivative

$$-\frac{8}{\left(x + x\left(\frac{x}{3} - \frac{e^4}{3}\right)\right)^2} + 3x$$

command

```
integrate((3*x^3*exp(4)^3+(-9*x^4-27*x^3)*exp(4)^2+(9*x^5+54*x^4+81*x^3+144)*exp(4)-
3*x^6-27*x^5-81*x^4-81*x^3-288*x-432)/(x^3*exp(4)^3+(-3*x^4-9*x^3)*exp(4)^2+(3*x^5+18*x^4+27*
x^6-9*x^5-27*x^4-27*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$3x - \frac{72}{(x^2 - xe^4 + 3x)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.41 Problem number 1697

$$\int \frac{25 - 20x + 5ex + 4x^2 + e^8 x^4 + e^4(10x^2 - 4x^3 - ex^3)}{e^9 x^5 + e(25x - 20x^2 + 4x^3) + e^5(10x^3 - 4x^4)} dx$$

Optimal antiderivative

$$5 + \ln(x) e^{-1} + \frac{x}{-2x + x^2 e^4 + 5}$$

command

```
integrate((x^4*exp(2)^4+(-x^3*exp(1)-4*x^3+10*x^2)*exp(2)^2+5*x*exp(1)+4*x^2-20*x+25)/(x^5*exp(4*x^4+10*x^3)*exp(1)*exp(2)^2+(4*x^3-20*x^2+25*x)*exp(1)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{(-1)} \log(|x|) + \frac{x}{x^2 e^4 - 2x + 5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.42 Problem number 1702

$$\int \frac{-16 - 32x - 16x^2 + e^{2x+e^{2x}} \left( 3e^{\frac{2x}{2+2x}} x + 6e^{\frac{x}{2+2x}} x^2 + 3x^3 \right) \left( -108x^2 - 288x^3 - 252x^4 - 72x^5 + e^{\frac{2x}{2+2x}} (-36 - 180x - 180x^2) \right)}{16x^2 + 32x^3 + 16x^4 + e^{2e^{2x}} \left( 3e^{\frac{2x}{2+2x}} x + 6e^{\frac{x}{2+2x}} x^2 + 3x^3 \right) (9 + 18x + 9x^2) + e^{e^{2x}} \left( 3e^{\frac{2x}{2+2x}} x + 6e^{\frac{x}{2+2x}} x^2 + 3x^3 \right)}$$

Optimal antiderivative

$$\frac{4}{4x + 3e^{3e^{2x}} \left( x + e^{\frac{x}{2x+2}} \right)^2 x}$$

command

```
integrate(((((-72*x^3-180*x^2-180*x-36)*exp(x/(2+2*x)))^2+(-144*x^4-432*x^3-468*x^2-144*x)*exp(x/(2+2*x))-72*x^5-252*x^4-288*x^3-108*x^2)*exp(x)^2*exp((3*x*exp(x/(2+2*x)))^2+6*x^2+16*x^2-32*x-16)/((9*x^2+18*x+9)*exp((3*x*exp(x/(2+2*x)))^2+6*x^2*exp(x/(2+2*x))+3*x^3)*exp(x)^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.43 Problem number 1725

$$\int \frac{\frac{e^4 x(2x+3x^3)}{2+x^2} + (4x + 2x^3) \log^2(2)}{\frac{e^8 x^2(8+4x^2)}{(2+x^2)^2} + \frac{e^4 x(16+8x^2) \log^2(2)}{2+x^2} + (8 + 4x^2) \log^4(2)} dx$$

Optimal antiderivative

$$\frac{x^2}{4 e^{4-\ln(x+\frac{2}{x})} + 4 \ln(2)^2}$$

command

`integrate(((3*x^3+2*x)*exp(-log((x^2+2)/x)+4)+(2*x^3+4*x)*log(2)^2)/((4*x^2+8)*exp(-log((x^2+2)/x)+4)^2+(8*x^2+16)*log(2)^2*exp(-log((x^2+2)/x)+4)+(4*x^2+8)*log(2)^4),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 x e^4 \log(2)^4 - 2 e^8 \log(2)^2 - x e^{12}}{4 \left( x^2 \log(2)^2 + x e^4 + 2 \log(2)^2 \right) \log(2)^6} + \frac{x^2 \log(2)^6 - x e^4 \log(2)^4}{4 \log(2)^8}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(x^3 + 2x) \log(2)^2 + (3x^3 + 2x) e^{(-\log(\frac{x^2+2}{x})+4)}}{4 \left( (x^2 + 2) \log(2)^4 + 2(x^2 + 2) e^{(-\log(\frac{x^2+2}{x})+4)} \log(2)^2 + (x^2 + 2) e^{(-2 \log(\frac{x^2+2}{x})+8)} \right)} dx$$

## 100.44 Problem number 1830

$$\int \frac{e^3 x^2 - 5e^8 x^3 + e^5(-e^6 + 10e^{11}x - 25e^{16}x^2) + (-e^3 x^2 + e^5(-e^6 + 10e^{11}x - 25e^{16}x^2)) \log(x)}{(-e^3 x^3 + 5e^8 x^4 + e^5(e^6 x - 10e^{11}x^2 + 25e^{16}x^3)) \log(x)} dx$$

Optimal antiderivative

$$\ln \left( \frac{\frac{x}{5x e^8 - e^3} + \frac{e^5}{x}}{\ln(x)} \right)$$

command

`integrate(((((-25*x^2*exp(4)^4+10*x*exp(3)*exp(4)^2-exp(3)^2)*exp(5)-x^2*exp(3))*log(x)+(-25*x^2*exp(4)^4+10*x*exp(3)*exp(4)^2-exp(3)^2)*exp(5)-5*x^3*exp(4)^2+x^2*exp(3))/((25*x^3*exp(10*x^2*exp(3)*exp(4)^2+x*exp(3)^2)*exp(5)+5*x^4*exp(4)^2-x^3*exp(3))/log(x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\log(x^2 + 5 x e^{13} - e^8) - \log(5 x e^5 - 1) - \log(x) - \log(\log(x))$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.45 Problem number 1909

$$\int \frac{-3600 + 480x + 164x^2 + e^4(3600 + 3120x + 76x^2 - 284x^3 + 4x^4) + e^8(180x^2 + 180x^3 + 45x^4)}{900x^2 - 120x^3 + 4x^4 + e^4(-1800x^2 - 660x^3 + 112x^4 - 4x^5) + e^8(900x^2 + 780x^3 + 109x^4 - 26x^5 + x^6)} dx$$

Optimal antiderivative

$$\frac{4}{\left(\frac{2}{2+x} - e^4\right)x} + \frac{x}{5 - \frac{x}{3}} + \frac{2}{15}$$

command

```
integrate(((45*x^4+180*x^3+180*x^2)*exp(4)^2+(4*x^4-284*x^3+76*x^2+3120*x+3600)*exp(4)+164*x^3600)/((x^6-26*x^5+109*x^4+780*x^3+900*x^2)*exp(4)^2+(-4*x^5+112*x^4-660*x^3-1800*x^2)*exp(4)120*x^3+900*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{45x^2e^4 + 4x^2 + 90xe^4 - 142x - 120}{x^3e^4 - 13x^2e^4 - 2x^2 - 30xe^4 + 30x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.46 Problem number 1911

$$\int \frac{2^{4+e^{3-x}+i\pi+x+\log(5-\log(5))}}{16 + e^{6-2x} + 8x + x^2 + e^{3-x}(8 + 2x) + (8 + 2e^{3-x} + 2x)(i\pi + \log(5 - \log(5))) + (i\pi + \log(5 - \log(5)))^2} \frac{1}{e^{2^{4+e^{3-x}+i\pi+x+\log(5-\log(5))}}} (-\log(2) + e^{3-x} \log(2)) dx$$

Optimal antiderivative

$$e^{\frac{\ln(2)}{e^{\ln(\ln(5)-5)+e^2}e^{1-x+4+x}}}$$

command

```
integrate((exp(2)*log(2)*exp(1-x)-log(2))*exp(log(2)/(log(log(5)-5)+exp(2)*exp(1-x)+4+x))*exp(5)+exp(2)*exp(1-x)+4+x))/(log(log(5)-5)^2+(2*exp(2)*exp(1-x)+2*x+8)*log(log(5)-5)+exp(2)^2*x)^2+(2*x+8)*exp(2)*exp(1-x)+x^2+8*x+16),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(2^{\left(\frac{1}{x+e^{(-x+3)}+\log(\log(5)-5)+4}\right)}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(e^{(-x+3)} \log(2) - \log(2)) 2^{\left(\frac{1}{x+e^{(-x+3)}+\log(\log(5)-5)+4}\right)} e^{\left(2^{\left(\frac{1}{x+e^{(-x+3)}+\log(\log(5)-5)+4}\right)}\right)}}{x^2 + 2(x+4)e^{(-x+3)} + 2(x+e^{(-x+3)}+4) \log(\log(5)-5) + \log(\log(5)-5)^2 + 8x + e^{(-2x+6)} + 16} dx$$



## 100.47 Problem number 1920

$$\int \frac{-3375x^3 - 2025x^4 - 405x^5 - 27x^6 + e^6(1 + 3x + 3x^2 + x^3) + e^4(-45x - 99x^2 - 63x^3 - 9x^4) + e^2(675x^2 + 945x^3 - 405x^4 - 2025x^5 - 3375x^6)}{x^2 \left( x - \ln(5) + \frac{(-\frac{1}{3} - \frac{x}{3})e^2}{x} + 5 \right)^2} dx$$

Optimal antiderivative

$$\frac{-2 + x}{x^2 \left( x - \ln(5) + \frac{(-\frac{1}{3} - \frac{x}{3})e^2}{x} + 5 \right)^2}$$

command

```
integrate((( -27*x+108)*log(5)+(-9*x+45)*exp(2)+81*x^2-81*x-540)/(27*x^3*log(5)^3+((27*x^3+27*81*x^4-405*x^3)*log(5)^2+((9*x^3+18*x^2+9*x)*exp(2)^2+(-54*x^4-324*x^3-270*x^2)*exp(2)+81*x^5+9*x^4-63*x^3-99*x^2-45*x)*exp(2)^2+(27*x^5+297*x^4+945*x^3+675*x^2)*exp(2)-27*x^6-405*x^5-2025*x^4-3375*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9(x-2)}{(3x^2 - xe^2 - 3x \log(5) + 15x - e^2)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.48 Problem number 1932

$$\int \frac{e^4(225 - 150x) + 1125e^2x^2}{e^4 - 30e^2x + 225x^2} dx$$

Optimal antiderivative

$$\frac{5(3-x)x}{\frac{1}{15} - xe^{-2}}$$

command

```
integrate((( -150*x+225)*exp(2)^2+1125*x^2*exp(2))/(exp(2)^2-30*exp(2)*x+225*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$5xe^2 + \frac{e^6 - 45e^4}{3(15x - e^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.49 Problem number 1952

$$\int \frac{32x + 16e^3x + 2e^6x}{1 + e^5 - 2x^2 + x^4 + e^{5/2}(-2 + 2x^2)} dx$$

Optimal antiderivative

$$\frac{(4 + e^3)^2}{1 - e^{\frac{5}{2}} - x^2}$$

command

`integrate((2*x*exp(3)^2+16*x*exp(3)+32*x)/(exp(5/4)^4+(2*x^2-2)*exp(5/4)^2+x^4-2*x^2+1),x, a1`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^{12} + 16e^9 + 96e^6 + 256e^3 + 256}{x^2e^6 + 8x^2e^3 + 16x^2 + e^{\frac{17}{2}} - e^6 + 8e^{\frac{11}{2}} - 8e^3 + 16e^{\frac{5}{2}} - 16}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.50 Problem number 2020

$$\int e^{-e-x} \left( e^{7+x} + e^{e-x} (e^{2+x} + e^e(1-x) + e^2(x-x^2)) \right) dx$$

Optimal antiderivative

$$(1 + e^2e^{-e}x) \left( e^{xe^{-x}} + e^5 \right)$$

command

`integrate((((1-x)*exp(exp(1))+exp(1)^2*exp(x)+(-x^2+x)*exp(1)^2)*exp(x/exp(x))+exp(1)^2*exp(5`

Giac 1.9.0-11 via sagemath 9.6 output

$$\left( xe^{(xe^{(-x)}-x-e+2)} + xe^{(-x-e+7)} + e^{(xe^{(-x)}-x)} \right) e^x$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\left( (x^2 - x)e^2 + (x - 1)e^e - e^{(x+2)} \right) e^{(xe^{(-x)})} - e^{(x+7)} e^{(-x-e)} dx$$

## 100.51 Problem number 2076

$$\int \frac{e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}} (-50 + (-100 - 10x^3 \log(x) + 20e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}} x^5 \log^3(x) + \log\left(5 - e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}}\right) (-5x^5 \log^3(x) + \log\left(5 - e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}}\right))}{\left(-100x^5 \log^3(x) + 20e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}} x^5 \log^3(x) + \log\left(5 - e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}}\right) (-5x^5 \log^3(x) + \log\left(5 - e^{\frac{25+10x^3 \log(x) + (-5x^4+x^6) \log^2(x)}{x^4 \log^2(x)}}\right)\right)}$$

Optimal antiderivative

$$\ln\left(\ln\left(\ln\left(5 - e^{\left(\frac{5}{x^2 \ln(x)} + x\right)^2 - 5}\right) + 20\right)\right)$$

command

```
integrate((2*x^6*log(x)^3-10*x^3*log(x)^2+(-10*x^3-100)*log(x)-50)*exp(((x^6-5*x^4)*log(x)^2+5*x^4)*log(x)^2+10*x^3*log(x)+25)/x^4/log(x)^2-5*x^5*log(x)^3*log(-exp(((x^6-5*x^4)*log(x)^2+10*x^3*log(x)+25)/x^4/log(x)^2)-100*x^5*log(x)^3)/log(log(-exp(((x^6-5*x^4)*log(x)^2+10*x^3*log(x)+25)/x^4/log(x)^2)+5)+20),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\log\left(\log\left(-e^{\left(\frac{x^6 \log(x)^2 - 5x^4 \log(x)^2 + 10x^3 \log(x) + 25}{x^4 \log(x)^2}\right)} + 5\right) + 20\right)\right)$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 100.52 Problem number 2118

$$\int \frac{-10e^8 - e^4 x^2 + x^6(-250 - 100x - 10x^2) + x^3(-75x - 30x^2 - 3x^3 + e^4(100 + 20x))}{5e^{12}x^2 + e^8x^3(-50x^2 - 10x^3) + e^4x^6(125x^2 + 50x^3 + 5x^4)} dx$$

Optimal antiderivative

$$\frac{\left(2 + \frac{x}{5} - \frac{x}{5\left(\frac{e^4}{5+x} - x^3\right)}\right) e^{-4}}{x}$$

command

```
integrate((( -10*x^2-100*x-250)*x^6+((20*x+100)*exp(4)-3*x^3-30*x^2-75*x)*x^3-10*exp(4)^2-x^2*exp(4))/((5*x^4+50*x^3+125*x^2)*exp(4)*x^6+(-10*x^3-50*x^2)*exp(4)^2*x^3+5*x^2*exp(4)^3),x)
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(10x^4 + 50x^3 + x^2 + 5x - 10e^4)e^{(-4)}}{5(x^5 + 5x^4 - xe^4)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.53 Problem number 2176

$$\int \frac{e^{12}(-e^5 + e^{10}(-1 - x^2))}{1 + e^5(2 + 2x - 2x^2) + e^{10}(1 + 2x - x^2 - 2x^3 + x^4)} dx$$

Optimal antiderivative

$$\frac{e^{12}}{x - \frac{e^{-5}+1}{x} - 1}$$

command

`integrate(((x^2-1)*exp(5)^2-exp(5))*exp(12)/((x^4-2*x^3-x^2+2*x+1)*exp(5)^2+(-2*x^2+2*x+2)*e`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^{17}}{x^2e^5 - xe^5 - e^5 - 1}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{((x^2 + 1)e^{10} + e^5)e^{12}}{(x^4 - 2x^3 - x^2 + 2x + 1)e^{10} - 2(x^2 - x - 1)e^5 + 1} dx$$

## 100.54 Problem number 2209

$$\int -\frac{20e^{1+e^{1+x}+e^{1+x}+x}}{49 + 14e^2 + e^4 + e^{2e^{1+x}} + e^{e^{1+x}}(14 + 2e^2)} dx$$

Optimal antiderivative

$$\frac{20}{7 + e^{e^{1+x}} + e^2}$$

command

`integrate(-20*exp(1+x)*exp(exp(1+x))*exp(exp(exp(1+x)))/(exp(exp(exp(1+x)))^2+(2*exp(2)+14)*e`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{20}{e^2 + e^{\left(e^{(x+1)}\right)} + 7}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.55 Problem number 2215

$$\int \frac{-x^6 + 120x^7 + 1112x^8 + 3210x^9 + 3504x^{10} + 1300x^{11} + 150x^{12} + e^9(-1 + 50x^2 + 780x^3 + 2904x^4 + 1300x^5 + \dots}{\dots}$$

Optimal antiderivative

$$\left(x + 5 + \frac{3}{x + \frac{e^3}{x}}\right)^2 (5x^2 + x)^2 - \ln(-x)$$

command

```
integrate(((150*x^6+1300*x^5+2904*x^4+780*x^3+50*x^2-1)*exp(3)^3+(450*x^8+3900*x^7+9612*x^6+63*x^2)*exp(3)^2+(450*x^10+3900*x^9+10212*x^8+8820*x^7+3336*x^6+570*x^5+33*x^4)*exp(3)+150*x^11*x^6)/(x*exp(3)^3+3*x^3*exp(3)^2+3*x^5*exp(3)+x^7),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25x^6 + 260x^5 + 876x^4 + 1070x^3 - 150x^2e^3 + 556x^2 - 810xe^3 + 120x + 3(270x^3e^6 - 70x^3e^3 - 50x^2e^9 + 327x^2e^6 - 6x^2e^3 + 270xe^9 - 40xe^6 - 50e^{12} + 252e^9 - 3e^6)}{(x^2 + e^3)^2} - \log(|x|)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.56 Problem number 2217

$$\int -\frac{5}{50x + 20e^{25}x \log(x) + 2e^{50}x \log^2(x)} dx$$

Optimal antiderivative

$$\frac{\ln(x)}{-10 - 2e^{25} \ln(x)}$$

command

```
integrate(-5/(2*x*exp(25)^2*log(x)^2+20*x*exp(25)*log(x)+50*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5e^{(-25)}}{2(e^{25} \log(x) + 5)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.57 Problem number 2232

$$\int \frac{32e^8 + 128x^4 + e^4(24 + 128x^2)}{9x^3 + 16e^8x^3 + 48x^5 + 64x^7 + e^4(24x^3 + 64x^5)} dx$$

Optimal antiderivative

$$-\frac{4}{\frac{3x}{2x + \frac{e^4}{x}} + 4x^2}$$

command

```
integrate((32*exp(4)^2+(128*x^2+24)*exp(4)+128*x^4)/(16*x^3*exp(4)^2+(64*x^5+24*x^3)*exp(4)+6
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4(2x^2 + e^4)}{8x^4 + 4x^2e^4 + 3x^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.58 Problem number 2289

$$\int \frac{39 - 9 \log(4 + e^2)}{169 - 78e^2x + 9e^4x^2 + (-78 + 18e^2x) \log(4 + e^2) + 9 \log^2(4 + e^2)} dx$$

Optimal antiderivative

$$\frac{x}{\frac{13}{3} - \ln(4 + e^2) - e^2x}$$

command

```
integrate((-9*log(4+exp(2))+39)/(9*log(4+exp(2))^2+(18*exp(2)*x-78)*log(4+exp(2))+9*x^2*exp(2)
78*exp(2)*x+169),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(3 \log(e^2 + 4) - 13)e^{(-2)}}{3xe^2 + 3 \log(e^2 + 4) - 13}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.59 Problem number 2423**

$$\int \frac{(16 - 12x^2 + 4x^3 + e^3(-16x^2 - 16x^3 + 28x^4 - 8x^5) + e^3(-16x - 16x^2 + 28x^3 - 8x^4) \log(x)) \log \left( \log^2 \left( e^{-e^3(2} \right. \right.}{(x^2 + x \log(x))}$$

Optimal antiderivative

$$(-2 + x)^2 \ln \left( \ln \left( (x + \ln(x)) e^{-(x^2+x+2)e^3} \right)^2 \right)^2$$

command

```
integrate((((2*x^2-4*x)*log(x)+2*x^3-4*x^2)*log((x+log(x))/exp((x^2+x+2)*exp(3))))*log(log((x+
8*x^4+28*x^3-16*x^2-16*x)*exp(3)*log(x)+(-8*x^5+28*x^4-16*x^3-16*x^2)*exp(3)+4*x^3-
12*x^2+16)*log(log((x+log(x))/exp((x^2+x+2)*exp(3)))^2))/(x*log(x)+x^2)/log((x+log(x))/exp((x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & x^2 \log \left( x^4 e^6 + 2 x^3 e^6 - 2 x^2 e^3 \log(x + \log(x)) + 5 x^2 e^6 - 2 x e^3 \log(x + \log(x)) \right. \\ & \quad \left. + 4 x e^6 - 4 e^3 \log(x + \log(x)) + \log(x + \log(x))^2 + 4 e^6 \right)^2 \\ & \quad - 4 x \log \left( x^4 e^6 + 2 x^3 e^6 - 2 x^2 e^3 \log(x + \log(x)) + 5 x^2 e^6 - 2 x e^3 \log(x + \log(x)) \right. \\ & \quad \left. + 4 x e^6 - 4 e^3 \log(x + \log(x)) + \log(x + \log(x))^2 + 4 e^6 \right)^2 \\ & \quad + 4 \log \left( x^4 e^6 + 2 x^3 e^6 - 2 x^2 e^3 \log(x + \log(x)) + 5 x^2 e^6 - 2 x e^3 \log(x + \log(x)) \right. \\ & \quad \left. + 4 x e^6 - 4 e^3 \log(x + \log(x)) + \log(x + \log(x))^2 + 4 e^6 \right)^2 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.60 Problem number 2440**

$$\int \frac{40e^5 + 550x - 200x^2}{-33275x^4 + 18150x^5 - 3300x^6 + 200x^7 + e^{10}(-5324 + 2904x - 528x^2 + 32x^3) + e^5(26620x^2 - 14520x^3 + 264}$$

Optimal antiderivative

$$\frac{1}{\left(x^2 - \frac{2e^5}{5}\right) (-2x + 11)^2}$$

command

`integrate((40*exp(5)-200*x^2+550*x)/((32*x^3-528*x^2+2904*x-5324)*exp(5)^2+(-160*x^5+2640*x^4+14520*x^3+26620*x^2)*exp(5)+200*x^7-3300*x^6+18150*x^5-33275*x^4),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25(220x + 8e^5 + 605)}{(5x^2 - 2e^5)(64e^{10} - 9680e^5 + 366025)} - \frac{20(220x + 8e^5 - 1815)}{(2x - 11)^2(64e^{10} - 9680e^5 + 366025)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.61 Problem number 2443

$$\int \frac{5 + e^3(-1 - 4x) - 6x - x^2}{75 + 3e^6 + 30x^2 + 3x^4 + e^3(-30 - 6x^2)} dx$$

Optimal antiderivative

$$\frac{\frac{13}{3} + \frac{2}{3}x^2 + \frac{1}{3}x}{5 + x^2 - e^3}$$

command

`integrate(((4*x-1)*exp(3)-x^2-6*x+5)/(3*exp(3)^2+(-6*x^2-30)*exp(3)+3*x^4+30*x^2+75),x, algo`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x + 2e^3 + 3}{3(x^2 - e^3 + 5)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.62 Problem number 2504

$$\int \frac{e^4(-4 - 2x) - 11x^2 - 8e^{15}x^2 - e^{20}x^2 - 8x^3 - x^4 + e^8(4 + 2x) + e^5(-4e^4 + 4e^8 - 28x^2 - 8x^3) + e^{10}(-e^4 + e^8)}{16x^2 + 8e^{15}x^2 + e^{20}x^2 + 8x^3 + x^4 + e^{10}(24x^2 + 2x^3) + e^5(32x^2 + 8x^3)}$$

Optimal antiderivative

$$\frac{x + \frac{e^4 - e^8 - x}{x}}{(e^5 + 2)^2 + x} - x$$

command



`integrate((-x^2*exp(5)^4-8*x^2*exp(5)^3+(exp(4)^2-exp(4)-2*x^3-23*x^2)*exp(5)^2+(4*exp(4)^2-4*exp(4)-8*x^3-28*x^2)*exp(5)+(2*x+4)*exp(4)^2+(-2*x-4)*exp(4)-x^4-8*x^3-11*x^2)/(x^2*exp(5)^4`

Giac 1.9.0-11 via sagemath 9.6 output

$$-x - \frac{xe^{10} + 4xe^5 + 5x + e^8 - e^4}{x^2 + xe^{10} + 4xe^5 + 4x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.63 Problem number 2509

$$\int \frac{e^{-5x} \left( e^{2e^{-5x}} (-2e^{5x} - 10x) - e^{5x} x^3 \log(4) \right)}{x^3 \log(4)} dx$$

Optimal antiderivative

$$-x + \frac{e^{2e^{-5x}}}{2x^2 \ln(2)} - 5$$

command

`integrate(1/2*((-2*exp(5*x)-10*x)*exp(2/exp(5*x))-2*x^3*log(2)*exp(5*x))/x^3/log(2)/exp(5*x),`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2x^3 \log(2) - e^{(2e^{-5x})}}{2x^2 \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{\left( x^3 e^{(5x)} \log(2) + (5x + e^{(5x)}) e^{(2e^{-5x})} \right) e^{(-5x)}}{x^3 \log(2)} dx$$

### 100.64 Problem number 2578

$$\int \frac{-50e^4 - 2x^2 + 8x^3 + e^2(-20x + 60x^2)}{75e^4 + 30e^2x + 3x^2} dx$$

Optimal antiderivative

$$-\frac{2x}{3} + 3 + \frac{4x^3}{3(5e^2 + x)}$$

command

`integrate((-50*exp(2)^2+(60*x^2-20*x)*exp(2)+8*x^3-2*x^2)/(75*exp(2)^2+30*exp(2)*x+3*x^2),x,`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4}{3}x^2 - \frac{20}{3}xe^2 - \frac{2}{3}x - \frac{500e^6}{3(x+5e^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.65 Problem number 2602

$$\int \frac{e^{7x+x^2+8e^{7x+x^2}}x^3(-96x^2-224x^3-64x^4)}{e^{10}+e^{16e^{7x+x^2}}x^3+2e^{5+8e^{7x+x^2}}x^3} dx$$

Optimal antiderivative

$$\frac{4}{e^{8e^{(7+x)x^3}}+e^5}$$

command

`integrate((-64*x^4-224*x^3-96*x^2)*exp(x^2+7*x)*exp(8*x^3*exp(x^2+7*x))/(exp(8*x^3*exp(x^2+7*x`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4}{e^5+e^{\left(8x^3e^{(x^2+7x)}\right)}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.66 Problem number 2727

$$\int \frac{4ex + e^{1+x}(2+2x) + e^{\frac{2e-e^x}{e}}(-2e+2e^xx)}{e^{1+\frac{3(2e-e^x)}{e}}x^3 - e^{1+3x}x^3 - 3e^{1+2x}x^4 - 3e^{1+x}x^5 - ex^6 + e^{\frac{2(2e-e^x)}{e}}(-3e^{1+x}x^3 - 3ex^4) + e^{\frac{2e-e^x}{e}}(3e^{1+2x}x^3 + 6e^{1+x}}$$

Optimal antiderivative

$$\frac{1}{x^2(e^{2-e^xe^{-1}} - e^x - x)^2}$$

command

```
integrate(((2*exp(x)*x-2*exp(1))*exp((-exp(x)+2*exp(1))/exp(1))+(2+2*x)*exp(1)*exp(x)+4*x*exp(x)+2*exp(1))/exp(1))^3+(-3*x^3*exp(1)*exp(x)-3*x^4*exp(1))*exp((-exp(x)+2*exp(1))/exp(1))*exp(x)+2*exp(1))/exp(1)-x^3*exp(1)*exp(x)^3-3*x^4*exp(1)*exp(x)^2-3*x^5*exp(1)*exp(x)-x^6*exp(1)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.67 Problem number 2742

$$\int \frac{32 + 2e^{10} + 512x^3 + 128x^4 + 128x^8 + e^5(-16 - 32x^4)}{64 - 64x + 16x^2 + e^{10}x^2 - 128x^5 + 64x^6 + 64x^{10} + e^5(16x - 8x^2 - 16x^6)} dx$$

Optimal antiderivative

$$e^2 - \frac{2}{x - \frac{4}{4x^4 + 2 - \frac{e^5}{2}}}$$

command

```
integrate((2*exp(5)^2+(-32*x^4-16)*exp(5)+128*x^8+128*x^4+512*x^3+32)/(x^2*exp(5)^2+(-16*x^6-8*x^2+16*x)*exp(5)+64*x^10+64*x^6-128*x^5+16*x^2-64*x+64),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(8x^4 - e^5 + 4)}{8x^5 - xe^5 + 4x - 8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.68 Problem number 2762

$$\int \frac{-16x^2 + 8x^3 + 8x^4 + e^2(2 + 8x + 8x^2) + e(-9 - 4x - 16x^2 - 16x^3)}{2x^2 + 8x^3 + 8x^4 + e^2(2 + 8x + 8x^2) + e(-4x - 16x^2 - 16x^3)} dx$$

Optimal antiderivative

$$\frac{3}{2\left(\frac{x}{3} - \frac{e}{3}\right)\left(2 + \frac{1}{x}\right)} + 3 + x$$

command

```
integrate(((8*x^2+8*x+2)*exp(1)^2+(-16*x^3-16*x^2-4*x-9)*exp(1)+8*x^4+8*x^3-16*x^2)/((8*x^2+8*x^3-16*x^2-4*x)*exp(1)+8*x^4+8*x^3+2*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x + \frac{9x}{2(2x^2 - 2xe + x - e)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.69 Problem number 2781

$$\int \frac{-2e^3 + e^6(-4x + 10x^2) + e^6(-2 + 20x - 50x^2) \log(4)}{x^2 + 2e^3x^3 + e^6x^4 + (2x - 10x^2 + e^3(4x^2 - 20x^3) + e^6(2x^3 - 10x^4)) \log(4) + (1 - 10x + 25x^2 + e^3(2x - 20x^2)) \log(4)}$$

Optimal antiderivative

$$\frac{2}{(e^{-3} + x) \left( \frac{x}{1-5x} + 2 \ln(2) \right)}$$

command

```
integrate((2*(-50*x^2+20*x-2)*exp(3)^2*log(2)+(10*x^2-4*x)*exp(3)^2-2*exp(3))/(4*((25*x^4-10*x^3+x^2)*exp(3)^2+(50*x^3-20*x^2+2*x)*exp(3)+25*x^2-10*x+1)*log(2)^2+2*((-10*x^4+2*x^3)*exp(3)+20*x^3+4*x^2)*exp(3)-10*x^2+2*x)*log(2)+x^4*exp(3)^2+2*x^3*exp(3)+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(5xe^3 - e^3)}{10x^2e^3 \log(2) - x^2e^3 - 2xe^3 \log(2) + 10x \log(2) - x - 2 \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.70 Problem number 2785

$$\int \frac{12x^8 - 8x^9 - 12x^{10} + e^{10}(6x^3 - 4x^4 - 6x^5) + (12x^8 - 4x^9 - 4x^{10} + e^{10}(-24x^3 + 8x^4 + 8x^5)) \log(3x - x^2 - x^3)}{(e^{24}(-3 + x + x^2) + e^{14}(-12x^5 + 4x^6 + 4x^7) + e^4(-12x^{10} + 4x^{11} + 4x^{12})) \log^2(3x - x^2 - x^3)} dx$$

Optimal antiderivative

$$\frac{2e^{-4}}{\left(\frac{e^{10}}{x^4} + 2x\right) \ln(x(-x^2 - x + 3))}$$

command

```
integrate((((8*x^5+8*x^4-24*x^3)*exp(5)^2-4*x^10-4*x^9+12*x^8)*log(-x^3-x^2+3*x)+(-6*x^5-4*x^4+6*x^3)*exp(5)^2-12*x^10-8*x^9+12*x^8)/((x^2+x-3)*exp(4)*exp(5)^4+(4*x^7+4*x^6-12*x^5)*exp(4)*exp(5)^2+(4*x^12+4*x^11-12*x^10)*exp(4))/log(-x^3-x^2+3*x)^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2x^4}{2x^5e^4 \log(-x^3 - x^2 + 3x) + e^{14} \log(-x^3 - x^2 + 3x)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.71 Problem number 2800

$$\int \frac{-12 + 4e^2}{1444 + 4e^4 + e^2(152 - 44x) - 836x + 121x^2} dx$$

Optimal antiderivative

$$\frac{-4 + x}{19 - \frac{11x}{2} + e^2}$$

command

```
integrate((4*exp(2)-12)/(4*exp(2)^2+(-44*x+152)*exp(2)+121*x^2-836*x+1444),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4(e^2 - 3)}{11(11x - 2e^2 - 38)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.72 Problem number 2904**

$$\int \frac{-4e^2 + 118098x - 16x^2 + e(-39366 + 16x)}{e^2x^3 - 4ex^4 + 4x^5} dx$$

Optimal antiderivative

$$\frac{\frac{19683}{e-2x} + \frac{x^2}{2} + 2}{x^2}$$

command

`integrate((-4*exp(1)^2+(16*x-39366)*exp(1)-16*x^2+118098*x)/(x^3*exp(1)^2-4*x^4*exp(1)+4*x^5)`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{78732 e^{(-2)}}{2x - e} + \frac{(39366x + 2e^2 + 19683e)e^{(-2)}}{x^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.73 Problem number 2928**

$$\int \frac{e^{-7x/3}((192 + e^{7x/3}(-36 - 12x + 12x^2)) \log^3(x) + (-112x + e^{7x/3}(-3x + 6x^2)) \log^4(x))}{3x} dx$$

Optimal antiderivative

$$\ln(x)^4 \left( x^2 - 3 + 16e^{-\frac{7x}{3}} - x \right)$$

command

`integrate(1/3*(((6*x^2-3*x)*exp(7/3*x)-112*x)*log(x)^4+((12*x^2-12*x-36)*exp(7/3*x)+192)*log(x)^3))`

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & x^2 \log(3)^4 + 4x^2 \log(3)^3 \log\left(\frac{1}{3}x\right) + 6x^2 \log(3)^2 \log\left(\frac{1}{3}x\right)^2 \\ & + 4x^2 \log(3) \log\left(\frac{1}{3}x\right)^3 + x^2 \log\left(\frac{1}{3}x\right)^4 - x \log(3)^4 + 16e^{(-\frac{7}{3}x)} \log(3)^4 \\ & - 4x \log(3)^3 \log\left(\frac{1}{3}x\right) + 64e^{(-\frac{7}{3}x)} \log(3)^3 \log\left(\frac{1}{3}x\right) - 6x \log(3)^2 \log\left(\frac{1}{3}x\right)^2 \\ & + 96e^{(-\frac{7}{3}x)} \log(3)^2 \log\left(\frac{1}{3}x\right)^2 - 4x \log(3) \log\left(\frac{1}{3}x\right)^3 + 64e^{(-\frac{7}{3}x)} \log(3) \log\left(\frac{1}{3}x\right)^3 \\ & - x \log\left(\frac{1}{3}x\right)^4 + 16e^{(-\frac{7}{3}x)} \log\left(\frac{1}{3}x\right)^4 - 12 \log(3)^3 \log\left(\frac{1}{3}x\right) \\ & - 18 \log(3)^2 \log\left(\frac{1}{3}x\right)^2 - 12 \log(3) \log\left(\frac{1}{3}x\right)^3 - 3 \log\left(\frac{1}{3}x\right)^4 \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left( \left( 3(2x^2 - x)e^{\frac{7}{3}x} - 112x \right) \log(x)^4 + 12 \left( (x^2 - x - 3)e^{\frac{7}{3}x} + 16 \right) \log(x)^3 \right) e^{-\frac{7}{3}x}}{3x} dx$$

### 100.74 Problem number 2992

$$\int \frac{1}{-15625 + 46875x - 46875x^2 + 15625x^3 - 30e^{10}x^3 + e^{12}x^3 + e^2(-18750x + 37500x^2 - 18750x^3) + e^6(1500x^2 - 1500x^3)} dx$$

Optimal antiderivative

$$\frac{x}{\left( 5 - \frac{x(e^2 - 5 + 4 \ln(19))^2}{5} \right)^2}$$

command

```
integrate((-400*x*log(19)^2+(-200*exp(2)*x+1000*x)*log(19)-25*x*exp(2)^2+250*exp(2)*x-625*x-625)/(4096*x^3*log(19)^6+(6144*x^3*exp(2)-30720*x^3)*log(19)^5+(3840*x^3*exp(2)^2-38400*x^3*exp(2)+96000*x^3-19200*x^2)*log(19)^4+(1280*x^3*exp(2)^3-19200*x^3*exp(2)^2+(96000*19200*x^2)*exp(2)-160000*x^3+96000*x^2)*log(19)^3+(240*x^3*exp(2)^4-4800*x^3*exp(2)^3+(36000*7200*x^2)*exp(2)^2+(-120000*x^3+72000*x^2)*exp(2)+150000*x^3-180000*x^2+30000*x)*log(19)^2+(2600*x^3*exp(2)^4+(6000*x^3-1200*x^2)*exp(2)^3+(-30000*x^3+18000*x^2)*exp(2)^2+(75000*x^3-90000*x^2+15000*x)*exp(2)-75000*x^3+150000*x^2-75000*x)*log(19)+x^3*exp(2)^6-30*x^3*exp(2)^5+75*x^2)*exp(2)^4+(-2500*x^3+1500*x^2)*exp(2)^3+(9375*x^3-11250*x^2+1875*x)*exp(2)^2+(-18750*x^3+37500*x^2-18750*x)*exp(2)+15625*x^3-46875*x^2+46875*x-15625),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25x}{\left( 8xe^2 \log(19) + 16x \log(19)^2 + xe^4 - 10xe^2 - 40x \log(19) + 25x - 25 \right)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.75 Problem number 3042

$$\int \frac{e^{-\frac{3}{2}/x} \left( e^{\frac{3}{2}/x} (480x^3 - 320x^4) + e^{-\frac{3}{2}/x} \left( -3 + e^{\frac{3}{2}/x} \right) \left( -81 + 216x - 144x^2 + e^{\frac{3}{2}/x} (18x^2 - 48x^3 + 32x^4) \right) \right)}{18x^2 - 48x^3 + 32x^4} dx$$

Optimal antiderivative

$$e^{x-3} e^{-\frac{3}{2x}} - \frac{10x^2}{x - \frac{3}{4}}$$

command

```
integrate((((32*x^4-48*x^3+18*x^2)*exp(3/2/x)-144*x^2+216*x-81)*exp((x*exp(3/2/x)-3)/exp(3/2/x))+(-320*x^4+480*x^3)*exp(3/2/x))/(32*x^4-48*x^3+18*x^2)/exp(3/2/x),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{80x^2e^{(-\frac{3}{2x})} - 8xe^{\left(\frac{2x^2-6xe^{(-\frac{3}{2x})}-3}{2x}\right)} - 60xe^{(-\frac{3}{2x})} + 6e^{\left(\frac{2x^2-6xe^{(-\frac{3}{2x})}-3}{2x}\right)} + 45e^{(-\frac{3}{2x})}}{2\left(4xe^{(-\frac{3}{2x})} - 3e^{(-\frac{3}{2x})}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{\left(\left(144x^2 - 2(16x^4 - 24x^3 + 9x^2)e^{\left(\frac{3}{2x}\right)} - 216x + 81\right)e^{\left(\left(xe^{\left(\frac{3}{2x}\right)} - 3\right)e^{(-\frac{3}{2x})}\right)} + 160(2x^4 - 3x^3)e^{\left(\frac{3}{2x}\right)}\right)e^{(-\frac{3}{2x})}}{2(16x^4 - 24x^3 + 9x^2)} dx$$

### 100.76 Problem number 3164

$$\int \frac{e^4x + e^{\frac{256-32x^2+x^4}{e^4}}(e^4(-4+x) - 256x^2 + 64x^3 + 16x^4 - 4x^5) + e^4(4-x)\log(4-x)}{e^{5+\frac{2(256-32x^2+x^4)}{e^4}}(-4+x) + e^{5+\frac{256-32x^2+x^4}{e^4}}(8-2x)\log(4-x) + e^5(-4+x)\log^2(4-x)} dx$$

Optimal antiderivative

$$\frac{xe^{-1}}{e^{(x^2-16)^2e^{-4}} - \ln(4-x)}$$

command

```
integrate(((((-x+4)*exp(4)*log(-x+4))+((x-4)*exp(4)-4*x^5+16*x^4+64*x^3-256*x^2)*exp((x^4-32*x^2+256)/exp(4))+x*exp(4)))/((x-4)*exp(1)*exp(4)*log(-x+4)^2+(-2*x+8)*exp(1)*exp(4)*exp((x^4-32*x^2+256)/exp(4))*log(-x+4)+(x-4)*exp(1)*exp(4)*exp((x^4-32*x^2+256)/exp(4))^2),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError



### 100.77 Problem number 3227

$$\int \frac{32768 - 8192x^2 + 1536x^3 - 128x^4 - 128x^5 + 16x^6 - 2x^7 + \frac{e^{25+\log^2(2)}(2048-1536x+384)}{1024}}{-16384x^3 - 4096x^4 + 3072x^5 + 768x^6 - 192x^7 - 48x^8 + 4x^9 + x^{10} + \frac{e^{50+2\log^2(2)}(-64x^3+48x^4-12x^5+x^6)}{1048576} + \frac{e^{25+\log^2(2)}}{1048576}}$$

Optimal antiderivative

$$\frac{\left(\frac{4-x}{x} + \frac{4}{4-x}\right)^2}{e^{(\ln(2)-5)^2} + (4+x)^2}$$

command

```
integrate((( -64*x^3+384*x^2-1536*x+2048)*exp(log(2)^2-10*log(2)+25)-2*x^7+16*x^6-128*x^5-128*x^4+1536*x^3-8192*x^2+32768)/((x^6-12*x^5+48*x^4-64*x^3)*exp(log(2)^2-10*log(2)+25)^2+(2*8*x^7-64*x^6+256*x^5+512*x^4-2048*x^3)*exp(log(2)^2-10*log(2)+25)+x^10+4*x^9-48*x^8-192*x^7+768*x^6+3072*x^5-4096*x^4-16384*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{384 x e^{(2 \log(2)^2 - 20 \log(2) + 50)} + 24576 x e^{(\log(2)^2 - 10 \log(2) + 25)} + 589824 x + e^{(4 \log(2)^2 - 40 \log(2) + 100)} + 128 e^{(3 \log(2)^2 - 30 \log(2) + 75)}}{(x^2 + 8x + e^{(\log(2)^2 - 10 \log(2) + 25)} + 16)} \left( e^{(4 \log(2)^2 - 40 \log(2) + 100)} + 160 e^{(3 \log(2)^2 - 30 \log(2) + 75)} + 84 \right) - 32 \left( 12 x^3 e^{(2 \log(2)^2 - 20 \log(2) + 50)} + 768 x^3 e^{(\log(2)^2 - 10 \log(2) + 25)} + 18432 x^3 - x^2 e^{(3 \log(2)^2 - 30 \log(2) + 75)} - 184 x^2 e^{(2 \log(2)^2 - 20 \log(2) + 50)} \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.78 Problem number 3251

$$\int \frac{-4 + 6x - 2x^2 + (2x - x^2 + x \log(x)) \log(24 - 12x)}{(-20x^2 + 20x^3 - 5x^4 + (-10x^2 + 5x^3) \log(x)) \log(24 - 12x + 12 \log(x)) + (-40x + 40x^2 - 10x^3 + (-20x + 10x^2) \log(x))}$$

Optimal antiderivative

$$\frac{x}{5x + 5 \ln \left( \frac{1}{4 \ln(12 \ln(x) - 12x + 24)^2 (-2 + x)} \right)}$$

command

```
integrate(((((-2+x)*log(x)-x^2+4*x-4)*log(12*log(x)-12*x+24)*log(1/(4*x-8)/log(12*log(x)-12*x+24)^2)+(x*log(x)-x^2+2*x)*log(12*log(x)-12*x+24)-2*x^2+6*x-4)/(((5*x-10)*log(x)-5*x^2+20*x-20)*log(12*log(x)-12*x+24)*log(1/(4*x-8)/log(12*log(x)-12*x+24)^2)+((10*x^2-20*x)*log(x)-10*x^3+40*x^2-40*x)*log(12*log(x)-12*x+24)*log(1/(4*x-8)/log(12*log(x)-12*x+24)^2)+((5*x^3-10*x^2)*log(x)-5*x^4+20*x^3-20*x^2)*log(12*log(x)-12*x+24))),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.79 Problem number 3265

$$\int \frac{(-1 - 240x - 240e^{16}x) \log(5e^3)}{x^2 + 240x^3 + 14400x^4 + 14400e^{32}x^4 + e^{16}(240x^3 + 28800x^4)} dx$$

Optimal antiderivative

$$\frac{\ln(5e^3)}{(24(5 + 5e^{16})x + 1)x}$$

command

```
integrate((-240*x*exp(16)-240*x-1)*log(5*exp(3))/(14400*x^4*exp(16)^2+(28800*x^4+240*x^3)*exp
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log(5e^3)}{120x^2e^{16} + 120x^2 + x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.80 Problem number 3356

$$\int \frac{-1576x^4 - 620944x^4 + 370360x^5 + 83593x^6 + 8460x^7 + 324x^8 + e^{16}(2500 + 2000x + 600x^2 + 80x^3 + 4x^4) + e^{12}(-40000x^4 -$$

Optimal antiderivative

$$6 + \frac{5x}{2} - \left( \frac{(4x - e^4)(5+x)}{x} - x \right)^2$$

command

```
integrate(((−4*x^4−80*x^3−300*x^2)*exp(4)^2+(280*x^4+1600*x^3)*exp(4)+36*x^6−1576*x^4)/((4*x^48*x^5−1040*x^4−8400*x^3−30000*x^2−40000*x)*exp(4)^3+(216*x^6+5020*x^5+43552*x^4+167020*x^3+2432*x^7−10680*x^6−99112*x^5−408640*x^4−630400*x^3)*exp(4)+324*x^8+8460*x^7+83593*x^6+370360*x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2x^3}{18x^4 - 12x^3e^4 + 235x^3 + 2x^2e^8 - 140x^2e^4 + 788x^2 + 20xe^8 - 400xe^4 + 50e^8}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.81 Problem number 3377

$$\int -\frac{4 \log(2) \log(9)}{x^2 + (-16x - 32e^3x) \log(2) + (64 + 256e^3 + 256e^6) \log^2(2)} dx$$

Optimal antiderivative

$$\frac{2x \ln(3)}{1 - \frac{x}{4 \ln(2)} + 4e^3 + x}$$

command

```
integrate(−8*log(2)*log(3)/((256*exp(3)^2+256*exp(3)+64)*log(2)^2+(−32*x*exp(3)−16*x)*log(2)+
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{8 \log(3) \log(2)}{16e^3 \log(2) - x + 8 \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.82 Problem number 3389

$$\int \frac{(-80 - 256e^2 - 192x) \log(2)}{625x^2 + 20480e^6x^2 + 1500x^3 + 1200x^4 + 320x^5 + e^4(19200x^2 + 15360x^3) + e^2(6000x^2 + 9600x^3 + 3840x^4)} dx$$

Optimal antiderivative

$$e^2 + \frac{\ln(2)}{5 \left(x + \frac{5}{4} + 4e^2\right)^2 x}$$

command

```
integrate((-256*exp(2)-192*x-80)*log(2)/(20480*x^2*exp(2)^3+(15360*x^3+19200*x^2)*exp(2)^2+(3
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{16}{5} \left( \frac{8(2x + 16e^2 + 5)}{(4x + 16e^2 + 5)^2(256e^4 + 160e^2 + 25)} - \frac{1}{x(256e^4 + 160e^2 + 25)} \right) \log(2)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.83 Problem number 3415

$$\int \frac{-117 + 396x - 225x^2 - 36x^3 + 27x^4 + e^2(54 - 198x + 162x^2 - 36x^3)}{4 + e^4 + e^2(-4 - 2x) + 4x + x^2} dx$$

Optimal antiderivative

$$\frac{3(x^2 - 3x + 1)(3x^2 - 9x + 3)}{2 - e^2 + x}$$

command

```
integrate((( -36*x^3+162*x^2-198*x+54)*exp(2)+27*x^4-36*x^3-225*x^2+396*x-117)/(exp(2)^2+(-2*x-4)*exp(2)+x^2+4*x+4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$9x^3 + 9x^2e^2 - 72x^2 + 9xe^4 - 90xe^2 + 243x + \frac{9(e^8 - 14e^6 + 71e^4 - 154e^2 + 121)}{x - e^2 + 2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.84 Problem number 3426

$$\int \frac{e^{x - \frac{2 \log^2(4)}{-3 \log^2(4) + e^x(x^2 - 2x \log(3) \log(4) + \log^2(3) \log^2(4))}} ((-8x - 4x^2) \log^2(4) + (8 + 8x) \log(3) \log^3(4))}{9 \log^4(4) + e^x(-6x^2 \log^2(4) + 12x \log(3) \log^3(4) - 6 \log^2(3) \log^4(4)) + e^{2x}(x^4 - 4x^3 \log(3) \log(4) + 6x^2 \log^2(3))}$$

Optimal antiderivative

$$2 \ln(2) - 2e^{\frac{2}{3 - e^x \left( \ln(3) - \frac{x}{2 \ln(2)} \right)^2}}$$

command

```
integrate((-64*log(3)^2*log(2)^4+8*(8*x+8)*log(3)*log(2)^3+4*(-4*x^2-8*x)*log(2)^2)*exp(x)*exp(8*log(2)^2/((4*log(3)^2*log(2)^2-4*x*log(2)*log(3)+x^2)*exp(x)-12*log(2)^2))/((16*log(3)^4*1032*x*log(3)^3*log(2)^3+24*x^2*log(3)^2*log(2)^2-8*x^3*log(3)*log(2)+x^4)*exp(x)^2+(-96*log(3)^2*log(2)^4+96*x*log(3)*log(2)^3-24*x^2*log(2)^2)*exp(x)+144*log(2)^4),x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-2e^{\left(-\frac{8\log(2)^2}{4e^x\log(3)^2\log(2)^2-4xe^x\log(3)\log(2)+x^2e^x-12\log(2)^2}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$\int$

$$\frac{16\left(4\log(3)^2\log(2)^4-4(x+1)\log(3)\log(2)^3+(x^2+2x)\log(2)^2\right)e^{\left(x-\frac{1}{4\log(3)^2}\right)}}{144\log(2)^4+\left(16\log(3)^4\log(2)^4-32x\log(3)^3\log(2)^3+24x^2\log(3)^2\log(2)^2-8x^3\log(3)\log(2)+x^4\right)e^{(2x)}}$$

### 100.85 Problem number 3434

$$\int \frac{1}{2}e^{8-2x}(64x-112x^2+40x^3-4x^4+e^{-4+x}(16-24x+4x^2)+3^{2x}(-1+\log(3))+3^x(8-20x+4x^2+(8x-2x^2)\log(3)+e^{-4+x}(-2+2\log(3)))) dx$$

Optimal antiderivative

$$\left(1+\left(\frac{e^{x\ln(3)}}{2}-x^2+4x\right)e^{4-x}\right)^2$$

command

```
integrate(1/2*((log(3)-1)*exp(x*log(3)))^2+((2*log(3)-2)*exp(x-4)+(-2*x^2+8*x)*log(3)+4*x^2-20*x+8)*exp(x*log(3)))+(4*x^2-24*x+16)*exp(x-4)-4*x^4+40*x^3-112*x^2+64*x)/exp(x-4)^2,x, algor
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{-2(x^2-4x)e^{(-x+4)}+(x^4-8x^3+16x^2)e^{(-2x+8)}\left(x^2\log(3)^3-6x^2\log(3)^2-4x\log(3)^3+12x^2\log(3)+24x\log(3)^2-8x^2-48x\log(3)+32x\right)e^{(x\log(3)-2x+8)}}{\log(3)^3-6\log(3)^2+12\log(3)-8}+\frac{1}{4}e^{(2x\log(3)-2x+8)}+e^{(x\log(3)-x+4)}$$

Giac 1.7.0 via sagemath 9.3 output

$\int$

$$-\frac{1}{2}\left(4x^4-40x^3-2\left(2x^2+(\log(3)-1)e^{(x-4)}-(x^2-4x)\log(3)-10x+4\right)3^x+112x^2-3^{2x}(\log(3)-1)-4\right)$$

### 100.86 Problem number 3467

$$\int \frac{65536 + 256e^2 - 98304x^4 + 65536x^6 - 12288x^8 + e(-8192 + 2048x^4) + e^2(-16384x + 24576x^3 - 12288x^5 + 24576x^7)}{e^6 - 24e^4x + 192e^2x^2 - 512x^3}$$

Optimal antiderivative

$$\frac{4\left(\frac{e}{4} - (-x^2 + 2)\right)^2}{\left(x - \frac{e^2}{8}\right)^2}$$

command

```
integrate(((((-512*x^3+1024*x)*exp(1)+2048*x^7-12288*x^5+24576*x^3-16384*x)*exp(2)+256*exp(1)^8-8192)*exp(1)-12288*x^8+65536*x^6-98304*x^4+65536)/(exp(2)^3-24*x*exp(2)^2+192*x^2*exp(2)-512*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4x^6 + x^5e^2 + \frac{3}{16}x^4e^4 - 32x^4 + \frac{1}{32}x^3e^6 - 8x^3e^2 + \frac{5}{1024}x^2e^8 - \frac{3}{2}x^2e^4 - 2x^2e + 96x^2 + \frac{3}{4096}xe^{10} - \frac{1}{4}xe^6 - \frac{1}{2}xe^3 + 24xe^2 + \frac{64xe^{14} - 24576xe^{10} - 65536xe^7 + 3145728xe^6 + 8388608xe^3 - 134217728xe^2 - 7e^{16} + 2560e^{12} + 6144e^9 - 24576e^5}{65536(8x - e^2)^2}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.87 Problem number 3518

$$\int \frac{-162x^4 + 50ex^4 - 246x^5 - 4x^6 + e^9(2x - 50ex + 306x^2 + 4x^3) + e^6(54x^2 + 150ex^2 - 838x^3 - 12x^4) + e^3(306x^5 - 150x^3*exp(1) + 12x^5 + 778x^4 + 306x^3)*exp(3) + 50x^4*exp(1) - 4x^6 - 246x^5 - 162x^4}{25e^9 - 75e^6x + 75e^3x^2 - 25x^3}$$

Optimal antiderivative

$$x^2 \left( 4x - e + \left( \frac{2x}{-x + e^3} + \frac{x}{5} + \frac{1}{5} \right)^2 \right)$$

command

```
integrate(((((-50*x*exp(1)+4*x^3+306*x^2+2*x)*exp(3)^3+(150*x^2*exp(1)-12*x^4-838*x^3+54*x^2)*e^6+150*x^3*exp(1)+12*x^5+778*x^4+306*x^3)*exp(3)+50*x^4*exp(1)-4*x^6-246*x^5-162*x^4)/(25*exp(3)^3-75*x*exp(3)^2+75*x^2*exp(3)-25*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{25}x^4 + \frac{82}{25}x^3 - \frac{4}{5}x^2e^3 - x^2e + \frac{81}{25}x^2 - \frac{4}{5}xe^6 + \frac{36}{5}xe^3 - \frac{4(xe^{12} - 19xe^9 - e^{15} + 14e^{12})}{5(x - e^3)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(2x^6 + 123x^5 - 25x^4e + 81x^4 - (2x^3 + 153x^2 - 25xe + x)e^9 + (6x^4 + 419x^3 - 75x^2e - 27x^2)e^6 - (6x^5 + \dots)}{25(x^3 - 3x^2e^3 + 3xe^6 - e^9)} dx$$

### 100.88 Problem number 3593

$$\int \frac{4 + 12x + 5x^2 + e^4(-4 + 20x - 5x^2)}{32 + 32x - 24x^2 - 16x^3 + 8x^4 + e^8(32 - 32x + 8x^2) + e^4(64 - 48x^2 + 16x^3)} dx$$

Optimal antiderivative

$$\frac{-5x^2 + 4}{(-2 + x)(8x + 8e^4 + 8)} + 50$$

command

```
integrate((( -5*x^2+20*x-4)*exp(2)^2+5*x^2+12*x+4)/((8*x^2-32*x+32)*exp(2)^4+(16*x^3-48*x^2+64)*exp(2)^2+8*x^4-16*x^3-24*x^2+32*x+32),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5xe^4 - 5x - 10e^4 - 6}{8(x^2 + xe^4 - x - 2e^4 - 2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.89 Problem number 3658

$$\int \frac{152587890625x^8 + 48828125000x^9 + 6835937500x^{10} + 546875000x^{11} + 27343750x^{12} + 875000x^{13} + 17500x^{14} + \dots}{\dots} dx$$

Optimal antiderivative

$$\frac{3}{\frac{256e^{20}\left(6 + \frac{6}{x+25}\right)^4}{81x^4} + 4}$$

command

```
integrate((3072*x^11+629760*x^10+56398848*x^9+2882018304*x^8+91912863744*x^7+1873311897600*x^6+...),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{768(x^4 + 104x^3 + 4056x^2 + 70304x + 456976)e^{20}}{x^8 + 100x^7 + 3750x^6 + 62500x^5 + 1024x^4e^{20} + 390625x^4 + 106496x^3e^{20} + 4153344x^2e^{20} + 71991296xe^{20} + \dots}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.90 Problem number 3709

$$\int \frac{e^{-60+5e^x+5x+5x^2+5\log(x)} (-26 + e^x(2 - 2x) - 2x^2 + 2\log(x))}{720 + 5e^{2x} - 120x - 115x^2 + 10x^3 + 5x^4 + e^x(-120 + 10x + 10x^2) + (-120 + 10e^x + 10x + 10x^2)\log(x) + 5\log(x)}$$

Optimal antiderivative

$$e^{\frac{2x}{5\ln(x)+5e^x+5x^2+5x-60}}$$

command

```
integrate((2*log(x)+(2-2*x)*exp(x)-2*x^2-26)*exp(2*x/(5*log(x)+5*exp(x)+5*x^2+5*x-60))/(5*log(x)^2+(10*exp(x)+10*x^2+10*x-120)*log(x)+5*exp(x)^2+(10*x^2+10*x-120)*exp(x)+5*x^4+115*x^2-120*x+720),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(\frac{2x}{5(x^2+x+e^x+\log(x)-12)}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(x^2 + (x-1)e^x - \log(x) + 13)e^{\left(\frac{2x}{5(x^2+x+e^x+\log(x)-12)}\right)}}{5(x^4 + 2x^3 - 23x^2 + 2(x^2 + x - 12)e^x + 2(x^2 + x + e^x - 12)\log(x) + \log(x)^2 - 24x + e^{2x} + 144)} dx$$

## 100.91 Problem number 3720

$$\int \frac{4e^6x + 2x^3 - 4x^5 + 2x^7 + e^2(1 - 3x^2) + e^4(-2x^2 - 6x^6)}{x^2 - 2x^4 + 4e^8x^4 + x^6 + e^4(4x^3 - 4x^5)} dx$$

Optimal antiderivative

$$\frac{1 - \frac{e^2}{x} - x^4}{2xe^4 - x^2 + 1}$$

command

```
integrate((4*x*exp(2)^3+(-6*x^6-2*x^2)*exp(2)^2+(-3*x^2+1)*exp(2)+2*x^7-4*x^5+2*x^3)/(4*x^4*e^4*x^5+4*x^3)*exp(2)^2+x^6-2*x^4+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x^2 + 2xe^4 + \frac{8x^2e^{12} + 4x^2e^4 + 4xe^8 + e^2}{x^3 - 2x^2e^4 - x}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 100.92 Problem number 3734

$$\int \frac{e^{\frac{1 - \frac{x(2+x^4)^{220}}{e^{1100}}}{x^2}} \left( -4 - 2x^4 + \frac{(2+x^4)^{220}(2x-879x^5)}{e^{1100}} \right)}{2x^3 + x^7} dx$$

Optimal antiderivative

$$e^{\frac{\frac{1}{x} - e^{220 \ln(x^4+2) - 1100}}{x}}$$

command

```
integrate((( -879*x^5+2*x)*exp(220*log(x^4+2)-1100)-2*x^4-4)*exp((-x*exp(220*log(x^4+2)-1100)+1)/x^2)/(x^7+2*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.93 Problem number 3744

$$\int -\frac{12}{324 + e^2 - 36x + x^2 + e(-36 + 2x)} dx$$

Optimal antiderivative

$$\frac{4}{-6 + \frac{e}{3} + \frac{x}{3}} + 4$$

command

```
integrate(-12/(exp(1)^2+(2*x-36)*exp(1)+x^2-36*x+324),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{12}{x + e - 18}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.94 Problem number 3827

$$\int \frac{432x + 216e^3x^2 + e^6(-18900 + 36x^3) + e^9(3150x + 2x^4)}{216 + 108e^3x + 18e^6x^2 + e^9x^3} dx$$

Optimal antiderivative

$$\left(x - \frac{3150}{(x + 6e^{-3})^2}\right)x$$

command

`integrate(((2*x^4+3150*x)*exp(3)^3+(36*x^3-18900)*exp(3)^2+216*x^2*exp(3)+432*x)/(x^3*exp(3)^3`

Giac 1.9.0-11 via sagemath 9.6 output

$$x^2 - \frac{3150xe^6}{(xe^3 + 6)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(108x^2e^3 + (x^4 + 1575x)e^9 + 18(x^3 - 525)e^6 + 216x)}{x^3e^9 + 18x^2e^6 + 108xe^3 + 216} dx$$

## 100.95 Problem number 3866

$$\int \frac{-7938 - 11250e^8 - 18900x - 11250x^2 + e^4(18900 + 22500x) + e^{-\frac{50x}{-21+25e^4-25x}}(-882 - 1250e^8 - 1050x - 1250x^2)}{3969x^3 + 5625e^8x^3 + 9450x^4 + 5625x^5} dx$$

Optimal antiderivative

$$\frac{\left(e^{\frac{x}{25+x-e^4}} + 3\right)^2}{9x^2}$$

command

`integrate((( -1250*exp(4)^2+(1250*x+2100)*exp(4)-1250*x^2-1050*x-882)*exp(-25*x/(25*exp(4)-25*x-21))^2+(-7500*exp(4)^2+(11250*x+12600)*exp(4)-7500*x^2-9450*x-5292)*exp(-25*x/(25*exp(4)-25*x-21))-11250*exp(4)^2+(22500*x+18900)*exp(4)-11250*x^2-18900*x-7938)/(5625*x^3*exp(4)^2+(-11250*x^4-9450*x^3)*exp(4)+5625*x^5+9450*x^4+3969*x^3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{281250xe^8}{25x-25e^4+21} - \frac{472500xe^4}{25x-25e^4+21} + \frac{22050xe^{\left(\frac{50x}{25x-25e^4+21}\right)}}{25x-25e^4+21} - \frac{275625x^2e^{\left(\frac{50x}{25x-25e^4+21}\right)}}{(25x-25e^4+21)^2} + \frac{132300xe^{\left(\frac{25x}{25x-25e^4+21}\right)}}{25x-25e^4+21} - \frac{1653750x^2e^{\left(\frac{25x}{25x-25e^4+21}\right)}}{(25x-25e^4+21)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.96 Problem number 3906**

$$\int \frac{-18 - 18e^9 + 36x}{(8000 + 1200x - 1140x^2 - 119x^3 + 57x^4 + 3x^5 - x^6 + e^{27}(64 + 48x + 12x^2 + x^3) + e^{18}(960 + 528x + 36x^2 - 2$$

Optimal antiderivative

$$\frac{9}{\ln(5)^2 (4+x)^2 (x-5-e^9)^2}$$

command

```
integrate((-18*exp(9)+36*x-18)/((x^3+12*x^2+48*x+64)*exp(9)^3+(-3*x^4-21*x^3+36*x^2+528*x+960
141*x^3-348*x^2+1680*x+4800)*exp(9)-x^6+3*x^5+57*x^4-119*x^3-1140*x^2+1200*x+8000)/log(5)^2,x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9}{(x^2 - xe^9 - x - 4e^9 - 20)^2 \log(5)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.97 Problem number 3982**

$$\int \frac{-2ex^2 + e^{\frac{1}{2}(28+e^4)}(-2e^5 + 4ex)}{e^8x^2 - 2e^4x^3 + x^4} dx$$

Optimal antiderivative

$$\frac{2 \left( e^{\frac{e^4}{2}+14} - x \right) e}{x(e^4 - x)}$$

command

```
integrate((( -2*exp(1)*exp(4)+4*x*exp(1))*exp(1/2*exp(4)+14)-2*x^2*exp(1))/(x^2*exp(4)^2-
2*x^3*exp(4)+x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( xe - e^{\left(\frac{1}{2}e^4+15\right)} \right)}{x^2 - xe^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.98 Problem number 3986**

$$\int \frac{e^4(480x - 144x^2 - 120x^3 + 12x^4) + e^2(1600x^2 - 640x^3 - 1200x^4 + 160x^5)}{25600x^2 + 19200x^4 + 4800x^6 + 400x^8 + e^4(576 + 432x^2 + 108x^4 + 9x^6) + e^2(7680x + 5760x^3 + 1440x^5 + 120x^7)} dx$$

Optimal antiderivative

$$\frac{x^2(5-x)}{\left(\frac{3}{4} + 5xe^{-2}\right)(x^2+4)^2}$$

command

```
integrate(((12*x^4-120*x^3-144*x^2+480*x)*exp(2)^2+(160*x^5-1200*x^4-640*x^3+1600*x^2)*exp(2)^2)/(25600*x^2+19200*x^4+4800*x^6+400*x^8+e^4*(576+432*x^2+108*x^4+9*x^6)+e^2*(7680*x+5760*x^3+1440*x^5+120*x^7)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{720(3e^8 + 100e^6)}{(20x + 3e^2)(81e^8 + 28800e^4 + 2560000)} - \frac{4(27x^3e^8 + 900x^3e^6 - 135x^2e^8 + 1440x^2e^6 + 128000x^2e^2 - 19200xe^4 - 640000xe^2 + 2880e^6 + 96000e^4)}{(x^2 + 4)^2(81e^8 + 28800e^4 + 2560000)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.99 Problem number 4005**

$$\int \frac{4(5+i\pi)^2 - 2e^8x^2 + 4e^{16}x^4 + (5+i\pi)(-2-8e^8x^2)}{(5+i\pi)^2 - 2e^8(5+i\pi)x^2 + e^{16}x^4} dx$$

Optimal antiderivative

$$\left(4 - \frac{2}{\ln(-e^5) - x^2e^8}\right)x$$

command

```
integrate((4*log(-exp(5))^2+(-8*x^2*exp(4)^2-2)*log(-exp(5))+4*x^4*exp(4)^4-2*x^2*exp(4)^2)/(exp(5)^2-2*x^2*exp(4)^2*log(-exp(5))+x^4*exp(4)^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4x + \frac{2x}{x^2e^8 - \log(-e^5)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.100 Problem number 4044

$$\int \frac{-3 + 3e^2 - 6x - 3 \log(9e^6)}{x^2 + e^4x^2 + 2x^3 + x^4 + e^2(-2x^2 - 2x^3) + (2x^2 - 2e^2x^2 + 2x^3) \log(9e^6) + x^2 \log^2(9e^6)} dx$$

Optimal antiderivative

$$\frac{3}{(x + 1 - e^2 + \ln(9e^6))x}$$

command

```
integrate((-3*log(9*exp(3)^2)+3*exp(2)-6*x-3)/(x^2*log(9*exp(3)^2)^2+(-2*x^2*exp(2)+2*x^3+2*x^2*x^3-2*x^2)*exp(2)+x^4+2*x^3+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3}{x^2 - xe^2 + x \log(9e^6) + x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.101 Problem number 4070

$$\int \frac{8x + 4e^{2x}x + e^x(-4 - 4x - 4x^2)}{4e^{3x}x - 4e^{2x}x^2 + (-4e^{2x}x + 4e^xx^2) \log(x^2) + (e^xx - x^2) \log^2(x^2) + (-8e^{2x}x + 8e^xx^2 + (4e^xx - 4x^2) \log(x^2))} dx$$

Optimal antiderivative

$$\frac{1}{\frac{\ln(x^2)}{2} + \ln(x - e^x) - e^x}$$

command

```
integrate((4*x*exp(x)^2+(-4*x^2-4*x-4)*exp(x)+8*x)/((4*exp(x)*x-4*x^2)*log(x-exp(x))^2+((4*exp(4*x^2)*log(x^2)-8*x*exp(x)^2+8*exp(x)*x^2)*log(x-exp(x))+exp(x)*x-x^2)*log(x^2)^2+(-4*x*exp(x)^2+4*exp(x)*x^2)*log(x^2)+4*x*exp(x)^3-4*exp(x)^2*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{2e^x - \log(x^2) - 2 \log(x - e^x)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4(xe^{(2x)} - (x^2 + x + 1)e^x + 2x)}{4x^2e^{(2x)} + (x^2 - xe^x) \log(x^2)^2 + 4(x^2 - xe^x) \log(x - e^x)^2 - 4xe^{(3x)} - 4(x^2e^x - xe^{(2x)}) \log(x^2) - 4(2x^2e^x -$$

## 100.102 Problem number 4108

$$\int \frac{81 + e^4(-14155776 - 110592x - 216x^2) + e^{15}(-201326592 - 2359296x - 9216x^2 - 12x^3) + e^{20}(4294967296 + \dots)}{\dots}$$

Optimal antiderivative

$$\frac{5}{\frac{4}{3} - \left(\frac{e^3}{3} - \frac{e^{-2}}{256+x}\right)^2}$$

command

```
integrate(((270*x^2+138240*x+17694720)*exp(1)^6*exp(3)+(-810*x-207360)*exp(1)^4)/((x^4+1024*x^3-9216*x^2-2359296*x-201326592)*exp(1)^6*exp(3)^3+((-24*x^4-24576*x^3-9437184*x^2-1610612736*x-103079215104)*exp(1)^8+(54*x^2+27648*x+3538944)*exp(1)^4)*exp(3)^2+((144*x^3+110108*x-27648)*exp(1)^2)*exp(3)+(144*x^4+147456*x^3+56623104*x^2+9663676416*x+618475290624)*exp(216*x^2-110592*x-14155776)*exp(1)^4+81),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{135(2xe^5 + 512e^5 - 3)}{(x^2e^{10} - 12x^2e^4 + 512xe^{10} - 6xe^5 - 6144xe^4 + 65536e^{10} - 1536e^5 - 786432e^4 + 9)(e^6 - 12)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.103 Problem number 4175

$$\int \frac{e^{\frac{-50x-2x^2}{-43-53x-2x^2+e^x(50x+2x^2)}}(2150 + 172x + 6x^2 + e^x(2500x^2 + 200x^3 + 4x^4))}{1849 + 4558x + 2981x^2 + 212x^3 + 4x^4 + e^x(-4300x - 5472x^2 - 412x^3 - 8x^4) + e^{2x}(2500x^2 + 200x^3 + 4x^4)} dx$$

Optimal antiderivative

$$e^{\frac{x}{1-e^x x+x+\frac{-7+x}{2x+50}}}$$

command

```
integrate(((4*x^4+200*x^3+2500*x^2)*exp(x)+6*x^2+172*x+2150)*exp((-2*x^2-50*x)/((2*x^2+50*x)*2*x^2-53*x-43))/((4*x^4+200*x^3+2500*x^2)*exp(x)^2+(-8*x^4-412*x^3-5472*x^2-4300*x)*exp(x)+4*
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(-\frac{2x^2}{2x^2e^x-2x^2+50xe^x-53x-43} - \frac{50x}{2x^2e^x-2x^2+50xe^x-53x-43}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(3x^2 + 2(x^4 + 50x^3 + 625x^2)e^x + 86x + 1075)e^{\left(\frac{2(x^2+25x)}{2x^2-2(x^2+25x)e^x+53x+43}\right)}}{4x^4 + 212x^3 + 2981x^2 + 4(x^4 + 50x^3 + 625x^2)e^{(2x)} - 4(2x^4 + 103x^3 + 1368x^2 + 1075x)e^x + 4558x + 1849}$$

## 100.104 Problem number 4339

$$\int \frac{e^{e^x}(5 - 10x + e^x(25 - 5x + 5x^2) - 80x^2 \log^3(x^2) + (-30x^2 + 10e^x x^3) \log^4(x^2) - 80x^3 \log^7(x^2) + (-20x^3 + 5e^x x^4) \log^8(x^2) + 4x^7 \log^{12}(x^2))}{25 - 10x + 11x^2 - 2x^3 + x^4 + (20x^3 - 4x^4 + 4x^5) \log^4(x^2) + (10x^4 - 2x^5 + 6x^6) \log^8(x^2) + 4x^7 \log^{12}(x^2)}$$

Optimal antiderivative

$$1 - \frac{x}{5} + \frac{e^{e^x}}{5 \left(x + \ln(x^2)^4 x^2\right)^2}$$

command

```
integrate(((5*exp(x)*x^4-20*x^3)*log(x^2)^8-80*x^3*log(x^2)^7+(10*exp(x)*x^3-30*x^2)*log(x^2)^6+80*x^2*log(x^2)^5+(5*x^2-5*x+25)*exp(x)-10*x+5)*exp(exp(x))/(x^8*log(x^2)^16+4*x^7*log(x^2)^15+2*x^5+10*x^4)*log(x^2)^8+(4*x^5-4*x^4+20*x^3)*log(x^2)^4+x^4-2*x^3+11*x^2-10*x+25),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{10 e^{(x+e^x)}}{x^4 e^x \log(x^2)^8 + 2 x^3 e^x \log(x^2)^4 + x^2 e^x - x e^x + 5 e^x}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.105 Problem number 4383

$$\int \frac{10 + 10e^{4x/5} + e^{3x/5}(40 - 40x) - 40x + 60x^2 - 40x^3 + 10x^4 + e^{2x/5}(60 - 120x + 60x^2) + e^{x/5}(40 - 120x + 120x^2)}{x^2}$$

Optimal antiderivative

$$\frac{2 \left( \ln(x) + \left(x - e^{\frac{x}{5}} - 1\right)^4 \right) \ln(x)}{x}$$

command

```
integrate(1/5*(-10*log(x)^2+((8*x-10)*exp(1/5*x)^4+(-24*x^2+24*x-40)*exp(1/5*x)^3+(24*x^3+12*60)*exp(1/5*x)^2+(-8*x^4-56*x^3+96*x^2+8*x-40)*exp(1/5*x)+30*x^4-80*x^3+60*x^2+10)*log(x)+10*40*x+40)*exp(1/5*x)^3+(60*x^2-120*x+60)*exp(1/5*x)^2+(-40*x^3+120*x^2-120*x+40)*exp(1/5*x)+10*40*x^3+60*x^2-40*x+10)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \left( x^4 \log(5) - 4 x^3 e^{\left(\frac{1}{5}x\right)} \log(5) + x^4 \log\left(\frac{1}{5}x\right) - 4 x^3 e^{\left(\frac{1}{5}x\right)} \log\left(\frac{1}{5}x\right) - 4 x^3 \log(5) + 6 x^2 e^{\left(\frac{2}{5}x\right)} \log(5) + 12 x^2 e^{\left(\frac{1}{5}x\right)} \log(5) \right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.106 Problem number 4399**

$$\int \frac{-2 + x - \log\left(\frac{5}{2}\right) + e^x(2 - x + \log\left(\frac{5}{2}\right)) - 4 \log(2 - x + \log\left(\frac{5}{2}\right))}{2 - x + \log\left(\frac{5}{2}\right)} dx$$

Optimal antiderivative

$$e^x + 2 \ln\left(\ln\left(\frac{5}{2}\right) + 2 - x\right)^2 + 2e^3 - x$$

command

```
integrate((-4*log(log(5/2)+2-x)+(log(5/2)+2-x)*exp(x)-log(5/2)+x-2)/(log(5/2)+2-x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \log\left(-x + \log\left(\frac{5}{2}\right) + 2\right)^2 - x + e^x + \log\left(\frac{5}{2}\right) + 2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(x - \log\left(\frac{5}{2}\right) - 2)e^x - x + \log\left(\frac{5}{2}\right) + 4 \log(-x + \log\left(\frac{5}{2}\right) + 2) + 2}{x - \log\left(\frac{5}{2}\right) - 2} dx$$

**100.107 Problem number 4455**

$$\int \frac{-2 + e^2 + (-4 - e^4 + e^2(4 - 4x) + 8x - 4x^2) \log(4)}{(4 + e^4 - 8x + 4x^2 + e^2(-4 + 4x)) \log(4)} dx$$

Optimal antiderivative

$$2 + \frac{x}{2 \ln(2)(2x - 2 + e^2)} - x + e^{-3}$$

command

```
integrate(1/2*(2*(-exp(2)^2+(-4*x+4)*exp(2)-4*x^2+8*x-4)*log(2)+exp(2)-2)/(exp(2)^2+(-4+4*x)*exp(2)+4*x^2-8*x+4)/log(2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{4x \log(2) + \frac{e^2 - 2}{2x + e^2 - 2}}{4 \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError



## 100.108 Problem number 4483

$$\int \frac{e^{1+6x}(30e^{8-2x} - 20e^{4-x})}{16 + 25e^{8-2x} - 40e^{4-x}} dx$$

Optimal antiderivative

$$\frac{e^{6x+1}}{5 - 4e^{-4+x}}$$

command

`integrate((30*exp(-x+4)^2-20*exp(-x+4))*exp(6*x+1)/(25*exp(-x+4)^2-40*exp(-x+4)+16),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{1024} \left( 256 e^{25} + 320 e^{(-x+29)} + 400 e^{(-2x+33)} + 500 e^{(-3x+37)} + 625 e^{(-4x+41)} \right) e^{(5x-20)} + \frac{3125 e^{25}}{1024 (5 e^{(-x+4)} - 4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.109 Problem number 4504

$$\int e^{\frac{2+4e^{e^x} + \frac{e^{2+4e^{e^x}}}{(3+x)^2}}{(3+x)^2}} \frac{(2+2x)^2 (4+e^{e^x+x} (12+16x+4x^2))}{(3+x)^2 (3+4x+x^2)} dx$$

Optimal antiderivative

$$e^{\frac{4(1+x)^2 e^{2+4e^{e^x}}}{(3+x)^2}} - 5$$

command

`integrate(((4*x^2+16*x+12)*exp(x)*exp(exp(x))+4)*exp(2*exp(exp(x))+log((2+2*x)/(3+x))+1)^2*exp(x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left( \frac{4 \left( x^2 e^{(4e^{e^x}+2)} + 2xe^{(4e^{e^x}+2)} + e^{(4e^{e^x}+2)} \right)}{x^2+6x+9} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4 \left( (x^2 + 4x + 3) e^{(x+e^x)} + 1 \right) e^{\left( e^{(4e^{e^x}+2) \log\left(\frac{2(x+1)}{x+3}\right)+2} + 4e^{e^x} + 2 \log\left(\frac{2(x+1)}{x+3}\right) + 2 \right)}}{x^2 + 4x + 3} dx$$

## 100.110 Problem number 4543

$$\int \frac{3125x^2 - 6250x^3 + 3125x^4 + e^4(3125x^3 - 6250x^4 + 3125x^5) + e^8(1250x^4 - 2500x^5 + 1250x^6) + e^{12}(250x^5 - 500x^6 + 250x^7)}{(5 + xe^4)^4(1 - x - (-x^2 + x)e^{-5})} dx$$

Optimal antiderivative

$$\frac{x}{(5 + xe^4)^4(1 - x - (-x^2 + x)e^{-5})}$$

command

```
integrate((((4*x^2-3*x)*exp(4)+5)*exp(5)^2+((-5*x^3+4*x^2)*exp(4)-5*x^2)*exp(5))/(((x^7-2*x^6+x^5)*exp(4)^5+(25*x^6-50*x^5+25*x^4)*exp(4)^4+(250*x^5-500*x^4+250*x^3)*exp(4)^3+(1250*2500*x^3+1250*x^2)*exp(4)^2+(3125*x^3-6250*x^2+3125*x)*exp(4)+3125*x^2-6250*x+3125)*exp(5)^2+2*x^8+4*x^7-2*x^6)*exp(4)^5+(-50*x^7+100*x^6-50*x^5)*exp(4)^4+(-500*x^6+1000*x^5-500*x^4)*exp(4)^3+2500*x^5+5000*x^4-2500*x^3)*exp(4)^2+(-6250*x^4+12500*x^3-6250*x^2)*exp(4)-6250*x^3+12500*x^2-6250*x)*exp(5)+(x^9-2*x^8+x^7)*exp(4)^5+(25*x^8-50*x^7+25*x^6)*exp(4)^4+(250*x^7-500*x^6+250*2500*x^5+1250*x^4)*exp(4)^2+(3125*x^5-6250*x^4+3125*x^3)*exp(4)+3125*x^4-6250*x^3+3125*x^2), x
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^3e^{52} + x^3e^{47} + 20x^3e^{43} + x^3e^{42} + 20x^3e^{38} + 150x^3e^{34} - 625x^3e^{21} + x^2e^{52} + 20x^2e^{48} + x^2e^{47} + 40x^2e^{43} + 400x^2e^{36} + xe^{31} + 20xe^{27} + xe^{26} + 20xe^{22} + 20xe^{17} + 20xe^{12} + 20xe^7 + 20xe^2 + 20x}{(x^2 - xe^5 - x + e^5)(e^{52} + 20e^{48} + 150e^{44} + 20e^{43} + 500e^{40} + 400e^{39} + 625e^{36} + 3000e^{35} + 150e^{34} + 10000e^{31} + 10000e^{27} + 10000e^{22} + 10000e^{17} + 10000e^{12} + 10000e^7 + 10000e^2 + 10000)} + \frac{20x^3e^{43} + x^3e^{42} + 20x^3e^{38} + 150x^3e^{34} - 625x^3e^{21} + x^2e^{52} + 20x^2e^{48} + x^2e^{47} + 40x^2e^{43} + 400x^2e^{36} + xe^{31} + 20xe^{27} + xe^{26} + 20xe^{22} + 20xe^{17} + 20xe^{12} + 20xe^7 + 20xe^2 + 20x}{(xe^4 + 5)^4(e^{52} + 20e^{48} + 150e^{44} + 20e^{43} + 500e^{40} + 400e^{39} + 625e^{36} + 3000e^{35} + 150e^{34} + 10000e^{31} + 10000e^{27} + 10000e^{22} + 10000e^{17} + 10000e^{12} + 10000e^7 + 10000e^2 + 10000)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.111 Problem number 4555

$$\int \frac{16 + 144x + 36x^2 + e^2x^2 + e(-8x - 20x^2)}{16 + 16x + 4x^2 + e^2x^2 + e(-8x - 4x^2)} dx$$

Optimal antiderivative

$$\frac{16x}{\frac{4+x}{x} - e + 1} + 2 + x$$

command

```
integrate((x^2*exp(1)^2+(-20*x^2-8*x)*exp(1)+36*x^2+144*x+16)/(x^2*exp(1)^2+(-4*x^2-8*x)*exp(1)+4*x^2+16*x+16), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{xe^2 - 20xe + 36x}{e^2 - 4e + 4} - \frac{256}{(xe - 2x - 4)(e^2 - 4e + 4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.112 Problem number 4701

$$\int \frac{240x + 768x^4}{25 - 320x^3 + 64e^2x^4 + 1024x^6 + e(80x^2 - 512x^5) + (-80x^2 - 128ex^4 + 512x^5)\log(4) + 64x^4\log^2(4)} dx$$

Optimal antiderivative

$$\frac{3}{2 \ln(2) - e - \frac{5}{8x^2} + 4x}$$

command

```
integrate((768*x^4+240*x)/(256*x^4*log(2)^2+2*(-128*x^4*exp(1)+512*x^5-80*x^2)*log(2)+64*x^4*512*x^5+80*x^2)*exp(1)+1024*x^6-320*x^3+25),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{24x^2}{32x^3 - 8x^2e + 16x^2\log(2) - 5}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.113 Problem number 4704

$$\int \frac{2e^{3+8e^5}x}{e^9 - 15e^6x + 75e^3x^2 - 125x^3} dx$$

Optimal antiderivative

$$\frac{x^2e^{8e^5}}{(5x - e^3)^2}$$

command

```
integrate(2*x*exp(3)*exp(4*exp(5))^2/(exp(3)^3-15*x*exp(3)^2+75*x^2*exp(3)-125*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(10x - e^3)e^{(8e^5+3)}}{25(5x - e^3)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2xe^{(8e^5+3)}}{125x^3 - 75x^2e^3 + 15xe^6 - e^9} dx$$

## 100.114 Problem number 4775

$$\int \frac{x^2}{x^2 - 2x^4 + x^6 + 8\log(3) + (3x^2 - 6x^4 + 3x^6 + 24\log(3)) \log\left(\frac{1}{64}(x^4 - 4x^6 + 6x^8 - 4x^{10} + x^{12} + (16x^2 - 32x^4 - \dots)\right)} dx$$

Optimal antiderivative

$$\frac{x^2}{\left(\ln\left(\left(\ln(3) + \frac{(-x^3+x)^2}{8}\right)^2\right) + 1\right)^2}$$

command

```
integrate(((16*x*log(3)+2*x^7-4*x^5+2*x^3)*log(log(3)^2+1/64*(16*x^6-32*x^4+16*x^2)*log(3)+1/16*x^10+3/32*x^8-1/16*x^6+1/64*x^4)+16*x*log(3)-22*x^7+28*x^5-6*x^3)/((8*log(3)+x^6-2*x^4+x^2)*log(log(3)^2+1/64*(16*x^6-32*x^4+16*x^2)*log(3)+1/64*x^12-1/16*x^10+3/32*x^8-1/16*x^6+1/64*x^4)^3+(24*log(3)+3*x^6-6*x^4+3*x^2)*log(log(3)^2+1/64*(16*x^6-32*x^4+16*x^2)*log(3)+1/64*x^12-1/16*x^10+3/32*x^8-1/16*x^6+1/64*x^4)^2+(24*log(3)+3*x^6-6*x^4+3*x^2)*log(log(3)^2+1/64*(16*x^6-32*x^4+16*x^2)*log(3)+1/64*x^12-1/16*x^10+3/32*x^8-1/16*x^6+1/64*x^4)+8*log(3)+x^6-2*x^4+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$108 x^5 \log(2)^2 - 36 x^5 \log(2) \log\left(x^{12} - 4 x^{10} + 6 x^8 + 16 x^6 \log(3) - 4 x^6 - 32 x^4 \log(3) + x^4 + 16 x^2 \log(3) + 64\right)$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.115 Problem number 4793

$$\int \frac{e^{\frac{1}{2}(-1-10x)}(-17-85x) + e^x(-3x^2 + e^{\frac{1}{2}(-1-10x)}(3+18x))}{3e^{-1-10x} - 6e^{\frac{1}{2}(-1-10x)}x + 3x^2} dx$$

Optimal antiderivative

$$\frac{x\left(\frac{17}{3} - e^x\right)}{x - e^{-5x - \frac{1}{2}}}$$

command

```
integrate((((18*x+3)*exp(-5*x-1/2)-3*x^2)*exp(x)+(-85*x-17)*exp(-5*x-1/2))/(3*exp(-5*x-1/2)^2-6*x*exp(-5*x-1/2)+3*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{x e^{(6x+\frac{1}{2})}}{x e^{(5x+\frac{1}{2})} - 1} + \frac{170}{3 \left( (10x+1)e^{(5x+\frac{1}{2})} - e^{(5x+\frac{1}{2})} - 10 \right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{3 \left( x^2 - (6x+1)e^{(-5x-\frac{1}{2})} \right) e^x + 17(5x+1)e^{(-5x-\frac{1}{2})}}{3 \left( x^2 - 2xe^{(-5x-\frac{1}{2})} + e^{(-10x-1)} \right)} dx$$

**100.116 Problem number 4795**

$$\int \frac{-45 - 9e^2x}{900x^3 + 240e^2x^4 + 16e^4x^5} dx$$

Optimal antiderivative

$$\frac{1}{4x^2 \left( 10 + \frac{4e^2x}{3} \right)}$$

command

`integrate((-9*exp(2)*x-45)/(16*x^5*exp(2)^2+240*x^4*exp(2)+900*x^3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{e^4}{150(2xe^2 + 15)} - \frac{2xe^2 - 15}{600x^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.117 Problem number 4846**

$$\int \frac{e^{\frac{15+e^{-4+x}(-15+3x)}{-12+e^{-3x}}} \left( 45 + e^{-4+x}(-81 + 3e + e^{-4+x}(180 + 9x - 9x^2 + e(-15 + 3x))) \right)}{144 + e^2 + e(-24 - 6x) + 72x + 9x^2} dx$$

Optimal antiderivative

$$e^{\frac{e^{-4+x}(5-x)-5}{4+x-\frac{e}{3}}}$$

command

integrate((((3\*x-15)\*exp(1)-9\*x^2+9\*x+180)\*exp(x-4)+3\*exp(1)-81)\*exp(exp(x-4))+45)\*exp(((3\*x-15)\*exp(exp(x-4))+15)/(exp(1)-3\*x-12))/(exp(1)^2+(-6\*x-24)\*exp(1)+9\*x^2+72\*x+144),x, algorithm

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(-\frac{3xe^{(x-4)}}{3x-e+12} + \frac{15e^{(x-4)}}{3x-e+12} - \frac{15}{3x-e+12}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{3\left(\left(3x^2 - (x-5)e - 3x - 60\right)e^{(x-4)} - e + 27\right)e^{(e^{(x-4)})} - 15\right)e^{\left(-\frac{3\left((x-5)e^{(x-4)} + 5\right)}{3x-e+12}\right)}}{9x^2 - 6(x+4)e + 72x + e^2 + 144} dx$$

**100.118 Problem number 4884**

$$\int \frac{-2e^{5/2} - 3e^5}{4x^2 + e^5(121 + 66x + 9x^2) + e^{5/2}(44x + 12x^2)} dx$$

Optimal antiderivative

$$\frac{1}{11 + 2xe^{-\frac{5}{2}} + 3x} - 4$$

command

integrate((-3\*exp(5/2)^2-2\*exp(5/2))/((9\*x^2+66\*x+121)\*exp(5/2)^2+(12\*x^2+44\*x)\*exp(5/2)+4\*x^2),x, algorithm

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3e^5 + 2e^{\frac{5}{2}}}{\left(3xe^{\frac{5}{2}} + 2x + 11e^{\frac{5}{2}}\right)\left(3e^{\frac{5}{2}} + 2\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.119 Problem number 4972

$$\int -\frac{440}{576 + 4e^{16} - 2640x + 3025x^2 + e^8(-96 + 220x)} dx$$

Optimal antiderivative

$$\frac{4}{\frac{55x}{2} + e^8 - 12}$$

command

```
integrate(-440/(4*exp(8)^2+(220*x-96)*exp(8)+3025*x^2-2640*x+576),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8}{55x + 2e^8 - 24}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.120 Problem number 4998

$$\int \frac{-225 - 30x - 4x^2 - 6x^3 - x^4 + e^{12}(-625 - 250x - 25x^2) + e^9(-2000 - 525x + 50x^2 + 15x^3) + e^3(-1225 - 500x - 2350) \cdot \exp(3)^2 + (-2x^4 + 4x^3 + 40x^2 - 215x - 1200) \cdot \exp(3) - x^4 - 6x^3 - 4x^2 - 30x - 225}{225x + 240x^2 + 94x^3 + 16x^4 + x^5 + e^{12}(625x + 250x^2 + 25x^3) + e^9(2000x + 1150x^2 + 200x^3 + 10x^4) + e^6(2350x + 1200x^2 + 150x^3 + 50x^4) + e^3(1225x + 500x^2 + 2350x^3 + 1200x^4) + e^3(-1225 - 500x - 2350)} dx$$

Optimal antiderivative

$$\frac{\frac{x}{3+x+e^3(5+x)} + 2 + x}{e^3 + 1 + \frac{x}{5}} - \ln(x)$$

command

```
integrate((( -25*x^2-250*x-625)*exp(3)^4+(15*x^3+50*x^2-525*x-2000)*exp(3)^3+(-x^4+25*x^3+115*500*x-2350)*exp(3)^2+(-2*x^4+4*x^3+40*x^2-215*x-1200)*exp(3)-x^4-6*x^3-4*x^2-30*x-225)/((25*x^3+250*x^2+625*x)*exp(3)^4+(10*x^4+200*x^3+1150*x^2+2000*x)*exp(3)^3+(x^5+40*x^4+400*x^3+1600*x^2+1500*x+500)*exp(3)^2+(5*x^4+20*x^3+200*x^2+1200*x+1200)*exp(3)+(-x^4-6*x^3-4*x^2-30*x-225)*exp(3)+(-1225-500*x-2350)*exp(3)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{5(5xe^6 + 8xe^3 + 2x + 25e^6 + 30e^3 + 9)}{x^2e^3 + x^2 + 5xe^6 + 15xe^3 + 8x + 25e^6 + 40e^3 + 15} - \log(|x|)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.121 Problem number 5042

$$\int -\frac{84e^2}{6480 + 80e^2 + 360x + 5x^2 + e(1440 + 40x)} dx$$

Optimal antiderivative

$$e^2 \left( 10 + \frac{4}{\frac{5x}{21} + \frac{60}{7} + \frac{20e}{21}} \right)$$

command

```
integrate(-84*exp(2)/(80*exp(1)^2+(40*x+1440)*exp(1)+5*x^2+360*x+6480),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{84e^2}{5(x+4e+36)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.122 Problem number 5062

$$\int \frac{32x + 26x^2 + \dots}{256 + 832x + 708x^2 - 140x^3 - 279x^4 + 40x^5 + 38x^6 - 12x^7 + x^8 + e^{12}(1 + 4x + 6x^2 + 4x^3 + x^4) + e^9(-16 - \dots)}$$

Optimal antiderivative

$$\frac{x^2}{2x + (1+x)^2(e^3 + x - 4)^2}$$

command

```
integrate(((2*x^2+2*x)*exp(3)^2+(-2*x^4-14*x^2-16*x)*exp(3)-2*x^5+6*x^4+26*x^2+32*x)/((x^4+4*40*x^3-80*x^2-60*x-16)*exp(3)^3+(6*x^6-24*x^5-60*x^4+124*x^3+398*x^2+340*x+96)*exp(3)^2+(4*x^32*x^6+24*x^5+248*x^4-76*x^3-872*x^2-864*x-256)*exp(3)+x^8-12*x^7+38*x^6+40*x^5-279*x^4-140*x^3+708*x^2+832*x+256),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2}{x^4 + 2x^3e^3 - 6x^3 + x^2e^6 - 4x^2e^3 + x^2 + 2xe^6 - 14xe^3 + 26x + e^6 - 8e^3 + 16}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 100.123 Problem number 5130

$$\int \frac{x + x^{\frac{3e^3+4x^2}{x}} \left( 3e^{e^3+4x^2} + e^{e^3+4x^2} (-3 + 24x^2) \log(x) \right)}{x^2} dx$$

Optimal antiderivative

$$\ln(x) + e^{\frac{3e^3+4x^2 \ln(x)}{x}}$$

command

`integrate((((24*x^2-3)*exp(exp(3)+4*x^2)*log(x)+3*exp(exp(3)+4*x^2))*exp(3*exp(exp(3)+4*x^2)*`

`Giac 1.9.0-11 via sagemath 9.6 output`

$$x^{\frac{3e^{(4x^2+e^3)}}{x}} + \log(x)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{3 \left( (8x^2 - 1)e^{(4x^2+e^3)} \log(x) + e^{(4x^2+e^3)} \right) x^{\frac{3e^{(4x^2+e^3)}}{x}} + x}{x^2} dx$$

## 100.124 Problem number 5234

$$\int \frac{e^e \frac{2(4+x \log(2x+4x^2+2x^3))}{\log(2x+4x^2+2x^3)} + 4e \frac{4+x \log(2x+4x^2+2x^3)}{\log(2x+4x^2+2x^3)} x+4x^2 \left( (8x^2 + 8x^3) \log^2(2x + 4x^2 + 2x^3) + e \frac{2(4+x \log(2x+4x^2+2x^3))}{\log(2x+4x^2+2x^3)} (-8 - \dots) \right)}{(x+x^2)^2} dx$$

Optimal antiderivative

$$e^{\left( 2x + e^{\frac{4}{\ln((x^2+x)(2x+2))} + x} \right)^2}$$

command

`integrate((((2*x^2+2*x)*log(2*x^3+4*x^2+2*x)^2-24*x-8)*exp((x*log(2*x^3+4*x^2+2*x)+4)/log(2*x^3+4*x^2+2*x))+8*x^3+8*x^2)*log(2*x^3+4*x^2+2*x)+`

`Giac 1.9.0-11 via sagemath 9.6 output`

$$e^{\left( 4x^2 + 4xe^{\left( x + \frac{4}{\log(2x^3+4x^2+2x)} \right)} + e^{\left( 2x + \frac{8}{\log(2x^3+4x^2+2x)} \right)} \right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 100.127 Problem number 5504

$$\int \frac{e^9(-10 - 100x^3) + e^{10}(-25 - 50x - 250x^3 - 125x^4)}{4e^8x^2 + e^9(20x^2 + 20x^3) + e^{10}(25x^2 + 50x^3 + 25x^4)} dx$$

Optimal antiderivative

$$\frac{-5x^3 + 1}{\left(\frac{2e^4e^{-5}}{5} + x + 1\right)x}$$

command

```
integrate((( -125*x^4-250*x^3-50*x-25)*exp(5)^2+(-100*x^3-10)*exp(4)*exp(5))/((25*x^4+50*x^3+2
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-5x - \frac{(25xe^2 + 20xe + 4x - 5e^2)e^{(-1)}}{5x^2e + 5xe + 2x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.128 Problem number 5513

$$\int \frac{-4 + e^{24}(256 - 4x^2) + e^{48}(-4096 - 6x + 128x^2 - x^4)}{4 + e^{24}(-256 + 4x^2) + e^{48}(4096 - 128x^2 + x^4)} dx$$

Optimal antiderivative

$$\frac{3}{x^2 - 64 + 2e^{-24}} - \frac{2}{5} - x$$

command

```
integrate((( -x^4+128*x^2-6*x-4096)*exp(24)^2+(-4*x^2+256)*exp(24)-4)/((x^4-128*x^2+4096)*exp(
256)*exp(24)+4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-x + \frac{3e^{24}}{x^2e^{24} - 64e^{24} + 2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.129 Problem number 5534

$$\int -\frac{128x}{4096e^{32} + 128e^{16}x^2 + x^4} dx$$

Optimal antiderivative

$$\frac{1}{e^{16} + \frac{x^2}{64}}$$

command

```
integrate(-128*x/(4096*exp(16)^2+128*x^2*exp(16)+x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{64}{x^2 + 64e^{16}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.130 Problem number 5560

$$\int \frac{-45 - 3x^2 + e^{10}(-15 - x^2) + e^x(-15 - 15x + x^2 + x^3) + e^{e^5}(-9 - 3e^{10} + e^x(-3 - 3x + x^2)) + (-45 + 6x + 3x^2 + e^{10}(-15 - x^2) + e^x(-15 - 15x + x^2 + x^3) + e^{e^5}(-9 - 3e^{10} + e^x(-3 - 3x + x^2)))}{-45 + 6x + 3x^2 + e^{e^5}(-9 + e^{10}(-3 + x) + e^x(-15 - 15x + x^2 + x^3) + e^{e^5}(-9 - 3e^{10} + e^x(-3 - 3x + x^2)))} dx$$

Optimal antiderivative

$$x + x \ln \left( \frac{e^x + e^{10} + 3}{(-3 + x)(x + e^{e^5} + 5)} \right)$$

command

```
integrate(((((-3+x)*exp(x)+(-3+x)*exp(5)^2+3*x-9)*exp(exp(5))+(x^2+2*x-15)*exp(x)+(x^2+2*x-15)*exp(5)^2+3*x^2+6*x-45)*log((exp(x)+exp(5)^2+3)/((-3+x)*exp(exp(5))+x^2+2*x-15))+((x^2-3*x-3)*exp(x)-3*exp(5)^2-9)*exp(exp(5))+(x^3+x^2-15*x-15)*exp(x)+(-x^2-15)*exp(5)^2-3*x^2-45)/(((((-3+x)*exp(x)+(-3+x)*exp(5)^2+3*x-9)*exp(exp(5))+(x^2+2*x-15)*exp(x)+(x^2+2*x-15)*exp(5)^2+3*x^2+6*x-45)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x \log \left( \frac{e^{20} + 3e^{10} + e^{(x+10)}}{x^2e^{10} + 2xe^{10} + xe^{(e^5+10)} - 15e^{10} - 3e^{(e^5+10)}} \right) + x$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{3x^2 + (x^2 + 15)e^{10} - (x^3 + x^2 - 15x - 15)e^x - ((x^2 - 3x - 3)e^x - 3e^{10} - 9)e^{(e^5)} - (3x^2 + (x^2 + 2x - 15)e^{10})e^{e^5}}{3x^2 + (x^2 + 2x - 15)e^{10} + (x^2 + 2x - 15)e^x + ((x^2 - 3x - 3)e^x - 3e^{10} - 9)e^{(e^5)} - (3x^2 + (x^2 + 2x - 15)e^{10})e^{e^5}}$$

## 100.131 Problem number 5568

$$\int \frac{663552x - 2097152x^5}{6561 + 41472x^2 + 107008x^4 + 65536e^{10}x^4 + 131072x^6 + 65536x^8 + e^5(41472x^2 + 131072x^4 + 131072x^6)} dx$$

Optimal antiderivative

$$\frac{16}{\frac{81}{256x^2} + 1 + x^2 + e^5}$$

command

`integrate((-2097152*x^5+663552*x)/(65536*x^4*exp(5)^2+(131072*x^6+131072*x^4+41472*x^2)*exp(5`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4096 x^2}{256 x^4 + 256 x^2 e^5 + 256 x^2 + 81}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.132 Problem number 5601

$$\int \frac{(16 + 20x) \log\left(\frac{1}{8}(8x + 5x^2)\right)}{8x + 5x^2} dx$$

Optimal antiderivative

$$\ln\left(\frac{5}{8}x^2 + x\right)^2$$

command

`integrate((20*x+16)*log(5/8*x^2+x)/(5*x^2+8*x),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\frac{5}{8}x^2 + x\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4(5x + 4) \log\left(\frac{5}{8}x^2 + x\right)}{5x^2 + 8x} dx$$

## 100.133 Problem number 5621

$$\int \frac{20 + 34e^2 - 68ex + 34x^2}{5e^2 - 10ex + 5x^2} dx$$

Optimal antiderivative

$$\frac{34x}{5} + \frac{4}{e-x} - 1$$

command

`integrate((34*exp(1)^2-68*x*exp(1)+34*x^2+20)/(5*exp(1)^2-10*x*exp(1)+5*x^2),x, algorithm="gi`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{34}{5}x - \frac{4}{x-e}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.134 Problem number 5667

$$\int \frac{12150 - 450e^3 + 2025x - 27x^3}{729x^3 + e^6x^3 + 162x^4 + 9x^5 + e^3(-54x^3 - 6x^4)} dx$$

Optimal antiderivative

$$\frac{3(5-x)(5+x)}{x^2 \left( \frac{e^3}{3} - 9 - x \right)}$$

command

`integrate((-450*exp(3)-27*x^3+2025*x+12150)/(x^3*exp(3)^2+(-6*x^4-54*x^3)*exp(3)+9*x^5+162*x^`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9(e^6 - 54e^3 + 504)}{(3x - e^3 + 27)(e^6 - 54e^3 + 729)} + \frac{225(3x + e^3 - 27)}{x^2(e^6 - 54e^3 + 729)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.135 Problem number 5707

$$\int \frac{4 \frac{16x}{4e^x + 20x - 4x^2 + e^4 x^2} (e^x(64 - 64x) \log(4) + (64x^2 - 16e^4 x^2) \log(4))}{16e^{2x} + 400x^2 - 160x^3 + 16x^4 + e^8 x^4 + e^x(160x - 32x^2 + 8e^4 x^2) + e^4(40x^3 - 8x^4)} dx$$

Optimal antiderivative

$$e^{\frac{8 \ln(2)}{x e^4 + \frac{x(5-x)+e^x}{x}}}$$

command

```
integrate(((2*(-64*x+64)*log(2)*exp(x)+2*(-16*x^2*exp(4)+64*x^2)*log(2))*exp(32*x*log(2)/(4*exp(4*x^2+20*x)))/(16*exp(x)^2+(8*x^2*exp(4)-32*x^2+160*x)*exp(x)+x^4*exp(4)^2+(-8*x^4+40*x^3)*exp(160*x^3+400*x^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2 \frac{32x}{x^2 e^4 - 4x^2 + 20x + 4e^x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{32(4(x-1)e^x \log(2) + (x^2 e^4 - 4x^2) \log(2)) 2 \frac{32x}{x^2 e^4 - 4x^2 + 20x + 4e^x}}{x^4 e^8 + 16x^4 - 160x^3 + 400x^2 - 8(x^4 - 5x^3)e^4 + 8(x^2 e^4 - 4x^2 + 20x)e^x + 16e^{(2x)}} dx$$

## 100.136 Problem number 5790

$$\int \frac{x^2 + e^{10} x^2 - 2x^3 + 2x^4 + x^6 + e^{-147+x}(-1 + e^5(-1+x) + x - 3x^2 + x^3) + e^5(2x^2 + 2x^4)}{x^2 + e^{10} x^2 + 2x^4 + x^6 + e^5(2x^2 + 2x^4)} dx$$

Optimal antiderivative

$$\frac{x + e^{x-147}}{(1 + x^2 + e^5)x} + x$$

command

```
integrate((((-1+x)*exp(5)+x^3-3*x^2+x-1)*exp(x-147)+x^2*exp(5)^2+(2*x^4+2*x^2)*exp(5)+x^6+2*x^2*x^3+x^2)/(x^2*exp(5)^2+(2*x^4+2*x^2)*exp(5)+x^6+2*x^4+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^6 e^{147} + 2x^4 e^{152} + 2x^4 e^{147} + 2x^3 e^{147} + x^2 e^{157} + 2x^2 e^{152} + x^2 e^{147} + 2x^2 e^x + 2x e^{152} + 2x e^{147} + 2e^{(x+5)} + 2e^x}{x^5 e^{147} + 2x^3 e^{152} + 2x^3 e^{147} + x e^{157} + 2x e^{152} + x e^{147}}$$

Giac 1.7.0 via sagemath 9.3 output

*sage<sub>0</sub>x*

## 100.137 Problem number 5800

$$\int \frac{3x^2 + 8x^3 + x^4 + e^2(5 + 2x - x^2) + e^6(e^2x^2 + 2x^3)}{1 + 8x + 18x^2 + e^{12}x^2 + 8x^3 + x^4 + e^6(2x + 8x^2 + 2x^3)} dx$$

Optimal antiderivative

$$\frac{(5 + x)e^2 + x^2}{e^6 + 4 + x + \frac{1}{x}}$$

command

`integrate((x^2*exp(2)+2*x^3)*exp(3)^2+(-x^2+2*x+5)*exp(2)+x^4+8*x^3+3*x^2)/(x^2*exp(3)^4+(2*`

Giac 1.9.0-11 via sagemath 9.6 output

$$x + \frac{xe^{12} - xe^8 + 8xe^6 + xe^2 + 15x + e^6 - e^2 + 4}{x^2 + xe^6 + 4x + 1}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.138 Problem number 5839

$$\int \frac{132 \log(e^{-x}(-22 + e^x))}{-22 + e^x} dx$$

Optimal antiderivative

$$3 \ln(1 - 22e^{-x})^2$$

command

`integrate(132*log((exp(x)-22)/exp(x))/(exp(x)-22),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$3 \log\left((e^x - 22)e^{(-x)}\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{132 \log((e^x - 22)e^{(-x)})}{e^x - 22} dx$$



## 100.139 Problem number 5861

$$\int \frac{-18 + \sqrt{e}(6 - 4x) + 12x + 4x^2}{9 + e - 6x + 7x^2 - 2x^3 + x^4 + \sqrt{e}(-6 + 2x - 2x^2)} dx$$

Optimal antiderivative

$$\frac{2(-3 + x)x}{x^2 - x + 3 - e^{\frac{1}{2}}} - 2$$

command

```
integrate(((6-4*x)*exp(1/2)+4*x^2+12*x-18)/(exp(1/2)^2+(-2*x^2+2*x-6)*exp(1/2)+x^4-2*x^3+7*x^2-6*x+9),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2\left(2x - e^{\frac{1}{2}} + 3\right)}{x^2 - x - e^{\frac{1}{2}} + 3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2\left(2x^2 - (2x - 3)e^{\frac{1}{2}} + 6x - 9\right)}{x^4 - 2x^3 + 7x^2 - 2(x^2 - x + 3)e^{\frac{1}{2}} - 6x + e + 9} dx$$

## 100.140 Problem number 5894

$$\int \frac{(-8x - 2x \log(\frac{1}{x})) \log(16 + 8 \log(\frac{1}{x}) + \log^2(\frac{1}{x})) + (8 + (64 + 16 \log(\frac{1}{x}))) \log^2(\frac{1}{x})}{(4x^3 + x^3 \log(\frac{1}{x})) \log(16 + 8 \log(\frac{1}{x}) + \log^2(\frac{1}{x})) + (8x^2 + 2x^2 \log(\frac{1}{x})) \log(16 + 8 \log(\frac{1}{x}) + \log^2(\frac{1}{x})) \log^2(\frac{1}{x})} dx$$

Optimal antiderivative

$$\frac{2}{x + \ln\left(\frac{\ln\left(\left(4 + \ln\left(\frac{1}{x}\right)\right)^2\right)}{x^4}\right)^2}$$

command

```
integrate((((16*log(1/x)+64)*log(log(1/x)^2+8*log(1/x)+16)+8)*log(log(log(1/x)^2+8*log(1/x)+16)+2*x*log(1/x)-8*x)*log(log(1/x)^2+8*log(1/x)+16))/((x*log(1/x)+4*x)*log(log(1/x)^2+8*log(1/x)+16))),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{16 \log(x)^2 - 8 \log(x) \log\left(\log\left(\log(x)^2 - 8 \log(x) + 16\right)\right) + \log\left(\log\left(\log(x)^2 - 8 \log(x) + 16\right)\right)^2} + x$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.141 Problem number 5980

$$\int \frac{e^4(44 - 28x + 111x^2 - 90x^3 + e^6(4 - 12x + 9x^2))}{484x^2 - 308x^3 + 709x^4 - 210x^5 + 225x^6 + e^{12}(4x^2 - 12x^3 + 9x^4) + e^6(88x^2 - 160x^3 + 102x^4 - 90x^5)} dx$$

Optimal antiderivative

$$\frac{e^4}{\left(5x - e^6 + \frac{16}{2x - \frac{4}{3}} + 1\right)x}$$

command

```
integrate(((9*x^2-12*x+4)*exp(3)^2-90*x^3+111*x^2-28*x+44)*exp(4)/((9*x^4-12*x^3+4*x^2)*exp(3)+90*x^5+102*x^4-160*x^3+88*x^2)*exp(3)^2+225*x^6-210*x^5+709*x^4-308*x^3+484*x^2),x, algorithm
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(3x - 2)e^4}{15x^3 - 3x^2e^6 - 7x^2 + 2xe^6 + 22x}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.142 Problem number 6107

$$\int \frac{100x \log(x) + 100x \log^2(x) + (2x^4 + 2e^x x^4) \log^4(x)}{625 + (250x^2 - 50e^x x^2 - 50x^3) \log^2(x) + (25x^4 + e^{2x} x^4 - 10x^5 + x^6 + e^x(-10x^4 + 2x^5)) \log^4(x)} dx$$

Optimal antiderivative

$$\frac{2}{5 + \frac{25}{x^2 \ln(x)^2} - e^x - x}$$

command

```
integrate(((2*exp(x)*x^4+2*x^4)*log(x)^4+100*x*log(x)^2+100*x*log(x))/((exp(x)^2*x^4+(2*x^5-10*x^4)*exp(x)+x^6-10*x^5+25*x^4)*log(x)^4+(-50*exp(x)*x^2-50*x^3+250*x^2)*log(x)^2+625),x, a
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.143 Problem number 6111

$$\int \frac{-2x^2 - 12x^5 + e^{10}(-2 - 12x^3) + e^5(-4x - 18x^2 - 24x^4)}{e^{10}x^2 + 2e^5x^3 + x^4} dx$$

Optimal antiderivative

$$\frac{2x + 2 - 6\left(x + \frac{3}{e^5 + x}\right)x^2}{x}$$

command

`integrate((( -12*x^3-2)*exp(5)^2+(-24*x^4-18*x^2-4*x)*exp(5)-12*x^5-2*x^2)/(x^2*exp(5)^2+2*x^3`

Giac 1.9.0-11 via sagemath 9.6 output

$$-6x^2 + \frac{2(9xe^5 + x + e^5)}{x^2 + xe^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.144 Problem number 6193

$$\int \frac{36 + 48x + 16x^2 + 4e^3x^2 + e^{3/2}(24x + 16x^2) + (2x^2 + e^{3/2}x^2) \log(3)}{9x^2 + 12x^3 + 4x^4 + e^3x^4 + e^{3/2}(6x^3 + 4x^4)} dx$$

Optimal antiderivative

$$-2 - \frac{4}{x} - \frac{\ln(3)}{3 + xe^{\frac{3}{2}} + 2x}$$

command

`integrate(((x^2*exp(3/2)+2*x^2)*log(3)+4*x^2*exp(3/2)^2+(16*x^2+24*x)*exp(3/2)+16*x^2+48*x+36`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4xe^{\frac{3}{2}} + x \log(3) + 8x + 12}{x^2e^{\frac{3}{2}} + 2x^2 + 3x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.145 Problem number 6197**

$$\int \frac{1}{81} e^{\frac{1}{81}(81e^{-1+2x} + e^{-1+x}(81x^2 + 11250x^5 + 390625x^8))} (162e^{-1+2x} + e^{-1+x}(162x + 81x^2 + 56250x^4 + 11250x^5 + 3125000x^7 + 390625x^8)) dx$$

Optimal antiderivative

$$e^{\left(\left(\frac{625}{9}x^4 + x\right)^2 + e^x\right)e^{-1+x}}$$

command

```
integrate(1/81*(162*exp(-1+x)*exp(x)+(390625*x^8+3125000*x^7+11250*x^5+56250*x^4+81*x^2+162*x
1+x))*exp(exp(-1+x)*exp(x)+1/81*(390625*x^8+11250*x^5+81*x^2)*exp(-1+x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(\frac{390625}{81}(x-1)^8 e^{(x-1)} + \frac{3125000}{81}(x-1)^7 e^{(x-1)} + \frac{10937500}{81}(x-1)^6 e^{(x-1)} + \frac{21886250}{81}(x-1)^5 e^{(x-1)} + \frac{27400000}{81}(x-1)^4 e^{(x-1)} + \frac{21987500}{81}(x-1)^3 e^{(x-1)}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

**100.146 Problem number 6209**

$$\int \frac{1}{8} (5x + 15x^2 + 10x^3 + e^{2x}(30x + 45x^2) + e^x(-25x - 55x^2 - 15x^3) + (e^{2x}(-25x - 30x^2) + e^x(10x + 20x^2 + 5x^3)) \log(2x) + e^{2x}(5x + 5x^2) \log^2(2x)) dx$$

Optimal antiderivative

$$\frac{5(x + 1 - e^x(3 - \ln(2x)))^2 x^2}{16}$$

command

```
integrate(1/8*(5*x^2+5*x)*exp(x)^2*log(2*x)^2+1/8*((-30*x^2-25*x)*exp(x)^2+(5*x^3+20*x^2+10*x
15*x^3-55*x^2-25*x)*exp(x)+5/4*x^3+15/8*x^2+5/8*x,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\begin{aligned} & \frac{5}{16} x^2 e^{(2x)} \log(2x)^2 + \frac{5}{16} x^4 + \frac{5}{8} x^3 - \frac{5}{8} x^2 e^x - \frac{5}{32} (2x - 1) e^{(2x)} \log(2x) + \frac{5}{16} x^2 \\ & + \frac{15}{32} (6x^2 - 2x + 1) e^{(2x)} + \frac{15}{16} x e^{(2x)} - \frac{5}{8} (3x^3 + 2x^2 + x - 1) e^x + \frac{5}{8} x e^x \\ & - \frac{5}{32} \left( (12x^2 - 2x + 1) e^{(2x)} - 4(x^3 + x^2) e^x \right) \log(2x) - \frac{15}{32} e^{(2x)} - \frac{5}{8} e^x \end{aligned}$$

Giac 1.7.0 via sagemath 9.3 output

$$\begin{aligned} & \int \frac{5}{8} (x^2 + x) e^{(2x)} \log(2x)^2 + \frac{5}{4} x^3 + \frac{15}{8} x^2 + \frac{15}{8} (3x^2 + 2x) e^{(2x)} \\ & - \frac{5}{8} (3x^3 + 11x^2 + 5x) e^x - \frac{5}{8} \left( (6x^2 + 5x) e^{(2x)} - (x^3 + 4x^2 + 2x) e^x \right) \log(2x) + \frac{5}{8} x dx \end{aligned}$$

## 100.147 Problem number 6216

$$\int \frac{144 + 1152x + 2160x^2 + 9e^{20}x^2 - 600x^3 - 60x^4 + 12x^5 - 144e^3x^5 + x^6 + 9e^4x^6 + e^{15}(-144x^2 + 36ex^3) + e^{10}}{}$$

Optimal antiderivative

$$\frac{5}{\frac{4-\frac{x^3}{3}}{x} + (e^5 + ex - 4)^2 - 2x}$$

command

```
integrate((-90*x^2*exp(1)*exp(5)-90*x^3*exp(1)^2+360*x^2*exp(1)+30*x^3+90*x^2+180)/(9*x^2*exp(144*x^2)*exp(5)^3+(54*x^4*exp(1)^2-432*x^3*exp(1)-6*x^4-36*x^3+864*x^2+72*x)*exp(5)^2+(36*x^5-432*x^4*exp(1)^2+(-12*x^5-72*x^4+1728*x^3+144*x^2)*exp(1)+48*x^4+288*x^3-2304*x^2-576*x)*exp(5)+9*x^6*exp(1)^4-144*x^5*exp(1)^3+(-6*x^6-36*x^5+864*x^4+72*x^3)*exp(1)^2+(48*x^5-2304*x^3-576*x^2)*exp(1)+x^6+12*x^5-60*x^4-600*x^3+2160*x^2+1152*x+144),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{15x}{3x^3e^2 - x^3 + 6x^2e^6 - 24x^2e - 6x^2 + 3xe^{10} - 24xe^5 + 48x + 12}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.148 Problem number 6258

$$\int \frac{36864 + e^7 + 384ex + e^2x^2}{36864 + 384ex + e^2x^2} dx$$

Optimal antiderivative

$$\frac{e^5}{x^2 - 192e^{-1} - (1+x)x} + x$$

command

```
integrate((exp(1)^2*exp(5)+x^2*exp(1)^2+384*x*exp(1)+36864)/(x^2*exp(1)^2+384*x*exp(1)+36864)
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x - \frac{e^6}{xe + 192}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.149 Problem number 6277

$$\int -\frac{9e^{\frac{16}{e^2}}}{81e^8x + 18e^4x \log(x) + x \log^2(x)} dx$$

Optimal antiderivative

$$\frac{e^{16e^{-2}}}{\frac{\ln(x)}{9} + e^4}$$

command

`integrate(-9*exp(16/exp(1)^2)/(x*log(x)^2+18*x*exp(4)*log(x)+81*x*exp(4)^2),x, algorithm="gia`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{9e^{(16e^{(-2)})}}{9e^4 + \log(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.150 Problem number 6278

$$\int \frac{14e^4x - 324x^4 + 27x^6 + 10206x^9 + e^2(-4 + 3x^2 - 756x^5)}{e^4 - 54e^2x^4 + 729x^8} dx$$

Optimal antiderivative

$$\left(\frac{x}{27x^4 - e^2} - 7\right)(-x^2 + 4)$$

command

`integrate((14*x*exp(1)^4+(-756*x^5+3*x^2-4)*exp(1)^2+10206*x^9+27*x^6-324*x^4)/(exp(1)^4-54*x^4*exp(1)^2+729*x^8),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$7x^2 - \frac{x^3 - 4x}{27x^4 - e^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{10206x^9 + 27x^6 - 324x^4 + 14xe^4 - (756x^5 - 3x^2 + 4)e^2}{729x^8 - 54x^4e^2 + e^4} dx$$

## 100.151 Problem number 6321

$$\int \frac{9 - 6e + e^2 + (5 - 2e)e^x + e^{2x}}{9 - 6e + e^2 + (6 - 2e)e^x + e^{2x}} dx$$

Optimal antiderivative

$$4 + x + \frac{1}{e^x + 3 - e}$$

command

```
integrate((exp(x)^2+(-2*exp(1)+5)*exp(x)+exp(1)^2-6*exp(1)+9)/(exp(x)^2+(-2*exp(1)+6)*exp(x)+6*exp(1)+9),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x - \frac{1}{e - e^x - 3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.152 Problem number 6397

$$\int \frac{-1476225 + 25e^{15} + 4920714x - 6561024x^2 + 4374000x^3 - 1458000x^4 + 194400x^5 + e^{12}(-1125 + 750x) + e^9}{-1476225 + 25e^{15} + 4920750x - 6561000x^2 + 4374000x^3 - 1458000x^4 + 194400x^5 + e^{12}(-1125 + 750x) + e^9}$$

Optimal antiderivative

$$x + \frac{2x^2}{25(e^3 + 6x - 9)^4}$$

command

```
integrate((25*exp(3)^5+(750*x-1125)*exp(3)^4+(9000*x^2-27000*x+20250)*exp(3)^3+(54000*x^3-243000*x^2+364500*x-182250)*exp(3)^2+(162000*x^4-972000*x^3+2187000*x^2-2186996*x+820125)*exp(3)^1+1458000*x^4+4374000*x^3-6561024*x^2+4920714*x-1476225)/(25*exp(3)^5+(750*x-1125)*exp(3)^4+(9000*x^2+20250)*exp(3)^3+(54000*x^3-243000*x^2+364500*x-182250)*exp(3)^2+(162000*x^4-972000*x^3+2187000*x^2-2187000*x+820125)*exp(3)+194400*x^5-1458000*x^4+4374000*x^3-6561000*x^2+4920750*x-1476225),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x + \frac{2x^2}{25(6x + e^3 - 9)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.153 Problem number 6398

$$\int \frac{360x + 270x^2 + e^{12}x^2 - 3e^8x^3 - x^5 + e^4(-120 + 3x^4)}{-120x^2 - 135x^3 + e^{12}x^3 - 3e^8x^4 - x^6 + e^4(120x + 135x^2 + 3x^5)} dx$$

Optimal antiderivative

$$\ln \left( x + \frac{\frac{120}{x} + 135}{(e^4 - x)^2} \right)$$

command

```
integrate((x^2*exp(2)^6-3*x^3*exp(2)^4+(3*x^4-120)*exp(2)^2-x^5+270*x^2+360*x)/(x^3*exp(2)^6-3*x^4*exp(2)^4+(3*x^5+135*x^2+120*x)*exp(2)^2-x^6-135*x^3-120*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log(x^4 - 2x^3e^4 + x^2e^8 + 135x + 120) - 2 \log(|x - e^4|) - \log(|x|)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.154 Problem number 6461

$$\int \frac{7040 + 4096x + 810x^2 + 67x^3 + 2x^4 + e^9(7 + 2x) + e^6(210 + 81x + 6x^2) + e^3(2104 + 1020x + 141x^2 + 6x^3)}{1000 + e^9 + 300x + 30x^2 + x^3 + e^6(30 + 3x) + e^3(300 + 60x + 3x^2)} dx$$

Optimal antiderivative

$$\left( 7 + \frac{4}{(10 + e^3 + x)^2} + x \right) x$$

command

```
integrate(((2*x+7)*exp(3)^3+(6*x^2+81*x+210)*exp(3)^2+(6*x^3+141*x^2+1020*x+2104)*exp(3)+2*x^4)/(1000+e^9+300*x+30*x^2+x^3+e^6*(30+3*x)+e^3*(300+60*x+3*x^2)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x^2 + 7x + \frac{4x}{(x + e^3 + 10)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2x^4 + 67x^3 + 810x^2 + (2x + 7)e^9 + 3(2x^2 + 27x + 70)e^6 + (6x^3 + 141x^2 + 1020x + 2104)e^3 + 4096x + 7040}{x^3 + 30x^2 + 3(x + 10)e^6 + 3(x^2 + 20x + 100)e^3 + 300x + e^9 + 1000} dx$$



**100.155 Problem number 6521**

$$\int \frac{150 + e(-6 - 4x) + 100x + 36x^2 + 16x^3}{5625x^2 + 3750x^3 + 1525x^4 + 600x^5 + 136x^6 + 24x^7 + 4x^8 + e^2(9x^2 + 6x^3 + x^4) + e(-450x^2 - 300x^3 - 86x^4 - \dots)}$$

Optimal antiderivative

$$-\frac{2}{x^2 \left(2x + \frac{25-e}{x}\right) (3+x)}$$

command

```
integrate(((−4*x−6)*exp(1)+16*x^3+36*x^2+100*x+150)/((x^4+6*x^3+9*x^2)*exp(1)^2+(−4*x^6−24*x^5−86*x^4−300*x^3−450*x^2)*exp(1)+4*x^8+24*x^7+136*x^6+600*x^5+1525*x^4+3750*x^3+56
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{2}{2x^4 + 6x^3 + 25x^2 - (x^2 + 3x)e + 75x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.156 Problem number 6534**

$$\int \frac{e^4 + 16x^4 + e^2(8x + 8x^2)}{3e^4 + 24e^2x^2 + 48x^4} dx$$

Optimal antiderivative

$$\frac{x}{3} + \frac{x}{3x + \frac{3e^2}{4x}} - 7$$

command

```
integrate((exp(2)^2+(8*x^2+8*x)*exp(2)+16*x^4)/(3*exp(2)^2+24*x^2*exp(2)+48*x^4),x, algorithm
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{3}x - \frac{e^2}{3(4x^2 + e^2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.157 Problem number 6594

$$\int \frac{(75 + 3e^3) \log(3)}{625 + e^6 + 50x + x^2 + e^3(50 + 2x)} dx$$

Optimal antiderivative

$$\frac{3x \ln(3)}{x + e^3 + 25}$$

command

```
integrate((3*exp(3)+75)*log(3)/(exp(3)^2+(2*x+50)*exp(3)+x^2+50*x+625),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{3(e^3 + 25) \log(3)}{x + e^3 + 25}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.158 Problem number 6625

$$\int \frac{752 + 256e^3}{2209 + 256e^6 + e^3(1504 - 2560x) - 7520x + 6400x^2} dx$$

Optimal antiderivative

$$\frac{x}{\frac{47}{16} - 5x + e^3} - 1$$

command

```
integrate((256*exp(3)+752)/(256*exp(3)^2+(-2560*x+1504)*exp(3)+6400*x^2-7520*x+2209),x, algor
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{16e^3 + 47}{5(80x - 16e^3 - 47)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.159 Problem number 6700

$$\int \frac{-e^{1+x}x^2 + 2e^{10+e^2+e^{8-x^2}-x^2}x^3 + (2e^{2+e^2+e^{8-x^2}}x + 2e^{1+x}x) \log(e^{2+e^2+e^{8-x^2}} + e^{1+x})}{(e^{2+e^2+e^{8-x^2}} + e^{1+x}) \log^2(e^{2+e^2+e^{8-x^2}} + e^{1+x})} dx$$

Optimal antiderivative

$$\frac{x^2}{\ln(e^{e^{-x^2+8}+e^2+2} + e^{1+x})}$$

command

```
integrate(((2*x*exp(exp(-x^2+8)+exp(2)+2)+2*x*exp(1+x))*log(exp(exp(-x^2+8)+exp(2)+2)+exp(1+x)
x^2+8)*exp(exp(-x^2+8)+exp(2)+2)-x^2*exp(1+x))/(exp(exp(-x^2+8)+exp(2)+2)+exp(1+x))/log(exp(e
x^2+8)+exp(2)+2)+exp(1+x))^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2x^3 e^{(-x^2+e^2+e^{(-x^2+8)+10})} - x^2 e^{(x+1)} + 2 \left( x e^{(x+1)} + x e^{(e^2+e^{(-x^2+8)+2})} \right) \log \left( e^{(x+1)} + e^{(e^2+e^{(-x^2+8)+2})} \right)}{\left( e^{(x+1)} + e^{(e^2+e^{(-x^2+8)+2})} \right) \log \left( e^{(x+1)} + e^{(e^2+e^{(-x^2+8)+2})} \right)^2} dx$$

## 100.160 Problem number 6722

$$\int \frac{960 + 480x + 2520x^2 + 360e^3x^2}{400 - 320x - 16x^2 - 248x^3 + 116x^4 + 28x^5 + 49x^6 + e^6x^6 + e^3(-40x^3 + 16x^4 + 4x^5 + 14x^6)} dx$$

Optimal antiderivative

$$\frac{12}{\left(\frac{2}{x} - \frac{(7+e^3)x^2}{10} - \frac{4}{5} - \frac{x}{5}\right)x}$$

command

```
integrate((360*x^2*exp(3)+2520*x^2+480*x+960)/(x^6*exp(3)^2+(14*x^6+4*x^5+16*x^4-40*x^3)*exp(
248*x^3-16*x^2-320*x+400),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{120}{x^3e^3 + 7x^3 + 2x^2 + 8x - 20}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{120(3x^2e^3 + 21x^2 + 4x + 8)}{x^6e^6 + 49x^6 + 28x^5 + 116x^4 - 248x^3 - 16x^2 + 2(7x^6 + 2x^5 + 8x^4 - 20x^3)e^3 - 320x + 400} dx$$

## 100.161 Problem number 6725

$$\int \frac{e^{5/2}(50x - 25x^2)}{16x^4 + e^5(10000 - 20000x + 10000x^2) + e^{5/2}(-800x^2 + 800x^3)} dx$$

Optimal antiderivative

$$\frac{x}{-16x + \frac{400(1-x)e^{\frac{5}{2}}}{x}}$$

command

`integrate((-25*x^2+50*x)*exp(5/2)/((10000*x^2-20000*x+10000)*exp(5/2)^2+(800*x^3-800*x^2)*exp`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25(x-1)e^{\frac{5}{2}}}{16\left(x^2 + 25xe^{\frac{5}{2}} - 25e^{\frac{5}{2}}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 100.162 Problem number 6847

$$\int \frac{-244140000 - e^{12} - 585938000x - 644531700x^2 - 429687600x^3 - 193359382x^4 - 61875000x^5 - 144375000x^6 - 488280000x + 2e^{12}x + 1171874000x^2 + 1289062200x^3 + 859374960x^4 + 386718748x^5 + 123750000x^6 + 288750000x^7}{(2x \exp(4)^3 + (6x^5 + 120x^4 + 900x^3 + 3000x^2 + 2100x + 21000)x^5 - 131250x^4 - 525000x^3 - 1312500x^2 - 1875000x - 1171874) \exp(4) - x^{12} - 60x^{11} - 1650x^{10} - 27500x^9 - 309375x^8 - 2475000x^7 - 14437500x^6 - 61875000x^5 - 193359382x^4 - 429687600x^3 - 644531700x^2 - 585938000x - 244140000)} dx$$

Optimal antiderivative

$$\frac{\ln\left(\frac{x^4}{((5+x)^4 + e^4)^2 - x}\right)}{2} - 3$$

command

`integrate((-exp(4)^3+(-3*x^4-60*x^3-450*x^2-1500*x-1875)*exp(4)^2+(-3*x^8-120*x^7-2100*x^6-21000*x^5-131250*x^4-525000*x^3-1312500*x^2-1875000*x-1171874)*exp(4)-x^12-60*x^11-1650*x^10-27500*x^9-309375*x^8-2475000*x^7-14437500*x^6-61875000*x^5-193359382*x^4-429687600*x^3-644531700*x^2-585938000*x-244140000)/(2*x*exp(4)^3+(6*x^5+120*x^4+900*x^3+3000*x^2+2100*x+21000)x^5-131250*x^4-525000*x^3-1312500*x^2-1875000*x-1171874)*exp(4)-x^12-60*x^11-1650*x^10-27500*x^9-309375*x^8-2475000*x^7-14437500*x^6-61875000*x^5-193359382*x^4-429687600*x^3-644531700*x^2-585938000*x-244140000)`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1}{2} \log(x^4 + 20x^3 + 150x^2 + 500x + e^4 + 626) + \log(x^4 + 20x^3 + 150x^2 + 500x + e^4 + 625) - \frac{1}{2} \log(x^4 + 20x^3 + 150x^2 + 500x + e^4 + 624) - \frac{1}{2} \log(|x|)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.163 Problem number 6850

$$\int -\frac{288e^{15}x^2}{e^{30} + 32e^{15}x^3 + 256x^6} dx$$

Optimal antiderivative

$$\frac{6x}{x + 16e^{-3}e^{-12}x^4}$$

command

```
integrate(-288*x^2*exp(3)^5/(exp(3)^10+32*x^3*exp(3)^5+256*x^6),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{6e^{15}}{16x^3 + e^{15}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.164 Problem number 6858

$$\int \frac{-12 + 6x - 2x^6 + e^2(-20x^4 - 40x^5 - 20x^6) + e^4(-10x^2 - 40x^3 - 60x^4 - 40x^5 - 10x^6) + e^5(2x + 10x^2 + 20x^3)}{-x^5 + e^2(-10x^3 - 20x^4 - 10x^5) + e^4(-5x - 20x^2 - 30x^3 - 20x^4 - 5x^5) + e^5(1 + 5x + 10x^2)} dx$$

Optimal antiderivative

$$11 + x^2 - \frac{3 - 2x}{(x - (1 + x)e)^4}$$

command

```
integrate(((2*x^6+10*x^5+20*x^4+20*x^3+10*x^2+2*x)*exp(1)^5+(-10*x^6-40*x^5-60*x^4-40*x^3-10*x^2)*exp(1)^4+(20*x^6+60*x^5+60*x^4+20*x^3)*exp(1)^3+(-20*x^6-40*x^5-20*x^4)*exp(1)^2+6*x+14)*exp(1)-2*x^6+6*x-12)/((x^5+5*x^4+10*x^3+10*x^2+5*x+1)*exp(1)^5+(-5*x^5-20*x^4-30*x^3-20*x^2-5*x)*exp(1)^4+(10*x^5+30*x^4+30*x^3+10*x^2)*exp(1)^3+(-10*x^5-20*x^4-10*x^3)*exp(1)^2+(5*x^5+5*x^4)*exp(1)-x^5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2e^{10} - 10x^2e^9 + 45x^2e^8 - 120x^2e^7 + 210x^2e^6 - 252x^2e^5 + 210x^2e^4 - 120x^2e^3 + 45x^2e^2 - 10x^2e + x^2}{e^{10} - 10e^9 + 45e^8 - 120e^7 + 210e^6 - 252e^5 + 210e^4 - 120e^3 + 45e^2 - 10e + 1} + \frac{2x - 3}{(xe - x + e)^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.165 Problem number 6885

$$\int \frac{x^{-\frac{25}{-84+8x+x^2+e^{-4-2x+2x^2}x^2+e^{-2-x+x^2}x(8+2x)}} \left( 2100 - 200x - 25x^2 + (200x + 50x^2) \log(x) + e^{-4-2x+2x^2} x^2 (-25 + (5 \right.}{7056x - 1344x^2 - 104x^3 + 16x^4 + x^5 + e^{-8-4x+4x^2} x^5 + e^{-6-3x+3x^2} x^3 (16x + 4x^2) + e^{-4-2x}}$$

Optimal antiderivative

$$\frac{\ln(x)}{e^{4-\frac{(e^{\ln(x)+x^2-x-2+4+x}) \left( \frac{e^{\ln(x)+x^2-x-2}}{5} + \frac{4}{5} + \frac{x}{5} \right)}{5}}$$

command

```
integrate((((100*x^2-50*x+50)*log(x)-25)*exp(log(x)+x^2-x-2)^2+((100*x^3+350*x^2-100*x+200)*1
50*x-200)*exp(log(x)+x^2-x-2)+(50*x^2+200*x)*log(x)-25*x^2-200*x+2100)*exp(-25*log(x)/(exp(lo
x-2)^2+(2*x+8)*exp(log(x)+x^2-x-2)+x^2+8*x-84)))/(x*exp(log(x)+x^2-x-2)^4+(4*x^2+16*x)*exp(log
x-2)^3+(6*x^3+48*x^2-104*x)*exp(log(x)+x^2-x-2)^2+(4*x^4+48*x^3-208*x^2-1344*x)*exp(log(x)+x^
x-2)+x^5+16*x^4-104*x^3-1344*x^2+7056*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{x^{x^2} e^{(2x^2-2x-4)} + 2x^2 e^{(x^2-x-2)} + x^2 + 8x e^{(x^2-x-2)} + 8x - 84}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.166 Problem number 6911

$$\int \frac{50x^2 + 20x^3 - 10x^4 + e^4(-100x - 30x^2) + e^2(-100x^2 - 40x^3) + (-10x^2 - 4x^3 + 2x^4 + e^4(20x + 6x^2) + e^2(20x + 6x^2))}{e^8 + 4e^6x + x^2 - 2x^3 + x^4 + e^4(-2x + 6x^2) + e^2(-4x^2 + 4x^3)}$$

Optimal antiderivative

$$\frac{2(2 \ln(3) - 5) x^2 (5 + x)}{(x + e^2)^2 - x}$$

command

```
integrate((2*((6*x^2+20*x)*exp(2)^2+(8*x^3+20*x^2)*exp(2)+2*x^4-4*x^3-10*x^2)*log(3)+(-
30*x^2-100*x)*exp(2)^2+(-40*x^3-100*x^2)*exp(2)-10*x^4+20*x^3+50*x^2)/(exp(2)^4+4*x*exp(2)^3+
2*x)*exp(2)^2+(4*x^3-4*x^2)*exp(2)+x^4-2*x^3+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4x \log(3) - 10x + \frac{2(6xe^4 \log(3) - 28xe^2 \log(3) - 15xe^4 + 70xe^2 + 12x \log(3) + 4e^6 \log(3) - 12e^4 \log(3) - 30x - 10e^6 + 30e^4)}{x^2 + 2xe^2 - x + e^4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.167 Problem number 6953**

$$\int \frac{12x^2 + 3e^2x^2 - 10x^3 + 4x^4 + e^3(8x + 2e^2x - 5x^2)}{16 + e^4 - 40x + 57x^2 - 40x^3 + 16x^4 + e^2(8 - 10x + 8x^2)} dx$$

Optimal antiderivative

$$\frac{x^2(e^3 + x)}{4 + 4x^2 - 5x + e^2}$$

command

```
integrate(((2*exp(2)*x-5*x^2+8*x)*exp(3)+3*x^2*exp(2)+4*x^4-10*x^3+12*x^2)/(exp(2)^2+(8*x^2-10*x+8)*exp(2)+16*x^4-40*x^3+57*x^2-40*x+16),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4}x + \frac{20xe^3 - 4xe^2 + 9x - 4e^5 - 16e^3 - 5e^2 - 20}{16(4x^2 - 5x + e^2 + 4)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4x^4 - 10x^3 + 3x^2e^2 + 12x^2 - (5x^2 - 2xe^2 - 8x)e^3}{16x^4 - 40x^3 + 57x^2 + 2(4x^2 - 5x + 4)e^2 - 40x + e^4 + 16} dx$$

**100.168 Problem number 7061**

$$\int \frac{384x + 128e^{32}x + 256x^2}{9 + e^{64} + 24x + 16x^2 + e^{32}(6 + 8x)} dx$$

Optimal antiderivative

$$\frac{16x^2}{\frac{3}{4} + \frac{e^{32}}{4} + x}$$

command

```
integrate((128*x*exp(16)^2+256*x^2+384*x)/(exp(16)^4+(8*x+6)*exp(16)^2+16*x^2+24*x+9),x, algo
```

Giac 1.9.0-11 via sagemath 9.6 output

$$16x + \frac{4(e^{64} + 6e^{32} + 9)}{4x + e^{32} + 3}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.169 Problem number 7195

$$\int \frac{-6e^5 - 6x + (-6x - 6x^2 - 2x^6 + e^5(-6x - 2x^5)) \log\left(\frac{2}{x}\right) + (6e^5x^4 + 6x^5) \log\left(\frac{2}{x}\right) \log\left(\frac{5 \log\left(\frac{2}{x}\right)}{e^5 + x}\right) + (-6e^5x^3 - 6x^4) \log\left(\frac{2}{x}\right) \log^2\left(\frac{5 \log\left(\frac{2}{x}\right)}{e^5 + x}\right)}{(-e^5x^4 - x^5) \log\left(\frac{2}{x}\right) + (3e^5x^3 + 3x^4) \log\left(\frac{2}{x}\right) \log\left(\frac{5 \log\left(\frac{2}{x}\right)}{e^5 + x}\right) + (-3e^5x^2 - 3x^3) \log\left(\frac{2}{x}\right) \log^2\left(\frac{5 \log\left(\frac{2}{x}\right)}{e^5 + x}\right)}$$

Optimal antiderivative

$$x^2 - \frac{3}{\left(\ln\left(\frac{5 \ln\left(\frac{2}{x}\right)}{e^5 + x}\right) - x\right)^2}$$

command

```
integrate(((2*x^2*exp(5)+2*x^3)*log(2/x)*log(5*log(2/x)/(exp(5)+x))^3+(-6*x^3*exp(5)-6*x^4)*log(2/x)*log(5*log(2/x)/(exp(5)+x))^2+(6*x^4*exp(5)+6*x^5)*log(2/x)*log(5*log(2/x)/(exp(5)+x))-2*x^5-6*x)*exp(5)-2*x^6-6*x^2-6*x)*log(2/x)-6*exp(5)-6*x)/((x*exp(5)+x^2)*log(2/x)*log(5*log(2/x)/(exp(5)+x))-3*x^2*exp(5)-3*x^3)*log(2/x)*log(5*log(2/x)/(exp(5)+x))^2+(3*x^3*exp(5)+3*x^4)*log(2/x)*log(5*log(2/x)/(exp(5)+x))-x^5)*log(2/x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.170 Problem number 7256

$$\int \frac{-2 - 25x + e^{x^2}(-27 + 4x^2) + 27 \log(x) + (-e^{x^2} - x + \log(x)) \log\left(\frac{e^{2x^2} + 2e^{x^2}x + x^2 + (-2e^{x^2} - 2x) \log(x) + \log^2(x)}{x^2}\right)}{-e^{x^2}x^2 - x^3 + x^2 \log(x)}$$

Optimal antiderivative

$$5 - \frac{25 + \ln\left(\frac{(e^{x^2} - \ln(x) + x)^2}{x^2}\right)}{x}$$

command

```
integrate(((log(x)-exp(x^2)-x)*log((log(x)^2+(-2*exp(x^2)-2*x)*log(x)+exp(x^2)^2+2*exp(x^2)*x-27)*exp(x^2)-25*x-2)/(x^2*log(x)-x^2*exp(x^2)-x^3)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{\log\left(x^2 + 2xe^{(x^2)} - 2x \log(x) - 2e^{(x^2)} \log(x) + \log(x)^2 + e^{(2x^2)}\right) - 2 \log(x) + 25}{x}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 100.171 Problem number 7282

$$\int \frac{-2592x^2 + 144x^3 + 576x^5 + e^4(-18 + 4x^3) + e^2(432x - 96x^4)}{3e^4x^3 - 72e^2x^4 + 432x^5} dx$$

Optimal antiderivative

$$\frac{4x}{3} + \frac{4}{-12x + e^2} + \frac{3}{x^2}$$

command

```
integrate(((4*x^3-18)*exp(2)^2+(-96*x^4+432*x)*exp(2)+576*x^5+144*x^3-2592*x^2)/(3*x^3*exp(2)
72*x^4*exp(2)+432*x^5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4}{3}x - \frac{4}{12x - e^2} + \frac{3}{x^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.172 Problem number 7297

$$\int \frac{64 + e^2(40 - 40x) - 64x + 56x^2}{64 + 25e^4 + e^2(80 - 140x) - 224x + 196x^2} dx$$

Optimal antiderivative

$$\frac{x}{1 - \frac{5(e^2 - 2x)}{2(2x - 4)}}$$

command

```
integrate((( -40*x+40)*exp(2)+56*x^2-64*x+64)/(25*exp(2)^2+(-140*x+80)*exp(2)+196*x^2-
224*x+64),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2}{7}x + \frac{5(5e^4 - 12e^2 - 32)}{49(14x - 5e^2 - 8)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.173 Problem number 7312**

$$\int \frac{5e^{2x/5}x^2 \log(16) + (-60 + 15x) \log(16) + e^{2x/5}(-8x^2 + 2x^3) \log(16) \log(4 - x) + (20 - 5x) \log(16) \log(2x^3)}{-20x^2 + 5x^3} dx$$

Optimal antiderivative

$$4 \left( e^{\frac{2x}{5}} \ln(4 - x) + \frac{\ln(2x^3)}{x} \right) \ln(2)$$

command

```
integrate((4*(-5*x+20)*log(2)*log(2*x^3)+4*(2*x^3-8*x^2)*log(2)*exp(1/5*x)^2*log(-x+4)+20*x^2*log(2)*exp(1/5*x)^2+4*(15*x-60)*log(2))/(5*x^3-20*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$4 e^{\left(\frac{2}{5}x\right)} \log(2) \log(-x + 4) + \frac{4 \log(2)^2}{x} + \frac{4 \log(2) \log\left((x - 4)^3 + 12(x - 4)^2 + 48x - 128\right)}{x}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.174 Problem number 7353**

$$\int \frac{75e^{4+x}}{625e^2 + 81e^{2x} + 450e^{1+x}} dx$$

Optimal antiderivative

$$\frac{e^3}{3 + \frac{25e e^{-x}}{3}}$$

command

```
integrate(75*exp(1)*exp(3)*exp(x)/(81*exp(x)^2+450*exp(1)*exp(x)+625*exp(1)^2),x, algorithm="
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25 e^5}{3 (25 e^2 + 9 e^{(x+1)})}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.175 Problem number 7479

$$\int \frac{-25e^2x^2 + e^4(-200x + 50x^2)}{4x^2 - 4x^3 + x^4 + e^4(64 - 64x + 16x^2) + e^2(32x - 32x^2 + 8x^3)} dx$$

Optimal antiderivative

$$\frac{x^2}{(-xe^{-2} - 4)\left(\frac{4}{25} - \frac{2x}{25}\right)}$$

command

```
integrate((50*x^2-200*x)*exp(2)^2-25*x^2*exp(2))/((16*x^2-64*x+64)*exp(2)^2+(8*x^3-32*x^2+32*x)*exp(2)+x^4-4*x^3+4*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{25(2xe^4 - xe^2 - 4e^4)}{x^2 + 4xe^2 - 2x - 8e^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.176 Problem number 7551

$$\int \frac{e^5 + 4 \log\left(\frac{-1+7\log(2)}{\log(2)}\right)}{8e^{10} - 8e^5x + 2x^2 + (32e^5 - 16x) \log\left(\frac{-1+7\log(2)}{\log(2)}\right) + 32 \log^2\left(\frac{-1+7\log(2)}{\log(2)}\right)} dx$$

Optimal antiderivative

$$\frac{2}{8 - \frac{4(-e^5+x)}{\ln\left(7 - \frac{1}{\ln(2)}\right) + \frac{x}{4}}}$$

command

```
integrate((4*log((7*log(2)-1)/log(2))+exp(5))/(32*log((7*log(2)-1)/log(2))^2+(32*exp(5)-16*x)*log((7*log(2)-1)/log(2))+8*exp(5)^2-8*x*exp(5)+2*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^5 + 4 \log\left(\frac{7 \log(2)-1}{\log(2)}\right)}{2\left(x - 2e^5 - 4 \log\left(\frac{7 \log(2)-1}{\log(2)}\right)\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.177 Problem number 7560

$$\int \frac{e^{-x} \left( e^{10e^{-x}} (-2e^x - 10x) - e^{e^x + 2x} x^3 + e^{5e^{-x}} (4e^x + 10x) + e^x (-6 - x^3) \right)}{x^3} dx$$

Optimal antiderivative

$$2 + \frac{(1 - e^{5e^{-x}})^2 + 2}{x^2} - x - e^{e^x}$$

command

```
integrate((-x^3*exp(x)^2*exp(exp(x))+(-2*exp(x)-10*x)*exp(5/exp(x))^2+(4*exp(x)+10*x)*exp(5/exp(x)-6)*exp(x))/exp(x)/x^3,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(x^3 e^x + x^2 e^{(x+e^x)} - e^{(x+10e^{-x})} + 2e^{(x+5e^{-x})} - 3e^x) e^{-x}}{x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{(x^3 e^{(2x+e^x)} + (x^3 + 6)e^x + 2(5x + e^x)e^{(10e^{-x})} - 2(5x + 2e^x)e^{(5e^{-x})}) e^{-x}}{x^3} dx$$

## 100.178 Problem number 7561

$$\int \frac{7}{20} \sqrt[5]{-2e^3 x^2 + 3e^6 x^2} dx$$

Optimal antiderivative

$$\frac{x(x e^3 (3(e^3 - 1)x + x))^{\frac{1}{5}}}{4}$$

command

```
integrate(7/20*(3*x^2*exp(3)^2-2*x^2*exp(3))^(1/5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{4} (3x^2 e^6 - 2x^2 e^3)^{\frac{1}{5}} x$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{7}{20} (3x^2 e^6 - 2x^2 e^3)^{\frac{1}{5}} dx$$

## 100.179 Problem number 7805

$$\int \frac{-7 - e^{12} + 36x - 111x^2 + 66x^3 + 12x^4 - 24x^5 + 53x^6 - 42x^7 + 20x^9 - 15x^{10} + 6x^{11} - x^{12} + e^8(-9 - 3x^2 + 6x^3 + 3x^4)}{1 + e^{12} + 3x^2 - 6x^3 + 6x^4 - 12x^5 + 19x^6 - 18x^7 + 18x^8 - 20x^9 + 15x^{10} - 6x^{11} + x^{12} + e^8(3 + 3x^2 - 6x^3 + 3x^4)} dx$$

Optimal antiderivative

$$x^2 - \left( x + \frac{3}{1 + e^4 + (x^2 - x)^2} \right)^2 - x$$

command

```
integrate((-exp(4)^3+(-3*x^4+6*x^3-3*x^2-9)*exp(4)^2+(-3*x^8+12*x^7-18*x^6+12*x^5+3*x^4-6*x^2-15)*exp(4)-x^12+6*x^11-15*x^10+20*x^9-42*x^7+53*x^6-24*x^5+12*x^4+66*x^3-111*x^2+36*x-7)/(exp(4)^3+(3*x^4-6*x^3+3*x^2+3)*exp(4)^2+(3*x^8-12*x^7+18*x^6-12*x^5+9*x^4-12*x^3+6*x^2+3)6*x^11+15*x^10-20*x^9+18*x^8-18*x^7+19*x^6-12*x^5+6*x^4-6*x^3+3*x^2+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-x - \frac{3(2x^5 - 4x^4 + 2x^3 + 2xe^4 + 2x + 3)}{(x^4 - 2x^3 + x^2 + e^4 + 1)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.180 Problem number 7835

$$\int \frac{-1 + e^4(-5 - 2x^2) + e^8(10x + 5x^2 - x^4) + e^{2+x}(5e^4 + e^8(-10x + 5x^2))}{5e^4 + 10e^8x^2 + 5e^{12}x^4} dx$$

Optimal antiderivative

$$\frac{e^{2+x} - x - 1}{1 + x^2e^4} - \frac{xe^{-4}}{5}$$

command

```
integrate((((5*x^2-10*x)*exp(4)^2+5*exp(4))*exp(2+x)+(-x^4+5*x^2+10*x)*exp(4)^2+(-2*x^2-5)*exp(4)-1)/(5*x^4*exp(4)^3+10*x^2*exp(4)^2+5*exp(4)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{x^3e^4 + 10xe^4 + x + 10e^4 - 5e^{(x+6)}}{5(x^2e^8 + e^4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.181 Problem number 7917

$$\int \frac{270 + 90e^{13/2} - 90x^2}{9 + e^{13} + 12x - 4e^{39/4}x + 10x^2 + 4x^3 + x^4 + e^{13/2}(6 + 4x + 6x^2) + e^{13/4}(-12x - 8x^2 - 4x^3)} dx$$

Optimal antiderivative

$$\frac{90x}{2x + \left(x - e^{\frac{13}{4}}\right)^2 + 3} + 1$$

command

```
integrate((90*exp(13/4)^2-90*x^2+270)/(exp(13/4)^4-4*x*exp(13/4)^3+(6*x^2+4*x+6)*exp(13/4)^2+4*x^3-8*x^2-12*x)*exp(13/4)+x^4+4*x^3+10*x^2+12*x+9),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{90x}{x^2 - 2xe^{\frac{13}{4}} + 2x + e^{\frac{13}{2}} + 3}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{90\left(x^2 - e^{\frac{13}{2}} - 3\right)}{x^4 + 4x^3 + 10x^2 - 4xe^{\frac{39}{4}} + 2(3x^2 + 2x + 3)e^{\frac{13}{2}} - 4(x^3 + 2x^2 + 3x)e^{\frac{13}{4}} + 12x + e^{13} + 9} dx$$

## 100.182 Problem number 7950

$$\int \frac{e^{-\frac{6}{e^{5x} - x^4 - 2x^3 \log(5) - x^2 \log^2(5) + (-2x^3 - 2x^2 \log(5)) \log(x) - x^2 \log^2(5)}}}{e^{2e^{5x}} + x^8 + 4x^7 \log(5) + 6x^6 \log^2(5) + 4x^5 \log^3(5) + x^4 \log^4(5) + (4x^7 + 12x^6 \log(5) + 12x^5 \log^2(5) + 4x^4 \log^3(5) + 4x^3 \log^4(5) + 4x^2 \log^5(5) + 4x \log^6(5) + \log^7(5))} dx$$

Optimal antiderivative

$$4 - \frac{6}{e^{x^2(\ln(5x)+x)^2 - e^{5x}}}$$

command

```
integrate((-30*exp(5*x)*exp(exp(5*x))+12*x*log(x)^2+(24*x*log(5)+36*x^2+12*x)*log(x)+12*x*log(3)/(exp(exp(5*x))-x^2*log(x)^2+(-2*x^2*log(5)-2*x^3)*log(x)-x^2*log(5)^2-2*x^3*log(5)-x^4))^2/(exp(exp(5*x))^2+(-2*x^2*log(x)^2+(-4*x^2*log(5)-4*x^3)*log(x)-2*x^2*log(5)^2-4*x^3*log(5)-2*x^4)*exp(exp(5*x))+x^4*log(x)^4+(4*x^4*log(5)+4*x^5)*log(x)^3+(6*x^4*log(5)^2+4*x^3*log(5)^2+4*x^2*log(5)^2+4*x*log(5)^2+log(5)^2)+log(5)^7),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-e^{\left(\frac{6}{x^4 + 2x^3 \log(5) + x^2 \log(5)^2 + 2x^3 \log(x) + 2x^2 \log(5) \log(x) + x^2 \log(x)^2 - e^{(5x)}}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{6 \left( 4x^3 + 2x \log(5)^2 + 2x \log(x)^2 + 2x^2 + 2(3 \log(5) \log(x) + \log(5) \log(x)^2) \right)}{x^8 + 4x^7 \log(5) + 6x^6 \log(5)^2 + 4x^5 \log(5)^3 + x^4 \log(5)^4 + x^4 \log(x)^4 + 4(x^5 + x^4 \log(5)) \log(x)^3 + 6(x^6 + 2x^5 \log(5) + x^4 \log(5)^2 + x^3 \log(5) \log(x) + x^2 \log(5) \log(x)^2 + x \log(5) \log(x)^3 + \log(5) \log(x)^4)}$$

**100.183 Problem number 7956**

$$\int \frac{-216 + e^9 (e(-1080 + 540x) + e^3(-1080 + 540x)) + e^{18} (e^4(-3600 + 3600x - 900x^2) + e^2(-1800 + 1800x - 900x^2))}{(x^2 - 2x - \frac{6xe^{-9}}{5(e+e^3)})^2}$$

Optimal antiderivative

$$\frac{x^2}{\left(x^2 - 2x - \frac{6xe^{-9}}{5(e+e^3)}\right)^2}$$

command

```
integrate((-250*exp(3)^3-750*exp(1)*exp(3)^2-750*exp(1)^2*exp(3)-250*exp(1)^3)*exp(9)^3/((125*x^2+1500*x-1000)*exp(3)^3+(375*x^3-2250*x^2+4500*x-3000)*exp(1)*exp(3)^2+(375*x^3-2250*x^2+4500*x-3000)*exp(1)^2*exp(3)+(125*x^3-750*x^2+1500*x-1000)*exp(1)^3)*exp(9)^3+((-450*x^2+1800*x-1800)*exp(3)^2+(-900*x^2+3600*x-3600)*exp(1)*exp(3)+(-450*x^2+1800*x-1800)*exp(1)^2)*exp(9)^2+((540*x-1080)*exp(3)+(540*x-1080)*exp(1))*exp(9)-216),x, algorithm="maxima")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{25 (e^9 + 3e^7 + 3e^5 + e^3)e^{27}}{(5xe^{12} + 5xe^{10} - 10e^{12} - 10e^{10} - 6)^2(e^{12} + e^{10})}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{250 (e^9 + 3e^7 + 3e^5 + e^3)e^{27}}{125 ((x^3 - 6x^2 + 12x - 8)e^9 + 3(x^3 - 6x^2 + 12x - 8)e^7 + 3(x^3 - 6x^2 + 12x - 8)e^5 + (x^3 - 6x^2 + 12x - 8)e^3)}$$

## 100.184 Problem number 7960

$$\int -\frac{1}{e^{10} + 2e^5x + x^2} dx$$

Optimal antiderivative

$$\frac{1}{e^5 + x}$$

command

```
integrate(-1/(exp(5)^2+2*x*exp(5)+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{x + e^5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.185 Problem number 7969

$$\int \frac{-320 + 480x^2 + 320x^3 + 60x^4 + e^{\frac{4}{5}(40+3x)}(-20 + 48x) + e^{\frac{3}{5}(40+3x)}(160 - 288x - 144x^2) + e^{\frac{2}{5}(40+3x)}(-480 + 5}{}$$

Optimal antiderivative

$$\left(2 - e^{\frac{3x}{5}+8} + x\right)^4 \left(\ln(x)^2 + \frac{4}{x}\right)$$

command

```
integrate(1/5*((12*x^2*exp(3/5*x+8)^4+(-36*x^3-92*x^2)*exp(3/5*x+8)^3+(36*x^4+204*x^3+264*x^2
12*x^5-132*x^4-384*x^3-336*x^2)*exp(3/5*x+8)+20*x^5+120*x^4+240*x^3+160*x^2)*log(x)^2+(10*x*e
40*x^2-80*x)*exp(3/5*x+8)^3+(60*x^3+240*x^2+240*x)*exp(3/5*x+8)^2+(-40*x^4-240*x^3-
480*x^2-320*x)*exp(3/5*x+8)+10*x^5+80*x^4+240*x^3+320*x^2+160*x)*log(x)+(48*x-20)*exp(3/5*x+8
144*x^2-288*x+160)*exp(3/5*x+8)^3+(144*x^3+696*x^2+576*x-480)*exp(3/5*x+8)^2+(-48*x^4-
448*x^3-1056*x^2-384*x+640)*exp(3/5*x+8)+60*x^4+320*x^3+480*x^2-320)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out



## 100.186 Problem number 7981

$$\int \frac{-5124800 + 1310720e^{15}x^3 - 131072e^{20}x^4 + e^5(1923011209 + 15350400x + 2560000x^2 + e^5(-36840960x - 24568320x^2 - 4096000x^3) + e^{10}(22113792x^2 + 14744$$

Optimal antiderivative

$$\frac{5 + x}{3 + x - \frac{3}{16(2xe^5 - 5)(8xe^5 - 20)}}$$

command

```
integrate((-131072*x^4*exp(5)^4+1310720*x^3*exp(5)^3+(-4917504*x^2-7680*x)*exp(5)^2+(8199680*5124800)/((65536*x^6+393216*x^5+589824*x^4)*exp(5)^4+(-655360*x^5-3932160*x^4-5898240*x^3)*exp(5)+4096000*x^3-24568320*x^2-36840960*x)*exp(5)+2560000*x^2+15350400*x+23011209),x, algorithm="gi
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{512x^2e^{10} - 2560xe^5 + 3203}{256x^3e^{10} + 768x^2e^{10} - 1280x^2e^5 - 3840xe^5 + 1600x + 4797}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.187 Problem number 8012

$$\int \frac{-4x + 3x^2 + e^3(2x - x^2) + e^5(4 - 12x + 13x^2 - 6x^3 + x^4 + e^6(1 - 2x + x^2) + e^3(-4 + 10x - 8x^2 + 2x^3))}{4 - 12x + 13x^2 - 6x^3 + x^4 + e^6(1 - 2x + x^2) + e^3(-4 + 10x - 8x^2 + 2x^3)} dx$$

Optimal antiderivative

$$x \left( \frac{x^2}{(2 - x - e^3)(x^2 - x)} + e^5 \right)$$

command

```
integrate((((x^2-2*x+1)*exp(3)^2+(2*x^3-8*x^2+10*x-4)*exp(3)+x^4-6*x^3+13*x^2-12*x+4)*exp(5)+x^2+2*x)*exp(3)+3*x^2-4*x)/((x^2-2*x+1)*exp(3)^2+(2*x^3-8*x^2+10*x-4)*exp(3)+x^4-6*x^3+13*x^2-12*x+4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$xe^5 + \frac{xe^3 - 3x - e^3 + 2}{x^2 + xe^3 - 3x - e^3 + 2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.188 Problem number 8026

$$\int \frac{e^x (1280 - 288x - 144x^2)}{6400 + 1440x^2 + 960x^3 + 241x^4 + e^{40}x^4 + 108x^5 + 54x^6 + 12x^7 + x^8 + e^{30}(-12x^4 - 4x^5) + e^{20}(160x^2 + 54x^4)}$$

Optimal antiderivative

$$\frac{e^x}{\frac{x^2(x-e^{10}+3)^2}{16} + 5 - \ln(2)}$$

command

```
integrate((-256*log(2)+(16*x^2-32*x)*exp(10)^2+(-32*x^3+192*x)*exp(10)+16*x^4+32*x^3-144*x^2-288*x+1280)*exp(x)/(256*log(2)^2+(-32*x^2*exp(10)^2+(64*x^3+192*x^2)*exp(10)-32*x^4-192*x^3-288*x^2-2560)*log(2)+x^4*exp(10)^4+(-4*x^5-12*x^4)*exp(10)^3+(6*x^6+36*x^5+54*x^4*x^7-36*x^6-108*x^5-108*x^4-320*x^3-960*x^2)*exp(10)+x^8+12*x^7+54*x^6+108*x^5+241*x^4+960*x^3)
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{32 e^x}{x^4 - 2x^3e^{10} + 6x^3 + x^2e^{20} - 6x^2e^{10} + 9x^2 - 16\log(2) + 80}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.189 Problem number 8040

$$\int \frac{-5x^2 + e^x(5x - x^2 + x^3) + (5x^2 - 5e^xx^2) \log\left(\frac{x}{3}\right) + (-12x + 4x^2 + (60 - 20x) \log\left(\frac{x}{3}\right)) \log(3x^2) + (-15 + 11x) \log^2(3x^2)}{e^{2x}x^2 - 2e^xx^3 + x^4 + (6x^2 - 2x^3 + e^x(-6x + 2x^2)) \log^2(3x^2) + (9 - 6x + x^2) \log^4(3x^2)}$$

Optimal antiderivative

$$\frac{5 \ln\left(\frac{x}{3}\right) - x}{e^x - \frac{\ln(3x^2)^2(3-x)}{x} - x}$$

command

```
integrate((( -15*log(1/3*x)-x^2+11*x-15)*log(3*x^2)^2+((-20*x+60)*log(1/3*x)+4*x^2-12*x)*log(3*x^2)+(-5*exp(x)*x^2+5*x^2)*log(1/3*x)+(x^3-x^2+5*x)*exp(x)-5*x^2)/((x^2-6*x+9)*log(3*x^2)^4+((2*x^2-6*x)*exp(x)-2*x^3+6*x^2)*log(3*x^2)^2+exp(x)^2*x^2-2*exp(x)*x^3+x^4))
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.190 Problem number 8091

$$\int -\frac{5e^{3+e^x+x}}{25 + 10e^{3+e^x} + e^{6+2e^x}} dx$$

Optimal antiderivative

$$\frac{5}{5 + e^3 e^{e^x}}$$

command

`integrate(-5*exp(3)*exp(x)*exp(exp(x))/(exp(3)^2*exp(exp(x))^2+10*exp(3)*exp(exp(x))+25), x, a`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5}{e^{(e^x+3)} + 5}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.191 Problem number 8173

$$\int \frac{(-x - 16ex^2 - 64x^4) \log(x) + (-e^2 - x - 8ex^2 - 16x^4 + (e^2 + x + 8ex^2 + 16x^4) \log(x)) \log(e^2 + x + 8ex^2 + 16x^4)}{(e^2 + x + 8ex^2 + 16x^4)}$$

Optimal antiderivative

$$x + \frac{\frac{x}{\ln(x+(e+4x^2)^2)} - x}{\ln(x)}$$

command

`integrate((((exp(1)^2+8*x^2*exp(1)+16*x^4+x)*log(x)^2+(-exp(1)^2-8*x^2*exp(1)-16*x^4-x)*log(x)+exp(1)^2+8*x^2*exp(1)+16*x^4+x)*log(exp(1)^2+8*x^2*exp(1)+16*x^4+x)^2+((exp(1)^2+8*x^2*exp(1)-16*x^4-x)*log(exp(1)^2+8*x^2*exp(1)+16*x^4+x)+(-16*x^2*exp(1)-64*x^4-x)*log(x))/(exp(1)^2+8*x^2*exp(1)+16*x^4+x)/log(x)^2/log(exp(1)^2+8*x^2*exp(1)+16*x^4+x)`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x \log(16x^4 + 8x^2e + x + e^2) \log(x) - x \log(16x^4 + 8x^2e + x + e^2) + x}{\log(16x^4 + 8x^2e + x + e^2) \log(x)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.192 Problem number 8184

$$\int \frac{-2e^8 + 2x^4}{e^{12} + x^3 - 3x^4 + 3x^5 - x^6 + e^8(3x - 3x^2) + e^4(3x^2 - 6x^3 + 3x^4)} dx$$

Optimal antiderivative

$$\ln(2 \ln(2)) + \left( \frac{x}{(1-x)x + e^4} - 1 \right)^2$$

command

```
integrate((-2*exp(4)^2+2*x^4)/(exp(4)^3+(-3*x^2+3*x)*exp(4)^2+(3*x^4-6*x^3+3*x^2)*exp(4)-x^6+3*x^5-3*x^4+x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2x^3 - x^2 - 2xe^4}{(x^2 - x - e^4)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2(x^4 - e^8)}{x^6 - 3x^5 + 3x^4 - x^3 + 3(x^2 - x)e^8 - 3(x^4 - 2x^3 + x^2)e^4 - e^{12}} dx$$

## 100.193 Problem number 8211

$$\int \frac{e^4 x^2 + e^4(8ex + 5e^2 x^2) + e^6 x^2 \log(4) - e^6 x^2 \log(2)}{e^2 x^2 + e^2(8ex + 10e^2 x^2) + e^4(16 + 40ex + 25e^2 x^2) + (2e^4 x^2 + e^4(8ex + 10e^2 x^2)) \log(4) + e^6 x^2 \log^2(4) + (-2e^4 x^2 + e^4(8ex + 10e^2 x^2)) \log(2) - 2e^6 x^2 \log(2) \log(4)} dx$$

Optimal antiderivative

$$\frac{x}{2 \ln(2) + e^{-2} + \frac{4e^{-1}}{x} + 5 - \ln(3e^2)}$$

command

```
integrate((-x^2*exp(1)^2*exp(2)^2*log(3*exp(2))+2*x^2*exp(1)^2*exp(2)^2*log(2)+(5*x^2*exp(1)^4*x^2*exp(1)^2*exp(2)^2*log(2)+(-10*x^2*exp(1)^2-8*x*exp(1))*exp(2)^2-2*x^2*exp(1)^2*exp(2))*
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2xe^4 \log(2) - xe^4 \log(3e^2) + 5xe^4 + xe^2}{4e^4 \log(2)^2 - 4e^4 \log(2) \log(3e^2) + e^4 \log(3e^2)^2 + 20e^4 \log(2) + 4e^2 \log(2) - 10e^4 \log(3e^2) - 2e^2 \log(3e^2) + 2e^4} + \frac{(2xe^2 \log(2) - xe^2 \log(3e^2) + 5xe^2 + x + 4e)(4e^4 \log(2)^2 - 4e^4 \log(2) \log(3e^2) + e^4 \log(3e^2)^2 + 20e^4 \log(2))}{16e^4}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.194 Problem number 8257

$$\int \frac{-64 - 144x - 108x^2 - 27x^3 + e^3(-48 - 120x - 99x^2 - 27x^3) + e^6(-12 - 33x - 30x^2 - 9x^3) + e^9(-1 - 3x - 3x^2 - x^3)}{x^4 - \frac{x}{4 - 5xe^8 + e^3} + 1} dx$$

Optimal antiderivative

$$\frac{4}{\left(x - \frac{x}{4 - 5xe^8 + e^3} + 1\right)^2}$$

command

```
integrate((-1000*x^3*exp(4)^6+(600*x^2*exp(3)+2400*x^2)*exp(4)^4+(-120*x*exp(3)^2-920*x*exp(3)-1760*x)*exp(4)^2+8*exp(3)^3+88*exp(3)^2+320*exp(3)+384)/((125*x^6+375*x^5+375*x^4+75*x^3-225*x^2-225*x^3-75*x^2)*exp(3)-225*x^5-750*x^4-825*x^3-300*x^2)*exp(4)^4+((15*x^4+45*x^3-3*x^2-3*x-1)*exp(3)^3+(-9*x^3-30*x^2-33*x-12)*exp(3)^2+(-27*x^3-99*x^2-120*x-48)*exp(3)-27*x^3-108*x^2-144*x-64),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4(25x^2e^{16} - 10xe^{11} - 40xe^8 + e^6 + 8e^3 + 16)}{(5x^2e^8 + 5xe^8 - xe^3 - 3x - e^3 - 4)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.195 Problem number 8286

$$\int \frac{2e^{6-3e^x+3e^{2x-x^2}}x - 6e^{4-2e^x+2e^{2x-x^2}}x^2 - e^4x^3 - 2x^4 + e^{-e^x+e^{2x-x^2}}(3e^6x^2 + 6e^2x^3 + 2e^{6+x}x^3 + e^{6+2x-x^2}(-4x^3 + 12e^{2-e^x+e^{2x-x^2}}x^2 - 4x^3))}{4e^{6-3e^x+3e^{2x-x^2}} - 12e^{4-2e^x+2e^{2x-x^2}}x + 12e^{2-e^x+e^{2x-x^2}}x^2 - 4x^3} dx$$

Optimal antiderivative

$$\frac{\left(\frac{x^2}{(xe^{-2} - e^{e(2-x)x - e^x})^2} + x\right)x}{4}$$

command

```
integrate((2*x*exp(2)^3*exp(-exp(x)+exp(-x^2+2*x))^3-6*x^2*exp(2)^2*exp(-exp(x)+exp(-x^2+2*x))^2+(2*x^3*exp(2)^3*exp(x)+(4*x^4-4*x^3)*exp(2)^3*exp(-x^2+2*x)+3*x^2*exp(2)^3+6*x^3*exp(x)+exp(-x^2+2*x))-x^3*exp(2)^2-2*x^4)/(4*exp(2)^3*exp(-exp(x)+exp(-x^2+2*x))^3-12*x*exp(2)^2*exp(-exp(x)+exp(-x^2+2*x))^2+12*x^2*exp(2)*exp(-exp(x)+exp(-x^2+2*x))-4*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.196 Problem number 8346

$$\int \frac{e^{\frac{3(-x+x^2-x^3+(-x+x^2)\log(-1+x))}{x-x^2+(-1+x)\log(-1+x)}}}{(x^3 - 2x^4 + x^5 + (-2x^2 + 4x^3 - 2x^4)\log(-1+x) + (x - 2x^2 + x^3)\log^2(-1+x))} dx \quad -400$$

Optimal antiderivative

$$1 - \frac{4}{\left(\frac{\ln(x^2)}{5} - \frac{e^{\frac{x}{x-\ln(-1+x)}}(-1+x)^x}{5}\right)^2}$$

command

```
integrate((((200*x^3-400*x^2+200*x)*log(-1+x)^2+(-400*x^4+800*x^3-400*x^2+200*x)*log(-1+x)+200*x^5-400*x^4+200*x^2)*exp(((x^2-x)*log(-1+x)-x^3+x^2-x)/((-1+x)*log(-1+x)-x^2+x))+(-400*x^2+800*x-400)*log(-1+x)^2+(800*x^3-1600*x^2+800*x)*log(-1+x)-400*x^4+800*x^3-400*x^2)/(((x^3-2*x^2+x)*log(-1+x)^2+(-2*x^4+4*x^3-2*x^2)*log(-1+x)+x^5-2*x^4+x^3)*exp(((x^2-x)*log(-1+x)-x^3+x^2-x)/((-1+x)*log(-1+x)-x^2+x))^3+((-3*x^3+6*x^2-3*x)*log(-1+x)^2+(6*x^4-12*x^3+6*x^2)*log(-1+x)-3*x^5+6*x^4-3*x^3)*log(x^2)*exp(((x^2-x)*log(-1+x)-x^3+x^2-x)/((-1+x)*log(-1+x)-x^2+x))^2+((3*x^3-6*x^2+3*x)*log(-1+x)^2+(-6*x^4+12*x^3-6*x^2)*log(-1+x)+3*x^5-6*x^4+3*x^3)*log(x^2)^2*exp(((x^2-x)*log(-1+x)-x^3+x^2-x)/((-1+x)*log(-1+x)-x^2+x))+((-x^3+2*x^2-x)*log(-1+x)^2+(2*x^4-4*x^3+2*x^2)*log(-1+x)-x^5+2*x^4-x^3)*log(x^2)
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{100}{2e^{\left(\frac{x^3-x^2\log(x-1)-x^2+x\log(x-1)+x}{x^2-x\log(x-1)-x+\log(x-1)}\right)}\log(x^2) - \log(x^2)^2 - e^{\left(\frac{2(x^3-x^2\log(x-1)-x^2+x\log(x-1)+x)}{x^2-x\log(x-1)-x+\log(x-1)}\right)}}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.197 Problem number 8361

$$\int \frac{12x^2\log(2) + 3e^{2/x}x^2\log(2) + 3e^{\frac{2(x^3+x^3\log(2))}{\log(2)}}x^2\log(2) + e^{\frac{1}{x}}(5 + 12x^2)\log(2) + e^{\frac{x^3+x^3\log(2)}{\log(2)}}(15x^4 - 6e^{\frac{1}{x}}x^2\log(2))}{12x^2\log(2) + 12e^{\frac{1}{x}}x^2\log(2) + 3e^{2/x}x^2\log(2) + 3e^{\frac{2(x^3+x^3\log(2))}{\log(2)}}x^2\log(2) + e^{\frac{x^3+x^3\log(2)}{\log(2)}}(-12x^2\log(2))} dx$$

Optimal antiderivative

$$\frac{5}{3\left(e^{\frac{1}{x}} - e^{\frac{x^2(x\ln(2)+x)}{\ln(2)}} + 2\right)} + x$$

command

```
integrate((3*x^2*log(2)*exp((x^3*log(2)+x^3)/log(2))^2+(-6*x^2*log(2)*exp(1/x)+(15*x^4-
12*x^2)*log(2)+15*x^4)*exp((x^3*log(2)+x^3)/log(2))+3*x^2*log(2)*exp(1/x)^2+(12*x^2+5)*log(2)
6*x^2*log(2)*exp(1/x)-12*x^2*log(2)*exp((x^3*log(2)+x^3)/log(2))+3*x^2*log(2)*exp(1/x)^2+12*
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.198 Problem number 8377

$$\int \frac{3x^2 + 3x^4 - ex^4}{1 + 6x^2 + 4x^3 + 9x^4 + e^2x^4 + 12x^5 + 4x^6 + e(-2x^2 - 6x^4 - 4x^5)} dx$$

Optimal antiderivative

$$\frac{x}{\frac{1}{x^2} + 3 - e + 2x}$$

command

```
integrate((-x^4*exp(1)+3*x^4+3*x^2)/(x^4*exp(1)^2+(-4*x^5-6*x^4-2*x^2)*exp(1)+4*x^6+12*x^5+9*
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2e - 3x^2 - 1}{2(2x^3 - x^2e + 3x^2 + 1)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.199 Problem number 8396

$$\int \frac{-9x^4 - 2x^5 + e^{15}(288x - 152x^2 - 48x^3) + e^{20}(-148 + 184x - 33x^2 - 18x^3) + e^{10}(-215x^2 + 6x^3 + 12x^4) + e^5}{-8e^5x^3 + x^4 + e^{20}(16 - 24x + 9x^2) + e^{15}(-32x + 24x^2) + e^{10}(24x^2 - 6x^3)}$$

Optimal antiderivative

$$\frac{x}{3x - (xe^{-5} - 2)^2} + x - (5 + x)^2$$

command

integrate(((−18\*x^3−33\*x^2+184\*x−148)\*exp(5)^4+(−48\*x^3−152\*x^2+288\*x)\*exp(5)^3+(12\*x^4+6\*x^3−215\*x^2)\*exp(5)^2+(16\*x^4+72\*x^3)\*exp(5)−2\*x^5−9\*x^4)/((9\*x^2−24\*x+16)\*exp(5)^4+(24\*x^2−32\*x)\*exp(5)^3+(−6\*x^3+24\*x^2)\*exp(5)^2−8\*x^3\*exp(5)+x^4), x, algorithm="giac")

Giac 1.9.0-11 via sagemath 9.6 output

$$-x^2 - 9x - \frac{xe^{10}}{x^2 - 3xe^{10} - 4xe^5 + 4e^{10}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2x^5 + 9x^4 + (18x^3 + 33x^2 - 184x + 148)e^{20} + 8(6x^3 + 19x^2 - 36x)e^{15} - (12x^4 + 6x^3 - 215x^2)e^{10} - 8(2x^5 + 9x^4)}{x^4 - 8x^3e^5 + (9x^2 - 24x + 16)e^{20} + 8(3x^2 - 4x)e^{15} - 6(x^3 - 4x^2)e^{10}} dx$$

### 100.200 Problem number 8409

$$\int \frac{-36x^2 - 12x^3 - 18x^2 \log(5) + e^{80}x^{40}(492 + 240x + 246 \log(5))}{4 + 4x + x^2 + (4 + 2x) \log(5) + \log^2(5)} dx$$

Optimal antiderivative

$$\frac{6(e^{40 \ln(x)+80} - x^2) x}{x + 2 + \ln(5)}$$

command

integrate(((246\*log(5)+240\*x+492)\*exp(40\*log(x)+80)−18\*x^2\*log(5)−12\*x^3−36\*x^2)/(log(5)^2+(2

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.201 Problem number 8422

$$\int \frac{e^{2e^4}(512x - 1024x^2 + 768x^3 - 256x^4 + 32x^5 + e^8(-4 + x^2))}{e^{16}x^2 + 4096x^4 - 8192x^5 + 6144x^6 - 2048x^7 + 256x^8 + e^8(-128x^3 + 128x^4 - 32x^5)} dx$$

Optimal antiderivative

$$\frac{e^{2e^4}}{\left(\frac{e^8}{(-2+x)^2x} - 16\right) x^2}$$



command

```
integrate((x^2-4)*exp(4)^2+32*x^5-256*x^4+768*x^3-1024*x^2+512*x)*exp(2*exp(4))/(x^2*exp(4)^
32*x^5+128*x^4-128*x^3)*exp(4)^2+256*x^8-2048*x^7+6144*x^6-8192*x^5+4096*x^4),x, algorithm="g
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{(x^2 - 4x + 4)e^{(2e^4)}}{16x^4 - 64x^3 + 64x^2 - xe^8}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

### 100.202 Problem number 8454

$$\int \frac{16x - 16ex}{-1 + e^3 + 12x - 48x^2 + 64x^3 + e^2(-3 + 12x) + e(3 - 24x + 48x^2)} dx$$

Optimal antiderivative

$$2 - \frac{8}{\left(4 - \frac{1-e}{x}\right)^2}$$

command

```
integrate((-16*x*exp(1)+16*x)/(exp(1)^3+(12*x-3)*exp(1)^2+(48*x^2-24*x+3)*exp(1)+64*x^3-
48*x^2+12*x-1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{8xe - 8x + e^2 - 2e + 1}{2(4x + e - 1)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{16(xe - x)}{64x^3 - 48x^2 + 3(4x - 1)e^2 + 3(16x^2 - 8x + 1)e + 12x + e^3 - 1} dx$$

### 100.203 Problem number 8502

$$\int \frac{18x - 10x^3 + e^5(-12 - 2x^3)}{(e^{10}x^3 - 2e^5x^4 + x^5) \log\left(\frac{\log^2(5)}{4}\right)} dx$$

Optimal antiderivative

$$\frac{2x + 10 - \frac{6}{x^2}}{\ln\left(\frac{\ln(5)^2}{4}\right)(-e^5 + x)}$$

command

`integrate((-2*x^3-12)*exp(5)-10*x^3+18*x)/(x^3*exp(5)^2-2*x^4*exp(5)+x^5)/log(1/4*log(5)^2),`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2\left(\frac{(e^{15}+5e^{10}-3)e^{(-10)}}{x-e^5} + \frac{3(x+e^5)e^{(-10)}}{x^2}\right)}{\log\left(\frac{1}{4}\log(5)^2\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.204 Problem number 8692

$$\int \frac{e^{2x + \frac{e^{-x}}{-2x^2 + x^4}} (8x^3 - 8x^5 + 2x^7 + e^{-x}(4 + 2x - 4x^2 - x^3))}{4x^3 - 4x^5 + x^7} dx$$

Optimal antiderivative

$$e^{2x} e^{-\frac{e^{-x}}{-x^4 + 2x^2}}$$

command

`integrate((-x^3-4*x^2+2*x+4)*exp(-x)+2*x^7-8*x^5+8*x^3)*exp(2*x)/(x^7-4*x^5+4*x^3)/exp(-exp(-x)/(x^4-2*x^2)),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(\frac{2x^5-4x^3+e^{(-x)}}{x^4-2x^2}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(2x^7 - 8x^5 + 8x^3 - (x^3 + 4x^2 - 2x - 4)e^{(-x)})e^{\left(2x + \frac{e^{(-x)}}{x^4 - 2x^2}\right)}}{x^7 - 4x^5 + 4x^3} dx$$

## 100.205 Problem number 8749

$$\int \frac{3 + 2x + 3x^2 + e(-6 + 2x - 32x^2 + 6x^3) + e^2(3 - 4x + 32x^2 - 24x^3 + 3x^4)}{1 + e(-2 + 2x) + e^2(1 - 2x + x^2)} dx$$

Optimal antiderivative

$$x \left( 3 + x + x^2 \left( \frac{10x}{x - x^2 - e^{-1}x} + 1 \right) \right)$$

command

```
integrate(((3*x^4-24*x^3+32*x^2-4*x+3)*exp(1)^2+(6*x^3-32*x^2+2*x-6)*exp(1)+3*x^2+2*x+3)/((x^2*x+1)*exp(1)^2+(-2+2*x)*exp(1)+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$(x^3 e^6 - 9 x^2 e^6 - 7 x e^6 + 10 x e^5) e^{(-6)} - \frac{10 (e^3 - 3 e^2 + 3 e - 1) e^{(-2)}}{x e - e + 1}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.206 Problem number 8766

$$\int \frac{4e^4 + e^2(-1 - 8x) + 4x^2 + e^4(e^4 - 2e^2x + x^2)}{e^4 - 2e^2x + x^2} dx$$

Optimal antiderivative

$$x \left( \frac{1}{x - e^2} + 4 + e^4 \right)$$

command

```
integrate(((exp(2)^2-2*exp(2)*x+x^2)*exp(4)+4*exp(2)^2+(-8*x-1)*exp(2)+4*x^2)/(exp(2)^2-2*exp(2)*x+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x e^4 + 4 x + \frac{e^2}{x - e^2}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.207 Problem number 8826

$$\int \frac{1}{1 - 2x^4 + 8x^5 - 12x^6 + 8x^7 - x^8 - 8x^9 + 28x^{10} - 56x^{11} + 70x^{12} - 56x^{13} + 28x^{14} - 8x^{15} + x^{16} + (-32x^3 + 64x^4)}$$

Optimal antiderivative

$$\frac{3}{-1 + (x^2 - x - (4 + 4x) \ln(2))^4}$$

command

```
integrate((-3072*x^3-9216*x^2-9216*x-3072)*log(2)^4+(3840*x^4+6144*x^3-3072*x-768)*log(2)^3+
1728*x^5+2304*x^3-576*x)*log(2)^2+(336*x^6-576*x^5+384*x^3-144*x^2)*log(2)-24*x^7+84*x^6-
108*x^5+60*x^4-12*x^3)/((65536*x^8+524288*x^7+1835008*x^6+3670016*x^5+4587520*x^4+3670016*x^3+
131072*x^9-786432*x^8-1835008*x^7-1835008*x^6+1835008*x^4+1835008*x^3+786432*x^2+131072*x)*lo
458752*x^7-1146880*x^6-458752*x^5+458752*x^4+458752*x^3+114688*x^2)*log(2)^6+(-57344*x^11-
114688*x^10+114688*x^9+344064*x^8-344064*x^6-114688*x^5+114688*x^4+57344*x^3)*log(2)^5+(17920
71680*x^10+107520*x^8-71680*x^6+17408*x^4-2048*x^3-3072*x^2-2048*x-512)*log(2)^4+(-
3584*x^13+7168*x^12+7168*x^11-21504*x^10+21504*x^8-7168*x^7-7168*x^6+4096*x^5+1024*x^4-
1024*x^2-512*x)*log(2)^3+(448*x^14-1792*x^13+1792*x^12+1792*x^11-4480*x^10+1792*x^9+1792*x^8-
1792*x^7+256*x^6+384*x^4-192*x^2)*log(2)^2+(-32*x^15+192*x^14-448*x^13+448*x^12-448*x^10+448*
192*x^8+64*x^7-64*x^6+64*x^4-32*x^3)*log(2)+x^16-8*x^15+28*x^14-56*x^13+70*x^12-56*x^11+28*x^
8*x^9-x^8+8*x^7-12*x^6+8*x^5-2*x^4+1),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{3}{x^8 - 4x^7 + 6x^6 - 4x^5 + 256(x^4 + 4x^3 + 6x^2 + 4x) \log(2)^4 + x^4 - 256(x^5 + 2x^4 - 2x^2 - x) \log(2)^3 + 256 \log(2)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.208 Problem number 8869

$$\int \frac{e^{-400x^5 + (2000x^4 + 400x^5) \log(x)} (-5 + 4x + (-20 - 5x) \log(x))}{200x^7 + (-2000x^6 - 400x^7) \log(x) + (5000x^5 + 2000x^6 + 200x^7) \log^2(x)} dx$$

Optimal antiderivative

$$2e^{-\frac{1}{400x^4(x+(-x-5)\ln(x))}}$$

command

```
integrate((-5*x-20)*log(x)+4*x-5)/((200*x^7+2000*x^6+5000*x^5)*log(x)^2+(-400*x^7-
2000*x^6)*log(x)+200*x^7)/exp(-1/((400*x^5+2000*x^4)*log(x)-400*x^5)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2e^{\left(\frac{1}{400(x^5 \log(x) - x^5 + 5x^4 \log(x))}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{(5(x+4)\log(x) - 4x + 5)e^{\left(-\frac{1}{400(x^5 - (x^5 + 5x^4)\log(x))}\right)}}{200(x^7 + (x^7 + 10x^6 + 25x^5)\log(x)^2 - 2(x^7 + 5x^6)\log(x))} dx$$

**100.209 Problem number 8892**

$$\int \frac{8x^5 + e^8(-48 + 8x + 16x^2 + 2x^3) + e^4(192x - 80x^3 - 8x^4)}{-8x^4 + 4x^6 + e^8(-8 + 8x + 2x^2 - 4x^3 + x^4) + e^4(-16x^2 + 8x^3 + 8x^4 - 4x^5)} dx$$

Optimal antiderivative

$$\frac{4}{\frac{2}{3(x-2x^2e^{-4})} - \frac{1}{3}} + \ln(-x^2 + 2)$$

command

`integrate(((2*x^3+16*x^2+8*x-48)*exp(4)^2+(-8*x^4-80*x^3+192*x)*exp(4)+8*x^5)/((x^4-4*x^3+2*x^2+8*x-8)*exp(4)^2+(-4*x^5+8*x^4+8*x^3-16*x^2)*exp(4)+4*x^6-8*x^4),x, algorithm="gia`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{24e^4}{2x^2 - xe^4 + 2e^4} + \log(|x^2 - 2|)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.210 Problem number 9094**

$$\int \frac{-16e^{4x} + 288x - 576x^2 + e^{e^{5-x}}(e^{4x}(-1 - 9e^{5-x}) + 18x - 36x^2)}{144e^{4x} + 9e^{e^{5-x}+4x}} dx$$

Optimal antiderivative

$$\ln(16 + e^{e^{5-x}}) - \frac{x}{9} - \frac{1}{9} + x^2e^{-4x}$$

command

```
integrate(((((-9*exp(5-x)-1)*exp(4*x)-36*x^2+18*x)*exp(exp(5-x))-16*exp(4*x)-576*x^2+288*x)/(9*x))+144*exp(4*x)),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{1}{9} \left( 9(x-5)^2 e^{(-4x+20)} - (x-5)e^{20} + 90(x-5)e^{(-4x+20)} + 9e^{20} \log \left( e^{(e^{(-x+5)})} + 16 \right) + 225 e^{(-4x+20)} \right) e^{(-20)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{576x^2 + (36x^2 + (9e^{(-x+5)} + 1)e^{(4x)} - 18x)e^{(e^{(-x+5)})} - 288x + 16e^{(4x)}}{9(16e^{(4x)} + e^{(4x+e^{(-x+5)})})} dx$$

### 100.211 Problem number 9153

$$\int \frac{40 - 100x^2 - 4e^{32}x^4 + e^{16}(-16x + 40x^3)}{25x^2 - 10e^{16}x^3 + e^{32}x^4} dx$$

Optimal antiderivative

$$\frac{8}{x^2 \left( e^{16} - \frac{5}{x} \right)} - 4x$$

command

```
integrate((-4*x^4*exp(16)^2+(40*x^3-16*x)*exp(16)-100*x^2+40)/(x^4*exp(16)^2-10*x^3*exp(16)+2
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-4x + \frac{8}{x^2 e^{16} - 5x}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.212 Problem number 9157

$$\int \frac{150 - 30e - 225x}{100x^3 + 4e^2x^3 - 200x^4 + 100x^5 + e(-40x^3 + 40x^4)} dx$$

Optimal antiderivative

$$4 + \frac{5}{\left( \frac{4e}{3} + \frac{20x}{3} - \frac{20}{3} \right) x^2}$$

command

`integrate((-30*exp(1)-225*x+150)/(4*x^3*exp(1)^2+(40*x^4-40*x^3)*exp(1)+100*x^5-200*x^4+100*x`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{375}{4(5x + e - 5)(e^2 - 10e + 25)} - \frac{15(5x - e + 5)}{4x^2(e^2 - 10e + 25)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.213 Problem number 9163

$$\int \frac{2500e^{10} - 4x^2}{x^4 + e^{20}(390625 + 1250x + x^2) + e^{10}(1250x^2 + 2x^3)} dx$$

Optimal antiderivative

$$\frac{4}{\left(1 + \frac{625}{x}\right) e^{10} + x}$$

command

`integrate((2500*exp(5)^2-4*x^2)/((x^2+1250*x+390625)*exp(5)^4+(2*x^3+1250*x^2)*exp(5)^2+x^4),`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4x}{x^2 + xe^{10} + 625e^{10}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{4(x^2 - 625e^{10})}{x^4 + (x^2 + 1250x + 390625)e^{20} + 2(x^3 + 625x^2)e^{10}} dx$$

### 100.214 Problem number 9181

$$\int \frac{e^{\frac{-36+36ex-x^4-x^3 \log(e^e-x)}{-x^2+ex^3}} (72x + 72e^2x^3 - x^4 - 2x^5 + e(-144x^2 + x^5 + x^6)) + e^e(-72 - 72e^2x^2 + 2x^4 + e(144x -$$

$$-x^4 + 2ex^5 - e^2x^6 + e^e(x^3 - 2ex^4 + e^2x^5))}{-x^4 + 2ex^5 - e^2x^6 + e^e(x^3 - 2ex^4 + e^2x^5)}$$

Optimal antiderivative

$$e^{\frac{36}{x^2} - \frac{(x+\ln(e^e-x))x}{ex-1}}$$

command

```
integrate(((x^3*exp(exp(1))-x^4)*log(exp(exp(1))-x)+(-72*x^2*exp(1)^2+(-x^5+144*x)*exp(1)+2*x
72)*exp(exp(1))+72*x^3*exp(1)^2+(x^6+x^5-144*x^2)*exp(1)-2*x^5-x^4+72*x)*exp((-x^3*log(exp(ex
x)+36*x*exp(1)-x^4-36)/(x^3*exp(1)-x^2)))/((x^5*exp(1)^2-2*x^4*exp(1)+x^3)*exp(exp(1))-
x^6*exp(1)^2+2*x^5*exp(1)-x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e \left( -\frac{x^4}{x^3 e^{-x^2}} - \frac{x^3 \log(-x+e^e)}{x^3 e^{-x^2}} + \frac{36 x e}{x^3 e^{-x^2}} - \frac{36}{x^3 e^{-x^2}} \right)$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(2x^5 + x^4 - 72x^3e^2 - (x^6 + x^5 - 144x^2)e - (2x^4 - 72x^2e^2 - (x^5 - 144x)e - 72)e^e + (x^4 - x^3e^e) \log(-x + e^e))}{x^6e^2 - 2x^5e + x^4 - (x^5e^2 - 2x^4e + x^3)e^e} dx$$

**100.215 Problem number 9189**

$$\int -\frac{40e^{\frac{5}{4\log^2(x^2)} + \frac{5}{4\log^2(x^2)}}}{x \log^3(x^2)} dx$$

Optimal antiderivative

$$e^{\frac{5}{4\ln(x^2)^2} + 3\ln(2)}$$

command

```
integrate(-5*exp(5/4/log(x^2)^2)*exp(exp(5/4/log(x^2)^2)+3*log(2))/x/log(x^2)^3,x, algorithm=
```

Giac 1.9.0-11 via sagemath 9.6 output

$$8e \left( \frac{\left( \frac{5}{4\log(x^2)^2} \right) \log(x^2)^2 + 5}{4\log(x^2)^2} - \frac{5}{4\log(x^2)^2} \right)$$

Giac 1.7.0 via sagemath 9.3 output

*undef*



## 100.216 Problem number 9199

$$\int \frac{e^{4-2e^x} (2x + (2-2x^2) \log(4) + e^x (2x^2 + (2x-2x^3) \log(4)))}{x^2} + \frac{(-2x + 4x^2 \log(4)) \log(-x + (-1 + x^2) \log(4)) + \frac{e^{2-e^x} (2x-4x^2)}{-9x^2 + (-9x + 9x^3) \log(4)}}{-9x^2 + (-9x + 9x^3) \log(4)}$$

Optimal antiderivative

$$\frac{(e^{-\ln(x)-e^x+2} - \ln(2(x^2-1)\ln(2)-x)) \left( \frac{e^{-\ln(x)-e^x+2}}{3} - \frac{\ln(2(x^2-1)\ln(2)-x)}{3} \right)}{3}$$

command

```
integrate((((2*(-2*x^3+2*x)*log(2)+2*x^2)*exp(x)+2*(-2*x^2+2)*log(2)+2*x)*exp(-log(x)-exp(x)+2)^2+(((2*(2*x^3-2*x)*log(2)-2*x^2)*exp(x)+2*(2*x^2-2)*log(2)-2*x)*log(2*(x^2-1)*log(2)-x)-8*x^2*log(2)+2*x)*exp(-log(x)-exp(x)+2)+(8*x^2*log(2)-2*x)*log(2*(x^2-1)*log(2)-x))/(2*(9*x^3-9*x)*log(2)-9*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x^2 \log(2x^2 \log(2) - x - 2 \log(2))^2 - 2xe^{(-e^x+2)} \log(2x^2 \log(2) - x - 2 \log(2)) + e^{(-2e^x+4)}}{9x^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2((4x^2 \log(2) + ((x^2 - 2(x^3 - x) \log(2))e^x - 2(x^2 - 1) \log(2) + x) \log(2(x^2 - 1) \log(2) - x) - x)e^{(-e^x - \log(2))}}{9(x^2)}$$

## 100.217 Problem number 9327

$$\int \frac{e^{\frac{2(-198x-558x^2-18x^3+288x^4-72x^5+e^{2x}(-50x^2-10x^3+32x^4-8x^5)+e^x(60x+336x^2+36x^3-192x^4+48x^5))}{9+90x+189x^2-180x^3+36x^4+e^x(-30x-138x^2+120x^3-24x^4)+e^{2x}(25x^2-20x^3+4x^4)}}}{27+405x+1863x^2+1755x^3-3726x^4+1620x^5-216x^6+e^{2x}}$$

Optimal antiderivative

$$e^{\frac{36}{(x(-e^x+3)(-5+2x)-3)^2} - 4x - 4}$$

command

```
integrate(((((-32*x^6+240*x^5-600*x^4+500*x^3)*exp(x)^3+(288*x^6-2160*x^5+5256*x^4-3780*x^3-900*x^2)*exp(x)^2+(-864*x^6+6480*x^5-15336*x^4+9180*x^3+5040*x^2+612*x+360)*exp(x)+864*x^6-6480*x^5+14904*x^4-7020*x^3-7452*x^2-756*x-1188)*exp(((8*x^5+32*x^4-10*x^3-50*x^2)*exp(x)^2+192*x^4+36*x^3+336*x^2+60*x)*exp(x)-72*x^5+288*x^4-18*x^3-558*x^2-198*x)/((4*x^4-20*x^3+25*x^2+24*x^4+120*x^3-138*x^2-30*x)*exp(x)+36*x^4-180*x^3+189*x^2+90*x+9))^2/((8*x^6-60*x^5+150*x^4-125*x^3)*exp(x)^3+(-72*x^6+540*x^5-1314*x^4+945*x^3+225*x^2)*exp(x)^2+(216*x^6-1620*x^5+3834*x^4-2295*x^3-1296*x^2-135*x)*exp(x)-216*x^6+1620*x^5-3726*x^4+1755*x^3+1863*x^2+405*x+27)),x, algo
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(\frac{-3x - 4x^5 e^{(2x)} - 24x^5 e^x + 36x^5 - 4x^4 e^{(2x)} + 24x^4 e^x - 36x^4 - 55x^3 e^{(2x)} + 342x^3 e^x - 531x^3 + 100x^2 e^{(2x)} - 582x^2 e^x + 846x^2 - 120xe^x + 369x}{4x^4 e^{(2x)} - 24x^4 e^x + 36x^4 - 20x^3 e^{(2x)} + 120x^3 e^x - 180x^3 + 25x^2 e^{(2x)} - 138x^2 e^x + 189x^2 - 30xe^x + 90x + 9}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.218 Problem number 9331**

$$\int \frac{-32 \log(3) + 32 \log(3) \log\left(\frac{x}{3}\right) + (-8 + 16x) \log(3) \log^2\left(\frac{x}{3}\right) - 8 \log(3) \log^2\left(\frac{x}{3}\right) \log(x)}{\left(-4x \log\left(\frac{x}{3}\right) - x^2 \log^2\left(\frac{x}{3}\right) + x \log^2\left(\frac{x}{3}\right) \log(x)\right) \log^2\left(\frac{16x^2 + 8x^3 \log\left(\frac{x}{3}\right) + x^4 \log^2\left(\frac{x}{3}\right) + (-8x^2 \log\left(\frac{x}{3}\right) - 2x^3 \log^2\left(\frac{x}{3}\right)) \log(x) + x^2 \log\left(\frac{x}{3}\right)}{\log^2\left(\frac{x}{3}\right)}\right)}$$

Optimal antiderivative

$$\frac{4 \ln(3)}{\ln\left(x^2 \left(\frac{4}{\ln\left(\frac{x}{3}\right)} - \ln(x) + x\right)^2\right)}$$

command

```
integrate((-8*log(3)*log(1/3*x)^2*log(x)+(16*x-8)*log(3)*log(1/3*x)^2+32*log(3)*log(1/3*x)-32*log(3))/(x*log(1/3*x)^2*log(x)-x^2*log(1/3*x)^2-4*x*log(1/3*x))/log((x^2*log(1/3*x)^2*log(2*x^3*log(1/3*x)^2-8*x^2*log(1/3*x))*log(x)+x^4*log(1/3*x)^2+8*x^3*log(1/3*x)+16*x^2)/log(1/3
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(x^2 \log(3)^2 - 2x^2 \log(3) \log(x) - 2x \log(3)^2 \log(x) + x^2 \log(x)^2 + 4x \log(3) \log(x)^2 + \log(3)^2 \log(x)^2 - 2x\right)$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

**100.219 Problem number 9335**

$$\int \frac{112 + 8e^2 + e(-60 - 14x) + 56x + 7x^2}{64 + 4e^2 + e(-32 - 8x) + 32x + 4x^2} dx$$

Optimal antiderivative

$$\left(2 - \frac{x}{16 + 4x - 4e}\right) (4 + x) + \ln(\ln(5))$$

command

`integrate((8*exp(1)^2+(-14*x-60)*exp(1)+7*x^2+56*x+112)/(4*exp(1)^2+(-8*x-32)*exp(1)+4*x^2+32`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{7}{4}x - \frac{e^2 - 4e}{4(x - e + 4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.220 Problem number 9374

$$\int \frac{e^8(8 + 16x) \log(5)}{16 + e^{16} + 128x + 384x^2 + 512x^3 + 256x^4 + e^8(8 + 32x + 32x^2)} dx$$

Optimal antiderivative

$$\frac{\ln(5)}{\ln\left(e^{\frac{2e^8}{(4x+2)^2}}\right) + 2}$$

command

`integrate((16*x+8)*exp(4)^2*log(5)/(exp(4)^4+(32*x^2+32*x+8)*exp(4)^2+256*x^4+512*x^3+384*x^2`

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{e^8 \log(5)}{2(16x^2 + 16x + e^8 + 4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

### 100.221 Problem number 9385

$$\int \frac{e^{4+2e^{-\frac{1}{4+x}}+16\log^2(x)} \left( -32 + 16x + 2e^{-\frac{1}{4+x}}x - 2x^2 + (512 - 256x + 32x^2) \log(x) \right)}{16x^3 - 8x^4 + x^5} dx$$

Optimal antiderivative

$$\frac{e^{16\ln(x)^2} e^{2e^{\frac{1}{4-x}}+4}}{x^2} - 5$$

command

```
integrate(((32*x^2-256*x+512)*log(x)+2*x*exp(-1/(x-4))-2*x^2+16*x-32)*exp(exp(-1/(x-4)))+2)^2*exp(4*log(x)^2)^4/(x^5-8*x^4+16*x^3),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$e^{\left(\frac{64x \log(x)^2 + 8xe^{\left(-\frac{1}{x-4}\right)} - 256 \log(x)^2 - x - 32e^{\left(-\frac{1}{x-4}\right)}}{4(x-4)} + \frac{1}{x-4} + \frac{17}{4}\right)} x^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2\left(x^2 - xe^{\left(-\frac{1}{x-4}\right)} - 16(x^2 - 8x + 16)\log(x) - 8x + 16\right)e^{\left(16\log(x)^2 + 2e^{\left(-\frac{1}{x-4}\right)} + 4\right)}}{x^5 - 8x^4 + 16x^3} dx$$

### 100.222 Problem number 9448

$$\int \frac{(-3e^6 + e^3(-5 - 2x))\log(4) + (-3e^6 + e^3(-5 - 2x))\log(\log(4\log(2)))}{25x^2 + 9e^6x^2 + 10x^3 + x^4 + e^3(30x^2 + 6x^3)} dx$$

Optimal antiderivative

$$\frac{\ln(\ln(4\ln(2))) + 2\ln(2)}{x((5+x)e^{-3} + 3)}$$

command

```
integrate(((3*exp(3)^2+(-2*x-5)*exp(3))*log(log(4*log(2)))+2*(-3*exp(3)^2+(-2*x-5)*exp(3))*1
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4e^6 \log(2)^2 + 4e^6 \log(2) \log(\log(4\log(2))) + e^6 \log(\log(4\log(2)))^2}{2(3xe^6 + (x^2 + 5x)e^3) \log(2) + (3xe^6 + (x^2 + 5x)e^3) \log(\log(4\log(2)))}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.223 Problem number 9489

$$\int \frac{-3e^3x^2 + 3x^3 + e^{-2-2x}(-18e^3 + 18x) + e^{-1-x}(-15e^3x + 12x^2 - 3x^3) + (e^{-2-2x}(-18 + 18e^3) - 3x^2 + 3e^3x^2 - 6e^{4-2x}x^2 + 5e^{5-x}x^3 + e^6x^4 + (-12e^{1-2x}x^2 - 10e^{2-x}x^3 -$$

Optimal antiderivative

$$\frac{3 \ln(x) - 3x}{\left(\ln\left(\frac{x}{2+x e^{1+x}} + x\right) - e^3\right) x}$$

command

```
integrate(((((-18*exp(-1-x)^2-15*x*exp(-1-x)-3*x^2)*log(x)+18*exp(-1-x)^2+15*x*exp(-1-x)+3*x^2)*log((3*x*exp(-1-x)+x^2)/(2*exp(-1-x)+x))+((18*exp(3)-18)*exp(-1-x)^2+(15*x*exp(3)-12*x)*exp(-1-x)+3*x^2*exp(3)-3*x^2)*log(x)+(-18*exp(3)+18*x)*exp(-1-x)^2+(-15*x*exp(3)-3*x^3+12*x^2)*exp(-1-x)-3*x^2*exp(3)+3*x^3)/((6*x^2*exp(-1-x)^2+5*x^3*exp(-1-x)+x^4)*log((3*x*1-x)+x^2)/(2*exp(-1-x)+x))^2+(-12*x^2*exp(3)*exp(-1-x)^2-10*x^3*exp(3)*exp(-1-x)-2*x^4*exp(3)*1-x)+x^2)/(2*exp(-1-x)+x))+6*x^2*exp(3)^2*exp(-1-x)^2+5*x^3*exp(3)^2*exp(-1-x)+x^4*exp(3)^2),
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.224 Problem number 9497

$$\int \frac{144e^{34}}{16x^2 + e^{16}(-24e^2x - 24x^2) + e^{32}(9e^4 + 18e^2x + 9x^2)} dx$$

Optimal antiderivative

$$\frac{16x}{e^2 + x - \frac{4x e^{-16}}{3}}$$

command

```
integrate(144*exp(2)*exp(16)^2/((9*exp(2)^2+18*exp(2)*x+9*x^2)*exp(16)^2+(-24*exp(2)*x-24*x^2)*exp(16)+16*x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{144 e^{34}}{(3 x e^{16} - 4 x + 3 e^{18})(3 e^{16} - 4)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

## 100.225 Problem number 9580

$$\int \frac{-27 + e^6 + e^4(-9 - 3x) - 27x - 9x^2 - x^3 + 15x^4 + 3x^5 + e^2(27 + 18x + 3x^2 - 5x^4)}{-27 + e^6 + e^4(-9 - 3x) - 27x - 9x^2 - x^3 + e^2(27 + 18x + 3x^2)} dx$$

Optimal antiderivative

$$x - \frac{x^5}{(-e^2 + 3 + x)^2}$$

command

`integrate((exp(2)^3+(-3*x-9)*exp(2)^2+(-5*x^4+3*x^2+18*x+27)*exp(2)+3*x^5+15*x^4-x^3-9*x^2-27*x-27)/(exp(2)^3+(-3*x-9)*exp(2)^2+(3*x^2+18*x+27)*exp(2)-x^3-9*x^2-27*x-27),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{-x^3 - 2x^2e^2 + 6x^2 - 3xe^4 + 18xe^2 - 26x + 5xe^8 - 60xe^6 + 270xe^4 - 540xe^2 + 405x - 4e^{10} + 60e^8 - 360e^6 + 1080e^4 - 1620e^2 + 972}{(x - e^2 + 3)^2}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.226 Problem number 9605

$$\int \frac{-9x^3 - 18x^4 - 15x^5 - 4x^6 + e^4(27x^2 + 36x^3 + 25x^4 + 6x^5)}{e^{12} - 3e^8x + 3e^4x^2 - x^3} dx$$

Optimal antiderivative

$$\frac{x^3(3 + x)(x^2 + 2x + 3)}{(x - e^4)^2}$$

command

`integrate(((6*x^5+25*x^4+36*x^3+27*x^2)*exp(4)-4*x^6-15*x^5-18*x^4-9*x^3)/(exp(4)^3-3*x*exp(4)^2+3*x^2*exp(4)-x^3),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$x^4 + 2x^3e^4 + 5x^3 + 3x^2e^8 + 10x^2e^4 + 9x^2 + 4xe^{12} + 15xe^8 + 18xe^4 + 9x + \frac{6xe^{20} + 25xe^{16} + 36xe^{12} + 27xe^8 - 5e^{24} - 20e^{20} - 27e^{16} - 18e^{12}}{(x - e^4)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{4x^6 + 15x^5 + 18x^4 + 9x^3 - (6x^5 + 25x^4 + 36x^3 + 27x^2)e^4}{x^3 - 3x^2e^4 + 3xe^8 - e^{12}} dx$$

## 100.227 Problem number 9626

$$\int \frac{x^{\frac{5x^2}{15-8x+6x^2+(-8+2x)\log(4)+\log^2(4)}} (75x - 40x^2 + 30x^3 + (-40x + 10x^2) \log(4) + 5x \log^2(4) + (150x - 40x^2 + (-80x - 225 - 240x + 244x^2 - 96x^3 + 36x^4 + (-240 + 188x - 128x^2 + 24x^3) \log(4) + (94 - 48x + 16x^2) \log^2(4))}{225 - 240x + 244x^2 - 96x^3 + 36x^4 + (-240 + 188x - 128x^2 + 24x^3) \log(4) + (94 - 48x + 16x^2) \log^2(4)}$$

Optimal antiderivative

$$e^{\frac{\ln(x)x}{(x-4+2\frac{\ln(2)}{5x})^2-1}+x}$$

command

```
integrate(((40*x*log(2)^2+2*(10*x^2-80*x)*log(2)-40*x^2+150*x)*log(x)+20*x*log(2)^2+2*(10*x^2-40*x)*log(2)+30*x^3-40*x^2+75*x)*exp(5*x^2*log(x)/(4*log(2)^2+2*(2*x-8)*log(2)+6*x^2-8*x+15))/(16*log(2)^4+8*(4*x-16)*log(2)^3+4*(16*x^2-48*x+94)*log(2)^2+2*(24*x^3-128*x^2+188*x-240)*log(2)+36*x^4-96*x^3+244*x^2-240*x+225),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5x^2}{x^{6x^2+4x\log(2)+4\log(2)^2-8x-16\log(2)+15}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: TypeError

## 100.228 Problem number 9704

$$\int \frac{(9 + e - x^2 - 18x^4 + 8x^5 - 7x^8) (i\pi + \log(3))}{81 + e^2 - 108x + 54x^2 - 12x^3 + 109x^4 - 108x^5 + 36x^6 - 4x^7 + 54x^8 - 36x^9 + 6x^{10} + 12x^{12} - 4x^{13} + x^{16} + e}$$

Optimal antiderivative

$$\frac{x(\ln(3) + i\pi)}{(x^4 - x + 3)^2 + e}$$

command

```
integrate((exp(1)-7*x^8+8*x^5-18*x^4-x^2+9)*(log(3)+I*pi)/(exp(1)^2+(2*x^8-4*x^5+12*x^4+2*x^2-12*x+18)*exp(1)+x^16-4*x^13+12*x^12+6*x^10-36*x^9+54*x^8-4*x^7+36*x^6-108*x^5+109*x^4-12*x^3+54*x^2-108*x+81),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{(-i\pi - \log(3))x}{x^8 - 2x^5 + 6x^4 + x^2 - 6x + e + 9}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{(i\pi + \log(3))(7x^8 - 8x^5 + 18x^4 + x^2 - e - 9)}{x^{16} - 4x^{13} + 12x^{12} + 6x^{10} - 36x^9 + 54x^8 - 4x^7 + 36x^6 - 108x^5 + 109x^4 - 12x^3 + 54x^2 + 2(x^8 - 2x^5 + 6x^4 - 4x^2 + 3x - 1) + e}$$

## 100.229 Problem number 9755

$$\int \frac{40x + e^{5/x+x}(-20e^{5/x} + 8x + 4x^2) + e^{25x^2-50x^3+25x^4}(-4 - 200x^2 + 600x^3 - 400x^4)}{e^{50x^2-100x^3+50x^4}x^2 - 10e^{25x^2-50x^3+25x^4}x^3 + 25x^4 + e^{2e^{5/x}+2x}x^4 + e^{e^{5/x}+x}(-2e^{25x^2-50x^3+25x^4}x^3 + 10x^4)} dx$$

Optimal antiderivative

$$\frac{4}{x \left( e^{x^2(5x-5)^2} - x \left( e^{e^{5/x}+x} + 5 \right) \right)}$$

command

```
integrate((( -20*exp(5/x)+4*x^2+8*x)*exp(exp(5/x)+x)+(-400*x^4+600*x^3-200*x^2-4)*exp(25*x^4-50*x^3+25*x^2)+40*x)/(x^4*exp(exp(5/x)+x)^2+(-2*x^3*exp(25*x^4-50*x^3+25*x^2)+10*x^4)*exp(exp(50*x^3+25*x^2))^2-10*x^3*exp(25*x^4-50*x^3+25*x^2)+25*x^4), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

output too large to display

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.230 Problem number 9776

$$\int \frac{e^x e^{\frac{x}{x^2+x^2 \log(\frac{1}{x})}} (e^x(90 - 90x) + e^x(180 - 90x) \log(\frac{1}{x})) + e^{\frac{2e^x}{x^2+x^2 \log(\frac{1}{x})}} (e^x(-50 + 50x) + e^x(-100 + 50x) \log(\frac{1}{x}))}{9x^3 + 18x^3 \log(\frac{1}{x}) + 9x^3 \log^2(\frac{1}{x})} dx$$

Optimal antiderivative

$$\left( 3 - \frac{5 e^{\frac{e^x}{(x+x \ln(\frac{1}{x}))x}}}{3} \right)^2$$

command

```
integrate((((50*x-100)*exp(x)*log(1/x)+(50*x-50)*exp(x))*exp(exp(x)/(x^2*log(1/x)+x^2))^2+((-90*x+180)*exp(x)*log(1/x)+(-90*x+90)*exp(x))*exp(exp(x)/(x^2*log(1/x)+x^2)))/(9*x^3*log(1/x)^2+18*x^3*log(1/x)+9*x^3*log(1/x)^2), x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-10 e^{\left(-\frac{e^x}{x^2 \log(x)-x^2}\right)} + \frac{25}{9} e^{\left(-\frac{2e^x}{x^2 \log(x)-x^2}\right)}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{10 \left( 5 \left( (x-2)e^x \log\left(\frac{1}{x}\right) + (x-1)e^x \right) e^{\left(\frac{2e^x}{x^2 \log(\frac{1}{x})+x^2}\right)} - 9 \left( (x-2)e^x \log\left(\frac{1}{x}\right) + (x-1)e^x \right) e^{\left(\frac{e^x}{x^2 \log(\frac{1}{x})+x^2}\right)} \right)}{9 \left( x^3 \log\left(\frac{1}{x}\right)^2 + 2x^3 \log\left(\frac{1}{x}\right) + x^3 \right)} dx$$



## 100.231 Problem number 9830

$$\int \frac{-40x + 60x^5 + \frac{(-10+15x^4)^5(-100x-1350x^5)}{e}}{-8 + 12x^4 + \frac{(-10+15x^4)^5(-40+60x^4)}{e} + \frac{(-10+15x^4)^{10}(-50+75x^4)}{e^2}} dx$$

Optimal antiderivative

$$\frac{x^2}{e^{5 \ln(15x^4-10)-1} + \frac{2}{5}}$$

command

```
integrate((-1350*x^5-100*x)*exp(5*log(15*x^4-10)-1)+60*x^5-40*x)/((75*x^4-50)*exp(5*log(15*x^4-10)-1)^2+(60*x^4-40)*exp(5*log(15*x^4-10)-1)+12*x^4-8),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{5x^2e}{3796875x^{20} - 12656250x^{16} + 16875000x^{12} - 11250000x^8 + 3750000x^4 + 2e - 500000}$$

Giac 1.7.0 via sagemath 9.3 output

*undef*

## 100.232 Problem number 9875

$$\int \frac{20 - 12e^{10-8x} - 40x - 8x^2 + 24x^3 - 12x^4}{25 - 10x + 31x^2 - 36x^3 + 15x^4 - 18x^5 + 9x^6 + e^{10-8x}(e^6 - 6e^3x + 9x^2) + e^6(x^2 - 2x^3 + x^4) + e^3(-10x + 12x^2 - 6x^3 + 3x^4)}$$

Optimal antiderivative

$$\frac{4}{3x - \frac{5-x}{x^2-x-e^{5-4x}} - e^3}$$

command

```
integrate((-12*exp(-4*x+5)^2+(24*x^2-8*x-76)*exp(-4*x+5)-12*x^4+24*x^3-8*x^2-40*x+20)/((exp(3)-6*x*exp(3)+9*x^2)*exp(-4*x+5)^2+((-2*x^2+2*x)*exp(3)^2+(12*x^3-12*x^2+2*x-10)*exp(3)-18*x^4+18*x^3-6*x^2+30*x)*exp(-4*x+5)+(x^4-2*x^3+x^2)*exp(3)^2+(-6*x^5+12*x^4-8*x^3+12*x^2-10*x)*exp(3)+9*x^6-18*x^5+15*x^4-36*x^3+31*x^2-10*x+25),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{16 \left( (4x - 5)^2 + 24x - 16e^{(-4x+5)} - 25 \right)}{3(4x - 5)^3 - 4(4x - 5)^2e^3 + 33(4x - 5)^2 - 24(4x - 5)e^3 - 48(4x - 5)e^{(-4x+5)} + 484x - 20e^3 + 64e^{(-4x+8)} - 120}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

## 100.233 Problem number 9889

$$\int \frac{e^{-x} \left( e^{2e^{-x}x} (2500x^2 - 2500x^3) + e^x (1250e^5 + 300x^2 + 4x^3) + e^{e^{-x}x} (-7500x^2 - 100e^x x^2 + 7400x^3 + 100x^4) \right)}{625x^2} dx$$

Optimal antiderivative

$$\frac{2 \left( e^{x e^{-x}} - 3 - \frac{x}{25} \right)^2 x - 2 e^5}{x}$$

command

```
integrate(1/625*((-2500*x^3+2500*x^2)*exp(x/exp(x))^2+(-100*exp(x)*x^2+100*x^4+7400*x^3-7500*x^2)*exp(x/exp(x))+(1250*exp(5)+4*x^3+300*x^2)*exp(x))/exp(x)/x^2,x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left( x^3 e^{(-x)} - 50 x^2 e^{(x e^{(-x)} - x)} + 150 x^2 e^{(-x)} + 625 x e^{(2 x e^{(-x)} - x)} - 3750 x e^{(x e^{(-x)} - x)} - 625 e^{(-x+5)} \right) e^x}{625 x}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2 \left( 1250 (x^3 - x^2) e^{(2 x e^{(-x)})} - 50 (x^4 + 74 x^3 - x^2 e^x - 75 x^2) e^{(x e^{(-x)})} - (2 x^3 + 150 x^2 + 625 e^5) e^x \right) e^{(-x)}}{625 x^2} dx$$

## 100.234 Problem number 9890

$$\int \frac{4 - 2x + e^2(2x - x^2) + e^{\frac{x^2 + \log^2\left(\frac{2+e^2x}{x}\right)}} \left( -2x - 4x^2 + e^2(-x^2 - 2x^3) \right)}{16 + 24x + 12x^2 + 2x^3 + e^2(8x + 12x^2 + 6x^3 + x^4) + e^{\frac{3\left(x^2 + \log^2\left(\frac{2+e^2x}{x}\right)\right)}{x}} (2x^3 + e^2x^4) + e^{\frac{2\left(x^2 + \log^2\left(\frac{2+e^2x}{x}\right)\right)}{x}} (12x^3 + e^2x^4)} dx$$

Optimal antiderivative

$$\frac{x}{\left( x + e^{x + \frac{\ln\left(\frac{e^2x+2}{x}\right)^2}{x}} x + 2 \right)^2}$$

command

```
integrate((((2*exp(2)*x+4)*log((exp(2)*x+2)/x)^2+8*log((exp(2)*x+2)/x)+(-2*x^3-x^2)*exp(2)-4*x^2-2*x)*exp((log((exp(2)*x+2)/x)^2+x^2)/x)+(-x^2+2*x)*exp(2)+4-2*x)/((x^4*exp(2)+2*x^3)*exp(2)+16+24*x+12*x^2+2*x^3+e^2*(8*x+12*x^2+6*x^3+x^4))+e^(3*(x^2+log^2((2+e^2*x)/x))/x)*(2*x^3+e^2*x^4)+e^(2*(x^2+log^2((2+e^2*x)/x))/x)*(12*x^3+e^2*x^4),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{x}{x^2 e^{\left(\frac{2\left(x^2 + \log\left(\frac{x e^2 + 2}{x}\right)\right)^2}{x}\right)} + 2 x^2 e^{\left(\frac{x^2 + \log\left(\frac{x e^2 + 2}{x}\right)}{x}\right)} + x^2 + 4 x e^{\left(\frac{x^2 + \log\left(\frac{x e^2 + 2}{x}\right)}{x}\right)} + 4 x + 4}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out

### 100.235 Problem number 9903

$$\int \frac{90e^{53} - 120x^2}{9e^{106} + 100x^2 + 4e^2x^2 + 80x^3 + 16x^4 + e^{53}(60x + 12ex + 24x^2) + e(40x^2 + 16x^3)} dx$$

Optimal antiderivative

$$\frac{15}{2x + e + 5 + \frac{3e^3e^{50}}{2x}}$$

command

`integrate((90*exp(3)*exp(25)^2-120*x^2)/(9*exp(3)^2*exp(25)^4+(12*x*exp(1)+24*x^2+60*x)*exp(3`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{30x}{4x^2 + 2xe + 10x + 3e^{53}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{30(4x^2 - 3e^{53})}{16x^4 + 80x^3 + 4x^2e^2 + 100x^2 + 12(2x^2 + xe + 5x)e^{53} + 8(2x^3 + 5x^2)e + 9e^{106}} dx$$

### 100.236 Problem number 9911

$$\int \frac{-e^{2-2x}x^2 + e^{1-x+\frac{1}{4}(4+\log(4))}(-5+5x-2x^2) + e^{\frac{1}{2}(4+\log(4))}(-5-x^2)}{e^{2-2x}x^2 + e^{\frac{1}{2}(4+\log(4))}x^2 + 2e^{1-x+\frac{1}{4}(4+\log(4))}x^2} dx$$

Optimal antiderivative

$$\frac{5}{x + x e^{-1-\frac{\ln(2)}{2}} e^{1-x}} - x$$

command

`integrate((-x^2-5)*exp(1+1/2*log(2))^2+(-2*x^2+5*x-5)*exp(1-x)*exp(1+1/2*log(2))-x^2*exp(1-x)^2)/(x^2*exp(1+1/2*log(2))^2+2*x^2*exp(1-x)*exp(1+1/2*log(2))+x^2*exp(1-x)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2(2x - \log(2) - 4)^2 e^2 + (2x - \log(2) - 4)^2 e^{(-x + \frac{1}{2} \log(2) + 2)} + 2(2x - \log(2) - 4)e^2 \log(2) + (2x - \log(2) - 4)}{}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{x^2 e^{(-2x+2)} + (2x^2 - 5x + 5)e^{(-x+\frac{1}{2}\log(2)+2)} + (x^2 + 5)e^{(\log(2)+2)}}{2x^2 e^{(-x+\frac{1}{2}\log(2)+2)} + x^2 e^{(-2x+2)} + x^2 e^{(\log(2)+2)}} dx$$

**100.237 Problem number 9986**

$$\int -\frac{2e^{\frac{2x}{e^{10}(25+5x)+e^5(-50-10x)\log(4)+(25+5x)\log^2(4)}}}{e^{10}(25+10x+x^2)+e^5(-50-20x-2x^2)\log(4)+(25+10x+x^2)\log^2(4)} dx$$

Optimal antiderivative

$$5 - e^{\frac{x}{2(e^5 - 2 \ln(2))^2 (\frac{25}{4} + \frac{5x}{4})}}$$

command

`integrate(-2*exp(2*x/(4*(25+5*x)*log(2)^2+2*(-10*x-50)*exp(5)*log(2)+(25+5*x)*exp(5)^2))/(4*(2*x^2-20*x-50)*exp(5)*log(2)+(x^2+10*x+25)*exp(5)^2),x, algorithm="giac")`

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{4e^{\left(-\frac{2x}{5(4xe^5 \log(2) - 4x \log(2)^2 - xe^{10} + 20e^5 \log(2) - 20 \log(2)^2 - 5e^{10})}\right)} \log(2)^2 - 4e^{\left(-\frac{2x}{5(4xe^5 \log(2) - 4x \log(2)^2 - xe^{10} + 20e^5 \log(2) - 20 \log(2)^2 - 5e^{10})}\right)}}{4e^5 \log(2) - 4 \log(2)^2 - e^{10}}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2e^{\left(-\frac{2x}{5(4(x+5)e^5 \log(2) - 4(x+5) \log(2)^2 - (x+5)e^{10})}\right)}}{4(x^2 + 10x + 25)e^5 \log(2) - 4(x^2 + 10x + 25) \log(2)^2 - (x^2 + 10x + 25)e^{10}} dx$$

## 100.238 Problem number 10039

$$\int \frac{(-20 + 12x + (4 - 2x) \log(e^{8+x})) \log\left(\frac{1}{5}e^{-x}(-10x^2 + 2x^2 \log(e^{8+x}))\right)}{-5x + x \log(e^{8+x})} dx$$

Optimal antiderivative

$$\ln\left(\frac{2(\ln(e^3 e^{5+x}) - 5) x^2 e^{-x}}{5}\right)^2$$

command

```
integrate(((4-2*x)*log(exp(3)*exp(5+x))+12*x-20)*log(1/5*(2*x^2*log(exp(3)*exp(5+x))-10*x^2)/exp(x))/(x*log(exp(3)*exp(5+x))-5*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\log\left(\frac{2}{5}x^3 e^{(-x)} + \frac{6}{5}x^2 e^{(-x)}\right)^2$$

Giac 1.7.0 via sagemath 9.3 output

$$\int -\frac{2((x-2)\log(e^{(x+8)}) - 6x + 10)\log\left(\frac{2}{5}(x^2\log(e^{(x+8)}) - 5x^2)e^{(-x)}\right)}{x\log(e^{(x+8)}) - 5x} dx$$

## 100.239 Problem number 10074

$$\int \frac{16(4 - 12e^{16}x)^4}{81e^{64}x^4 \left(-25x + 75e^{16}x^2 + \frac{(4-12e^{16}x)^4(10x-30e^{16}x^2)}{81e^{64}x^4} + \frac{(4-12e^{16}x)^8(-x+3e^{16}x^2)}{6561e^{128}x^8}\right)} dx$$

Optimal antiderivative

$$\ln(15) + \frac{4}{5 - \left(\frac{4e^{-16}}{3x} - 4\right)^4}$$

command

```
integrate(16/81*(-12*x*exp(16)+4)^4/x^4/exp(16)^4/(1/6561*(3*x^2*exp(16)-x)*(-12*x*exp(16)+4)*30*x^2*exp(16)+10*x)*(-12*x*exp(16)+4)^4/x^4/exp(16)^4+75*x^2*exp(16)-25*x),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-\frac{1024(108x^3e^{112} - 54x^2e^{96} + 12xe^{80} - e^{64})e^{(-64)}}{251(20331x^4e^{64} - 27648x^3e^{48} + 13824x^2e^{32} - 3072xe^{16} + 256)}$$

Giac 1.7.0 via sagemath 9.3 output

Timed out



**100.242 Problem number 10188**

$$\int \frac{(-2x^2 + e^2(12 + 8x) + (-10x^2 + e^2(60 + 40x)) \log(5)) \log(\log(25))}{(4e^4 - 4e^2x + x^2) \log(5)} dx$$

Optimal antiderivative

$$\frac{\ln(2 \ln(5)) \left(5 + \frac{1}{\ln(5)}\right) x(3 + x)}{e^2 - \frac{x}{2}}$$

command

```
integrate((((40*x+60)*exp(2)-10*x^2)*log(5)+(8*x+12)*exp(2)-2*x^2)*log(2*log(5))/(4*exp(2)^2-4*exp(2)*x+x^2)/log(5),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$\frac{2 \left(5x \log(5) + x + \frac{2(10e^4 \log(5) + 15e^2 \log(5) + 2e^4 + 3e^2)}{x - 2e^2}\right) \log(2 \log(5))}{\log(5)}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.243 Problem number 10236**

$$\int \frac{20 - 20e^2 + 20ex - 5x^2}{4e^2 - 4ex + x^2} dx$$

Optimal antiderivative

$$\frac{20}{2e - x} - 5x - 4071$$

command

```
integrate((-20*exp(1)^2+20*x*exp(1)-5*x^2+20)/(4*exp(1)^2-4*x*exp(1)+x^2),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$-5x - \frac{20}{x - 2e}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError

**100.244 Problem number 10249**

$$\int \frac{618 - 164770x - 24360x^2 - 33206x^3 - 4820x^4 - 240x^5 - 4x^6 + e^{15}(20x + 4x^3) + e^{10}(-1212x - 60x^2 - 240x^3)}{-8000 + e^{15} + e^{10}(-60 - 3x) - 1200x - 60x^2 - x^3 + e^5(1200x^2 + 1200x + 1200)} dx$$

Optimal antiderivative

$$3 + \left( \frac{3}{20 - e^5 + x} + x^2 + 5 \right)^2$$

command

```
integrate(((4*x^3+20*x)*exp(5)^3+(-12*x^4-240*x^3-60*x^2-1212*x)*exp(5)^2+(12*x^5+480*x^4+48630)*exp(5)-4*x^6-240*x^5-4820*x^4-33206*x^3-24360*x^2-164770*x+618)/(exp(5)^3+(-3*x-60)*exp(5)^2+(3*x^2+120*x+1200)*exp(5)-x^3-60*x^2-1200*x-8000),x, algorithm="giac")
```

Giac 1.9.0-11 via sagemath 9.6 output

$$x^4 + 10x^2 + 6x + \frac{3(2xe^{10} - 80xe^5 + 810x - 2e^{15} + 120e^{10} - 2410e^5 + 16203)}{(x - e^5 + 20)^2}$$

Giac 1.7.0 via sagemath 9.3 output

$$\int \frac{2(2x^6 + 120x^5 + 2410x^4 + 16603x^3 + 12180x^2 - 2(x^3 + 5x)e^{15} + 6(x^4 + 20x^3 + 5x^2 + 101x)e^{10} - 3(2x^5 + 120x^4 + 1200x^3 + 1200x^2 + 1200x + 1200)e^5 + 1200x - 8000)}{x^3 + 60x^2 + 3(x + 20)e^{10} - 3(x^2 + 40x + 400)e^5 + 1200x - 8000} dx$$

**100.245 Problem number 10315**

$$\int \frac{4x + 2e^{40}x^2 + 2x^4 + e^{20}(-2 - 4x^3)}{e^{40}x^2 - 2e^{20}x^3 + x^4} dx$$

Optimal antiderivative

$$2x + \frac{2}{x(e^{20} - x)} - \frac{e^{\frac{4}{3}}}{3}$$

command

```
integrate((2*x^2*exp(5)^8+(-4*x^3-2)*exp(5)^4+2*x^4+4*x)/(x^2*exp(5)^8-2*x^3*exp(5)^4+x^4),x,
```

Giac 1.9.0-11 via sagemath 9.6 output

$$2x - \frac{2}{x^2 - xe^{20}}$$

Giac 1.7.0 via sagemath 9.3 output

Exception raised: NotImplementedError